

Steel Column Analysis and Design

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns



Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{KL^2}$$

$$r = \sqrt{\frac{I}{A}}$$

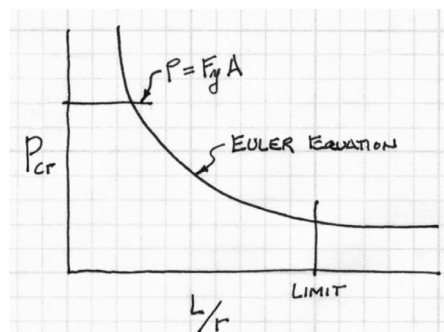
$$I = Ar^2$$

- A = Cross sectional area (in²)
- E = Modulus of elasticity of the material (lb/in²)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$



portrait by Emanuel Handmann, 1753



Analysis of Steel Columns

Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions



Analysis of Steel Columns

Short columns

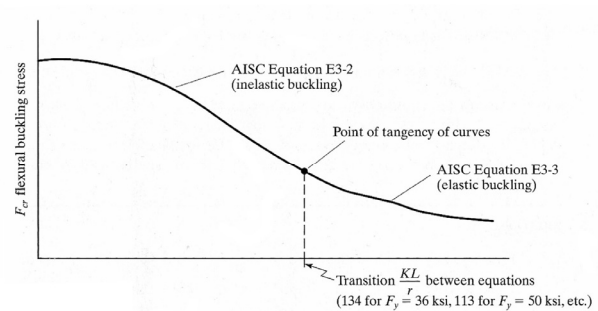
Fail by material crushing
Plastic behavior

Intermediate columns

Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

Long columns

Fail in Euler buckling
Elastic behavior



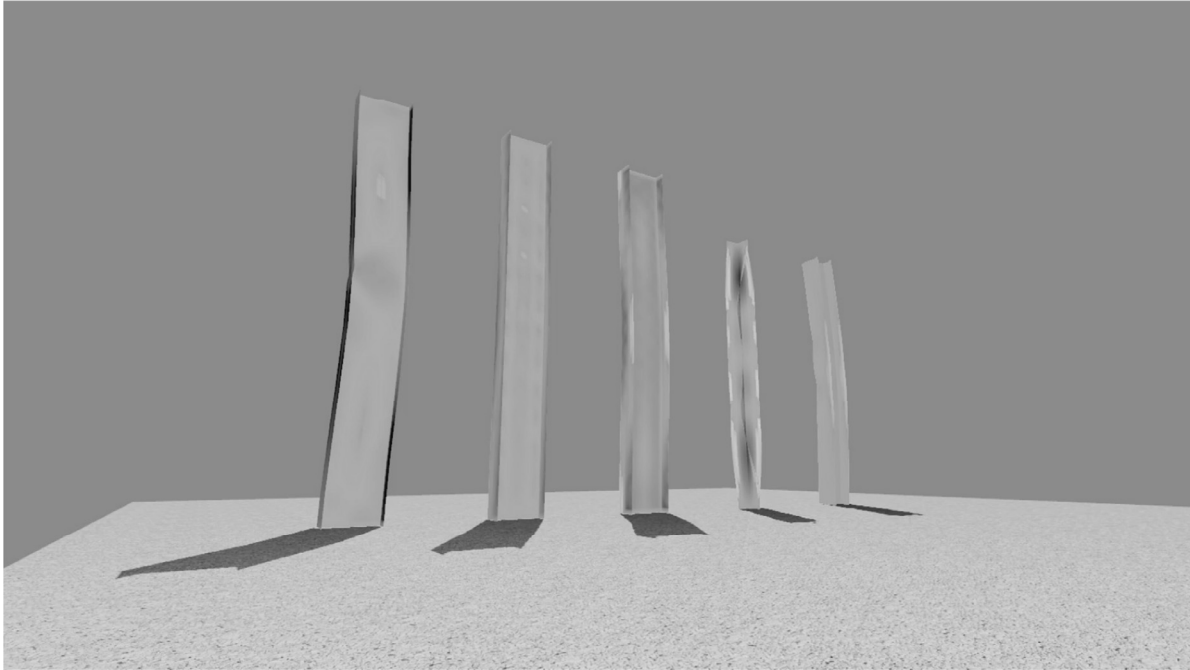
$$slenderness = \frac{KL}{r}$$

←—————→
short intermediate long

Failure Modes

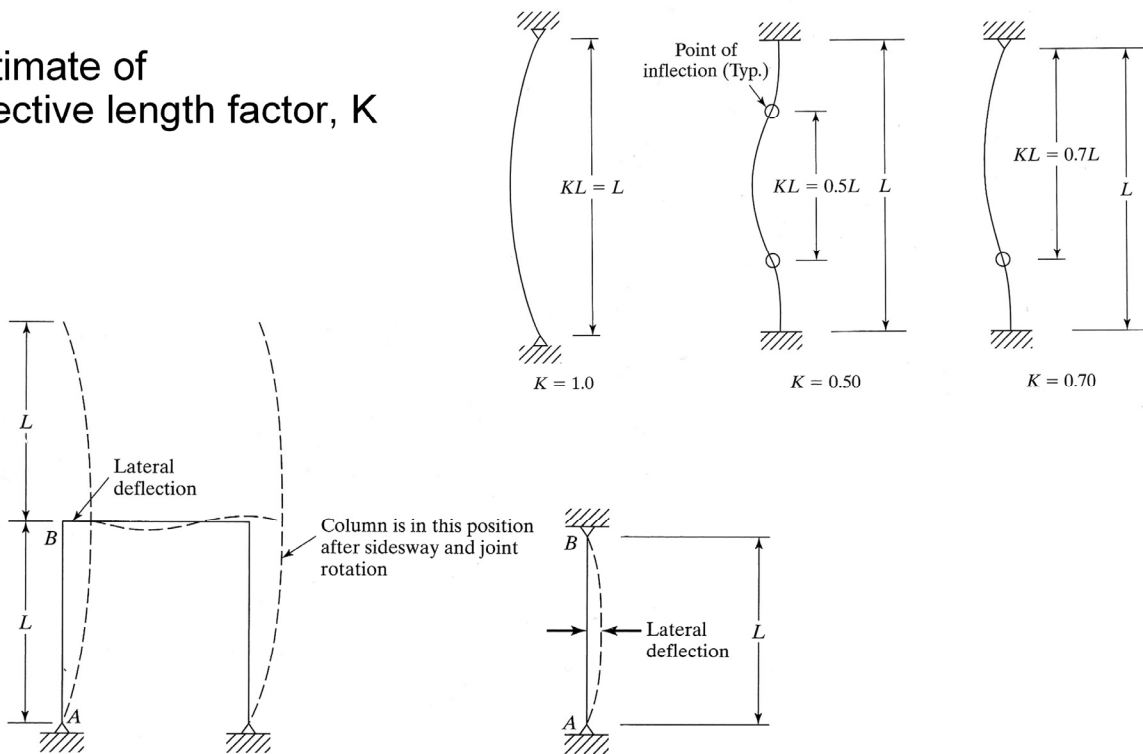
- Column 1: Strong axis flexural buckling
- Column 2: Web local buckling
- Column 3: Weak axis flexural buckling
- Column 4: Torsional buckling
- Column 5: Flange local buckling

“Dancing Columns”
Sherif El-Tawil



Analysis of Steel Columns

Estimate of effective length factor, K



Analysis of Steel Columns

Estimate of K:

TABLE C-A-7.1 Approximate Values of Effective Length Factor, K						
Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	<ul style="list-style-type: none"> Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free 					

Determining K factors by Alignment Charts

Sidesway Inhibited:
Braced frame
 $1.0 > K > 0.5$

Sidesway Uninhibited:
Un-braced frame
unstable $> K > 1.0$

More Pinned:
If I_c/L_c is large
and I_g/L_g is small
The connection is more pinned

More Fixed:
If I_c/L_c is small
and I_g/L_g is large
The connection is more fixed

Sidesway inhibited

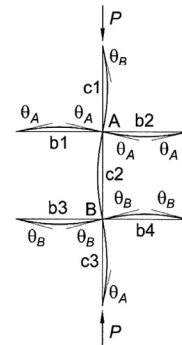
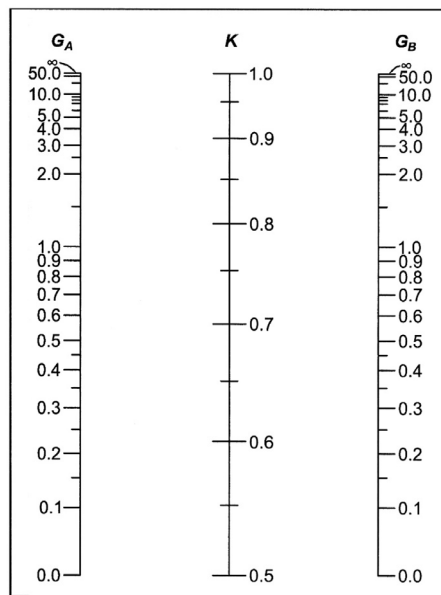


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).

$$G = \frac{\sum \left(\frac{EI}{L} \right)_{column}}{\sum \left(\frac{EI}{L} \right)_{beam}}$$

Determining K factors by Alignment Charts

Sidesway Inhibited:
 Braced frame
 $1.0 > K > 0.5$

Sidesway Uninhibited:
 Un-braced frame
 unstable $> K > 1.0$

More Pinned:
 If I_c/L_c is large
 and I_g/L_g is small
 The connection is more pinned

More Fixed:
 If I_c/L_c is small
 and I_g/L_g is large
 The connection is more fixed

Sidesway uninhibited

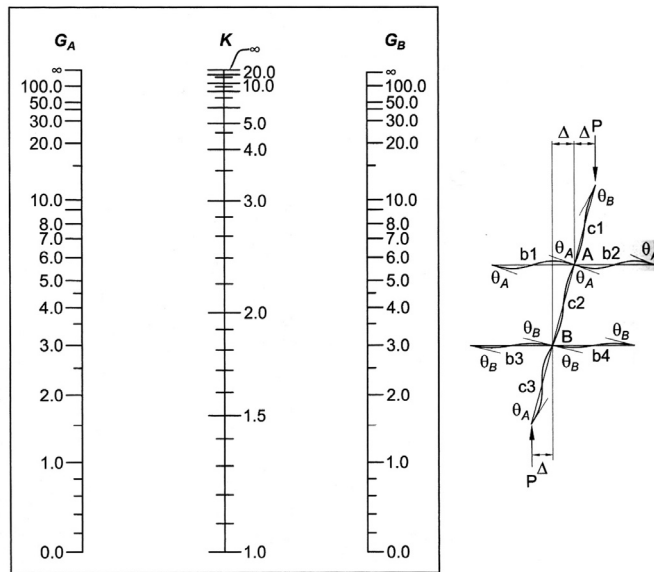


Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\sum \left(\frac{EI}{L} \right)_{column}}{\sum \left(\frac{EI}{L} \right)_{beam}}$$

Analysis of Steel Columns - LRFD

Euler equation:

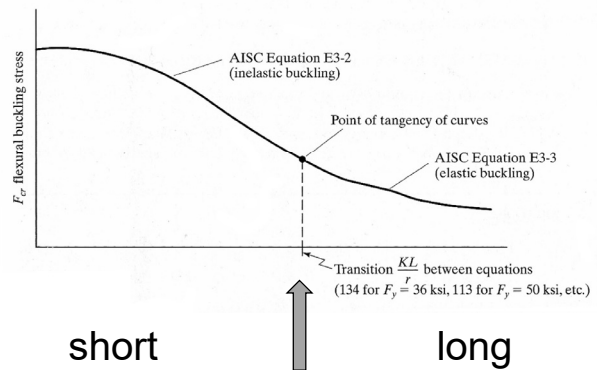
$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r} \right)^2}$$

Short & Intermediate Columns:

$$F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y$$

Long Columns:

$$F_{cr} = 0.877 F_e$$



Transition Slenderness $4.71 \sqrt{\frac{E}{F_y}}$

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

Analysis of Steel Columns pass / fail by LRFD



Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Factored load – P_u

Required:

- $P_u \leq \phi P_n$ (pass)

1. Calculate slenderness ratios. L_c/r , $L_c = KL$
The largest ratio governs.

2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness $4.71\sqrt{E/F_y}$
and determine column type (short or long)

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}} \right] F_y \quad \text{Short}$$

4. Calculate F_{cr} based on slenderness

5. Determine ϕP_n and compare to P_u

$$P_n = F_{cr} A_g \quad \phi = 0.9$$

6. If $P_u \leq \phi P_n$, then OK

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

Analysis of Steel Columns pass / fail by ASD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Factored Load – P_u

Required:

- $P_u \leq \phi P_n$ (pass)

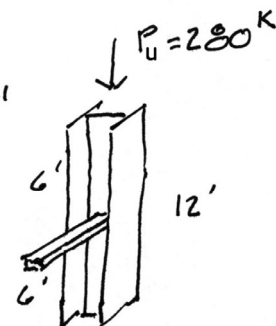
1. Calculate slenderness ratios.
The largest ratio governs.

2. Check slenderness ratio against
upper limit of 200 (recommended)

DATA :

$$\begin{aligned} W 8 \times 35 & \quad A-36 \\ r_x = 3.51'' & \quad F_y = 36 \text{ ksi} \\ r_y = 2.03'' & \\ A = 10.3 \text{ in}^2 & \end{aligned}$$

$$\begin{aligned} l_x = 12' & \quad l_y = 6' \\ K_x = K_y = 1.0 & \end{aligned}$$



X - X AXIS

$$\frac{K_x l_x}{r_x} = \frac{144''}{3.51''}$$

$$\underline{\underline{41.03}} < 200$$

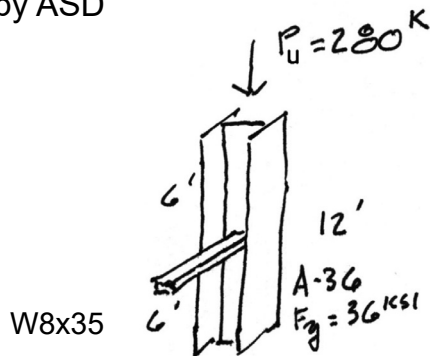
Y - Y AXIS

$$\frac{K_y l_y}{r_y} = \frac{72''}{2.03''}$$

$$35.47$$

Analysis of Steel Columns

pass / fail by ASD



$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$41 < 134 \therefore \text{SHORT}$$

Euler Equation

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 29000 \text{ KSI}}{41^2} = 170.2 \text{ KSI}$$

Short Column Equation

$$F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y = 0.9153 (36) = 32.95 \text{ KSI}$$

Column Strength

$$P_n = F_{cr} A_g = 32.95 \text{ KSI} \times 10.3 \text{ in}^2 = 339.39 \text{ K}$$

$$\phi P_n = 0.9 P_n = 0.9 (339.39) = 305.4 \text{ K}$$

$$P_u = 280 \text{ K} < 305.4 \text{ K} = \phi P_n \quad \checkmark \text{OK}$$

3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine ϕP_n and compare to P_u
6. If $P_u \leq \phi P_n$, then OK

Analysis of Steel Columns capacity by LRFD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y

Required:

- Max load capacity

1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine ϕP_n and Compute allowable capacity:
 $P_n = F_{cr} A_g \quad P_u = \phi P_n$

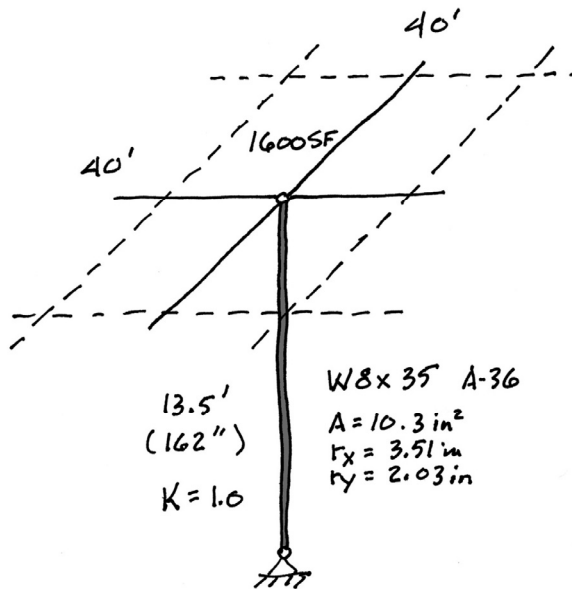
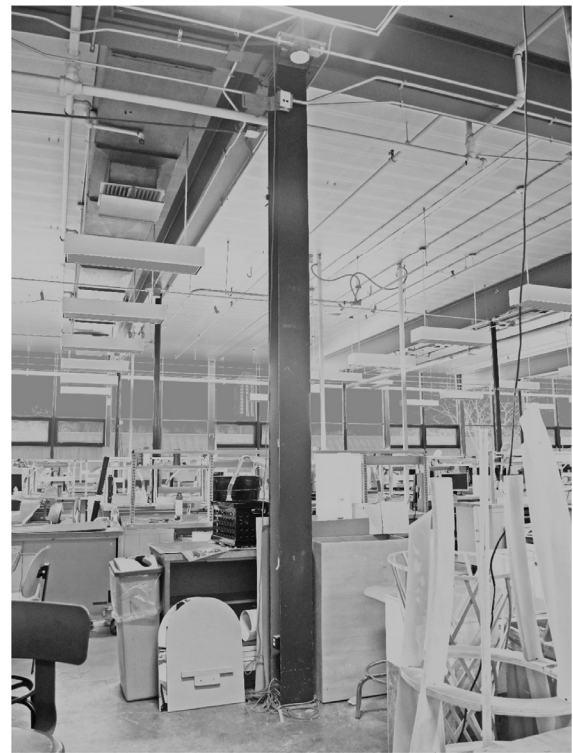


$$F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y \quad \text{Short}$$

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

Capacity Example 1

Free standing column
 Third floor studio space
 Supports roof load = 20 psf DL + SL
 snow \approx 15lbs / FT depth



Capacity Example 1

1. Calculate slenderness ratios.
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness

y-y Axis (controls)

$$\frac{K_y L_y}{r_y} = \frac{1(162'')}{2.03''} = 79.8 < 200 \checkmark$$

2. Check slenderness ratio against upper limit of 200 (recommended)

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$79.8 < 134 \therefore \text{SHORT}$$

3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)

Euler Buckling

$$F_e = \frac{\pi^2 E}{(K L/r)^2} = \frac{\pi^2 29000}{79.8^2} = 44.94 \text{ ksi}$$

Short Column Equation

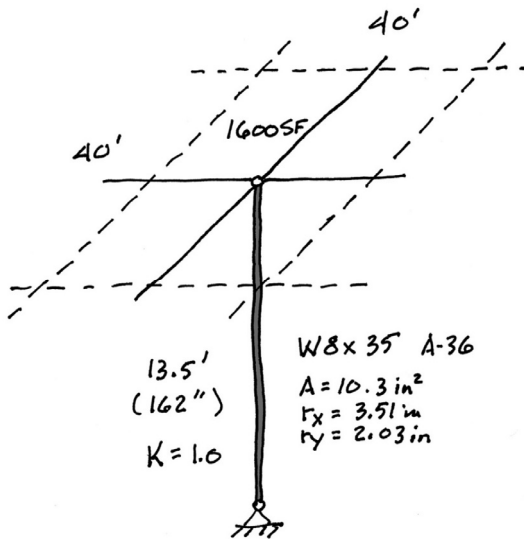
$$F_{cr} = \left[0.658 \frac{F_e}{F_y} \right] F_y = \left[0.7151 \right] 36 = 25.74 \text{ ksi}$$

Capacity Example 1

5. Determine ϕP_n and Compute allowable capacity: $P_u = \phi P_n$

DL = 20 psf

20 psf (1600 sf) = 32k on column



Column nominal strength

$$P_n = F_{cr} A_g = 25.74 \text{ ksi} \cdot 10.3 \text{ in}^2 = 265.1 \text{ k}$$

$$\phi P_n = 0.9(265) = 238.6 \text{ k} = P_u$$

Load capacity

$$P_u = 1.2(32) + 1.6(SL) = 238.6 \text{ k}$$

$$SL = 125.1 \text{ k}$$

$$\text{For } A_f = 40 \times 40 = 1600 \text{ SF}$$

$$SL = \frac{125100 \text{ k}}{1600 \text{ SF}} = 78.2 \text{ PSF}$$

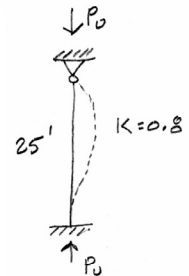
Capacity Example 2

long column – using equations

$$r_x = 3.51 \text{ in.}$$

$$r_y = 2.03 \text{ in.}$$

W8x35
 $F_y = 50 \text{ ksi}$
 $E = 29000 \text{ ksi}$
 $L = 25' \text{ (NO BRACINGS)}$



Slenderness y-y

$$\frac{K L_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$$

$$4.71 \sqrt{\frac{E}{F_y}} = 113 < 118.2 \quad \therefore \text{LONG}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{K L}{r}\right)^2} = \frac{\pi^2 29000}{118.2^2} = 20.47 \text{ ksi}$$

Long Column Equation

$$F_{cr} = 0.877(20.47) = 17.95 \text{ ksi}$$

Column strength

$$\phi P_n = \phi F_{cr} A_g = 0.9(17.95)(10.3) = 166.4 \text{ k}$$

Table G1 Buckling Length Coefficients, K_e

Buckling modes						
Theoretical K_e value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4
End condition code						
		Rotation fixed, translation fixed	Rotation free, translation fixed	Rotation fixed, translation free	Rotation free, translation free	

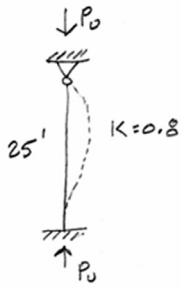
Capacity Example 2

long column – using table

$W8 \times 35$
 $F_y = 50 \text{ ksi}$
 $E = 29,000 \text{ ksi}$
 $L = 25'$ (NO BRACING)

r_y CONTROLS

$$KL = 0.8(25') = 20'$$



Shape	Table 4-1a (continued)												
	Available Strength in Axial Compression, kips												
	$F_y = 50 \text{ ksi}$												
W-Shapes													
lb/ft	W8x												
	67		58		48		40		35		31		
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	
Effective length, L_e (ft), with respect to least radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	8	508	763	439	660	361	543	298	448	262	394	232	348
	9	488	733	422	634	347	521	285	429	251	377	222	333
	10	467	701	403	606	331	497	272	409	239	359	211	317
	11	444	668	384	576	314	473	258	388	226	340	200	301
	12	421	633	363	546	297	447	243	366	213	321	189	283
	13	397	597	342	514	280	421	228	343	200	301	177	266
	14	373	560	321	482	262	394	213	321	187	281	165	248
	15	348	523	299	450	244	367	198	298	174	261	153	230
	16	324	487	278	418	226	340	183	275	160	241	141	212
	17	300	450	257	386	209	314	169	253	147	221	130	195
	18	276	415	236	355	192	288	154	232	135	203	118	178
	19	253	381	216	325	175	264	141	211	123	184	108	162
	20	231	347	197	296	159	239	127	191	111	166	97.2	146
22	191	287	163	244	132	198	105	158	91.5	138	80.3	121	
24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101	
26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5	
28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5	
30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9	
32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1	
34	79.9	120	68.1	102	55.1	82.8	44.0	66.1					

Design of Steel Columns with AISC Strength Tables

Data:

- Column – length
- Support conditions
- Material properties – F_y
- Applied load - P_{actual}

Required:

- Column Size

1. Enter table with height, $KL = L_c$
2. Read allowable load for each section to find the smallest adequate size.
3. **Tables assume weak axis buckling. If the strong axis controls the length must be divided by the ratio r_x/r_y**
4. Values stop in table (black line) at slenderness limit, $KL/r = 200$

Shape	Table 4-1a (continued)												
	Available Strength in Axial Compression, kips												
	$F_y = 50 \text{ ksi}$												
W-Shapes													
lb/ft	W8x												
	67		58		48		40		35		31		
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	
Effective length, L_e (ft), with respect to least radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
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	9	488	733	422	634	347	521	285	429	251	377	222	333
	10	467	701	403	606	331	497	272	409	239	359	211	317
	11	444	668	384	576	314	473	258	388	226	340	200	301
	12	421	633	363	546	297	447	243	366	213	321	189	283
	13	397	597	342	514	280	421	228	343	200	301	177	266
	14	373	560	321	482	262	394	213	321	187	281	165	248
	15	348	523	299	450	244	367	198	298	174	261	153	230
	16	324	487	278	418	226	340	183	275	160	241	141	212
	17	300	450	257	386	209	314	169	253	147	221	130	195
	18	276	415	236	355	192	288	154	232	135	203	118	178
	19	253	381	216	325	175	264	141	211	123	184	108	162
	20	231	347	197	296	159	239	127	191	111	166	97.2	146
22	191	287	163	244	132	198	105	158	91.5	138	80.3	121	
24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101	
26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5	
28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5	
30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9	
32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1	
34	79.9	120	68.1	102	55.1	82.8	44.0	66.1					

AISC Critical Stress Table

for previous example $Kl/r_y = 118.2$

TO FIND CAPACITY:

$$\phi F_{cr} = 16.2 \text{ ksi}$$

$$\phi P_n = P_u = \phi F_{cr} A_g$$

$$P_u = 16.2(10.3) = \underline{\underline{166.8 \text{ k}}}$$

Table 4-22 (continued)
Available Critical Stress for
Compression Members

$\frac{Kl}{r}$	$F_y = 35 \text{ ksi}$		$F_y = 36 \text{ ksi}$		$F_y = 42 \text{ ksi}$		$F_y = 46 \text{ ksi}$		$F_y = 50 \text{ ksi}$					
	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$				
	ASD	LFRD	ASD	LFRD	ASD	LFRD	ASD	LFRD	ASD	LFRD				
81	15.0	22.5	81	15.3	22.9	81	16.8	25.3	81	17.7	26.6	81	18.5	27.9
82	14.9	22.3	82	15.1	22.7	82	16.6	25.0	82	17.5	26.3	82	18.3	27.5
83	14.7	22.1	83	15.0	22.5	83	16.5	24.8	83	17.3	26.0	83	18.1	27.2
84	14.6	22.0	84	14.9	22.3	84	16.3	24.5	84	17.1	25.8	84	17.9	26.9
85	14.5	21.8	85	14.7	22.1	85	16.1	24.3	85	16.9	25.5	85	17.7	26.5
86	14.4	21.6	86	14.6	22.0	86	16.0	24.0	86	16.7	25.2	86	17.4	26.2
87	14.2	21.4	87	14.5	21.8	87	15.8	23.7	87	16.6	24.9	87	17.2	25.9
88	14.1	21.2	88	14.3	21.6	88	15.6	23.5	88	16.4	24.6	88	17.0	25.5
89	14.0	21.0	89	14.2	21.4	89	15.5	23.2	89	16.2	24.3	89	16.8	25.2
90	13.8	20.8	90	14.1	21.2	90	15.3	23.0	90	16.0	24.0	90	16.6	24.9
91	13.7	20.6	91	13.9	21.0	91	15.1	22.7	91	15.8	23.7	91	16.3	24.6
92	13.6	20.4	92	13.8	20.8	92	15.0	22.5	92	15.6	23.4	92	16.1	24.2
93	13.5	20.2	93	13.7	20.5	93	14.8	22.2	93	15.4	23.1	93	15.9	23.9
94	13.3	20.0	94	13.5	20.3	94	14.6	22.0	94	15.2	22.8	94	15.7	23.6
95	13.2	19.9	95	13.4	20.1	95	14.4	21.7	95	15.0	22.6	95	15.5	23.3
96	13.1	19.7	96	13.3	19.9	96	14.3	21.5	96	14.8	22.3	96	15.3	22.9
97	13.0	19.5	97	13.1	19.7	97	14.1	21.2	97	14.6	22.0	97	15.0	22.6
98	12.8	19.3	98	13.0	19.5	98	13.9	21.0	98	14.4	21.7	98	14.8	22.3
99	12.7	19.1	99	12.9	19.3	99	13.8	20.7	99	14.2	21.4	99	14.6	22.0
100	12.6	18.9	100	12.7	19.1	100	13.6	20.5	100	14.1	21.1	100	14.4	21.7
101	12.4	18.7	101	12.6	18.9	101	13.4	20.2	101	13.9	20.8	101	14.2	21.3
102	12.3	18.5	102	12.5	18.7	102	13.3	20.0	102	13.7	20.6	102	14.0	21.0
103	12.2	18.3	103	12.3	18.5	103	13.1	19.7	103	13.5	20.3	103	13.8	20.7
104	12.1	18.1	104	12.2	18.3	104	12.9	19.5	104	13.3	20.0	104	13.6	20.4
105	11.9	17.9	105	12.1	18.1	105	12.8	19.2	105	13.1	19.7	105	13.4	20.1
106	11.8	17.7	106	11.9	17.9	106	12.6	19.0	106	12.9	19.4	106	13.2	19.8
107	11.7	17.5	107	11.8	17.7	107	12.4	18.7	107	12.8	19.2	107	13.0	19.5
108	11.5	17.3	108	11.7	17.5	108	12.3	18.5	108	12.6	18.9	108	12.8	19.2
109	11.4	17.2	109	11.5	17.3	109	12.1	18.2	109	12.4	18.6	109	12.6	18.9
110	11.3	17.0	110	11.4	17.1	110	12.0	18.0	110	12.2	18.3	110	12.4	18.6
111	11.2	16.8	111	11.3	16.9	111	11.8	17.7	111	12.0	18.1	111	12.2	18.3
112	11.0	16.6	112	11.1	16.7	112	11.6	17.5	112	11.8	17.8	112	12.0	18.0
113	10.9	16.4	113	11.0	16.5	113	11.5	17.3	113	11.7	17.5	113	11.8	17.7
114	10.8	16.2	114	10.9	16.3	114	11.3	17.0	114	11.5	17.3	114	11.6	17.4
115	10.7	16.0	115	10.7	16.2	115	11.2	16.8	115	11.3	17.0	115	11.4	17.1
116	10.5	15.8	116	10.6	16.0	116	11.0	16.5	116	11.1	16.7	116	11.2	16.8
117	10.4	15.6	117	10.5	15.8	117	10.8	16.3	117	11.0	16.5	117	11.0	16.5
118	10.3	15.5	118	10.4	15.6	118	10.7	16.1	118	10.8	16.2	118	10.8	16.2
119	10.2	15.3	119	10.2	15.4	119	10.5	15.8	119	10.6	16.0	119	10.6	16.0
120	10.0	15.1	120	10.1	15.2	120	10.4	15.6	120	10.4	15.7	120	10.4	15.7

ASD LFRD
 $\Omega_c = 1.67$ $\phi_c = 0.90$

Steel Frame Construction



University of Michigan – North Quad

Steel Frame Construction

Messe Leipzig – 1996

Congress Centre – Gerkan, Marg und Partner

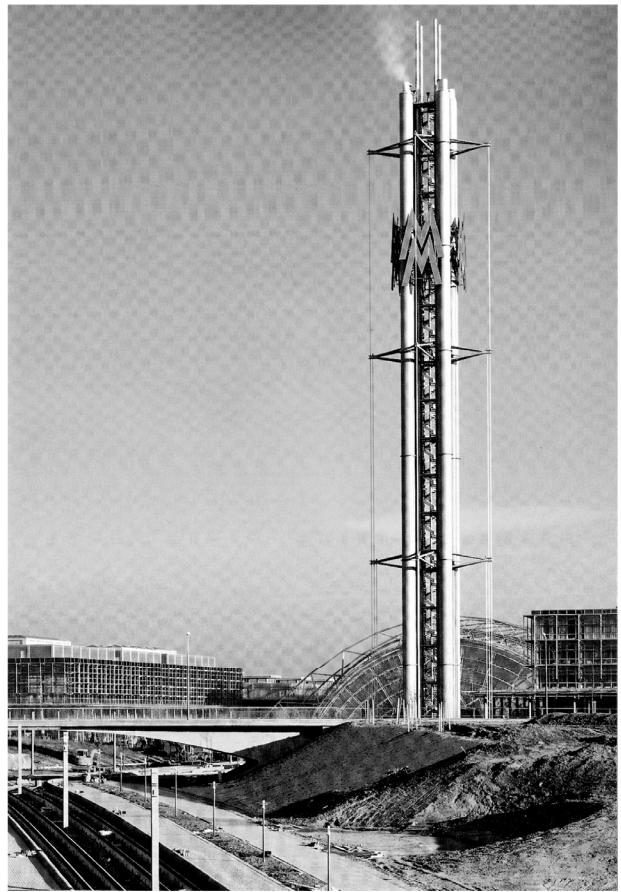
Glass Hall – Ian Ritchie Architects

Tower - Schlaich, Bergermann und Partner



Messe Leipzig - Glass Hall - Ian Ritchie Architects

University of Michigan, TCAUP



Messe Leipzig – Cable braced tower. Jörg Schlaich

Structures II

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Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

University of Michigan, TCAUP

Structures II

Slide 26 of 30

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Branching Columns (tree columns)



bridge in Pragsattel, Stuttgart, 1992
Schlaich, Bergemann und Partner

Branching Columns (tree columns)



Stuttgart Airport Terminal,
Gerkan, Marg und Partner
Schlaich, Bergemann und Partner