Architecture 324 Structures II

Steel Column Analysis and Design

- · Failure Modes
- · Effects of Slenderness
- · Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns



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Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{KL^2}$$

$$\underline{r} = \sqrt{\frac{I}{A}}$$

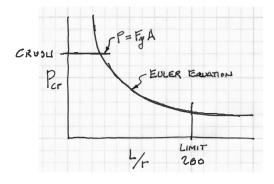
$$I = Ar^2$$

- A = Cross sectional area (in²)
- E = Modulus of elasticity of the material (lb/in²)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \le F_{cr}$$



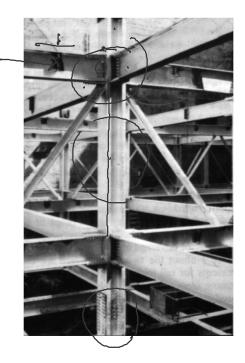
portrait by Emanuel Handmann,1753



Analysis of Steel Columns

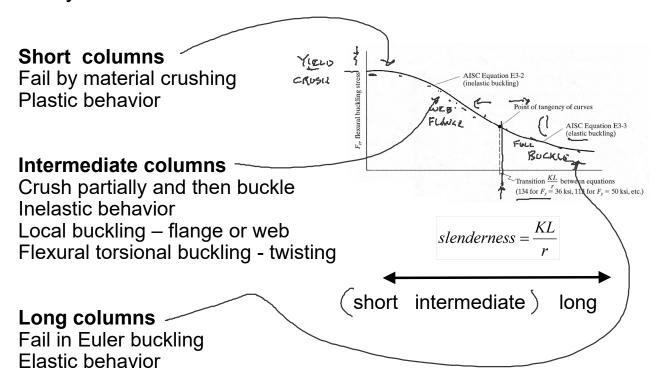
Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions



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Analysis of Steel Columns



Failure Modes

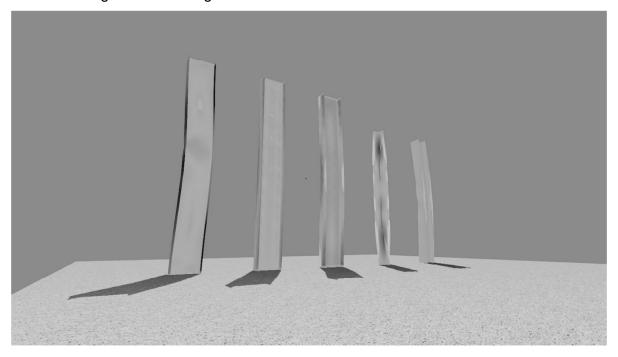
Column 1: Strong axis flexural buckling

Column 2: Web local buckling -

Column 3: Weak axis flexural buckling

Column 4: Torsional buckling
Column 5: Flange local buckling -

"Dancing Columns" Sherif El-Tawil

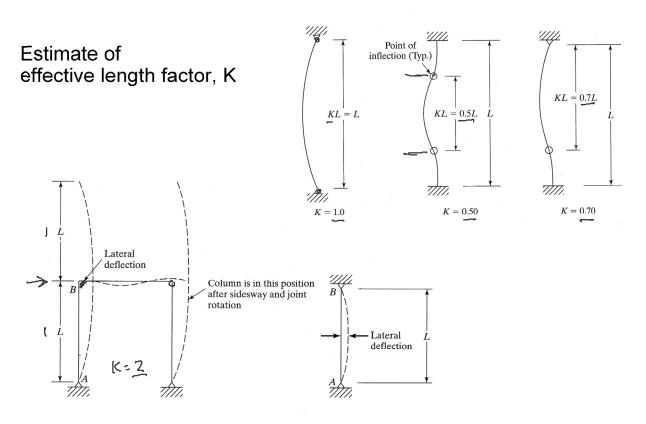


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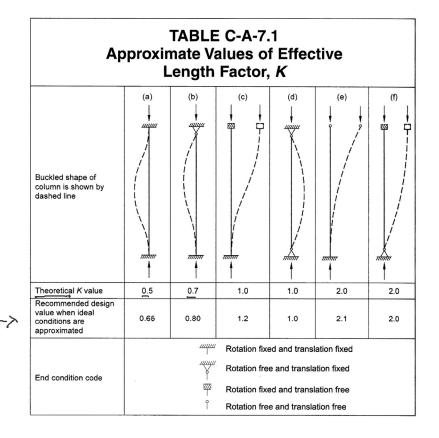
Analysis of Steel Columns



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Analysis of Steel Columns

Estimate of K:



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by Alignment Charts

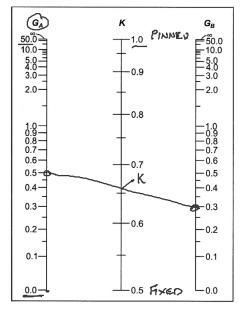
Sidesway Inhibited: Braced frame 1.0 > K > 0.5

Sidesway Uninhibited: Un-braced frame unstable > K > 1.0

More Pinned: If Ic/Lc is large and Ig/Lg is small The connection is more pinned

More Fixed: If Ic/Lc is small and Ig/Lg is large The connection is more fixed

Determining K factors (BRACED) Sidesway inhibited



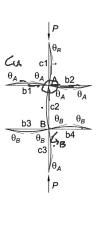


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).

$$G = \frac{\sum \left(\frac{EI}{L}\right)_{column}^{\checkmark}}{\sum \left(\frac{EI}{L}\right)_{beain}^{\checkmark}}$$

Determining K factors by Alignment Charts

Sidesway Inhibited: Braced frame 1.0 > K > 0.5

Sidesway Uninhibited: Un-braced frame unstable > K > 1.0

More Pinned: If Ic/Lc is large and Ig/Lg is small The connection is more pinned

More Fixed: If Ic/Lc is small and Ig/Lg is large The connection is more fixed

Sidesway uninhibited

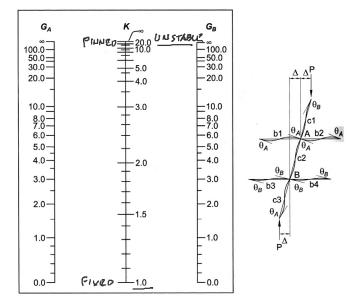


Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\sum \left(\frac{EI}{L}\right)_{column}}{\sum \left(\frac{EI}{L}\right)_{heam}}$$

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Analysis of Steel Columns - LRFD

Euler equation:

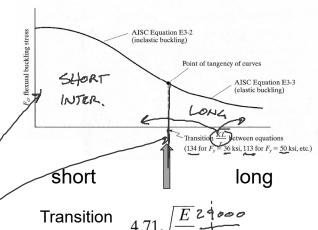
$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

Short & Intermediate Columns:

$$F_{cr} = \left[0.658 \right] F_{y}$$

Long Columns:

$$F_{cr} = 0.877 \widehat{F_e}$$



Slenderness

$$P_n = F_{cr}A_g$$

$$\phi_c P_n = \phi_c F_{cr}A_g$$

$$(\phi_c = 0.90)$$

Analysis of Steel Columns pass / fail by LRFD

Data:

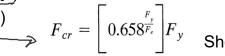
- Column size, length
- Support conditions
- Material properties Fy ✓
- Factored load Pu



 $Pu \leq \emptyset Pn (pass)$ CALC



- 1. Calculate slenderness ratios. L_c/r , $L_c = KL$ The largest ratio governs.
- 2. Check slenderness ratio against upper limit of 200 (recommended)
- 3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
- 4. Calculate F_{cr} based on slenderness
- 5. Determine øPn and compare to Pu $Pn = F_{cr} Ag$ $\phi = 0.9 \, \checkmark$
- 6. If $Pu \leq \emptyset Pn$, then OK



 $F_{cr} = 0.877 F_e$

Long

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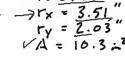
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Pu = 280 K

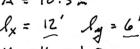
Analysis of Steel Columns pass / fail by ASD

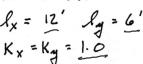
Data:

- Column size, length
- Support conditions
- Material properties Fy
- Factored Load Pu



DATA:







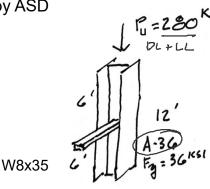


Required:

- Pu ≤ Ø Pn (pass)
- 1. Calculate slenderness ratios. The largest ratio governs.
- 2. Check slenderness ratio against upper limit of 200 (recommended)

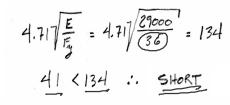
Analysis of Steel Columns

pass / fail by ASD



- 3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
- Calculate F_{cr} based on slenderness
- 5. Determine øPn and compare to Pu
- 6. If $Pu \le \emptyset Pn$, then OK

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Euler Equation

$$\frac{E}{E} = \frac{\pi^2 E}{\left(\frac{KL}{F}\right)^2} = \frac{\pi^2 29000 \, \text{Ks}}{4l^2} = 170.2 \, \text{Ks}$$

Short Column Equation

-PU=280K < 305,4K = PU OK LOAD- STRONGER

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Analysis of Steel Columns capacity by LRFD

Data:

- Column size, length
- Support conditions
- Material properties Fy

Required:

- Max load capacity
- Calculate slenderness ratios. The largest ratio governs.
- 2. Check slenderness ratio against upper limit of 200 (recommended)
- 3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
- 4. Calculate F_{cr} based on slenderness
- 5. Determine Pn and Compute allowable capacity: $Pn = F_{cr} A_a$ $Pu = \emptyset Pn$

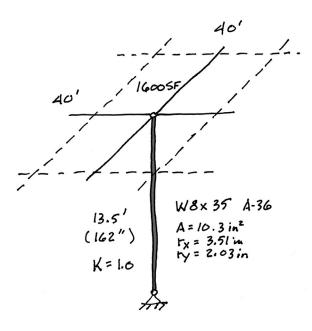


$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \quad \text{Shor}$$

$$F_{cr} = 0.877 F_{e}$$
 Long

Capacity Example 1

Free standing column
Third floor studio space
Supports roof load = 20 psf DL + SL
snow ≈ 15lbs / FT depth





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Capacity Example 1

- Calculate slenderness ratios.
 The largest ratio governs.
- Check slenderness ratio against upper limit of 200 (recommended)
- 3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
- 4. Calculate F_{cr} based on slenderness

$$\frac{y-y}{r_y} \stackrel{\text{Axis}}{=} \frac{(\text{controls})}{\frac{1(162^{\circ})}{r_y}} = \frac{1(162^{\circ})}{\frac{2.03^{\circ\prime\prime}}{}} = 79.8 < 200$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{(K_F)^2} = \frac{\pi^2 29000}{79.8^2} = 44.94 \text{ KSI}$$

Short Column Equation

$$\text{Fer} = \left[0.658^{\frac{r_y}{r_e}}\right] \text{ Fy } = \left[0.7151\right] 36 = 25.74 \text{ KSI}$$

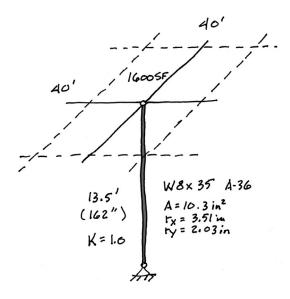
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Capacity Example 1

5. Determine øPn and Compute allowable capacity: Pu = øPn

DL = 20 psf 20 psf (1600 sf) = 32k on column



Column nominal strength

$$P_n = F_{cr} A_g = 25.74 \text{ KSI} 10.3 \text{ m}^2 = 265.1^K$$

 $\Phi_{RN} = 0.9(265) = 238.6^K = P_D$

Load capacity

For
$$A_T = 40 \times 40 = 1600 \text{ SF}$$

$$SL = \frac{125100}{1600 \text{ SF}} = 78.2 \text{ PSF}$$

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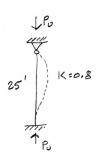
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Capacity Example 2 long column – using equations

$$r_x = 3.51 \text{ in.}$$

 $r_v = 2.03 \text{ in.}$

Table G1	Bucklii	ng Len	gth Coe	fficien	ts, K _e	
Buckling modes	Till the same of t			8	0	***
Theoretical $K_{\!e}$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0,65	0.80	1.2	1.0	2.10	2.4
End condition code	**************************************	Rotat Rotat	tion fixed tion free, tion fixed tion free,	translatio , translat	on fixed	



Slenderness y-y

$$\frac{K l_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$$

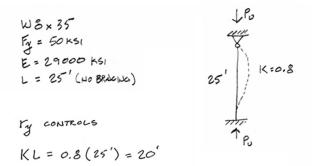
Euler Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{k\ell}{r}\right)^2} = \frac{\pi^2 29000}{118.2^2} = 20.47 \text{ KSI}$$

Long Column Equation

Column strength

Capacity Example 2 long column – using table



w	 B	-	Axia	al C	om	e Si pre -Sha	essi	_			<i>F_y</i> =	: 50 I	csi	
Sha	pe			,			W	3×						
lb.	ft	6		5		4		4		3		3		
Des	ign	P_n/Ω_o	$\phi_{\sigma}P_{n}$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_o	$\phi_c P_n$	P_n/Ω_o	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	φ _c P _i	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRF	
	0	590	886	512	769	422	634	350	526	308	463	273	411	
7	6	542	815	470	706	387	581	320	481	281	423	249	374	
į	7	526	790	455	685	375	563	309	465	272	409	241	362	
yra	8	508 488	763 733	439 422	660 634	361 347	543 521	298 285	448 429	262	394 377	232	348	
Effective length, L_c (ft), with respect to least radius of gyration, $r_{m{y}}$	10	467	701	403	606	347	497	285	429	239	359	211	317	
IIS (314			388	226		200	301	
adji	11 12	444 421	668 633	384 363	576 546	314 297	473 447	258 243	366	213	340 321	189	283	
st	13	397	597	342	514	280	421	228	343	200	301	177	266	
89	14	373	560	321	482	262	394	213	321	187	281	165	248	
윤	15	348	523	299	450	244	367	198	298	174	261	153	230	
bec	16	324	487	278	418	226	340	183	275	160	241	141	212	
8	17	300	450	257	386	209	314	169	253	147	221	130	195	
#	18	276	415	236	355	192	288	154	232	135	203	118	178	
·	19	253	381	216	325	175	264	141	211	123	104	108	162	
Ę	20	231	347	197	296	159	239	127	191	111	166	97.2	146	
, _L	22	191	287	163	244	132	198	105	158	91.5	138	80.3	121	
븅	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101	
<u>=</u>	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86	
ij.	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.	
<u>şe</u>	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64	
₩.	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.	
	34	79.9	120	68.1	102	55.1	82.8	44.0	66.1					
		Properties												
P _{wo} , kips		126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.	
Pwi, kip/i		19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.	
P_{wb} , kips		507	761	363	546	174	262	127	192	81.1	122	63.0	94.	
Pfb, kips		164	246	123	185	87.8	_	58.7	88.2	45.9	68.9	35.4	53	
L _ρ , ft L _r , ft		١.	7.49 47.6		7.42 11.6	,	7.35 35.2		7.21 29.9		7.17 27.0		7.18 24.8	
A_g , in. ²		_	19.7	-	7.1	_	14.1		1.7		10.3	-	9.13	
I_x , in. ⁴		2		22		18		14			27	11		
l _y , in. ⁴			38.6		5.1		60.9		19.1		42.6		37.1	
r_y , in.		1	2.12		2.10		2.08		2.04		2.03	'	2.02	
r _x /r _y			1.75		1.74		1.74		1.73		1.78		1.72	
$P_{\rm ex}L_c^2/10$		779		653		527		418		36		315		
$P_{ey}L_c^2/10$		25		215		174		14		12	20	106	30	
AS	D	LRF	D	Note: H	eavy line	indicates	L _c /r _y equ	ual to or g	reater th	an 200.				
$\Omega_c =$	1.67	φ _c = 0	0.90											

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Design of Steel Columns with AISC Strength Tables

Data:

- Column length
- Support conditions
- Material properties Fy
- Applied load Pactual

Required:

- Column Size
- 1. Enter table with height, KL = Lc
- 2. Read allowable load for each section to find the smallest adequate size.
- 3. Tables assume weak axis buckling. If the strong axis controls the length must be divided by the ratio rx/ry
- 4. Values stop in table (black line) at slenderness limit, KL/r = 200

DESIGN OF COMPRESSION MEMBERS

	_	,			om	e Si ipre	essi	•			F _y =	50 I	csi	
W	3				W	-Sha	pes							
Sha							W	_						
lb/	ft	6		5		4		4	-	3	-	3	-	
Des	ign	P_n/Ω_o	φ _e P _n	P_n/Ω_o	$\phi_c P_n$	P_n/Ω_o	¢₀Pn	P_n/Ω_o	φ ₀ P _n	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	Øc₽.	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRF	
	0	590	886	512	769	422	634	350	526	308	463	273	411	
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yra	9	488	763 733	439	634	347	521	298	448	251	377	232	333	
of G	10	467	701	403	606	331	497	272	409	239	359	211	317	
Effective length, L_c (ft), with respect to least radius of gyration, ℓ_F	11	444	668	384	576	314	473	258	388	226	340	200	301	
agi.	12	421	633	363	546	297	4/3	243	366	213	321	189	283	
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be	16	324	487	278	418	226	340	183	275	160	241	141	212	
82	17	300	450	257	386	209	314	169	253	147	221	130	195	
€	18	276	415	236	355	192	288	154	232	135	203	118	178	
š.	19	253	381	216	325	175	264	141	211	123	184	108	162	
Ę	20	231	347	197	296	159	239	127	191	111	166	97.2	146	
J, L	22	191	287	163	244	132	198	105	158	91.5	138	80.3	121	
튵	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101	
프	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86	
Ž.	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74	
g g	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64	
	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57	
	34	79.9	120	68.1	102	55.1	82.8	44.0	66.1					
						Propert	ies							
P _{wo} , kips		126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.	
P _w , kip/ii	1.	19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.	
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L_p , ft L_r , ft			7.49 47.6		7.42 11.6	,	7.35 35.2		7.21 29.9		7.17 27.0		7.18 24.8	
A_g , in. ²			19.7		7.1	_	4.1		11.7	_	10.3	<u> </u>	9.13	
I_{χ} , in. ⁴			72	22		18		14			27	11	10	
l _y , in. ⁴			88.6		75.1		60.9		49.1		42.6		37.1	
r_y , in.			2.12		2.10	'	2.08		2.04		2.03		2.02	
r _x /r _y			1.75		1.74		1.74		1.73		1.78		1.72	
$P_{ex}L_c^2/10$		77		653		527		418		360		315		
$P_{ey}L_c^2/10$		25		215		174		14		122	20	106	50	
ASI)	LRF	D	Note: H	eavy line	indicates	L _c /r _y eq	ual to or g	reater th	an 200.				

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	_	,	A۱	aila	able om	e Si pre	trer essi	nued ngth ion,	ı in	os	F _y =	= 50 k	si
W1	0				W	-Sha	pes						
Sha	<u> </u>						W1						
lb/	'ft	11		10	_	8		7		6		6	
Des	ign	P_n/Ω_c	φ _c P _n	P_n/Ω_c	φ _c P _n	P_n/Ω_c	φ _c P _n	P_n/Ω_c	φ _c P _n	P_n/Ω_c	φ _c P _n	P_n/Ω_c	$\phi_c P_n$
	_	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFI
ration, r _y	0 6 7 8 9	985 934 917 897 875	1480 1400 1380 1350 1310	877 831 815 797 777	1320 1250 1230 1200 1170	778 737 722 706 688	1170 1110 1090 1060 1030	680 643 630 615 599	966 946 925 900	596 563 552 539 525	895 846 829 810 789	530 500 490 479 466	796 752 737 719 700
Effective length, L_c (ft), with respect to least radius of gyration, ℓ_y	10 11 12 13 14 15	851 825 798 769 739 708	1280 1240 1200 1160 1110 1060	755 732 707 681 654 626	1130 1100 1060 1020 983 941	669 647 625 602 578 553	973 940 905 868 831	582 563 543 522 501 479	874 846 816 785 753 720	509 493 475 457 438 419	765 741 714 687 658 629	452 437 421 405 388 370	679 657 633 608 583 556
with respect to	16 17 18 19 20	677 645 613 580 548	1020 969 921 872 824	598 569 540 511 482	898 855 811 767 724	527 501 475 449 423	792 754 714 675 636	456 433 410 387 365	686 651 617 582 548	399 379 358 338 318	599 569 539 508 478	352 334 316 298 280	530 502 475 448 421
length, L_c (ft),	22 24 26 28 30	485 423 365 315 274	728 636 548 473 412	425 370 318 274 239	638 556 478 412 359	373 324 278 239 209	560 487 417 360 313	320 277 237 204 178	481 417 356 307 267	279 241 206 178 155	419 363 310 267 233	245 212 181 156 136	368 318 271 234 204
Effective	32 34 36 38 40	241 213 190 171 154	362 321 286 257 232	210 186 166 149 134	315 279 249 224 202	183 162 145 130 117	276 244 218 195 176	156 139 124 111 100	235 208 186 167 150	136 121 108 96.5 87.1	205 181 162 145 131	119 106 94.2 84.5 76.3	179 159 142 127 115
						Propert	ies						
P _{wo} , kips P _{wi} , kip/ii P _{wb} , kips P _{fb} , kips	n.	220 25.2 949 292	330 37.8 1430 439	184 22.7 690 235	275 34.0 1040 353	150 20.2 487 183	225 30.3 732 276	121 17.7 328 142	182 26.5 494 213	99.5 15.7 229 111	149 23.5 344 167	82.6 14.0 163 86.5	124 21. 245 130
L_p , ft L_r , ft			9.47 64.1		9.36 57.9		9.29 51.2		9.18 45.3		9.15 40.6		9.08 36.6
A_g , in. ² I_x , in. ⁴ I_y , in. ⁴ I_y , in. I_x/I_y I_x/I_y I_{ex}/I_y	r, ft g, in. ² c, in. ⁴ r, in. ⁴ r, in.		32.9 16 36 2.68 1.74 500	6 2 178	29.3 623 207 2.65 1.74 17800 5920		26.0 34 79 2.63 1.73 00	22.7 455 154 2.60 1.73 13000 4410		19.9 394 134 2.59 1.71 11300 3840		17.7 341 116 2.5 1.7 9760 3320	
ASI		LRF	-D										

<i>F</i> _y =	: 50 ks	si A	Ava	ailab	le S	(conti S tre r essi	ngth	in	6		
				١	W-Sh	apes				w	10
Sha	ape					W1	0×				
lb/	/ft	5	4	4	9	4	5	3	9	3	3
Des	ian	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	φ _c P _n	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	φ _c P _n	P_n/Ω_c	φ _c P _n
Duo	igii	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFI
	0	473	711	431	648	398	. 598	344	517	291	437
yration, ry	6 7 8 9	446 437 427 415	671 657 642 624	407 398 388 378	611 598 584 568	363 350 337 322	545 527 507 485	313 302 290 277	470 454 436 416	263 253 243 232	395 381 365 348
Effective length, L_c (ft), with respect to least radius of gyration, $r_{m{y}}$	10 11 12 13 14 15	403 389 375 361 345 330	585 564 542 519 495	366 354 341 327 313 299	550 532 512 492 471 449	307 291 274 256 239 222	461 437 411 385 359 333	263 249 234 219 203 188	396 374 352 329 306 283	220 207 194 181 168 155	330 311 292 272 253 233
with respect to	16 17 18 19 20	314 297 281 265 249	471 447 422 398 374	284 269 254 239 224	427 404 382 360 337	204 188 171 155 140	307 282 257 234 211	173 158 144 130 118	260 238 217 196 177	142 130 117 106 95.4	214 195 177 159 143
length, L_G (ft),	22 24 26 28 30	217 188 160 138 120	327 282 240 207 180	196 168 143 124 108	294 253 216 186 162	97.4 83.0 71.5 62.3	174 146 125 108 93.7	97.2 81.7 69.6 60.0 52.3	146 123 105 90.2 78.6	78.8 66.2 56.4 48.7 42.4	118 99. 84. 73. 63.
Effective	32 34 36 38 40	106 93.5 83.4 74.8 67.6	159 141 125 112 102	94.7 83.9 74.8 67.2 60.6	142 126 112 101 91.1	54.8	82.3	46.0	69.1	37.3	56.
					Prope	erties					
P _{wo} , kips P _{wi} , kip/i P _{wb} , kips P _{fb} , kips	n.	69.1 12.3 112 70.8	104 18.5 168 106	60.1 11.3 86.6 58.7	90.1 17.0 130 88.2	65.3 11.7 94.2 71.9	98.0 17.5 142 108	54.1 10.5 68.7 52.6	81.1 15.8 103 79.0	45.2 9.67 53.7 35.4	67.8 14.5 80.7 53.3
L_p , ft L_r , ft		3	9.04 33.6	3	8.97 31.6	2	7.10 6.9		6.99 4.2		6.85 1.8
A_g , in. ² I_x , in. ⁴ I_y , in. ⁴ I_y , in. I_y , in.	0 ⁴ , k-in. ²		5.8)3)3 2.56 1.71	27	14.4 72 93.4 2.54 1.71	1 24	3.3 8 53.4 2.01 2.15	1 20 4	1.5 9 5.0 1.98 2.16	17 3	9.71 1 6.6 1.94 2.16
AS	SD	LR	FD	Note: Hea	avy line ind	icates L _c /r _y	equal to o	r greater th	an 200.		
$\Omega_c =$	1.67	$\phi_c =$	0.90								

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AISC Critical Stress Table

Table 4-22 Available Critical Stress for Compression Members

= 35 I	KSI		$F_y = 36 \text{ k}$	SI	18.0	$F_y = 42 \text{ I}$	(Si		$F_y = 46 \text{ I}$	(Si		ksi	
$_{r}/\Omega_{c}$	φ _c F _{cr}		F_{cr}/Ω_c	$\phi_c F_{cr}$		F_{cr}/Ω_c	φ _c F _{cr}		F_{cr}/Ω_c	φ _c F _{cr}		F_{cr}/Ω_c	φ _c F _c
ksi	ksi	KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi
ASD	LRFD	1	ASD	LRFD	1	ASD	LRFD	1	ASD	LRFD	,	ASD	LRFD
1.0	31.5	15	21.6	32.4	1	25.1	37.8	1	27.5	41.4	1	29.9	45.0
1.0	31.5	2	21.6	32.4	2	25.1	37.8	2	27.5	41.4	2	29.9	45.0
0.9	31.5	3.	21.5	32.4	3	25.1	37.8	3	27.5	41.4	3	29.9	45.0
0.9	31.5	4	21.5	32.4	4	25.1	37.8	4	27.5	41.4	4	29.9	44.9
0.9	31.5	5	21.5	32.4	5	25.1	37.7	5	27.5	41.3	5	29.9	44.9
0.9	31.4	6	21.5	32.3	6	25.1	37.7	6	27.5	41.3	6	29.9	44.9
0.9	31.4	7	21.5	32.3	7	25.1	37.7	7	27.5	41.3	7	29.8	44.8
0.9	31.4	8	21.5	32.3	8	25.1	37.7	8	27.4	41.2	8	29.8	44.8
0.9	31.4	9	21.5	32.3	9	25.0	37.6	9	27.4	41.2	9	29.8	44.7
0.9	31.3	10	21.4	32.2	10	25.0	37.6	10	27.4	41.1	10	29.7	44.7
0.8	31.3	11	21.4	32.2	11	25.0	37.5	11	27.3	41.1	11	29.7	44.6
0.8	31.3	12	21.4	32.2	12	24.9	37.5	12	27.3	41.0	12	29.6	44.5
0.8	31.2	13	21.4	32.1	13	24.9	37.4	13	27.2	40.9	13	29.6	44.4
0.7	31.2	14	21.3	32.1	14	24.8	37.3	14	27.2	40.9	14	29.5	44.4
0.7	31.1	15	21.3	32.0	15	24.8	37.3	15	27.1	40.8	15	29.5	44.3
0.7	31.1	16	21.3	32.0	16	24.8	37.2	16	27.1	40.7	16	29.4	44.2
0.7	31.0	17	21.2	31.9	17	24.7	37.1	17	27.0	40.6	17	29.3	44.1
0.6	31.0	18	21.2	31.9	18	24.7	37.1	18	27.0	40.5	18	29.2	43.9
0.6	30.9	19	21.2	31.8	19	24.6	37.0	19	26.9	40.4	19	29.2	43.8
0.5	30.9	20	21.1	31.7	20	24.5	36.9	20	26.8	40.3	20	29.1	43.7
0.5	30.8	21	21.1	31.7	21	24.5	36.8	21	26.7	40.2	21	29.0	43.6
0.4	30.7	22	21.0	31.6	22	24.4	36.7	22	26.7	40.1	22	28.9	43.4
0.4	30.7	23	21.0	31.5	23	24.3	36.6	23	26.6	40.0	23	28.8	43.3
0.3	30.6	24	20.9	31.4	24	24.3	36.5	24	26.5	39.8	24	28.7	43.1
0.3	30.5	25	20.9	31.4	25	24.2	36.4	25	26.4	39.7	25	28.6	43.0
0.2	30.4	26	20.8	31.3	26	24.1	36.3	26	26.3	39.6	26	28.5	42.8
0.2	30.3	27	20.7	31.2	27	24.0	36.1	27	26.2	39.4	27	28.4	42.7
0.1	30.3	28	20.7	31.1	28	24.0	36.0	28	26.1	39.3	28	28.3	42.5
0.1	30.2	29	20.6	31.0	29	23.9	35.9	29	26.0	39.1	29	28.2	42.3
0.0	30.1	30	20.6	30.9	30	23.8	35.8	30	25.9	39.0	30	28.0	42.1
0.0	30.0	31	20.5	30.8	31	23.7	35.6	31	25.8	38.8	31	27.9	41.9
9.9	29.9	32	20.4	30.7	32	23.6	35.5	32	25.7	38.6	32	27.8	41.8
9.8	29.8	33	20.4	30.6	33	23.5	35.4	33	25.6	38.5	33	27.7	41.6
9.8	29.7	34	20.3	30.5	34	23.4	35.2	34	25.5	38.3	34	27.5	41.4
9.7	29.6	35	20.2	30.4	35	23.3	35.1	35	25.4	38.1	35	27.4	41.2
9.6	29.5	36	20.1	30.3	36	23.2	34.9	36	25.2	37.9	36	27.2	40.9
9.5	29.4	37	20.1	30.1	37	23.1	34.8	37	25.1	37.8	37	27.1	40.7
9.5	29.3	38	20.0	30.0	38	23.0	34.6	38	25.0	37.6	38	26.9	40.5
9.4	29.1	39	19.9	29.9	39	22.9	34.4	39	24.9	37.4	39	26.8	40.3
9.3	29.0	40	19.8	29.8	40	22.8	34.3	40	24.7	37.2	40	26.6	40.0
	LRFD	-	1	-		and the second	400	-	Tanana areas	Service of the last	-	and and ac	Q/r
9.5 9.4 9.3	ф	29.1 29.0	29.1 39 29.0 40 LRFD	29.3 38 20.0 29.1 39 19.9 29.0 40 19.8 LRFD	29.3 38 20.0 30.0 29.1 39 19.9 29.9 29.0 40 19.8 29.8 LRFD	29.3 38 20.0 30.0 38 29.1 39 19.9 29.9 39 29.0 40 19.8 29.8 40 LRFD	29.3 38 20.0 30.0 38 23.0 29.1 39 19.9 29.9 39 22.9 29.0 40 19.8 29.8 40 22.8 LRFD	29.3 38 20.0 30.0 38 23.0 34.6 29.1 39 19.9 29.9 39 22.9 34.4 29.0 40 19.8 29.8 40 22.8 34.3 LRFD	29.3 38 20.0 30.0 38 23.0 34.6 38 29.1 39 19.9 29.9 39 22.9 34.4 39 29.0 40 19.8 29.8 40 22.8 34.3 40 LRFD	29.3 38 20.0 30.0 38 23.0 34.6 38 25.0 29.1 39 19.9 29.9 39 22.9 34.4 39 24.9 29.0 40 19.8 29.8 40 22.8 34.3 40 24.7 LRFD	29.3 38 20.0 30.0 38 23.0 34.6 38 25.0 37.6 (29.1 39 19.9 29.9 39 22.9 34.4 39 24.9 37.4 29.0 40 19.8 29.8 40 22.8 34.3 40 24.7 37.2 LRFD	29.3 38 20.0 30.0 38 23.0 34.6 38 25.0 37.6 38 29.1 39 19.9 29.9 39 22.9 34.4 39 24.9 37.4 39 29.0 40 19.8 29.8 40 22.8 34.3 40 24.7 37.2 40 41.7	29.1 38 20.0 9.0 38 23.0 34.6 38 25.0 37.6 38 26.9 29.1 39 19.9 29.9 39 22.9 34.4 39 24.9 37.4 39 26.8 29.0 40 19.8 29.8 40 22.8 34.3 40 24.7 37.2 40 26.6 LRFD

for previous example $KI/r_y = 118.2$

Table 4-22 (continued) Available Critical Stress for Compression Members

	$F_y = 35 \text{ F}$	si		$F_y = 36 \text{ F}$	csi		$F_y = 42 $	si		$F_y = 46 \text{ I}$	csi		$F_y = 50 \text{ I}$	ksi
	F_{cr}/Ω_c	φ _c F _{cr}	w.	F_{cr}/Ω_c	φ _c F _{cr}	,,	F_{cr}/Ω_c	φ _c F _{cr}	w.	F_{cr}/Ω_c	φ _c F _{cr}		F_{cr}/Ω_c	ocFc.
KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi
1	ASD	LRFD	1	ASD	LRFD	10	ASD	LRFD	100	ASD	LRFD	1	ASD	LRFD
41	19.2	28.9	41	19.7	29.7	41	22.7	34.1	41	24.6	37.0	41	26.5	39.8
42	19.2	28.8	42	19.6	29.5	42	22.6	33.9	42	24.5	36.8	42	26.3	39.5
43	19.1	28.7	43	19.6	29.4	43	22.5	33.7	43	24.3	36.6	43	26.2	39.3
44	19.0	28.5	44	19.5	29.3	44	22.3	33.6	44	24.2	36.3	44	26.0	39.1
45	18.9	28.4	45	19.4 8	29.1	45	22.2	33.4	45	24.0	36.1	45	25.8	38.8
46	18.8	28.3	46	19.3	29.0	46	22.1	33.2	46	23.9	35.9	46	25.6	38.5
47	18.7	28.1	47	19.2	28.9	47	22.0	33.0	47	23.8	35.7	47	25.5	38.3
48	18.6	28.0	48	19.1	28.7	48	21.8	32.8	48	23.6	35.4	48	25.3	38.0
49	18.5	27.9	49	19.0	28.5	49	21.7	32.6	49	23.4	35.2	49	25.1	37.7
50	18.4	27.7	50	18.9	28.4	50	21.6	32.4	50	23.3	35.0	50	24.9	37.5
51	18.3	27.6	51	18.8	28.3	51	21.4	32.2	51	23.1	34.8	51	24.8	37.2
52	18.3	27.4	52	18.7	28.1	52	21.3	32.0	52	23.0	34.5	52	24.6	36.9
53	18.2	27.3	53	18.6	28.0	53	21.2	31.8	53	22.8	34.3	53	24.4	36.7
54	18.1	27.1	54	18.5	27.8	54	21.0	31.6	54	22.6	34.0	54	24.2	36.4
55	18.0	27.0	55	18.4	27.6	55	20.9	31.4	55	22.5	33.8	55	24.0	36.1
56	17.9	26.8	56	18.3	27.5	56	20.7	31.2	56	22.3	33.5	56	23.8	35.8
57	17.7	26.7	57	18.2	27.3	57	20.6	31.0	57	22.1	33.3	57	23.6	35.5
58	17.6	26.5	58	18.1	27.1	58	20.5	30.7	58	22.0 €	33.0	58	23.4	35.2
59	17.5 N	26.4	59	17.9	27.0	59	20.3	30.5	59	21.8	32.8	59	23.2	34.9
60	17.4	26.2	60	17.8	26.8	60	20.2	30.3	60	21.6	32.5	60	23.0	34.6
61	17.3	26.0	61	17.7	26.6	61	20.0	30.1	61	21.4	32.2	61	22.8	34.3
62	17.2	25.9	62	17.6	26.5	62	19.9	29.9	62	21.3	32.0	62	22.6	34.0
63	17.1	25.7	63	17.5	26.3	63	19.7	29.6	63	21.1	31.7	63	22.4	33.7
64	17.0	25.5	64	17.4	26:1	64	19.6	29.4	64	20.9	31.4	64	22.2	33.4
65	16.9	25.4	65	17.3	25.9	65	19.4	29.2	65	20.7	31.2	65	22.0	33.0
66	16.8	25.2	66	17.1	25.8	66	19.2	28.9	66	20.5	30.9	66	21.8	32.7
67	16.7	25.0	67	17.0	25.6	67	19.1	28.7	67	20.4	30.6	67	21.6	32.4
68	16.5	24.9	68	16.9	25.4	68	18.9	28.5	68	20.2	30.3	68	21.4	32.1
69	16.4	24.7	69	16.8	25.2	69	18.8	28.2	69	20.0	30.1	69	21.1	31.8
70	16.3	24.5	70	16.7	25.0	70	18.6	28.0	70	19.8	29.8	70	20.9	31.4
71	16.2	24.3	71	16.5	24.8	71	18.5	27.7	71	19.6	29.5	71	20.7	31.1
72	16.1.3	24.2	72	16.4	24.7	72	18.3	27.5	72	19.4	29.2	72	20.5	30.8
73	16.0	24.0	73	16.3	24.5	73	18.1	27.2	73	19.2	28.9	73	20.3	30.5
74	15.8	23.8	74	16.2	24.3	74	18.0	27.0	74	19.1	28.6	74	20.1	30.2
75	15.7	23.6	75	16.0	24.1	75	17.8	26.8	75	18.9	28.4	75	19.8	29.8
76	15.6	23.4	76	15.9	23.9	76	17.6	26.5	76	18.7	28.1	76	19.6	29.5
77	15.5	23.3	77	15.8	23.7	77	17.5	26.3	77	18.5	27.8	77	19.4	29.2
78	15.4	23.1	78	15.6	23.5	78	17.3	26.0	78	18.3	27.5	78	19.2	
79	15.2	22.9	79	15.5		79	17.1	25.8	79	18.1	27.2	79	19.0	28.5
80	15.1	22.7	80	15.4	23.1	80	17.0	25.5	80	17.9	26.9	80	18.8	28.2
_	100000000	LRFD	- 55		28	00				10000		00	12010N	_
A		c = 0.90	J											CPA

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Structures II

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AISC Critical Stress Table

for previous example $KI/r_y = 118.2$

TO FIND CAPACITY:

\$\Phi = 16.2 \quad \text{61}

\$\Phi = P_U = \Phi \text{Er Ag} \quad \text{PU} = 16.2 \quad \text{(10.3)} = \quad \quad \text{46.8} \quad \text{K}

Table 4-22 (continued) Available Critical Stress for Compression Members

	$F_y = 35 \text{ F}$	(Si		$F_y = 36 \text{ F}$	(Si		$F_y = 42 \text{ k}$	csi		$F_y = 46 \text{ I}$	ksi		$F_y = 50 \text{ i}$	(Si
in	F_{cr}/Ω_c	o _c F _{cr}		F_{cr}/Ω_c	φ _c F _{cr}		F_{cr}/Ω_c	φ _c F _{cr}	w.	F_{cr}/Ω_c	φ _c F _{cr}	w	F_{cr}/Ω_c	$\phi_c F_c$
KL	ksi	ksi	$\frac{KL}{r}$	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi	KL	ksi	ksi
1	ASD	LRFD	1	ASD	LRFD	1 '	ASD	LRFD	1	ASD	LRFD	1	ASD	LRF
81	15.0	22.5	81	15.3	22.9	81	16.8	25.3	81	17.7	26.6	81	18.5	27.9
82	14.9	22.3	82	15.1	22.7	82	16.6	25.0	82	17.5	26.3	82	18.3	27.5
83	14.7	22.1	83	15.0	22.5	83	16.5	24.8	83	17.3	26.0	83	18.1	27.2
84	14.6	22.0	84	14.9	22.3	84	16.3	24.5	84	17.1.0	25.8	84	17.9	26.9
85	14.5	21.8	85	14.7	22.1	85	16.1	24.3	85	16.9 8	25.5	85	17.7 8	26.5
86	14.4	21.6	86	14.6	22.0	86	16.0	24.0	86	16.7	25.2	86	17.4	26.2
87	14.2	21.4	87	14.5	21.8	87	15.8	23.7	87	16.6	24.9	87	17.2	25.9
88	14.1	21.2	88	14.3	21.6	88	15.6	23.5	88	16.4	24.6	88	17.0	25.5
89	14.0	21.0	89	14.2	21.4	89	15.5	23.2	89	16.2	24.3	89	16.8	25.2
90	13.8	20.8	90	14.1.2	21.2	90	15.3	23.0	90	16.0	24.0	90	16.6	24.9
91	13.7	20.6	91	13.9	21.0	91	15.1	22.7	91	15.8	23.7	91	16.3	24.6
92	13.6	20.4	92	13.8	20.8	92	15.0	22.5	92	15.6	23.4	92	16.1.5	24.2
93	13.5	20.2	93	13.7	20.5	93	14.8	22.2	93	15.4	23.1	93	15.9	23.9
94	13.3	20.0	94	13.5	20.3	94	14.6	22.0	94	15.2 8		94	15.7	23.6
95	13.2	19.9	95	13.4	20.1	95	14.4	21.7	95	15.0	22.6	95	15.5.8	23.3
96	13.1	19.7	96	13.3	19.9	96	14.3	21.5	96	14.8	22.3	96	15.3	22.9
97	13.0	19.5	97	13.1	19.7	97	14.1	21.2	97	14.6	22.0	97	15.0	22.6
98	12.8	19.3	98	13.0	19.5	98	13.9	21.0	98	14.4	21.7	98	14.8	22.3
99	12.7	19.1	99	12.9	19.3	99	13.8		99	14.2	21.4	99	14.6	22.0
100	12.6	18.9	100	12.7	19.1	100	13.6	20.5	100	14.1	21.1	100	14.4	21.7
101	12.4	18.7	101	12.6	18.9	101	13.4	20.2	101	13.9	20.8	101	14.2	21.3
102	12.3	18.5	102	12.5	18.7	102	13.3	20.0	102	13.7	20.6	102	14.0	21.0
103	12.2	18.3	103	12.3	18.5	103	13.1	19.7	103	13.5	20.3	103	13.8	20.7
104	12.1	18.1	104	12.2	18.3	104	12.9	19.5	104	13.3	20.0	104	13.6	20.4
105	11.9	17.9	105	12.1	18.1	105	12.8	19.2	105	13.1	19.7	105	13.4	20.1
106	11.8	17.7	106	11.9	17.9	106	12.6	19.0	106	12.9	19.4	106	13.2	19.8
107	11.7	17.5	107	11.8	17.7	107	12.4	18.7	107	12.8	19.2	107	13.0	19.5
108	11.5	17.3	108	11.7	17.5	108	12.3	18.5	108	12.6	18.9	108	12.8	19.2
109	11.4	17.2	109	11.5	17.3	109	12.1	18.2	109	12.4 3		109	12.6	18.9
110	11.3	17.0	110	11.4	17.1	110	12.0		110	12.2	18.3	110	12.4	18.6
111	11.2	16.8	111	11.3	16.9	111	11.8	17.7	111	12.0	18.1	111	12.2	18.3
112	11.0	16.6	112	11.1	16.7	112	11.6		112	11.8	17.8	112	12.0	18.0
113	10.9	16.4	113	11.0	16.5	113	11.5	17.3	113	11.7	17.5	113	11.8	17.7
114	10.8	16.2	114	10.9	16.3	114	11.3		114	11.5	17.3	114	11.6	
115	10.7	16.0	115	10.7.8	16.2	115	11.2	16.8	115	11.3	200	115	11.4	17.1
116	10.7	15.8	116	10.6	16.0	116	11.0	16.5	116	11.1	16.7	116	11.2	16.8
117	10.4	15.6	117	10.5	15.8	117	10.8	16.3	117	11.0	16.5	117	11.0	16.5
118	10.4	15.5	118	10.4	15.6	118	10.7	16.1	118	10.8	16.2	118	10.8	16.2
119	10.3		119	10.4	15.4	119	10.7	15.8	119	10.6	16.0	119	10.6	10.2
120	10.0	15.1	120	10.1	15.4	120	10.4	15.6	120	10.4	15.7	120	10.4	15.7
	1 SEC. C. C. C. C.		120	10.1	10.2	120	10.4	10.0	120	10.4	13.7	120	10.4	
1000	SD	LRFD												1124
$\Omega_c =$	1.67 ¢	c = 0.90												

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Steel Frame Construction



University of Michigan - North Quad

Steel Frame Construction

Messe Leipzig – 1996

Congress Centre – Gerkan, Marg und Partner Glass Hall – Ian Ritchie Architects Tower - Schlaich, Bergermann und Partner



Messe Leipzig - Glass Hall - Ian Ritchie Architects

University of Michigan, TCAUP



Messe Leipzig – Cable braced tower. Jörg Schlaich

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Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

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Steel Frame Construction



Messe Leipzig Glass Hall - Ian Ritchie Architects

Branching Columns (tree columns)





bridge in Pragsattel, Stuttgart, 1992 Schlaich, Bergermann und Partner





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Branching Columns (tree columns)

