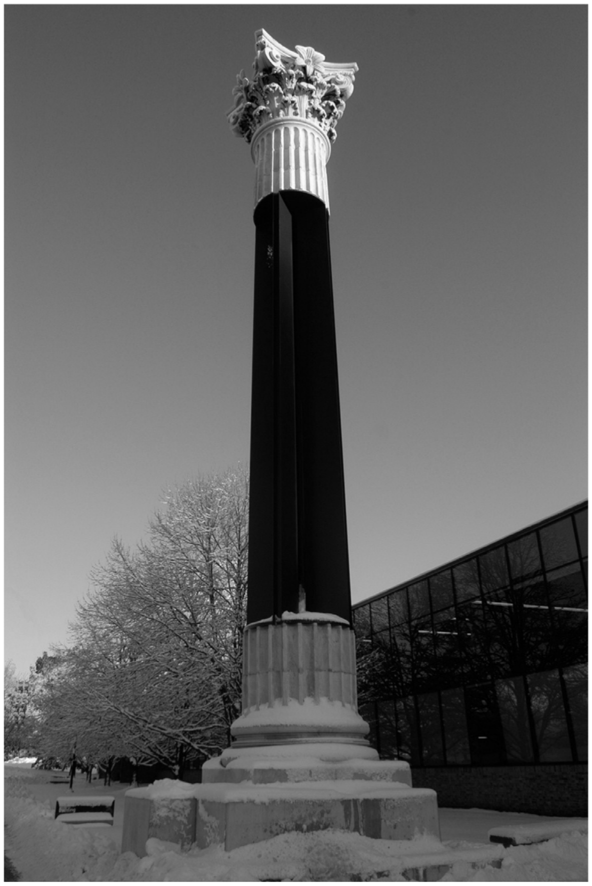
Architecture 324
Structures II

Steel Column
Analysis and Design



- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns
- Capacity Analysis of Steel Columns
- Design of Steel Columns

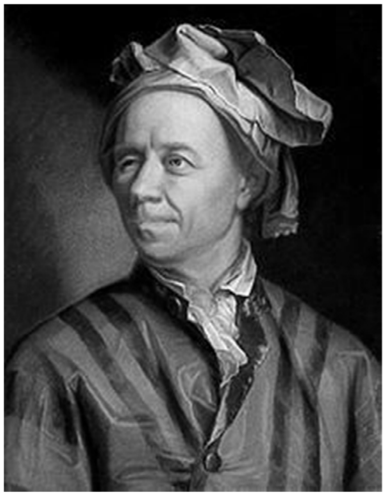
Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{KL^2}$$

$$r = \sqrt{\frac{I}{A}}$$

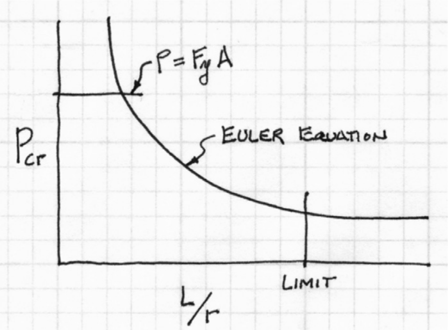
$$I = Ar^2$$



portrait by Emanuel Handmann,1753

- A = Cross sectional area (in^2)
- E = Modulus of elasticity of the material (lb/in^2)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$



Analysis of Steel Columns

Conditions of an Ideal Column

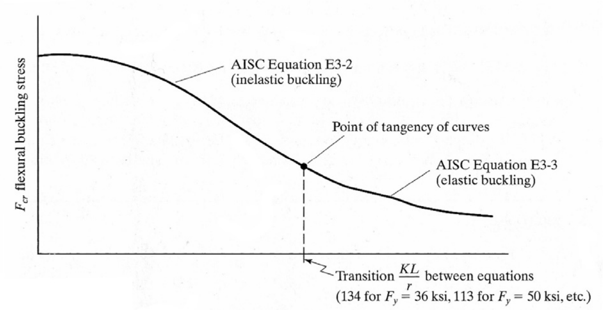
- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions

Analysis of Steel Columns

Short columns
Fail by material crushing
Plastic behavior

Intermediate columns
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

Long columns
Fail in Euler buckling
Elastic behavior



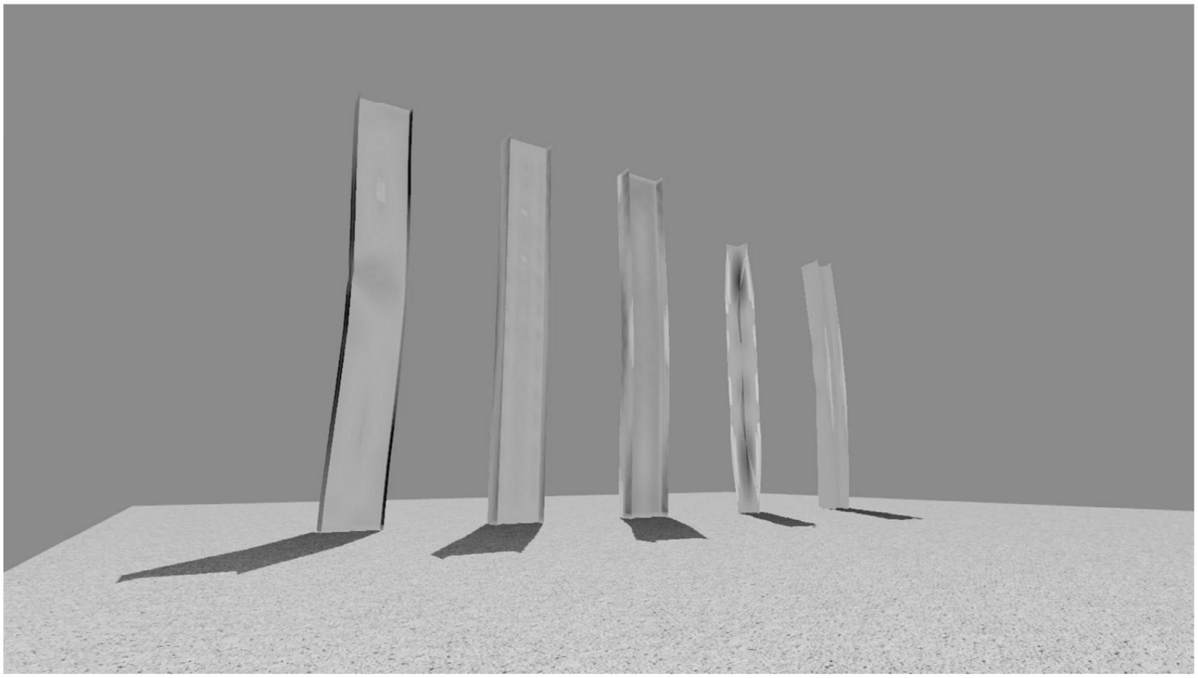
$$slenderness = \frac{KL}{r}$$

short intermediate long

Failure Modes

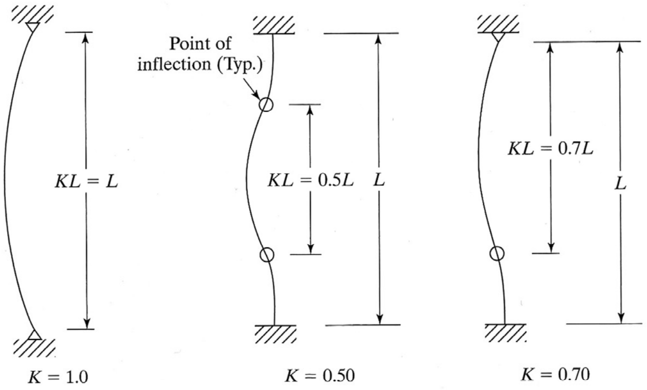
Column 1: Strong axis flexural buckling
Column 2: Web local buckling
Column 3: Weak axis flexural buckling
Column 4: Torsional buckling
Column 5: Flange local buckling

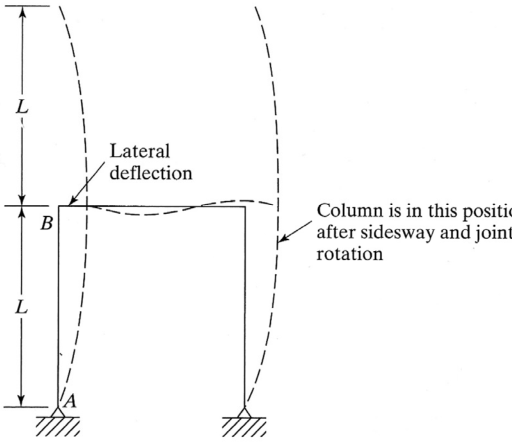
"Dancing Columns"
Sherif El-Tawil

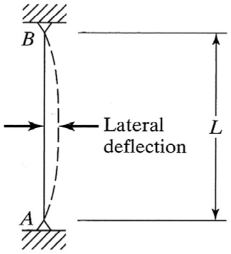


Analysis of Steel Columns

Estimate of effective length factor, K







Analysis of Steel Columns

Estimate of K:

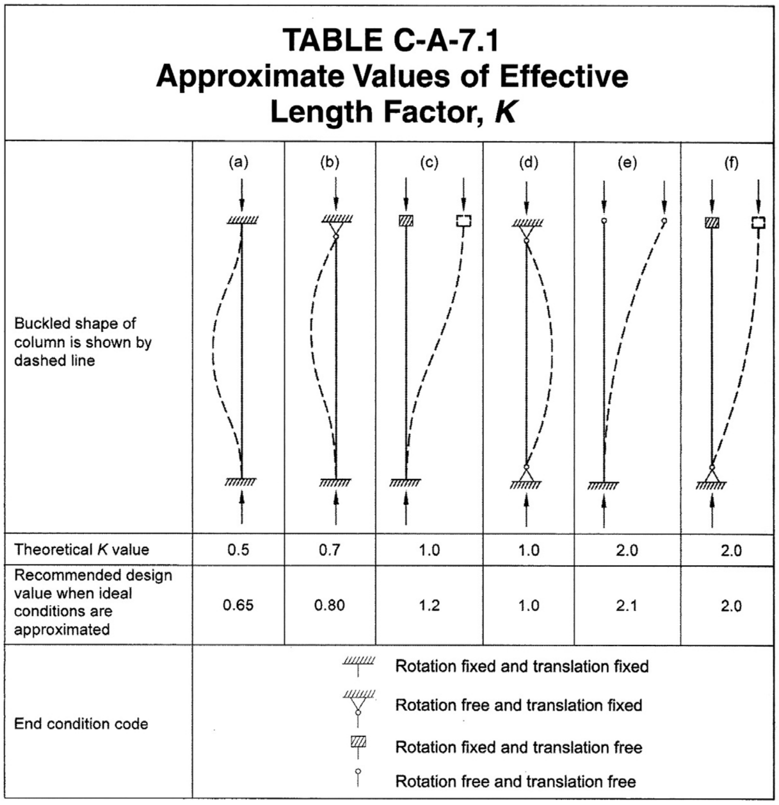


TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

Determining K factors
by Alignment Charts

Sidesway inhibited

Sidesway Inhibited:
  Braced frame
  1.0 > K > 0.5

Sidesway Uninhibited:
  Un-braced frame
  unstable > K > 1.0

More Pinned:
If Ic/Lc is large
and Ig/Lg is small
The connection is more pinned

More Fixed:
If Ic/Lc is small
and Ig/Lg is large
The connection is more fixed

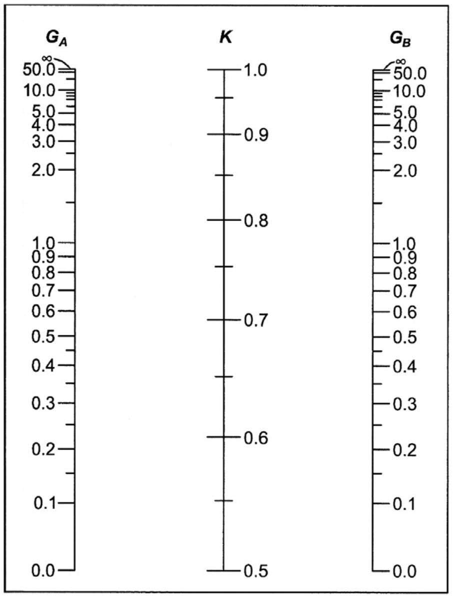
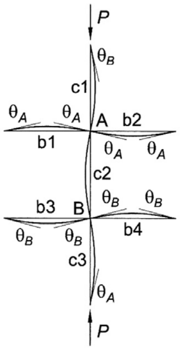


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma\left(\frac{EI}{L}\right)_{column}}{\Sigma\left(\frac{EI}{L}\right)_{beam}}$$

Determining K factors
by Alignment Charts

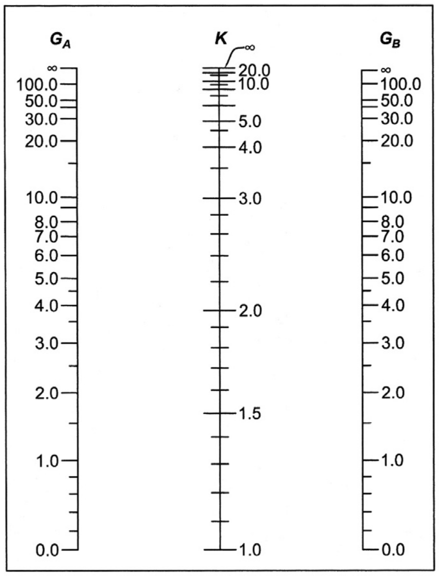
Sidesway Inhibited:
Braced frame
1.0 > K > 0.5

Sidesway Uninhibited:
Un-braced frame
unstable > K > 1.0

More Pinned:
If `Ic`/Lc is large
and `Ig`/Lg is small
The connection is more pinned

More Fixed:
If `Ic`/Lc is small
and `Ig`/Lg is large
The connection is more fixed

Sidesway uninhibited



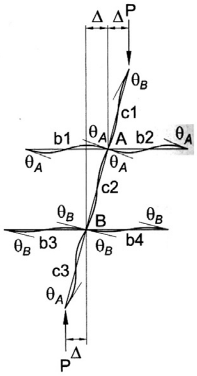


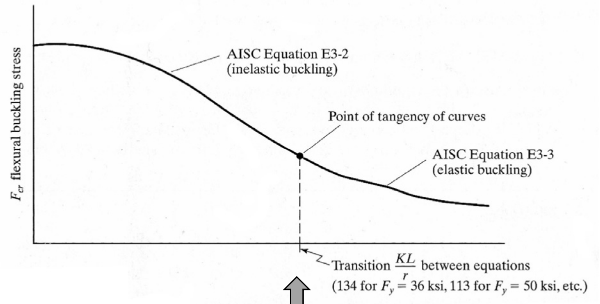
Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\Sigma\left(\frac{EI}{L}\right)_{column}}{\Sigma\left(\frac{EI}{L}\right)_{beam}}$$

Analysis of Steel Columns - LRFD

Euler equation:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$



Short & Intermediate Columns:

short long

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$

Transition Slenderness $4.71\sqrt{\frac{E}{F_y}}$

Long Columns:

$$F_{cr} = 0.877 F_e$$

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

Analysis of Steel Columns

pass / fail by LRFD

Data:

- Column – <u>size</u>, length
- Support conditions
- Material properties – Fy
- Factored load – Pu

Required:

- Pu ≦ ø Pn (pass)

1. Calculate slenderness ratios. L_c/r , $L_c = KL$
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$
   and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine øPn and compare to Pu
   $Pn = F_{cr}\ Ag$ ø = 0.9
6. If Pu ≦ ø Pn , then OK

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$ Short

$$F_{cr} = 0.877 F_e$$ Long

Analysis of Steel Columns

pass / fail by ASD

Data:

- Column – <u>size</u>, length
- Support conditions
- Material properties – Fy
- Factored Load – Pu

Required:

- Pu ≦ ø Pn (pass)

DATA :

W 8x35 A-36

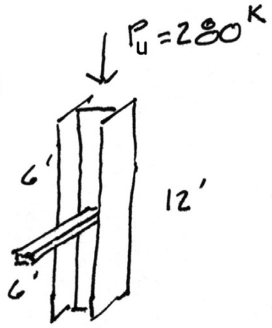
$r_x = 3.51''$ $F_y = 36^{ksi}$

$r_y = 2.03''$

$A = 10.3\ in^2$

$l_x = 12'$ $l_y = 6'$

$K_x = K_y = 1.0$



1. Calculate slenderness ratios.
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)

X - X AXIS

$$\frac{K_x l_x}{r_x} = \frac{144''}{3.51''}$$

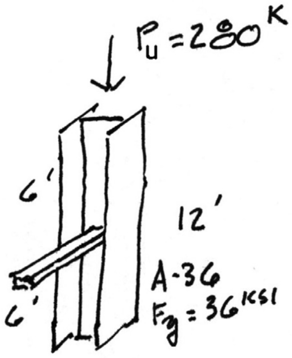
$\underline{41.03} < 200$

Y-Y AXIS

$$\frac{K_y l_y}{r_y} = \frac{72''}{2.03''}$$

35.47

Analysis of Steel Columns
pass / fail by ASD



W8x35

3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)

4. Calculate F_{cr} based on slenderness

5. Determine øPn and compare to Pu

6. If Pu ≦ ø Pn , then OK

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$41 < 134 \therefore \text{SHORT}$$

Euler Equation

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2\, 29000^{KSI}}{41^2} = 170.2^{KSI}$$

Short Column Equation

$$F_{cr} = \left[.658^{\frac{F_y}{F_e}}\right] F_Y = 0.9153\,(36) = 32.95\ \text{KSI}$$

Column Strength

$$P_n = F_{cr} A_g = 32.95\ \text{KSI} \times 10.3\ \text{in}^2 = 339.39^K$$

$$\phi P_n = 0.9\, P_n = 0.9\,(339.39) = 305.4^K$$

$$P_U = 280^K < 305.4^K = \phi P_n \quad \checkmark \text{OK}$$

Analysis of Steel Columns
capacity by LRFD

Data:
- Column – <u>size</u>, length
- Support conditions
- Material properties – Fy

Required:
- Max load capacity

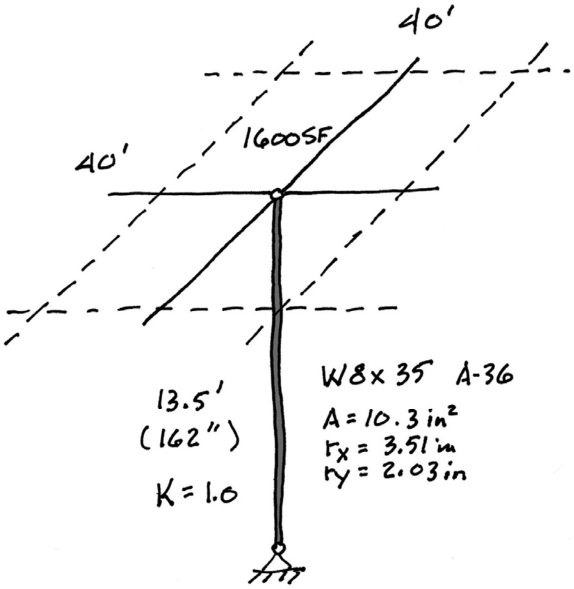
1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine øPn and Compute allowable capacity:
   $Pn = F_{cr} A_g$ Pu = øPn

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$ Short

$$F_{cr} = 0.877 F_e$$ Long

Capacity Example 1

Free standing column
Third floor studio space
Supports roof load = 20 psf DL + SL
snow ≈ 15lbs / FT depth



Capacity Example 1

1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness

y-y AXIS (CONTROLS)

$$\frac{K_y l_y}{r_y} = \frac{1\,(162")}{2.03"} = 79.8 < 200 \checkmark$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$79.8 < 134 \quad \therefore \text{SHORT}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2\, 29000}{79.8^2} = 44.94 \text{ KSI}$$

Short Column Equation

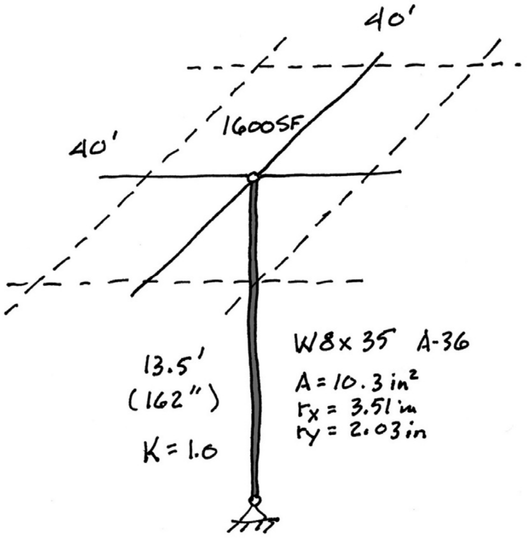
$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.7151\right] 36 = 25.74 \text{ KSI}$$

Capacity Example 1

5. Determine øPn and Compute allowable capacity: Pu = øPn

DL = 20 psf

20 psf (1600 sf) = 32k on column



Column nominal strength

$P_n = F_{cr} A_g = 25.74 \text{ KSI } 10.3 \text{ in}^2 = 265.1^K$

$\phi P_n = 0.9(265) = 238.6^K = P_U$

Load capacity

$P_U = 1.2(32) + 1.6(SL) = 238.6^K$

$SL = 125.1^K$

FOR $A_T = 40 \times 40 = 1600$ SF

$SL = \frac{125100^{\#}}{1600 SF} = 78.2$ PSF

Capacity Example 2
long column – using equations

Find the capacity for the 25 ft. column shown.

$r_x = 3.51$ in.

$r_y = 2.03$ in.

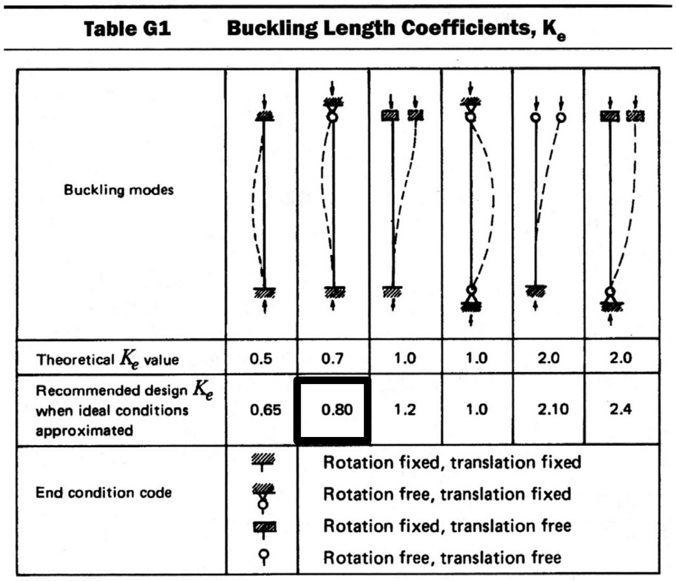


Table G1 Buckling Length Coefficients, K_e

Buckling modes						
Theoretical K_e value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0.65	$\boxed{0.80}$	1.2	1.0	2.10	2.4

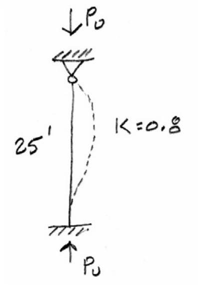
End condition code
- Rotation fixed, translation fixed
- Rotation free, translation fixed
- Rotation fixed, translation free
- Rotation free, translation free

W8x35

$F_y = 50$ KSI

$E = 29000$ KSI

$L = 25'$ (NO BRACING)



Slenderness y-y

$\frac{K l_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$

$4.71\sqrt{\frac{E}{F_y}} = 113 < 118.2 \quad \therefore$ LONG

Euler Buckling

$F_e = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2} = \frac{\pi^2 29000}{118.2^2} = 20.47$ KSI

Long Column Equation

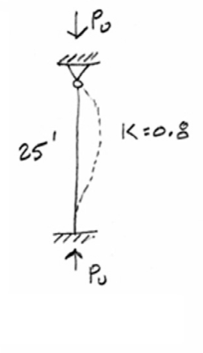
$F_{cr} = 0.877(20.47) = 17.95$ KSI

Column strength

$\phi P_n = \phi F_{cr} A_g = 0.9(\overset{KSI}{17.95})(\overset{in^2}{10.3}) = 166.4^K$

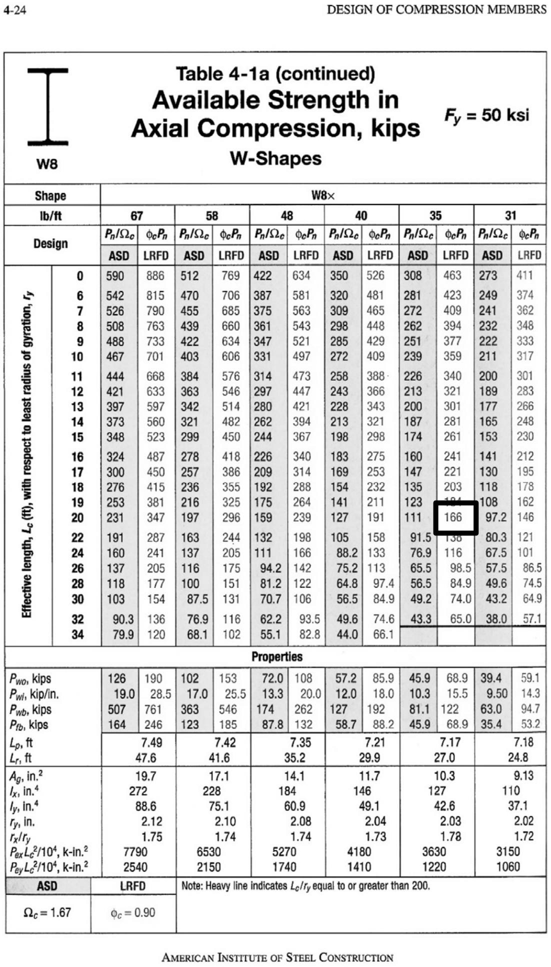
Capacity Example 2
long column – using table

W8x35
F_y = 50 KSI
E = 29000 KSI
L = 25' (NO BRACING)



r_y CONTROLS

KL = 0.8(25') = 20'



W8

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, L_c (ft), with respect to least radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	8	508	763	439	660	361	543	298	448	262	394	232	348
	9	488	733	422	634	347	521	285	429	251	377	222	333
	10	467	701	403	606	331	497	272	409	239	359	211	317
	11	444	668	384	576	314	473	258	388	226	340	200	301
	12	421	633	363	546	297	447	243	366	213	321	189	283
	13	397	597	342	514	280	421	228	343	200	301	177	266
	14	373	560	321	482	262	394	213	321	187	281	165	248
	15	348	523	299	450	244	367	198	298	174	261	153	230
	16	324	487	278	418	226	340	183	275	160	241	141	212
	17	300	450	257	386	209	314	169	253	147	221	130	195
	18	276	415	236	355	192	288	154	232	135	203	118	178
	19	253	381	216	325	175	264	141	211	123	[illegible]	108	162
	20	231	347	197	296	159	239	127	191	111	$\boxed{166}$	97.2	146
	22	191	287	163	244	132	198	105	158	91.5	138	80.3	121
	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101
	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5
	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5
	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9
	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1
	34	79.9	120	68.1	102	55.1	82.8	44.0	66.1				
Properties													
P_{wo}, kips		126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.1
P_{wi}, kip/in.		19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.3
P_{wb}, kips		507	761	363	546	174	262	127	192	81.1	122	63.0	94.7
P_{fb}, kips		164	246	123	185	87.8	132	58.7	88.2	45.9	68.9	35.4	53.2
L_p, ft		7.49		7.42		7.35		7.21		7.17		7.18	
L_r, ft		47.6		41.6		35.2		29.9		27.0		24.8	
A_g, in.^{2}		19.7		17.1		14.1		11.7		10.3		9.13	
I_x, in.^{4}		272		228		184		146		127		110	
I_y, in.^{4}		88.6		75.1		60.9		49.1		42.6		37.1	
r_y, in.		2.12		2.10		2.08		2.04		2.03		2.02	
r_x/r_y		1.75		1.74		1.74		1.73		1.78		1.72	
$P_{ex}L_c^2/10^4$, k-in.^{2}		7790		6530		5270		4180		3630		3150	
$P_{ey}L_c^2/10^4$, k-in.^{2}		2540		2150		1740		1410		1220		1060	
ASD	LRFD	Note: Heavy line indicates L_c/r_y equal to or greater than 200.											
$\Omega_c = 1.67$	$\phi_c = 0.90$												

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Design of Steel Columns with AISC Strength Tables

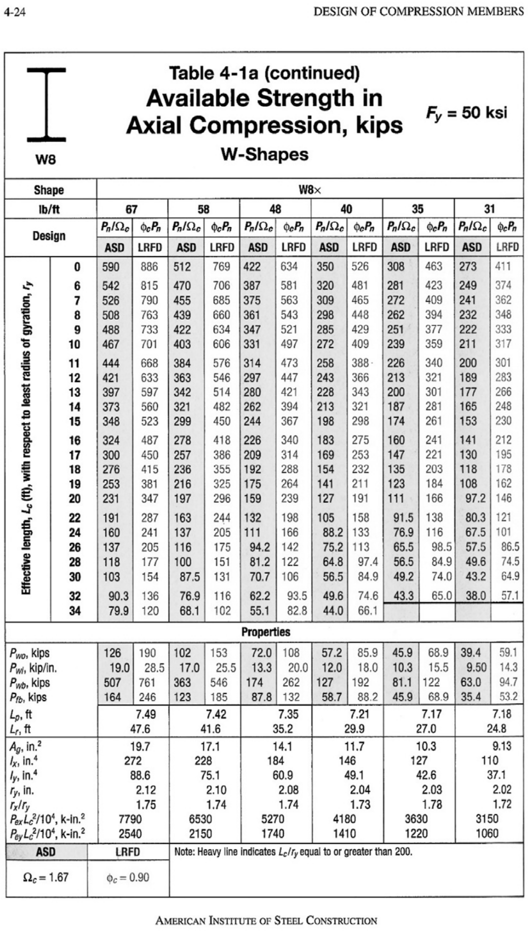
Data:

- Column – length
- Support conditions
- Material properties – F_y
- Applied design load - P_u

Required:

- Column Size

1. Enter table with height, KL = Lc
2. Read allowable load for each section to find the smallest adequate size.
3. **Tables assume weak axis buckling. If the strong axis controls the length must be divided by the ratio r_x/r_y**
4. Values stop in table (black line) at slenderness limit, KL/r = 200



4-24 DESIGN OF COMPRESSION MEMBERS

W8

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, L_c (ft), with respect to least radius of gyration, r_y	0	590	886	512	769	422	634	350	526	308	463	273	411
	6	542	815	470	706	387	581	320	481	281	423	249	374
	7	526	790	455	685	375	563	309	465	272	409	241	362
	8	508	763	439	660	361	543	298	448	262	394	232	348
	9	488	733	422	634	347	521	285	429	251	377	222	333
	10	467	701	403	606	331	497	272	409	239	359	211	317
	11	444	668	384	576	314	473	258	388	226	340	200	301
	12	421	633	363	546	297	447	243	366	213	321	189	283
	13	397	597	342	514	280	421	228	343	200	301	177	266
	14	373	560	321	482	262	394	213	321	187	281	165	248
	15	348	523	299	450	244	367	198	298	174	261	153	230
	16	324	487	278	418	226	340	183	275	160	241	141	212
	17	300	450	257	386	209	314	169	253	147	221	130	195
	18	276	415	236	355	192	288	154	232	135	203	118	178
	19	253	381	216	325	175	264	141	211	123	184	108	162
	20	231	347	197	296	159	239	127	191	111	166	97.2	146
	22	191	287	163	244	132	198	105	158	91.5	138	80.3	121
	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101
	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5
	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5
	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9
	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1
	34	79.9	120	68.1	102	55.1	82.8	44.0	66.1				
Properties													
P_{wo}, kips		126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.1
P_{wi}, kip/in.		19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.3
P_{wb}, kips		507	761	363	546	174	262	127	192	81.1	122	63.0	94.7
P_{fb}, kips		164	246	123	185	87.8	132	58.7	88.2	45.9	68.9	35.4	53.2
L_p, ft		7.49		7.42		7.35		7.21		7.17		7.18	
L_r, ft		47.6		41.6		35.2		29.9		27.0		24.8	
A_g, in.^{2}		19.7		17.1		14.1		11.7		10.3		9.13	
I_x, in.^{4}		272		228		184		146		127		110	
I_y, in.^{4}		88.6		75.1		60.9		49.1		42.6		37.1	
r_y, in.		2.12		2.10		2.08		2.04		2.03		2.02	
r_x/r_y		1.75		1.74		1.74		1.73		1.78		1.72	
$P_{ex}L_c^2/10^4$, k-in.^{2}		7790		6530		5270		4180		3630		3150	
$P_{ey}L_c^2/10^4$, k-in.^{2}		2540		2150		1740		1410		1220		1060	
ASD	LRFD	Note: Heavy line indicates L_c/r_y equal to or greater than 200.											
$\Omega_c = 1.67$	$\phi_c = 0.90$												

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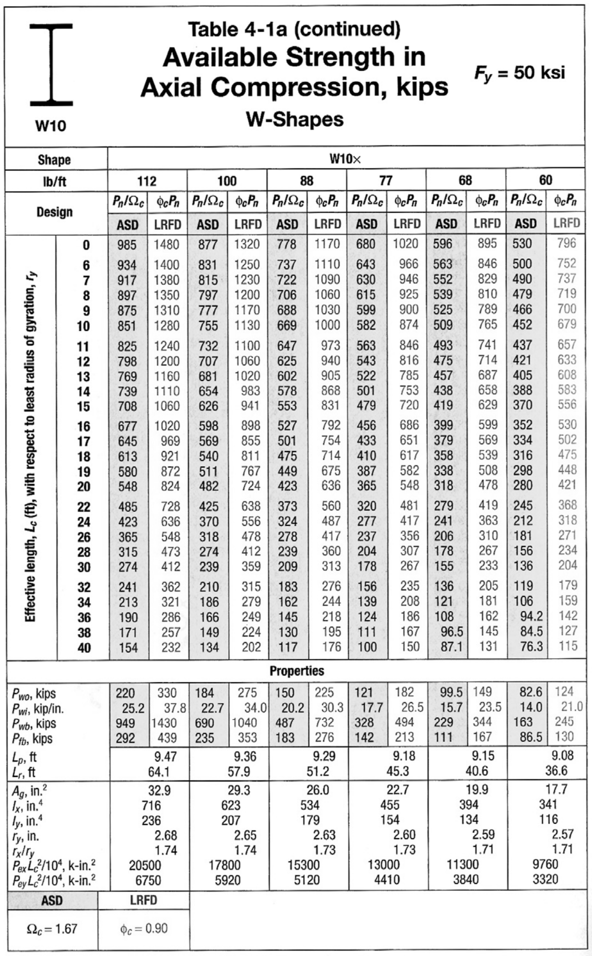


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes

W10

F_y = 50 ksi

Shape		W10×											
lb/ft		112		100		88		77		68		60	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, L_c (ft), with respect to least radius of gyration, r_y	0	985	1480	877	1320	778	1170	680	1020	596	895	530	796
	6	934	1400	831	1250	737	1110	643	966	563	846	500	752
	7	917	1380	815	1230	722	1090	630	946	552	829	490	737
	8	897	1350	797	1200	706	1060	615	925	539	810	479	719
	9	875	1310	777	1170	688	1030	599	900	525	789	466	700
	10	851	1280	755	1130	669	1000	582	874	509	765	452	679
	11	825	1240	732	1100	647	973	563	846	493	741	437	657
	12	798	1200	707	1060	625	940	543	816	475	714	421	633
	13	769	1160	681	1020	602	905	522	785	457	687	405	608
	14	739	1110	654	983	578	868	501	753	438	658	388	583
	15	708	1060	626	941	553	831	479	720	419	629	370	556
	16	677	1020	598	898	527	792	456	686	399	599	352	530
	17	645	969	569	855	501	754	433	651	379	569	334	502
	18	613	921	540	811	475	714	410	617	358	539	316	475
	19	580	872	511	767	449	675	387	582	338	508	298	448
	20	548	824	482	724	423	636	365	548	318	478	280	421
	22	485	728	425	638	373	560	320	481	279	419	245	368
	24	423	636	370	556	324	487	277	417	241	363	212	318
	26	365	548	318	478	278	417	237	356	206	310	181	271
	28	315	473	274	412	239	360	204	307	178	267	156	234
	30	274	412	239	359	209	313	178	267	155	233	136	204
	32	241	362	210	315	183	276	156	235	136	205	119	179
	34	213	321	186	279	162	244	139	208	121	181	106	159
	36	190	286	166	249	145	218	124	186	108	162	94.2	142
	38	171	257	149	224	130	195	111	167	96.5	145	84.5	127
	40	154	232	134	202	117	176	100	150	87.1	131	76.3	115
Properties													
P_{wo}, kips		220	330	184	275	150	225	121	182	99.5	149	82.6	124
P_{wi}, kip/in.		25.2	37.8	22.7	34.0	20.2	30.3	17.7	26.5	15.7	23.5	14.0	21.0
P_{wb}, kips		949	1430	690	1040	487	732	328	494	229	344	163	245
P_{fb}, kips		292	439	235	353	183	276	142	213	111	167	86.5	130
L_p, ft		9.47		9.36		9.29		9.18		9.15		9.08	
L_r, ft		64.1		57.9		51.2		45.3		40.6		36.6	
A_g, in.^{2}		32.9		29.3		26.0		22.7		19.9		17.7	
I_x, in.^{4}		716		623		534		455		394		341	
I_y, in.^{4}		236		207		179		154		134		116	
r_y, in.		2.68		2.65		2.63		2.60		2.59		2.57	
r_x/r_y		1.74		1.74		1.73		1.73		1.71		1.71	
$P_{ex}L_c^2/10^4$, k-in.^{2}		20500		17800		15300		13000		11300		9760	
$P_{ey}L_c^2/10^4$, k-in.^{2}		6750		5920		5120		4410		3840		3320	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

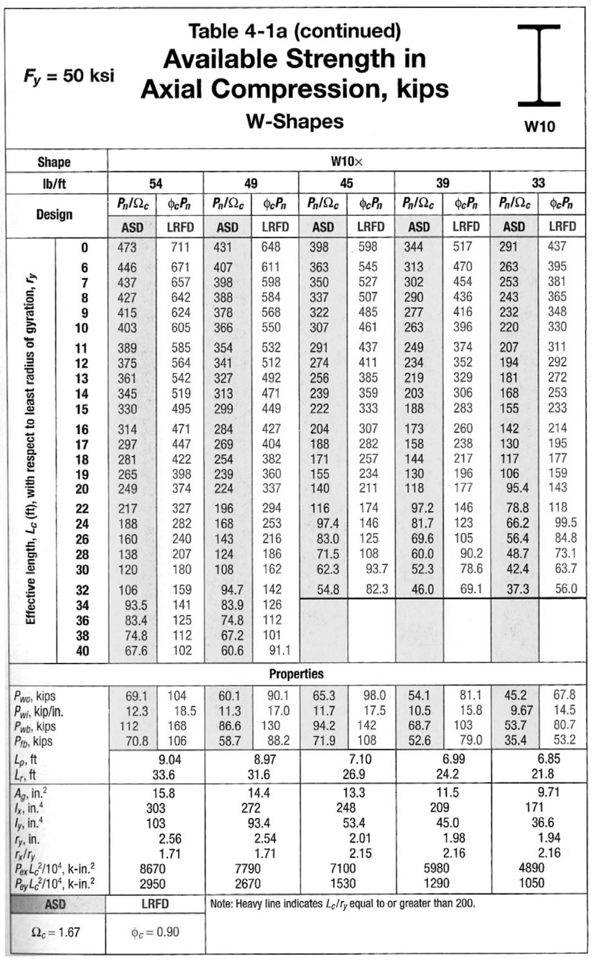


Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 50 ksi

W10

Shape		W10×									
lb/ft		54		49		45		39		33	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, L_c (ft), with respect to least radius of gyration, r_y	0	473	711	431	648	398	598	344	517	291	437
	6	446	671	407	611	363	545	313	470	263	395
	7	437	657	398	598	350	527	302	454	253	381
	8	427	642	388	584	337	507	290	436	243	365
	9	415	624	378	568	322	485	277	416	232	348
	10	403	605	366	550	307	461	263	396	220	330
	11	389	585	354	532	291	437	249	374	207	311
	12	375	564	341	512	274	411	234	352	194	292
	13	361	542	327	492	256	385	219	329	181	272
	14	345	519	313	471	239	359	203	306	168	253
	15	330	495	299	449	222	333	188	283	155	233
	16	314	471	284	427	204	307	173	260	142	214
	17	297	447	269	404	188	282	158	238	130	195
	18	281	422	254	382	171	257	144	217	117	177
	19	265	398	239	360	155	234	130	196	106	159
	20	249	374	224	337	140	211	118	177	95.4	143
	22	217	327	196	294	116	174	97.2	146	78.8	118
	24	188	282	168	253	97.4	146	81.7	123	66.2	99.5
	26	160	240	143	216	83.0	125	69.6	105	56.4	84.8
	28	138	207	124	186	71.5	108	60.0	90.2	48.7	73.1
	30	120	180	108	162	62.3	93.7	52.3	78.6	42.4	63.7
	32	106	159	94.7	142	54.8	82.3	46.0	69.1	37.3	56.0
	34	93.5	141	83.9	126						
	36	83.4	125	74.8	112						
	38	74.8	112	67.2	101						
	40	67.6	102	60.6	91.1						
Properties											
P_{wo}, kips		69.1	104	60.1	90.1	65.3	98.0	54.1	81.1	45.2	67.8
P_{wi}, kip/in.		12.3	18.5	11.3	17.0	11.7	17.5	10.5	15.8	9.67	14.5
P_{wb}, kips		112	168	86.6	130	94.2	142	68.7	103	53.7	80.7
P_{fb}, kips		70.8	106	58.7	88.2	71.9	108	52.6	79.0	35.4	53.2
L_p, ft		9.04		8.97		7.10		6.99		6.85	
L_r, ft		33.6		31.6		26.9		24.2		21.8	
A_g, in.^{2}		15.8		14.4		13.3		11.5		9.71	
I_x, in.^{4}		303		272		248		209		171	
I_y, in.^{4}		103		93.4		53.4		45.0		36.6	
r_y, in.		2.56		2.54		2.01		1.98		1.94	
r_x/r_y		1.71		1.71		2.15		2.16		2.16	
$P_{ex}L_c^2/10^4$, k-in.^{2}		8670		7790		7100		5980		4890	
$P_{ey}L_c^2/10^4$, k-in.^{2}		2950		2670		1530		1290		1050	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

Note: Heavy line indicates L_c/r_y equal to or greater than 200.

AISC Critical Stress Table

for previous example Kl/r_y = 118.2

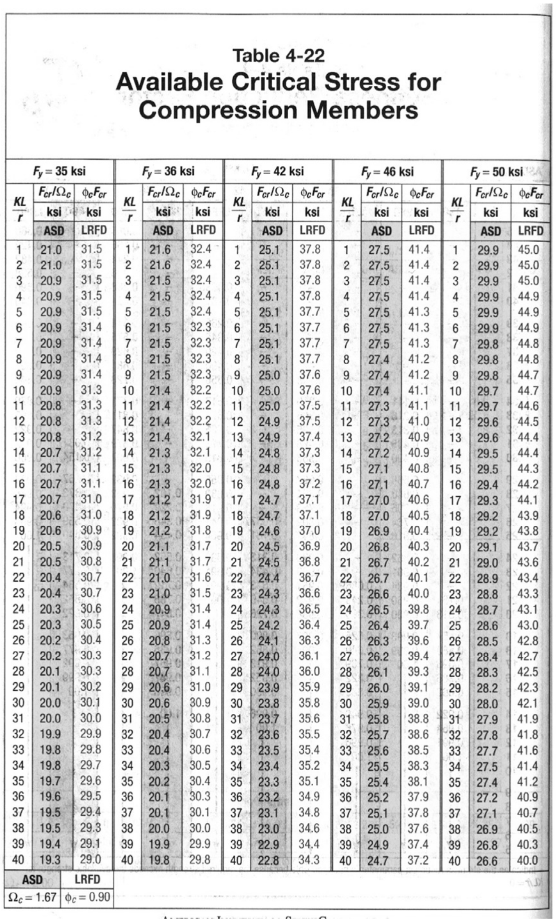


Table 4-22
Available Critical Stress for Compression Members

F_y = 35 ksi			F_y = 36 ksi			F_y = 42 ksi			F_y = 46 ksi			F_y = 50 ksi		
$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$
	ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi
	ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD
1	21.0	31.5	1	21.6	32.4	1	25.1	37.8	1	27.5	41.4	1	29.9	45.0
2	21.0	31.5	2	21.6	32.4	2	25.1	37.8	2	27.5	41.4	2	29.9	45.0
3	20.9	31.5	3	21.5	32.4	3	25.1	37.8	3	27.5	41.4	3	29.9	45.0
4	20.9	31.5	4	21.5	32.4	4	25.1	37.8	4	27.5	41.4	4	29.9	44.9
5	20.9	31.5	5	21.5	32.4	5	25.1	37.7	5	27.5	41.3	5	29.9	44.9
6	20.9	31.4	6	21.5	32.3	6	25.1	37.7	6	27.5	41.3	6	29.9	44.9
7	20.9	31.4	7	21.5	32.3	7	25.1	37.7	7	27.5	41.3	7	29.8	44.8
8	20.9	31.4	8	21.5	32.3	8	25.1	37.7	8	27.4	41.2	8	29.8	44.8
9	20.9	31.4	9	21.5	32.3	9	25.0	37.6	9	27.4	41.2	9	29.8	44.7
10	20.9	31.3	10	21.4	32.2	10	25.0	37.6	10	27.4	41.1	10	29.7	44.7
11	20.8	31.3	11	21.4	32.2	11	25.0	37.5	11	27.3	41.1	11	29.7	44.6
12	20.8	31.3	12	21.4	32.2	12	24.9	37.5	12	27.3	41.0	12	29.6	44.5
13	20.8	31.2	13	21.4	32.1	13	24.9	37.4	13	27.2	40.9	13	29.6	44.4
14	20.7	31.2	14	21.3	32.1	14	24.8	37.3	14	27.2	40.9	14	29.5	44.4
15	20.7	31.1	15	21.3	32.0	15	24.8	37.3	15	27.1	40.8	15	29.5	44.3
16	20.7	31.1	16	21.3	32.0	16	24.8	37.2	16	27.1	40.7	16	29.4	44.2
17	20.7	31.0	17	21.2	31.9	17	24.7	37.1	17	27.0	40.6	17	29.3	44.1
18	20.6	31.0	18	21.2	31.9	18	24.7	37.1	18	27.0	40.5	18	29.2	43.9
19	20.6	30.9	19	21.2	31.8	19	24.6	37.0	19	26.9	40.4	19	29.2	43.8
20	20.5	30.9	20	21.1	31.7	20	24.5	36.9	20	26.8	40.3	20	29.1	43.7
21	20.5	30.8	21	21.1	31.7	21	24.5	36.8	21	26.7	40.2	21	29.0	43.6
22	20.4	30.7	22	21.0	31.6	22	24.4	36.7	22	26.7	40.1	22	28.9	43.4
23	20.4	30.7	23	21.0	31.5	23	24.3	36.6	23	26.6	40.0	23	28.8	43.3
24	20.3	30.6	24	20.9	31.4	24	24.3	36.5	24	26.5	39.8	24	28.7	43.1
25	20.3	30.5	25	20.9	31.4	25	24.2	36.4	25	26.4	39.7	25	28.6	43.0
26	20.2	30.4	26	20.8	31.3	26	24.1	36.3	26	26.3	39.6	26	28.5	42.8
27	20.2	30.3	27	20.7	31.2	27	24.0	36.1	27	26.2	39.4	27	28.4	42.7
28	20.1	30.3	28	20.7	31.1	28	24.0	36.0	28	26.1	39.3	28	28.3	42.5
29	20.1	30.2	29	20.6	31.0	29	23.9	35.9	29	26.0	39.1	29	28.2	42.3
30	20.0	30.1	30	20.6	30.9	30	23.8	35.8	30	25.9	39.0	30	28.0	42.1
31	20.0	30.0	31	20.5	30.8	31	23.7	35.6	31	25.8	38.8	31	27.9	41.9
32	19.9	29.9	32	20.4	30.7	32	23.6	35.5	32	25.7	38.6	32	27.8	41.8
33	19.8	29.8	33	20.4	30.6	33	23.5	35.4	33	25.6	38.5	33	27.7	41.6
34	19.8	29.7	34	20.3	30.5	34	23.4	35.2	34	25.5	38.3	34	27.5	41.4
35	19.7	29.6	35	20.2	30.4	35	23.3	35.1	35	25.4	38.1	35	27.4	41.2
36	19.6	29.5	36	20.1	30.3	36	23.2	34.9	36	25.2	37.9	36	27.2	40.9
37	19.5	29.4	37	20.1	30.1	37	23.1	34.8	37	25.1	37.8	37	27.1	40.7
38	19.5	29.3	38	20.0	30.0	38	23.0	34.6	38	25.0	37.6	38	26.9	40.5
39	19.4	29.1	39	19.9	29.9	39	22.9	34.4	39	24.9	37.4	39	26.8	40.3
40	19.3	29.0	40	19.8	29.8	40	22.8	34.3	40	24.7	37.2	40	26.6	40.0

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

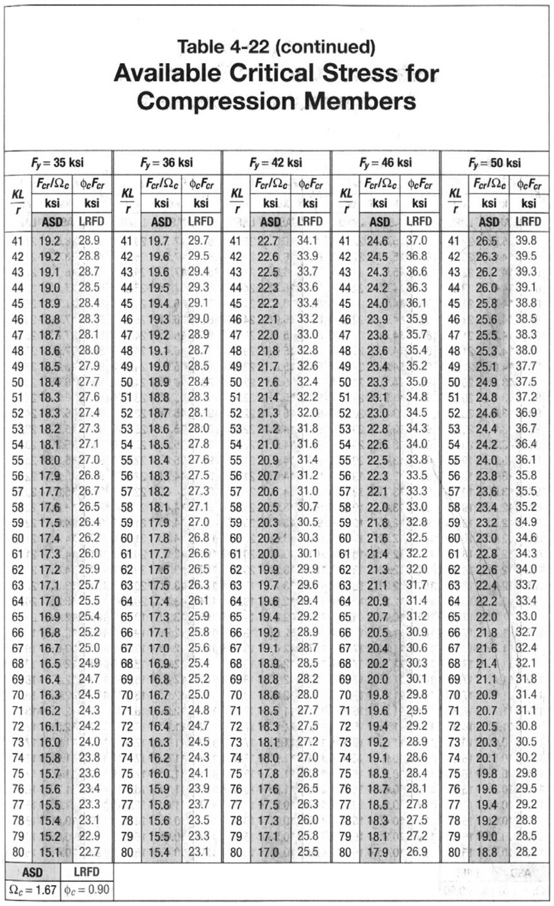


Table 4-22 (continued)
Available Critical Stress for Compression Members

F_y = 35 ksi			F_y = 36 ksi			F_y = 42 ksi			F_y = 46 ksi			F_y = 50 ksi		
$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$
	ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi
	ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD
41	19.2	28.9	41	19.7	29.7	41	22.7	34.1	41	24.6	37.0	41	26.5	39.8
42	19.2	28.8	42	19.6	29.5	42	22.6	33.9	42	24.5	36.8	42	26.3	39.5
43	19.1	28.7	43	19.6	29.4	43	22.5	33.7	43	24.3	36.6	43	26.2	39.3
44	19.0	28.5	44	19.5	29.3	44	22.3	33.6	44	24.2	36.3	44	26.0	39.1
45	18.9	28.4	45	19.4	29.1	45	22.2	33.4	45	24.0	36.1	45	25.8	38.8
46	18.8	28.3	46	19.3	29.0	46	22.1	33.2	46	23.9	35.9	46	25.6	38.5
47	18.7	28.1	47	19.2	28.9	47	22.0	33.0	47	23.8	35.7	47	25.5	38.3
48	18.6	28.0	48	19.1	28.7	48	21.8	32.8	48	23.6	35.4	48	25.3	38.0
49	18.5	27.9	49	19.0	28.5	49	21.7	32.6	49	23.4	35.2	49	25.1	37.7
50	18.4	27.7	50	18.9	28.4	50	21.6	32.4	50	23.3	35.0	50	24.9	37.5
51	18.3	27.6	51	18.8	28.3	51	21.4	32.2	51	23.1	34.8	51	24.8	37.2
52	18.3	27.4	52	18.7	28.1	52	21.3	32.0	52	23.0	34.5	52	24.6	36.9
53	18.2	27.3	53	18.6	28.0	53	21.2	31.8	53	22.8	34.3	53	24.4	36.7
54	18.1	27.1	54	18.5	27.8	54	21.0	31.6	54	22.6	34.0	54	24.2	36.4
55	18.0	27.0	55	18.4	27.6	55	20.9	31.4	55	22.5	33.8	55	24.0	36.1
56	17.9	26.8	56	18.3	27.5	56	20.7	31.2	56	22.3	33.5	56	23.8	35.8
57	17.7	26.7	57	18.2	27.3	57	20.6	31.0	57	22.1	33.3	57	23.6	35.5
58	17.6	26.5	58	18.1	27.1	58	20.5	30.7	58	22.0	33.0	58	23.4	35.2
59	17.5	26.4	59	17.9	27.0	59	20.3	30.5	59	21.8	32.8	59	23.2	34.9
60	17.4	26.2	60	17.8	26.8	60	20.2	30.3	60	21.6	32.5	60	23.0	34.6
61	17.3	26.0	61	17.7	26.6	61	20.0	30.1	61	21.4	32.2	61	22.8	34.3
62	17.2	25.9	62	17.6	26.5	62	19.9	29.9	62	21.3	32.0	62	22.6	34.0
63	17.1	25.7	63	17.5	26.3	63	19.7	29.6	63	21.1	31.7	63	22.4	33.7
64	17.0	25.5	64	17.4	26.1	64	19.6	29.4	64	20.9	31.4	64	22.2	33.4
65	16.9	25.4	65	17.3	25.9	65	19.4	29.2	65	20.7	31.2	65	22.0	33.0
66	16.8	25.2	66	17.1	25.8	66	19.2	28.9	66	20.5	30.9	66	21.8	32.7
67	16.7	25.0	67	17.0	25.6	67	19.1	28.7	67	20.4	30.6	67	21.6	32.4
68	16.5	24.9	68	16.9	25.4	68	18.9	28.5	68	20.2	30.3	68	21.4	32.1
69	16.4	24.7	69	16.8	25.2	69	18.8	28.2	69	20.0	30.1	69	21.1	31.8
70	16.3	24.5	70	16.7	25.0	70	18.6	28.0	70	19.8	29.8	70	20.9	31.4
71	16.2	24.3	71	16.5	24.8	71	18.5	27.7	71	19.6	29.5	71	20.7	31.1
72	16.1	24.2	72	16.4	24.7	72	18.3	27.5	72	19.4	29.2	72	20.5	30.8
73	16.0	24.0	73	16.3	24.5	73	18.1	27.2	73	19.2	28.9	73	20.3	30.5
74	15.8	23.8	74	16.2	24.3	74	18.0	27.0	74	19.1	28.6	74	20.1	30.2
75	15.7	23.6	75	16.0	24.1	75	17.8	26.8	75	18.9	28.4	75	19.8	29.8
76	15.6	23.4	76	15.9	23.9	76	17.6	26.5	76	18.7	28.1	76	19.6	29.5
77	15.5	23.3	77	15.8	23.7	77	17.5	26.3	77	18.5	27.8	77	19.4	29.2
78	15.4	23.1	78	15.6	23.5	78	17.3	26.0	78	18.3	27.5	78	19.2	28.8
79	15.2	22.9	79	15.5	23.3	79	17.1	25.8	79	18.1	27.2	79	19.0	28.5
80	15.1	22.7	80	15.4	23.1	80	17.0	25.5	80	17.9	26.9	80	18.8	28.2

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

American Institute of Steel Construction

AISC Critical Stress Table

for previous example $Kl/r_y = 118.2$

TO FIND CAPACITY:

$\phi F_{cr} = 16.2$ KSI

$\phi P_n = P_u = \phi F_{cr} A_g$

$P_u = 16.2(10.3) = \underline{166.8}^{K}$

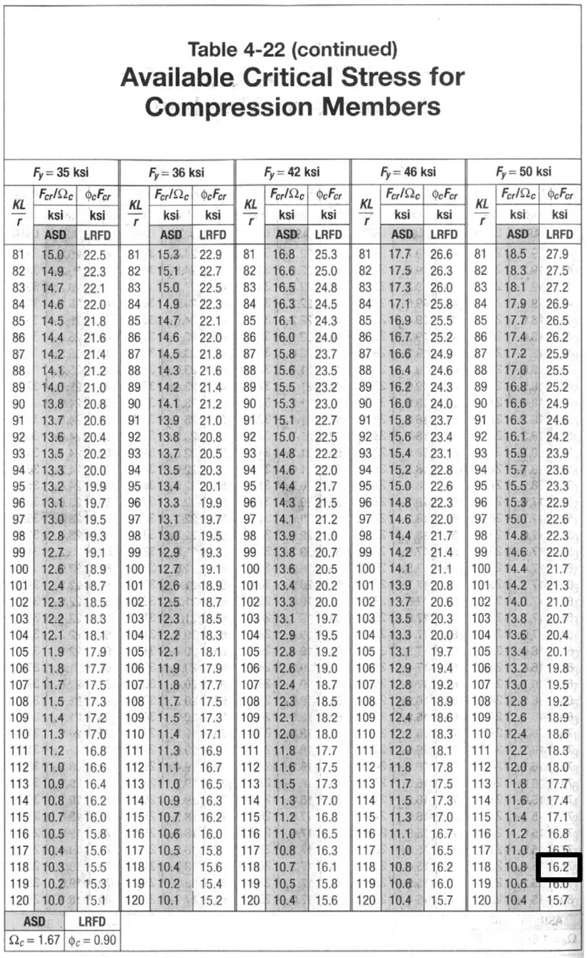


Table 4-22 (continued)

Available Critical Stress for Compression Members

	$F_y = 35$ ksi			$F_y = 36$ ksi			$F_y = 42$ ksi			$F_y = 46$ ksi			$F_y = 50$ ksi	
$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$	$\frac{KL}{r}$	F_{cr}/Ω_c	$\phi_c F_{cr}$
	ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi		ksi	ksi
	ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD		ASD	LRFD
81	15.0	22.5	81	15.3	22.9	81	16.8	25.3	81	17.7	26.6	81	18.5	27.9
82	14.9	22.3	82	15.1	22.7	82	16.6	25.0	82	17.5	26.3	82	18.3	27.5
83	14.7	22.1	83	15.0	22.5	83	16.5	24.8	83	17.3	26.0	83	18.1	27.2
84	14.6	22.0	84	14.9	22.3	84	16.3	24.5	84	17.1	25.8	84	17.9	26.9
85	14.5	21.8	85	14.7	22.1	85	16.1	24.3	85	16.9	25.5	85	17.7	26.5
86	14.4	21.6	86	14.6	22.0	86	16.0	24.0	86	16.7	25.2	86	17.4	26.2
87	14.2	21.4	87	14.5	21.8	87	15.8	23.7	87	16.6	24.9	87	17.2	25.9
88	14.1	21.2	88	14.3	21.6	88	15.6	23.5	88	16.4	24.6	88	17.0	25.5
89	14.0	21.0	89	14.2	21.4	89	15.5	23.2	89	16.2	24.3	89	16.8	25.2
90	13.8	20.8	90	14.1	21.2	90	15.3	23.0	90	16.0	24.0	90	16.6	24.9
91	13.7	20.6	91	13.9	21.0	91	15.1	22.7	91	15.8	23.7	91	16.3	24.6
92	13.6	20.4	92	13.8	20.8	92	15.0	22.5	92	15.6	23.4	92	16.1	24.2
93	13.5	20.2	93	13.7	20.5	93	14.8	22.2	93	15.4	23.1	93	15.9	23.9
94	13.3	20.0	94	13.5	20.3	94	14.6	22.0	94	15.2	22.8	94	15.7	23.6
95	13.2	19.9	95	13.4	20.1	95	14.4	21.7	95	15.0	22.6	95	15.5	23.3
96	13.1	19.7	96	13.3	19.9	96	14.3	21.5	96	14.8	22.3	96	15.3	22.9
97	13.0	19.5	97	13.1	19.7	97	14.1	21.2	97	14.6	22.0	97	15.0	22.6
98	12.8	19.3	98	13.0	19.5	98	13.9	21.0	98	14.4	21.7	98	14.8	22.3
99	12.7	19.1	99	12.9	19.3	99	13.8	20.7	99	14.2	21.4	99	14.6	22.0
100	12.6	18.9	100	12.7	19.1	100	13.6	20.5	100	14.1	21.1	100	14.4	21.7
101	12.4	18.7	101	12.6	18.9	101	13.4	20.2	101	13.9	20.8	101	14.2	21.3
102	12.3	18.5	102	12.5	18.7	102	13.3	20.0	102	13.7	20.6	102	14.0	21.0
103	12.2	18.3	103	12.3	18.5	103	13.1	19.7	103	13.5	20.3	103	13.8	20.7
104	12.1	18.1	104	12.2	18.3	104	12.9	19.5	104	13.3	20.0	104	13.6	20.4
105	11.9	17.9	105	12.1	18.1	105	12.8	19.2	105	13.1	19.7	105	13.4	20.1
106	11.8	17.7	106	11.9	17.9	106	12.6	19.0	106	12.9	19.4	106	13.2	19.8
107	11.7	17.5	107	11.8	17.7	107	12.4	18.7	107	12.8	19.2	107	13.0	19.5
108	11.5	17.3	108	11.7	17.5	108	12.3	18.5	108	12.6	18.9	108	12.8	19.2
109	11.4	17.2	109	11.5	17.3	109	12.1	18.2	109	12.4	18.6	109	12.6	18.9
110	11.3	17.0	110	11.4	17.1	110	12.0	18.0	110	12.2	18.3	110	12.4	18.6
111	11.2	16.8	111	11.3	16.9	111	11.8	17.7	111	12.0	18.1	111	12.2	18.3
112	11.0	16.6	112	11.1	16.7	112	11.6	17.5	112	11.8	17.8	112	12.0	18.0
113	10.9	16.4	113	11.0	16.5	113	11.5	17.3	113	11.7	17.5	113	11.8	17.7
114	10.8	16.2	114	10.9	16.3	114	11.3	17.0	114	11.5	17.3	114	11.6	17.4
115	10.7	16.0	115	10.7	16.2	115	11.2	16.8	115	11.3	17.0	115	11.4	17.1
116	10.5	15.8	116	10.6	16.0	116	11.0	16.5	116	11.1	16.7	116	11.2	16.8
117	10.4	15.6	117	10.5	15.8	117	10.8	16.3	117	11.0	16.5	117	11.0	16.5
118	10.3	15.5	118	10.4	15.6	118	10.7	16.1	118	10.8	16.2	118	10.8	16.2
119	10.2	15.3	119	10.2	15.4	119	10.5	15.8	119	10.6	16.0	119	10.6	16.0
120	10.0	15.1	120	10.1	15.2	120	10.4	15.6	120	10.4	15.7	120	10.4	15.7

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

Steel Frame Construction

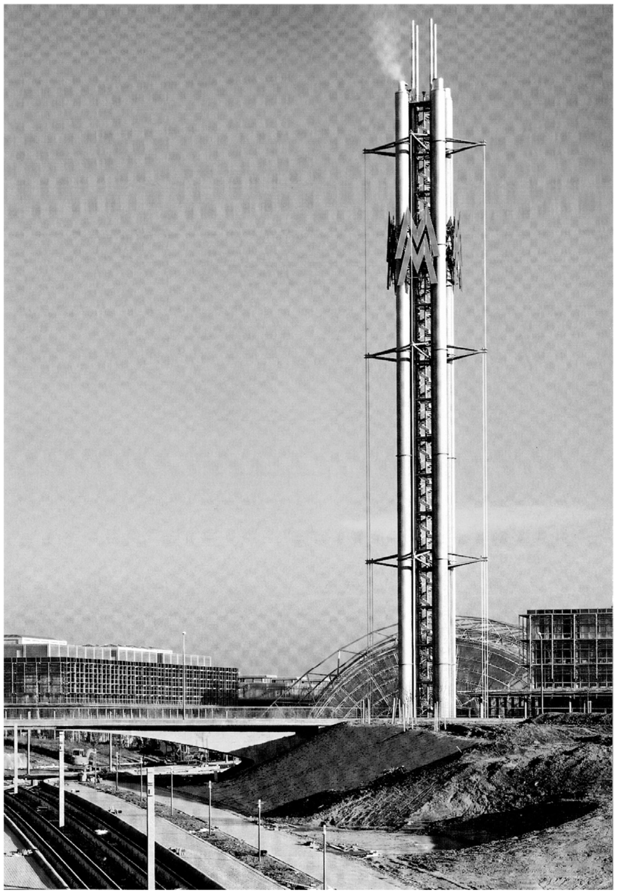
University of Michigan – North Quad

Steel Frame Construction

Messe Leipzig – 1996

Congress Centre – Gerkan, Marg und Partner
Glass Hall – Ian Ritchie Architects
Tower - Schlaich, Bergermann und Partner

Messe Leipzig - Glass Hall - Ian Ritchie Architects



Messe Leipzig – Cable braced tower. Jörg Schlaich

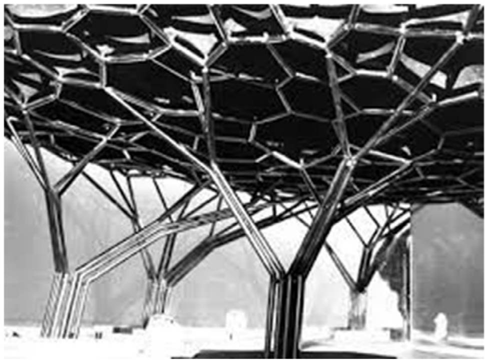
Steel Frame Construction

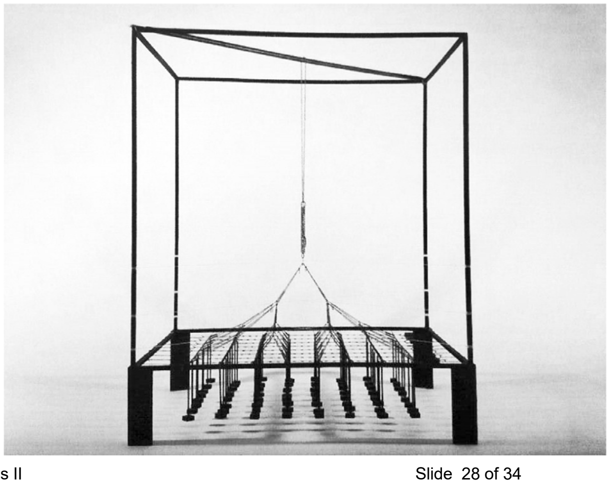
Messe Leipzig Glass Hall - Ian Ritchie Architects

Steel Frame Construction

Messe Leipzig Glass Hall - Ian Ritchie Architects

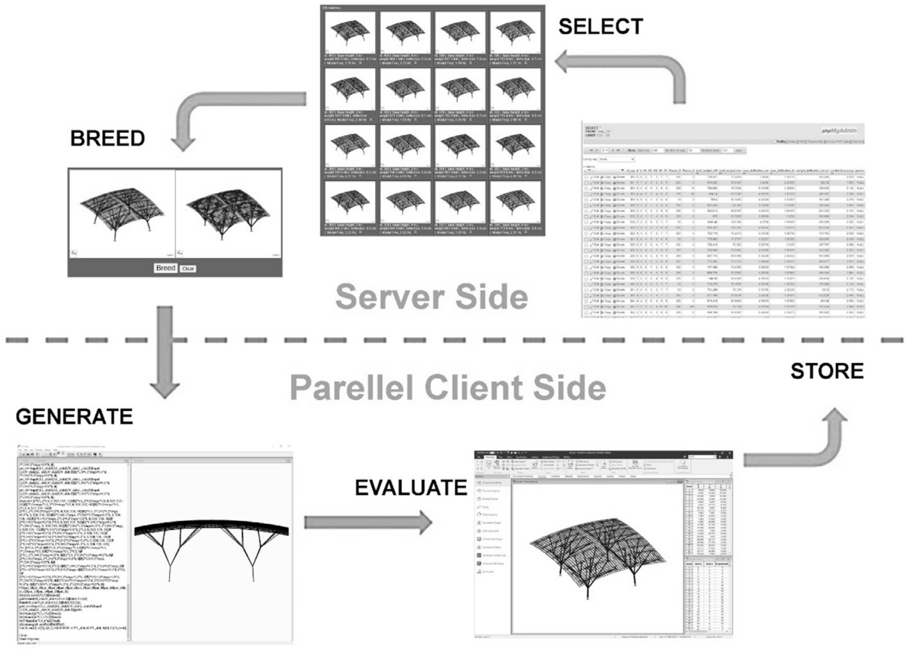
Branching Columns (tree columns)
Frei Otto





Branching Columns (tree columns) geometry optimization

ParaGen Cycle

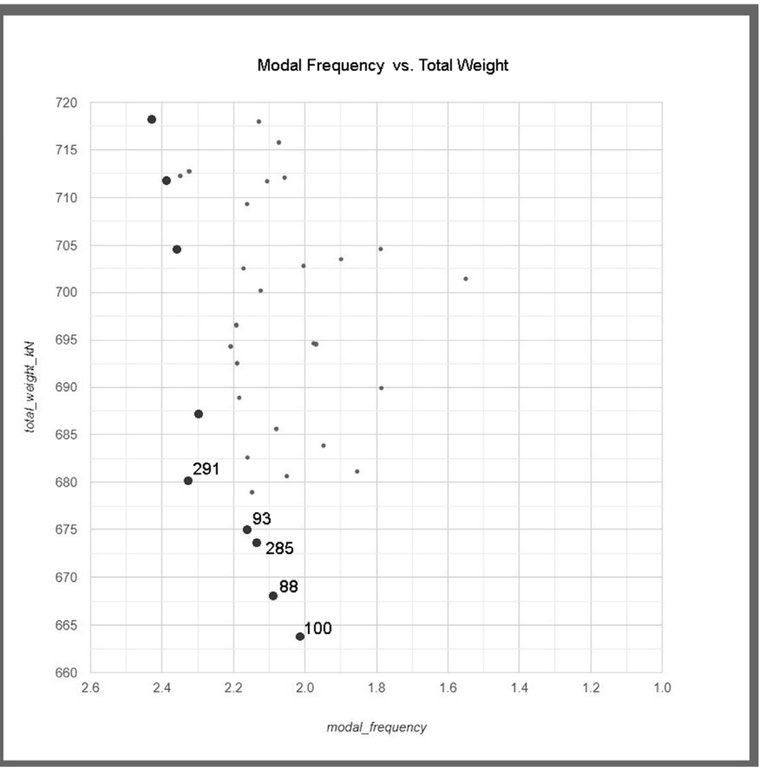


Branching Columns (tree columns) geometry optimization

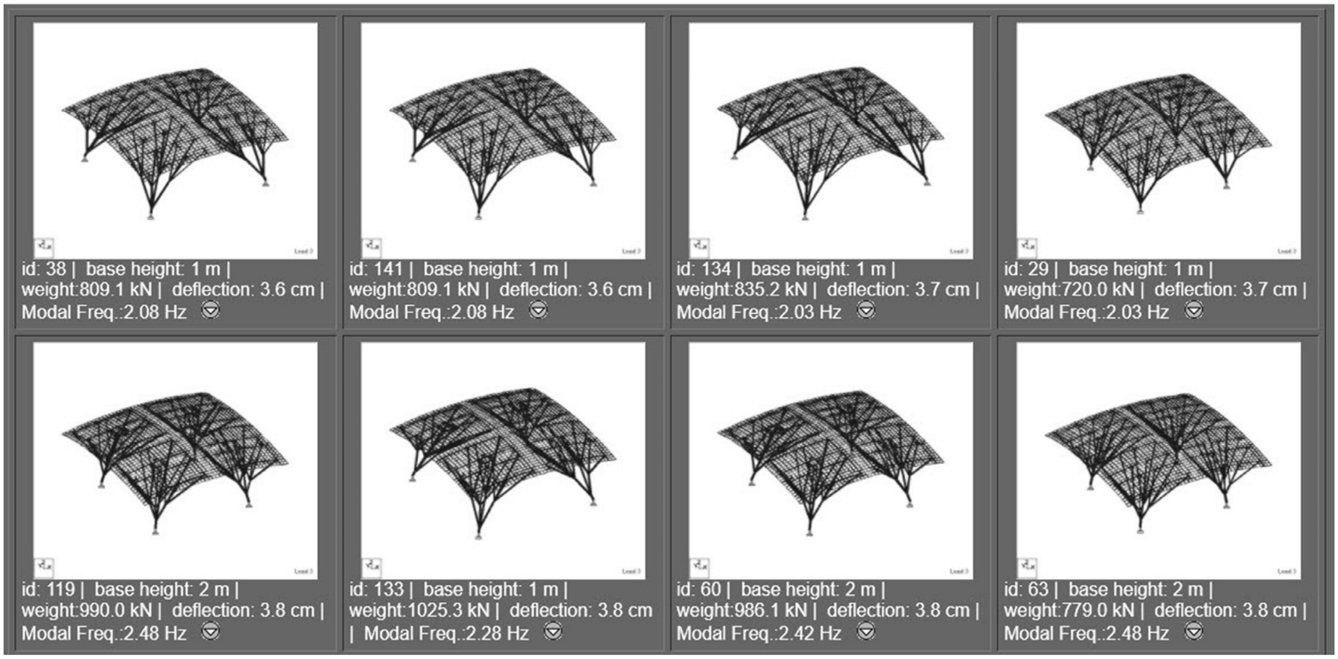
id: 93 | base height: 3 m | weight:674.8 kN | deflection: 5.4 cm | Modal Freq.:2.16 Hz

id: 285 | base height: 4 m | weight:673.7 kN | deflection: 6.2 cm | Modal Freq.:2.13 Hz

id: 88 | base height: 3 m | weight:668.0 kN | deflection: 5.5 cm | Modal Freq.:2.09 Hz



Branching Columns (tree columns)
geometry optimization



id: 38 | base height: 1 m | weight:809.1 kN | deflection: 3.6 cm | Modal Freq.:2.08 Hz

id: 141 | base height: 1 m | weight:809.1 kN | deflection: 3.6 cm | Modal Freq.:2.08 Hz

id: 134 | base height: 1 m | weight:835.2 kN | deflection: 3.7 cm | Modal Freq.:2.03 Hz

id: 29 | base height: 1 m | weight:720.0 kN | deflection: 3.7 cm | Modal Freq.:2.03 Hz

id: 119 | base height: 2 m | weight:990.0 kN | deflection: 3.8 cm | Modal Freq.:2.48 Hz

id: 133 | base height: 1 m | weight:1025.3 kN | deflection: 3.8 cm | Modal Freq.:2.28 Hz

id: 60 | base height: 2 m | weight:986.1 kN | deflection: 3.8 cm | Modal Freq.:2.42 Hz

id: 63 | base height: 2 m | weight:779.0 kN | deflection: 3.8 cm | Modal Freq.:2.48 Hz

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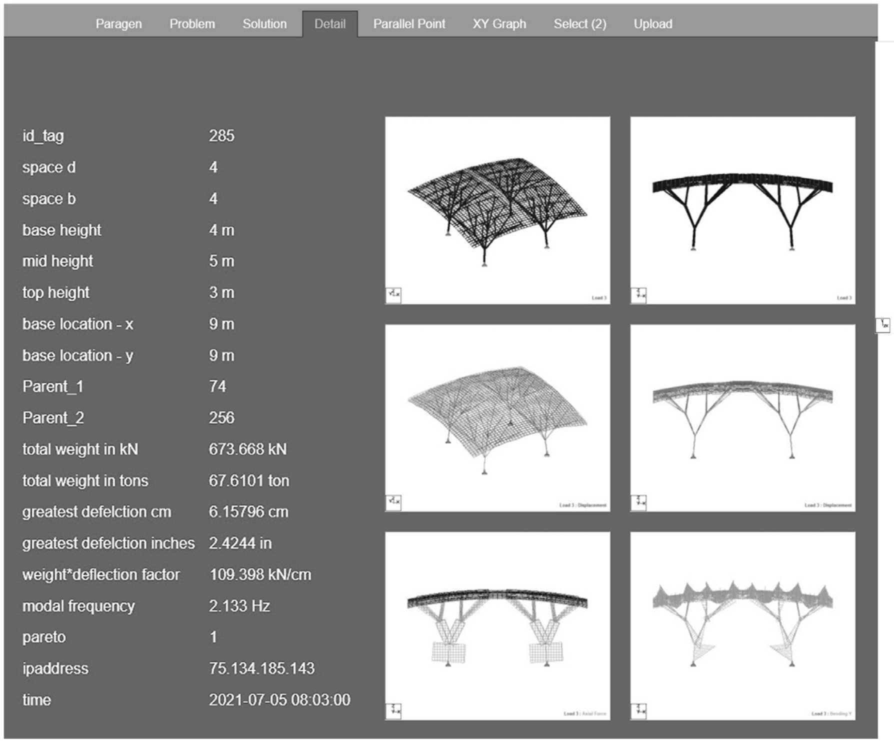
Branching Columns (tree columns)
geometry optimization



id_tag	285
space d	4
space b	4
base height	4 m
mid height	5 m
top height	3 m
base location - x	9 m
base location - y	9 m
Parent_1	74
Parent_2	256
total weight in kN	673.668 kN
total weight in tons	67.6101 ton
greatest defelction cm	6.15796 cm
greatest defelction inches	2.4244 in
weight*deflection factor	109.398 kN/cm
modal frequency	2.133 Hz
pareto	1
ipaddress	75.134.185.143
time	2021-07-05 08:03:00

Branching Columns (tree columns)

bridge in Pragsattel, Stuttgart, 1992
Schlaich, Bergermann und Partner

Branching Columns (tree columns)

Stuttgart Airport Terminal,
Gerkan, Marg und Partner
Schlaich, Bergermann und Partner