

Steel Column Analysis and Design

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns



Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{KL^2}$$

$$r = \sqrt{\frac{I}{A}}$$

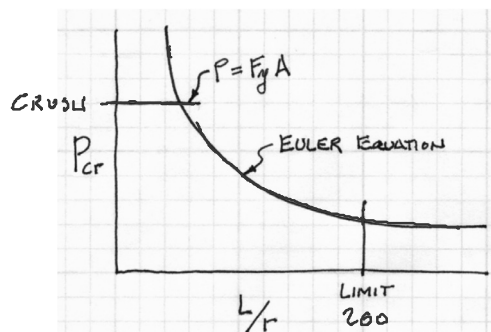
$$I = Ar^2$$

- A = Cross sectional area (in²)
- E = Modulus of elasticity of the material (lb/in²)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$



portrait by Emanuel Handmann, 1753



Analysis of Steel Columns

Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions

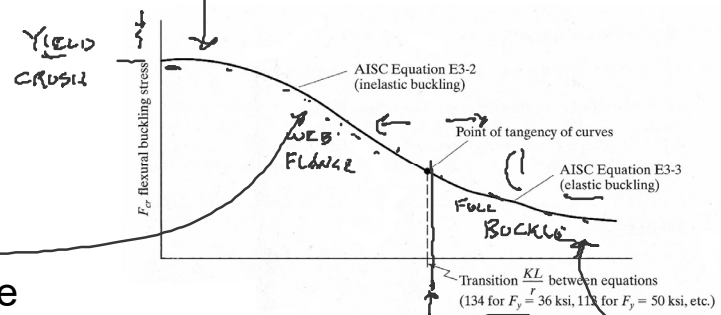


Analysis of Steel Columns

Short columns
Fail by material crushing
Plastic behavior

Intermediate columns
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

Long columns
Fail in Euler buckling
Elastic behavior



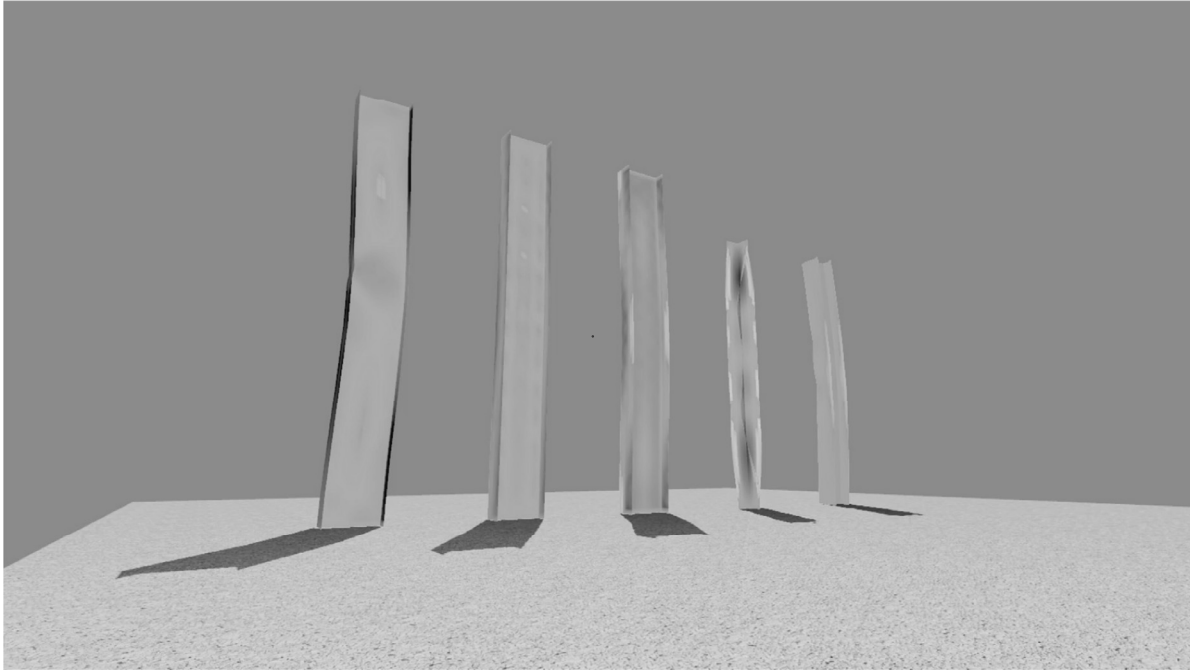
$$slenderness = \frac{KL}{r}$$

(short intermediate) long

Failure Modes

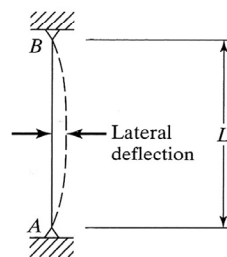
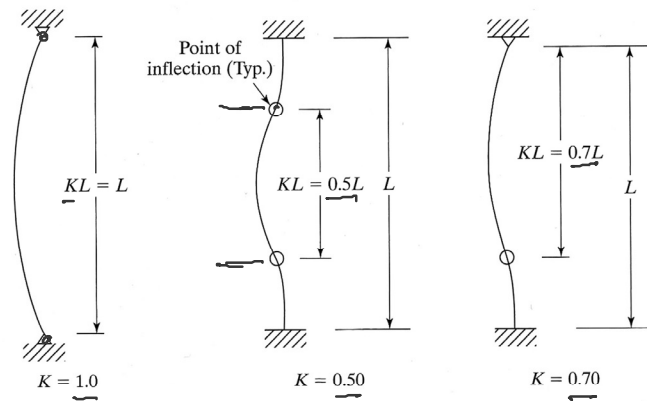
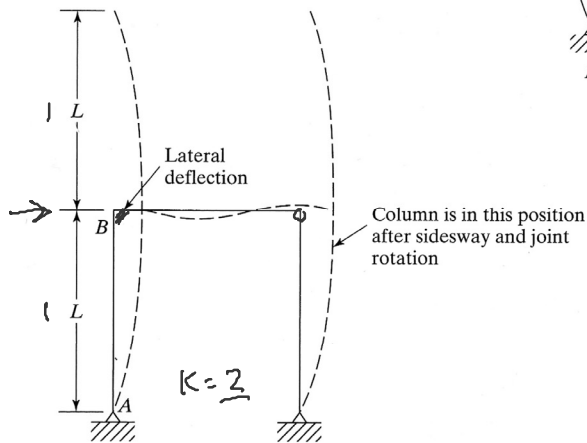
- Column 1: Strong axis flexural buckling
- Column 2: Web local buckling
- Column 3: Weak axis flexural buckling
- Column 4: Torsional buckling
- Column 5: Flange local buckling

“Dancing Columns”
Sherif El-Tawil



Analysis of Steel Columns

Estimate of effective length factor, K



Analysis of Steel Columns

Estimate of K:

TABLE C-A-7.1 Approximate Values of Effective Length Factor, K						
Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	<ul style="list-style-type: none"> Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free 					

Determining K factors by Alignment Charts (BRACED) Sidesway inhibited

Sidesway Inhibited:
Braced frame
 $1.0 > K > 0.5$

Sidesway Uninhibited:
Un-braced frame
unstable $> K > 1.0$

More Pinned:
If I_c/L_c is large
and I_g/L_g is small
The connection is more pinned

More Fixed:
If I_c/L_c is small
and I_g/L_g is large
The connection is more fixed

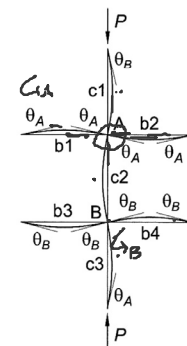
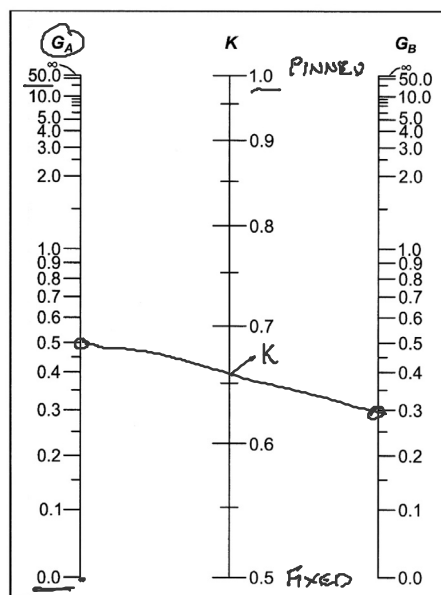


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).

$$G = \frac{\sum \left(\frac{EI}{L} \right)_{\text{column}}}{\sum \left(\frac{EI}{L} \right)_{\text{beam}}}$$

Determining K factors by Alignment Charts

Sidesway Inhibited:
 Braced frame
 $1.0 > K > 0.5$

Sidesway Uninhibited:
 Un-braced frame
 $unstable > K > 1.0$

More Pinned:
 If I_c/L_c is large
 and I_g/L_g is small
 The connection is more pinned

More Fixed:
 If I_c/L_c is small
 and I_g/L_g is large
 The connection is more fixed

Sidesway uninhibited

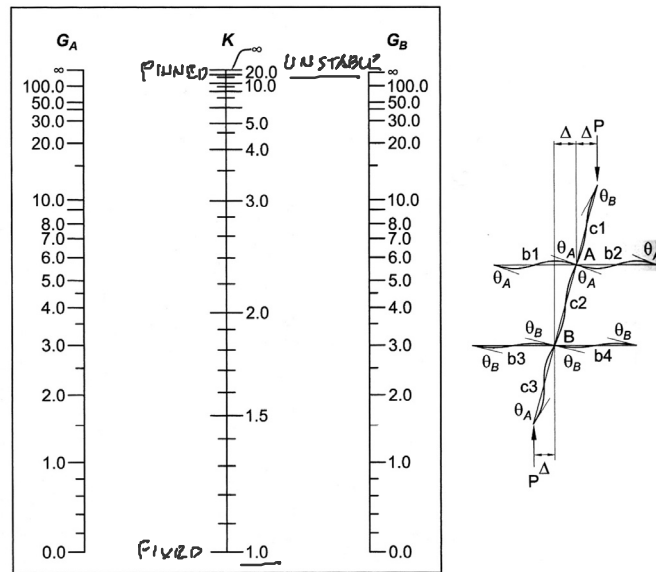


Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\sum \left(\frac{EI}{L} \right)_{column}}{\sum \left(\frac{EI}{L} \right)_{beam}}$$

Analysis of Steel Columns - LRFD

Euler equation:

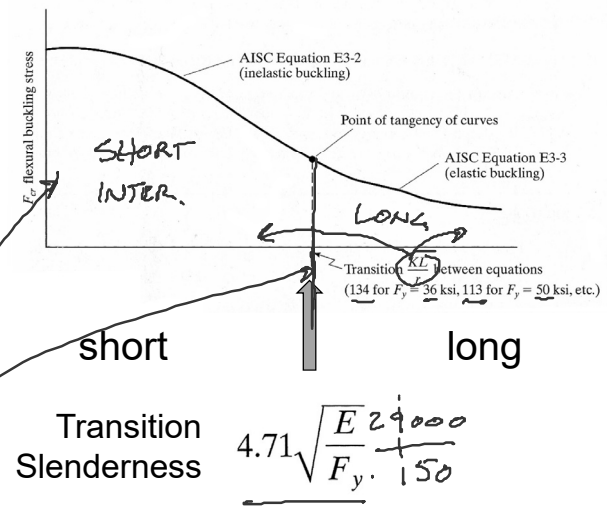
$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r} \right)^2}$$

Short & Intermediate Columns:

$$F_{cr} = \left[0.658 \left(\frac{F_y}{F_e} \right) \right] F_y$$

Long Columns:

$$F_{cr} = 0.877 F_e$$



$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

Analysis of Steel Columns

pass / fail by LRFD



Data:

- Column – size, length ✓
- Support conditions
- Material properties – F_y ✓
- Factored load – P_u

Required: ✓

- $P_u \leq \phi P_n$ (pass)
CALC

1. Calculate slenderness ratios. L_c/r , $L_c = KL$
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/F_y}$
and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
STRESS
5. Determine ϕP_n and compare to P_u
 $P_n = F_{cr} A_g$ $\phi = 0.9$ ✓
FORCE
6. If $P_u \leq \phi P_n$, then OK

$$F_{cr} = \left[0.658 \frac{F_y}{F_e} \right] F_y \quad \text{Short}$$

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

Analysis of Steel Columns

pass / fail by ASD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Factored Load – P_u

Required:

- $P_u \leq \phi P_n$ (pass)

1. Calculate slenderness ratios.
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)

DATA :

W 8x35
 $r_x = 3.51''$
 $r_y = 2.03''$
 $A = 10.3 \text{ in}^2$

AISC TABLE

A-36
 $F_y = 36 \text{ ksi}$

$P_u = 280 \text{ K}$

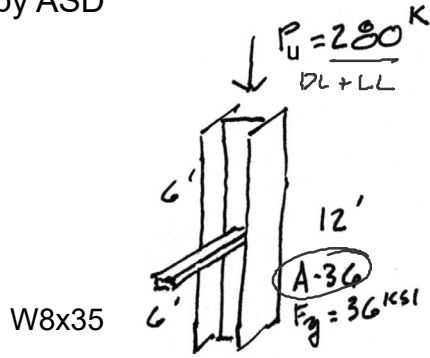
$l_x = 12'$ $l_y = 6'$
 $K_x = K_y = 1.0$

X-X AXIS
 $\frac{K_x l_x}{r_x} = \frac{144''}{3.51''} = 41.03 < 200$

Y-Y AXIS
 $\frac{K_y l_y}{r_y} = \frac{72''}{2.03''} = 35.47$

Analysis of Steel Columns

pass / fail by ASD



$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$41 < 134 \therefore \text{SHORT}$$

Euler Equation

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 29000 \text{ ksi}}{41^2} = 170.2 \text{ ksi}$$

Short Column Equation

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = 0.9153 \left(\frac{F_y}{36}\right) = 32.95 \text{ ksi}$$

Column Strength

$$P_n = F_{cr} A_g = 32.95 \text{ ksi} \times 10.3 \text{ in}^2 = 339.39 \text{ k}$$

STRENGTH

$$\phi P_n = 0.9 P_n = 0.9 (339.39) = 305.4 \text{ k}$$

$$\frac{P_u}{\text{Load}} = 280 \text{ k} < 305.4 \text{ k} = \phi P_n \quad \text{STRONGER} \quad \checkmark \text{OK}$$

3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine ϕP_n and compare to P_u
6. If $P_u \leq \phi P_n$, then OK

Analysis of Steel Columns capacity by LRFD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y

Required:

- Max load capacity

1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/F_y}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine ϕP_n and Compute allowable capacity:
 $P_n = F_{cr} A_g \quad P_u = \phi P_n$

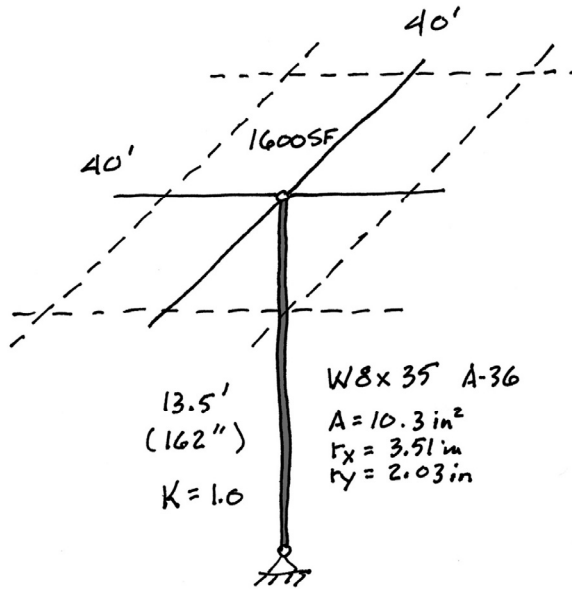
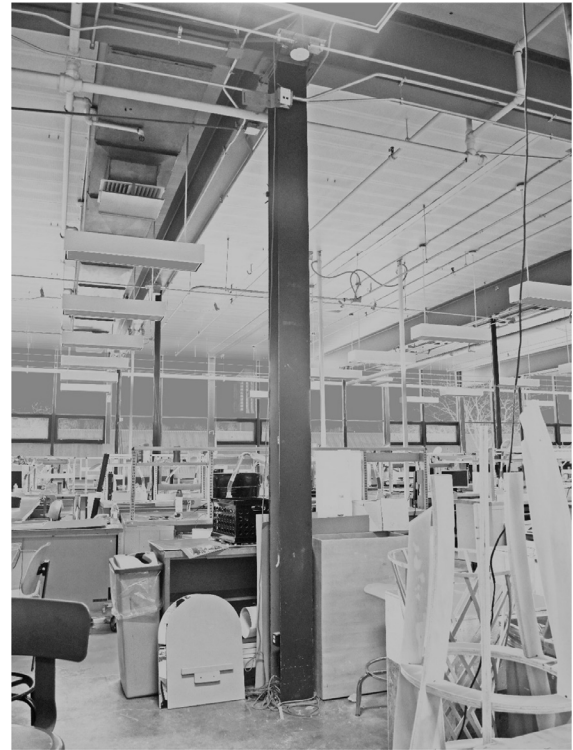


$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \quad \text{Short}$$

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

Capacity Example 1

Free standing column
 Third floor studio space
 Supports roof load = 20 psf DL + SL
 snow \approx 15lbs / FT depth



Capacity Example 1

1. Calculate slenderness ratios.
 The largest ratio governs.
2. Check slenderness ratio against
 upper limit of 200
 (recommended)
3. Calculate transition slenderness
 $4.71\sqrt{E/F_y}$ and determine
 column type (short or long)
4. Calculate F_{cr} based on
 slenderness

y-y Axis (controls)

$$\frac{K_y L_y}{r_y} = \frac{1(162'')}{2.03''} = 79.8 < 200 \checkmark$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$79.8 < 134 \therefore \text{SHORT}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{(K L/r)^2} = \frac{\pi^2 29000}{79.8^2} = 44.94 \text{ ksi}$$

Short Column Equation

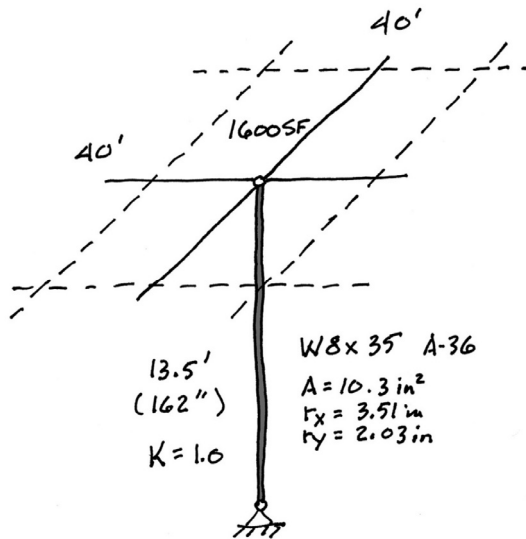
$$F_{cr} = \left[0.658 \frac{F_e}{F_y} \right] F_y = \left[0.7151 \right] 36 = 25.74 \text{ ksi}$$

Capacity Example 1

5. Determine ϕP_n and Compute allowable capacity: $P_u = \phi P_n$

DL = 20 psf

20 psf (1600 sf) = 32k on column



Column nominal strength

$$P_n = F_{cr} A_g = 25.74 \text{ ksi} \cdot 10.3 \text{ in}^2 = 265.1 \text{ k}$$

$$\phi P_n = 0.9(265) = 238.6 \text{ k} = P_u$$

Load capacity

$$P_u = 1.2(32) + 1.6(SL) = 238.6 \text{ k}$$

$$SL = 125.1 \text{ k}$$

$$\text{For } A_f = 40 \times 40 = 1600 \text{ SF}$$

$$SL = \frac{125100^*}{1600 \text{ SF}} = 78.2 \text{ PSF}$$

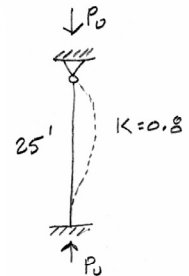
Capacity Example 2

long column – using equations

$$r_x = 3.51 \text{ in.}$$

$$r_y = 2.03 \text{ in.}$$

W8x35
F_y = 50 ksi
E = 29000 ksi
L = 25' (NO BRACINGS)



Slenderness y-y

$$\frac{K L_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$$

$$4.71 \sqrt{\frac{E}{F_y}} = 113 < 118.2 \quad \therefore \text{LONG}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{K L}{r}\right)^2} = \frac{\pi^2 29000}{118.2^2} = 20.47 \text{ ksi}$$

Long Column Equation

$$F_{cr} = 0.877(20.47) = 17.95 \text{ ksi}$$

Column strength

$$\phi P_n = \phi F_{cr} A_g = 0.9(17.95)(10.3) = 166.4 \text{ k}$$

Table G1 Buckling Length Coefficients, K_e

Buckling modes								
Theoretical K _e value	0.5	0.7	1.0	1.0	2.0	2.0		
Recommended design K _e when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4		
End condition code		Rotation fixed, translation fixed		Rotation free, translation fixed		Rotation fixed, translation free		Rotation free, translation free

AISC Critical Stress Table

for previous example $Kl/r_y = 118.2$

TO FIND CAPACITY:

$$\phi F_{cr} = 16.2 \text{ ksi}$$

$$\phi P_n = P_u = \phi F_{cr} A_g$$

$$P_u = 16.2(10.3) = \underline{\underline{166.8 \text{ K}}}$$

Table 4-22 (continued)
Available Critical Stress for
Compression Members

$\frac{Kl}{r}$	$F_y = 35 \text{ ksi}$		$F_y = 36 \text{ ksi}$		$F_y = 42 \text{ ksi}$		$F_y = 46 \text{ ksi}$		$F_y = 50 \text{ ksi}$					
	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	F_{cr}/Ω_c	$\phi_c F_{cr}$				
	ASD	LFRD	ASD	LFRD	ASD	LFRD	ASD	LFRD	ASD	LFRD				
81	15.0	22.5	81	15.3	22.9	81	16.8	25.3	81	17.7	26.6	81	18.5	27.9
82	14.9	22.3	82	15.1	22.7	82	16.6	25.0	82	17.5	26.3	82	18.3	27.5
83	14.7	22.1	83	15.0	22.5	83	16.5	24.8	83	17.3	26.0	83	18.1	27.2
84	14.6	22.0	84	14.9	22.3	84	16.3	24.5	84	17.1	25.8	84	17.9	26.9
85	14.5	21.8	85	14.7	22.1	85	16.1	24.3	85	16.9	25.5	85	17.7	26.5
86	14.4	21.6	86	14.6	22.0	86	16.0	24.0	86	16.7	25.2	86	17.4	26.2
87	14.2	21.4	87	14.5	21.8	87	15.8	23.7	87	16.6	24.9	87	17.2	25.9
88	14.1	21.2	88	14.3	21.6	88	15.6	23.5	88	16.4	24.6	88	17.0	25.5
89	14.0	21.0	89	14.2	21.4	89	15.5	23.2	89	16.2	24.3	89	16.8	25.2
90	13.8	20.8	90	14.1	21.2	90	15.3	23.0	90	16.0	24.0	90	16.6	24.9
91	13.7	20.6	91	13.9	21.0	91	15.1	22.7	91	15.8	23.7	91	16.3	24.6
92	13.6	20.4	92	13.8	20.8	92	15.0	22.5	92	15.6	23.4	92	16.1	24.2
93	13.5	20.2	93	13.7	20.5	93	14.8	22.2	93	15.4	23.1	93	15.9	23.9
94	13.3	20.0	94	13.5	20.3	94	14.6	22.0	94	15.2	22.8	94	15.7	23.6
95	13.2	19.9	95	13.4	20.1	95	14.4	21.7	95	15.0	22.6	95	15.5	23.3
96	13.1	19.7	96	13.3	19.9	96	14.3	21.5	96	14.8	22.3	96	15.3	22.9
97	13.0	19.5	97	13.1	19.7	97	14.1	21.2	97	14.6	22.0	97	15.0	22.6
98	12.8	19.3	98	13.0	19.5	98	13.9	21.0	98	14.4	21.7	98	14.8	22.3
99	12.7	19.1	99	12.9	19.3	99	13.8	20.7	99	14.2	21.4	99	14.6	22.0
100	12.6	18.9	100	12.7	19.1	100	13.6	20.5	100	14.1	21.1	100	14.4	21.7
101	12.4	18.7	101	12.6	18.9	101	13.4	20.2	101	13.9	20.8	101	14.2	21.3
102	12.3	18.5	102	12.5	18.7	102	13.3	20.0	102	13.7	20.6	102	14.0	21.0
103	12.2	18.3	103	12.3	18.5	103	13.1	19.7	103	13.5	20.3	103	13.8	20.7
104	12.1	18.1	104	12.2	18.3	104	12.9	19.5	104	13.3	20.0	104	13.6	20.4
105	11.9	17.9	105	12.1	18.1	105	12.8	19.2	105	13.1	19.7	105	13.4	20.1
106	11.8	17.7	106	11.9	17.9	106	12.6	19.0	106	12.9	19.4	106	13.2	19.8
107	11.7	17.5	107	11.8	17.7	107	12.4	18.7	107	12.8	19.2	107	13.0	19.5
108	11.5	17.3	108	11.7	17.5	108	12.3	18.5	108	12.6	18.9	108	12.8	19.2
109	11.4	17.2	109	11.5	17.3	109	12.1	18.2	109	12.4	18.6	109	12.6	18.9
110	11.3	17.0	110	11.4	17.1	110	12.0	18.0	110	12.2	18.3	110	12.4	18.6
111	11.2	16.8	111	11.3	16.9	111	11.8	17.7	111	12.0	18.1	111	12.2	18.3
112	11.0	16.6	112	11.1	16.7	112	11.6	17.5	112	11.8	17.8	112	12.0	18.0
113	10.9	16.4	113	11.0	16.5	113	11.5	17.3	113	11.7	17.5	113	11.8	17.7
114	10.8	16.2	114	10.9	16.3	114	11.3	17.0	114	11.5	17.3	114	11.6	17.4
115	10.7	16.0	115	10.7	16.2	115	11.2	16.8	115	11.3	17.0	115	11.4	17.1
116	10.5	15.8	116	10.6	16.0	116	11.0	16.5	116	11.1	16.7	116	11.2	16.8
117	10.4	15.6	117	10.5	15.8	117	10.8	16.3	117	11.0	16.5	117	11.0	16.5
118	10.3	15.5	118	10.4	15.6	118	10.7	16.1	118	10.8	16.2	118	10.8	16.2
119	10.2	15.3	119	10.2	15.4	119	10.5	15.8	119	10.6	16.0	119	10.6	16.0
120	10.0	15.1	120	10.1	15.2	120	10.4	15.6	120	10.4	15.7	120	10.4	15.7

ASD LFRD
 $\Omega_c = 1.67$ $\phi_c = 0.90$

Steel Frame Construction



University of Michigan – North Quad

Steel Frame Construction

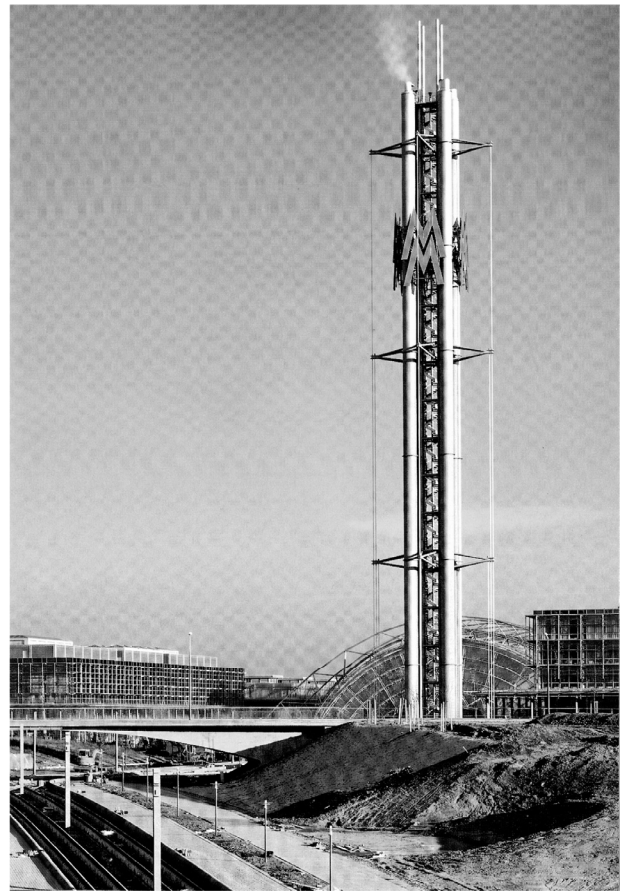
Messe Leipzig – 1996

Congress Centre – Gerkan, Marg und Partner
Glass Hall – Ian Ritchie Architects
Tower - Schlaich, Bergermann und Partner



Messe Leipzig - Glass Hall - Ian Ritchie Architects

University of Michigan, TCAUP



Messe Leipzig – Cable braced tower. Jörg Schlaich

Structures II

Slide 25 of 30

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

University of Michigan, TCAUP

Structures II

Slide 26 of 30

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Branching Columns (tree columns)



bridge in Pragsattel, Stuttgart, 1992
Schlaich, Bergemann und Partner

Branching Columns (tree columns)



Stuttgart Airport Terminal,
Gerkan, Marg und Partner
Schlaich, Bergemann und Partner