

Structural Continuity

- Continuity in Beams
- Deflection Method
- Slope Method
- Three-Moment Theorem

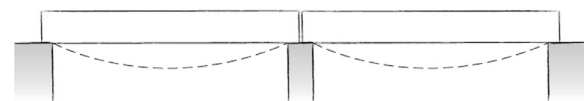


Millennium Bridge, London
Foster and Partners + Arup

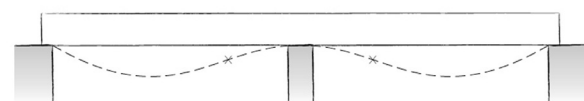
Photo by Ryan Donaghy

Continuous Beams

- Continuous over one or more supports
 - Most common in monolithic concrete
 - Steel: continuous or with moment connections
 - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
 - Cannot be solved by the three equations of statics alone
 - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports



two spans - simply supported



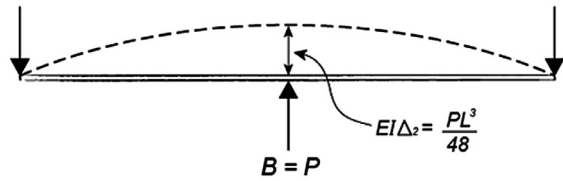
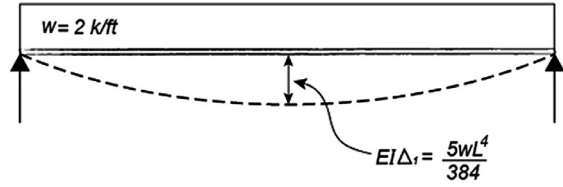
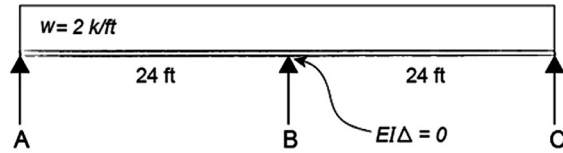
two spans - continuous

Deflection Method

- Two continuous, symmetric spans
- Symmetric Load

Procedure:

1. Remove the center support, and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.

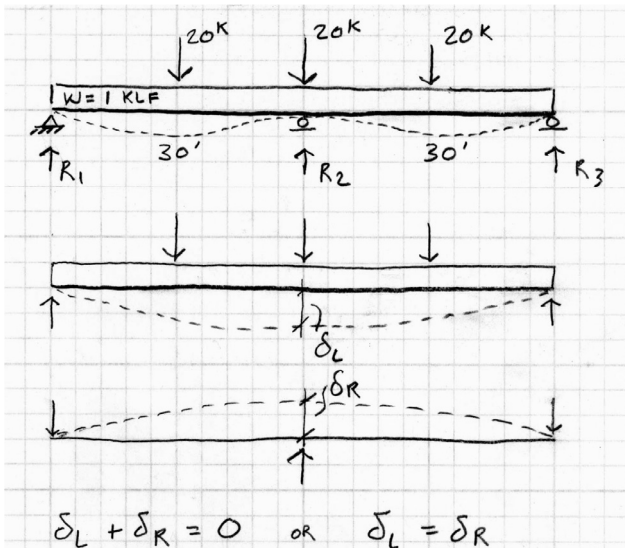


$$EI\Delta_1 + EI\Delta_2 = 0$$

Deflection Method - Example:

Given: Two symmetric spans with symmetric loading as shown.

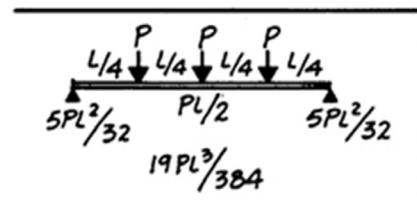
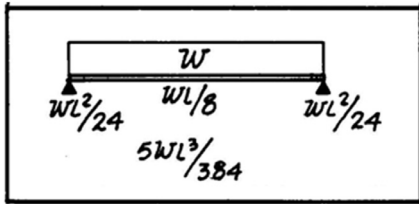
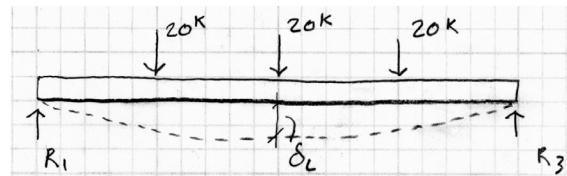
Find: All three reactions



MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT
NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"

Deflection Method

1. Remove the center support, and calculate the center deflection for each load case as a simple span.

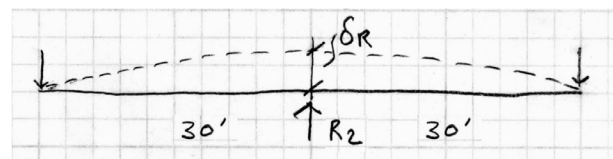


$$\delta_L = \frac{5wl^4}{384EI} + \frac{19Pl^3}{384EI} = \frac{5(1)(60)^4 + 19(20)(60)^3}{384EI}$$

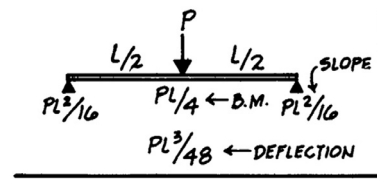
$$\delta_L = 382500/EI$$

Deflection Method – Example

2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.



3. Solve the resulting equation for the center reaction force. (upward point load)

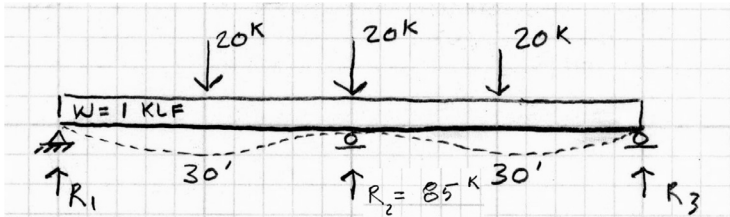


$$\delta_L = 382500/EI = \delta_R = \frac{R_2 l^3}{48EI} = R_2 4500/EI$$

$$R_2 = 85 \text{ K}$$

Deflection Method – Example

4. Calculate the remaining two end reactions.

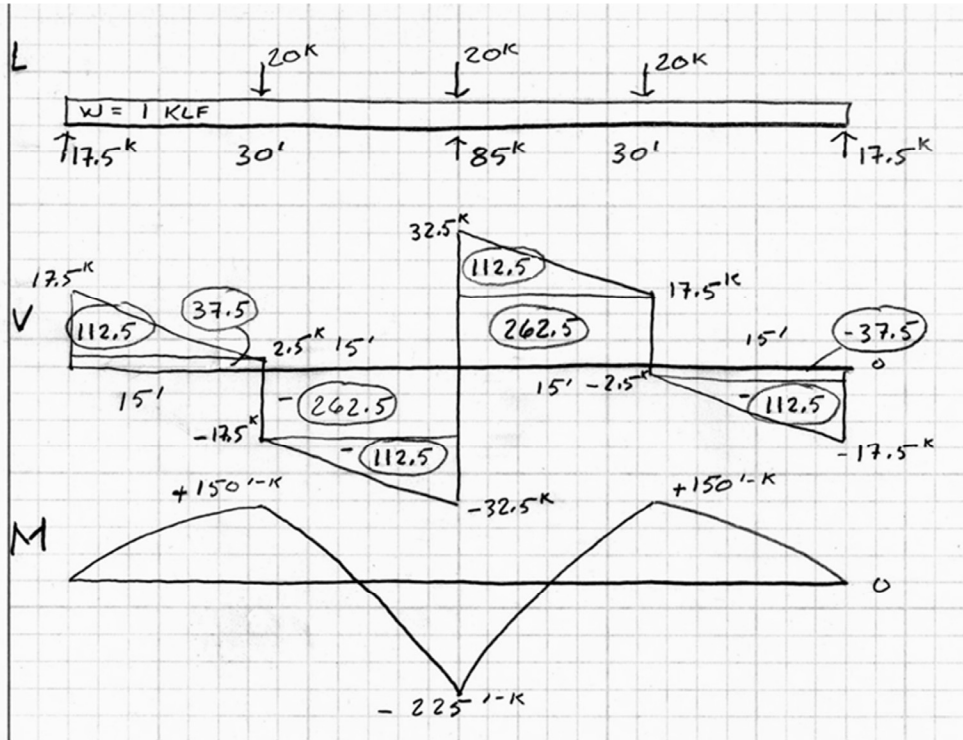


$$\sum F_v = 0 = R_1 + R_3 + 85 - 60 - 60 = 0$$

$$R_1 = R_3 = 17.5\text{ k}$$

Deflection Method - Example cont.:

5. Draw shear and moment diagrams as usual.

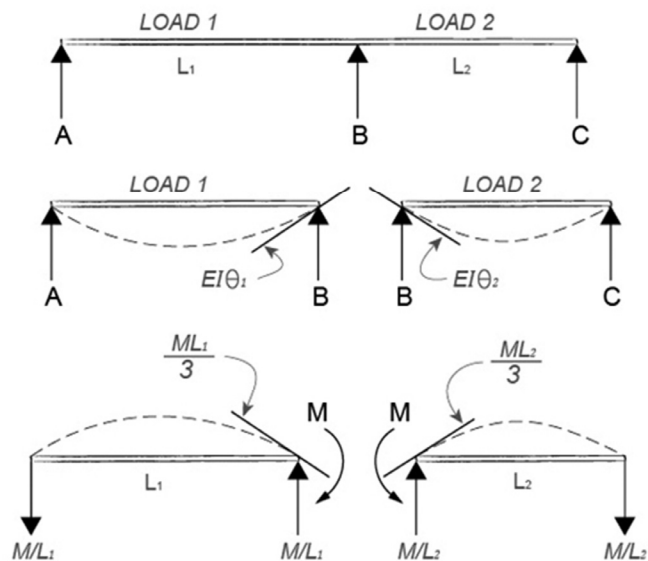


Slope Method

- Two continuous spans
- Non-symmetric loads and spans

Procedure:

1. Break the beam into two halves at the interior support, and calculate the interior slopes of the two simple spans.
2. Use the Slope Equation to solve for the negative interior moment.
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.
5. Draw the shear and moment diagrams as usual.

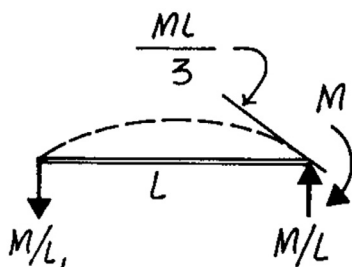


$$M = \frac{3}{L_1 + L_2} [EI\Theta_1 + EI\Theta_2]$$

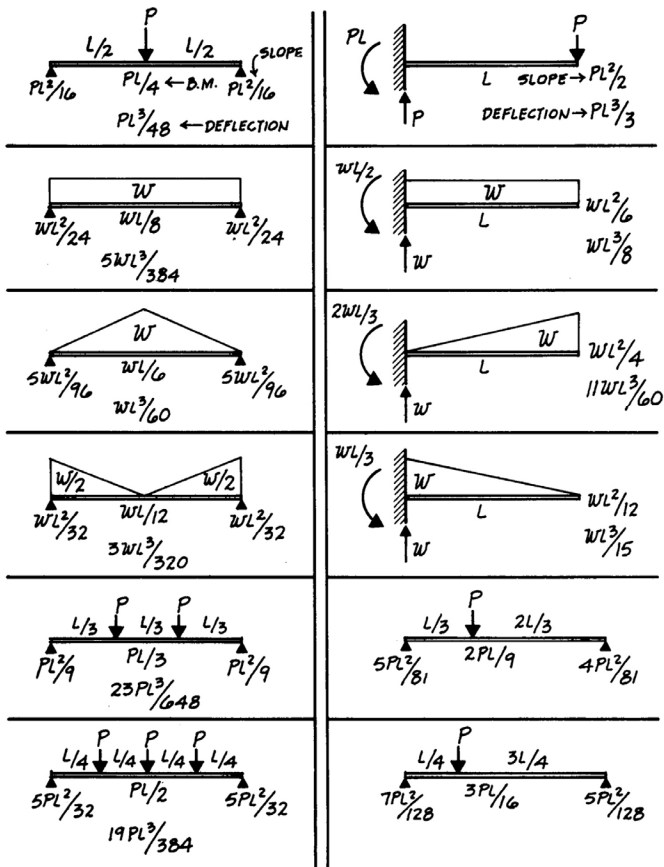
Slope Method

Slope equations:

$$M = \frac{3}{L_1 + L_2} [EI\Theta_1 + EI\Theta_2]$$



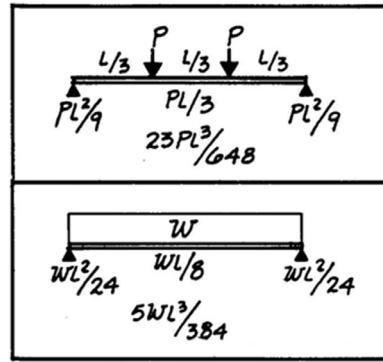
MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT
 NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"



Slope Method - Example

Given: Two non-symmetric spans with loading as shown.

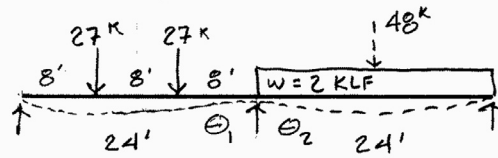
Find: All three reactions



1. Break the beam into two halves at the interior support, and calculate the interior slopes of the two simple spans.

2. Use the Slope Equation to solve for the negative interior moment.

$$M = \frac{3}{L_1 + L_2} [EI\theta_1 + EI\theta_2]$$



$$EI\theta_1 = \frac{PL^2}{9} = \frac{27(24)^2}{9} = 1728$$

$$EI\theta_2 = \frac{WL^2}{24} = \frac{48(24)^2}{24} = 1152$$

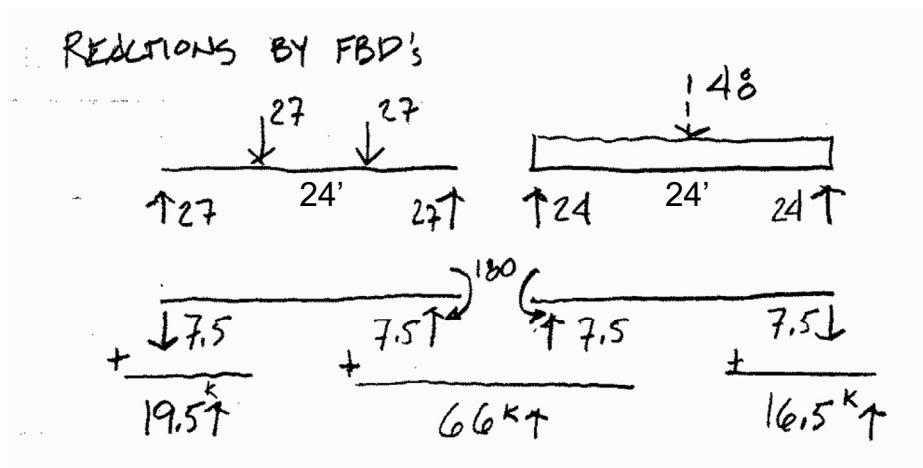
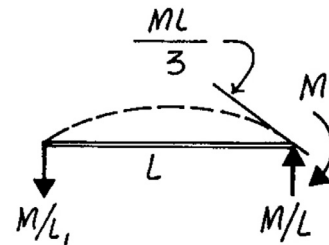
$$M = \frac{3}{L_1 + L_2} [EI\theta_1 + EI\theta_2] = \frac{3}{48} [2880]$$

$$M = 180 \text{ k'}$$

Example of Slope Method cont.:

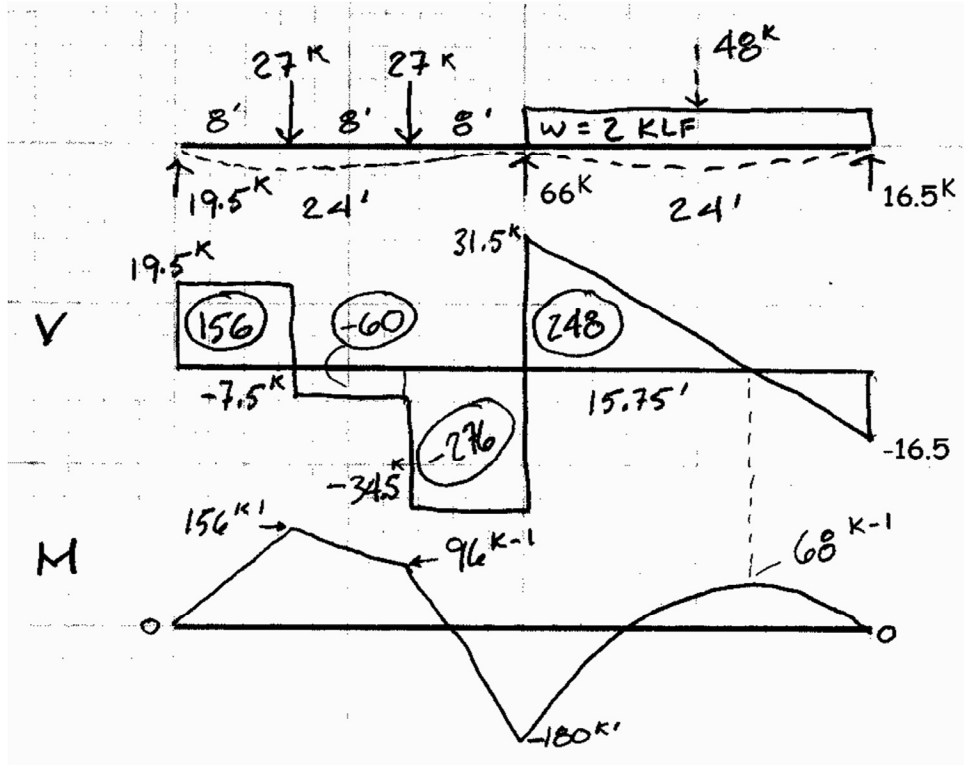
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.

4. Add all the reactions by superposition.



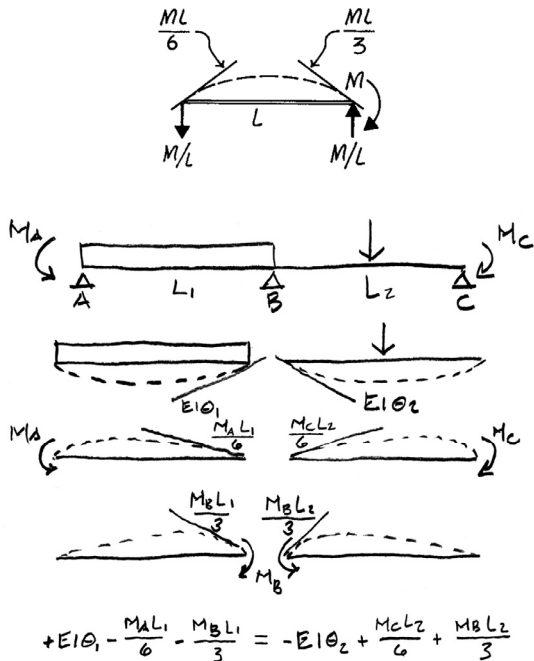
Example of Slope Method cont.:

5. Draw the shear and moment diagrams as usual.



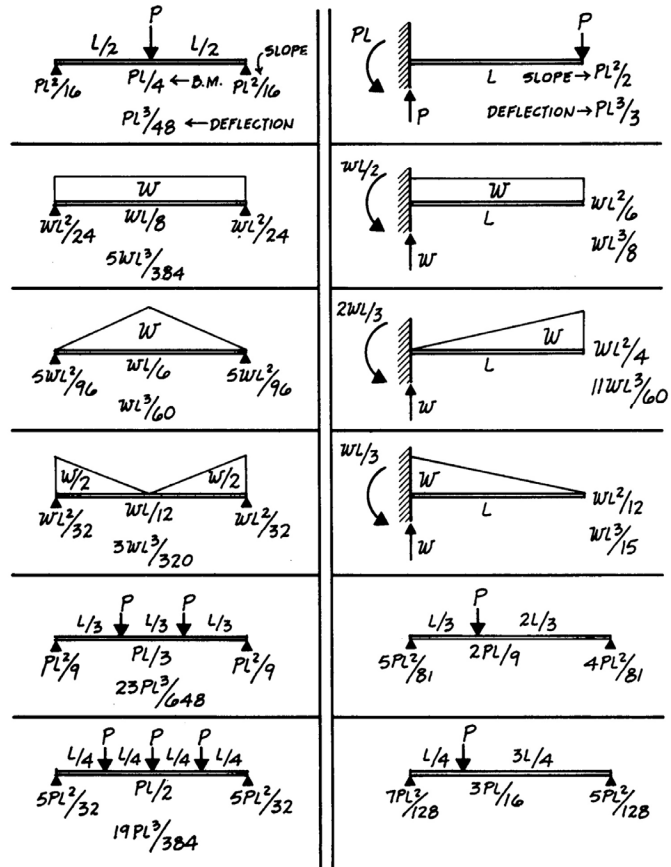
3-Moment Theorem

- Any number of continuous spans
- Non-Symmetric Load and Spans



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT
 NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"

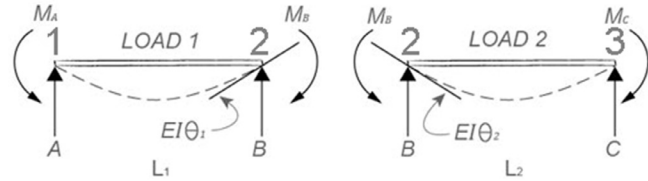
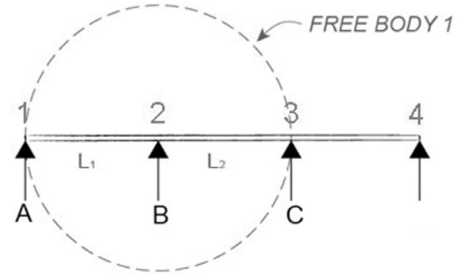


Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric

Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L_1 and L_2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.

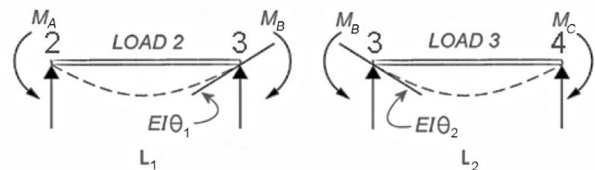
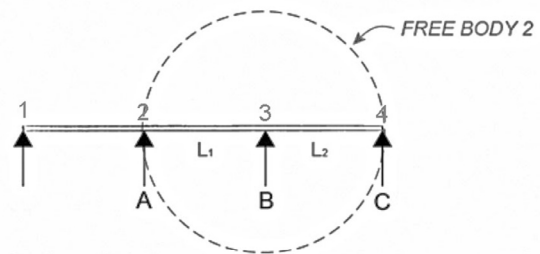


$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.

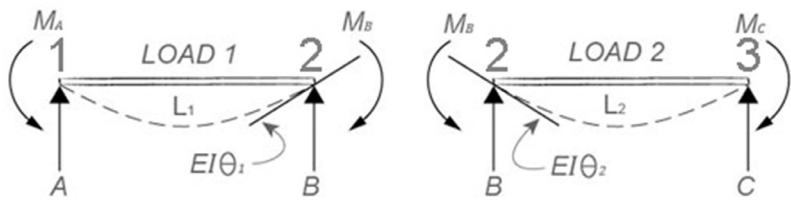


$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Three-Moment Theorem Example

Given: Three non-symmetric spans with loading as shown.

Find: All four reactions



1. Draw FBD
2. Label
3. Solve 3-moment equation

FIRST EQUATION (AT 1, 2 AND 3) :

$$M_A = 0 \quad L_1 = 12 \quad EI\theta_1 = \frac{wL^2}{24} = \frac{48(12)^2}{24} = 288$$

$$M_B = M_2 \quad L_2 = 30 \quad EI\theta_2 = \frac{5Pl^2}{81} = \frac{5(60)(30)^2}{81} = 3333.3$$

$$M_C = M_3 \quad L_1 + L_2 = 42 \quad EI\theta_2 = \frac{5Pl^2}{81} = \frac{5(60)(30)^2}{81} = 3333.3$$

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

$$0(12) + 2M_2(42) + M_3(30) = 6[288 + 3333.3]$$

$$M_2 = 258.667 - 0.35714 M_3$$

Three-Moment Theorem Example (cont.)

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

4. Move one span further and repeat the procedure.

SECOND EQUATION (AT 2, 3 AND 4) :

$$M_A = M_2 \quad L_1 = 30 \quad EI\theta_1 = \frac{4Pl^2}{81} = \frac{4(60)(30)^2}{81} = 2666.67$$

$$M_B = M_3 \quad L_2 = 15 \quad EI\theta_2 = \frac{wL^2}{24} = \frac{90(15)^2}{24} = 843.75$$

$$M_C = 0 \quad L_1 + L_2 = 45 \quad EI\theta_2 = \frac{wL^2}{24} = \frac{90(15)^2}{24} = 843.75$$

$$M_2(30) + 2M_3(45) + 0(15) = 6[2666.67 + 843.75]$$

$$M_2 = 702.084 - 3M_3$$

SIMULTANEOUS SOLUTION :

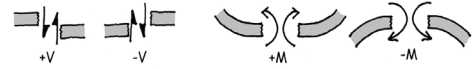
$$258.667 - 0.35714 M_3 = 702.084 - 3M_3$$

$$M_3 = 443.417 / 2.6428 = 167.779 \text{ K}^{-1}$$

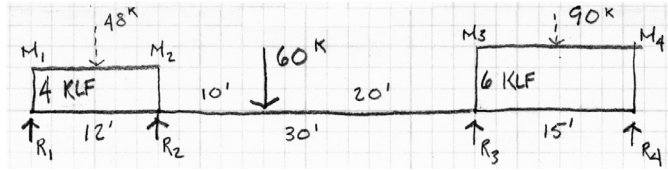
$$M_2 = 702.084 - 3(167.779) = 198.746 \text{ K}^{-1}$$

Three-Moment Theorem Example (cont.)

Sign convention



8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.



SOLVE REACTIONS WITH FBD'S

Span 1 (12')

$\sum M_{R_2} = R_1(12) - 48(6) + 198.7 = 0$
 $R_1 = 7.4378 \text{ k}$
 $\sum F_v = 7.4378 - 48 + V = 0$
 $V = 40.5622$

Span 2 (30')

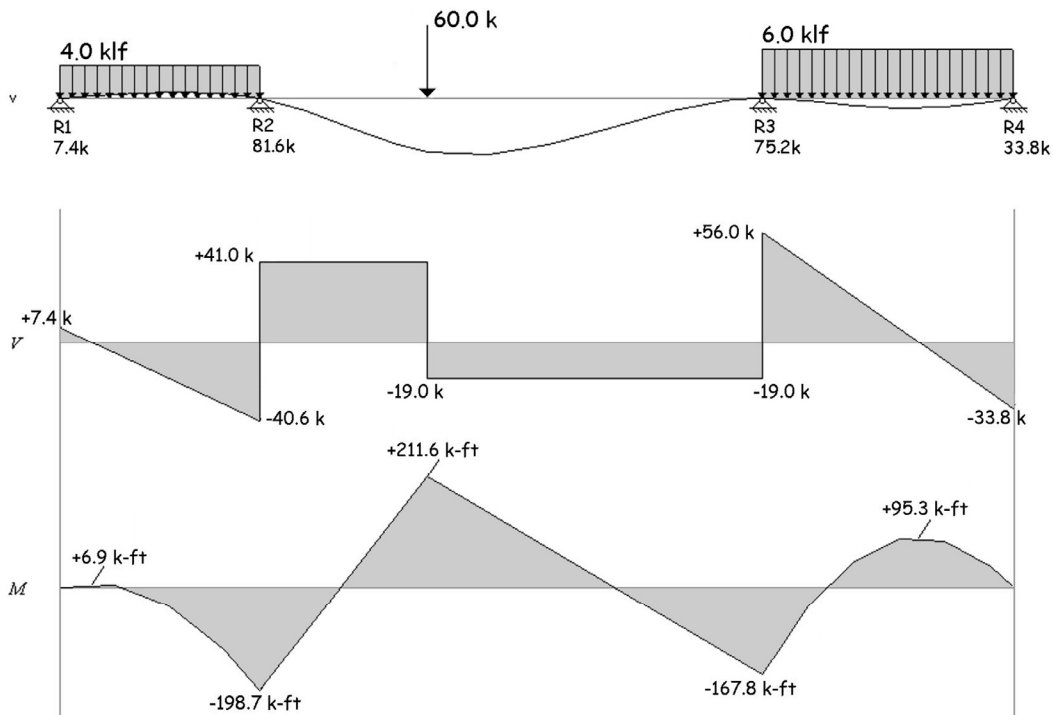
$\sum M_{R_3} = R_2(30) - 40.5622(30) - 60(20) - 198.75 + 167.78 = 0$
 $R_2 = 81.594 \text{ k}$
 $\sum F_v = -40.5622 + 81.594 - 60 + V = 0$
 $V = 18.967$

Span 3 (15')

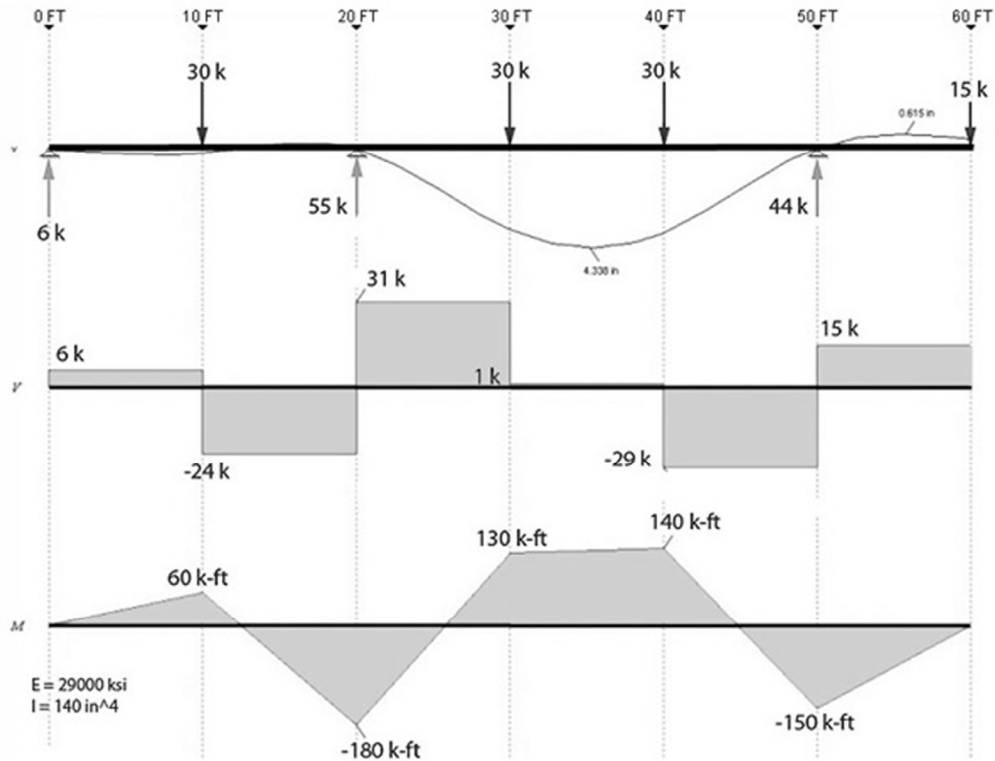
$\sum M_{R_4} = -167.78 - 18.967(15) + R_3(15) - 90(7.5) = 0$
 $R_3 = 75.1531 \text{ k}$
 $\sum M_{R_3} = -167.78 + 90(7.5) - R_4(15) = 0$
 $R_4 = 33.8147 \text{ k}$

Three-Moment Theorem Example (cont.)

9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.

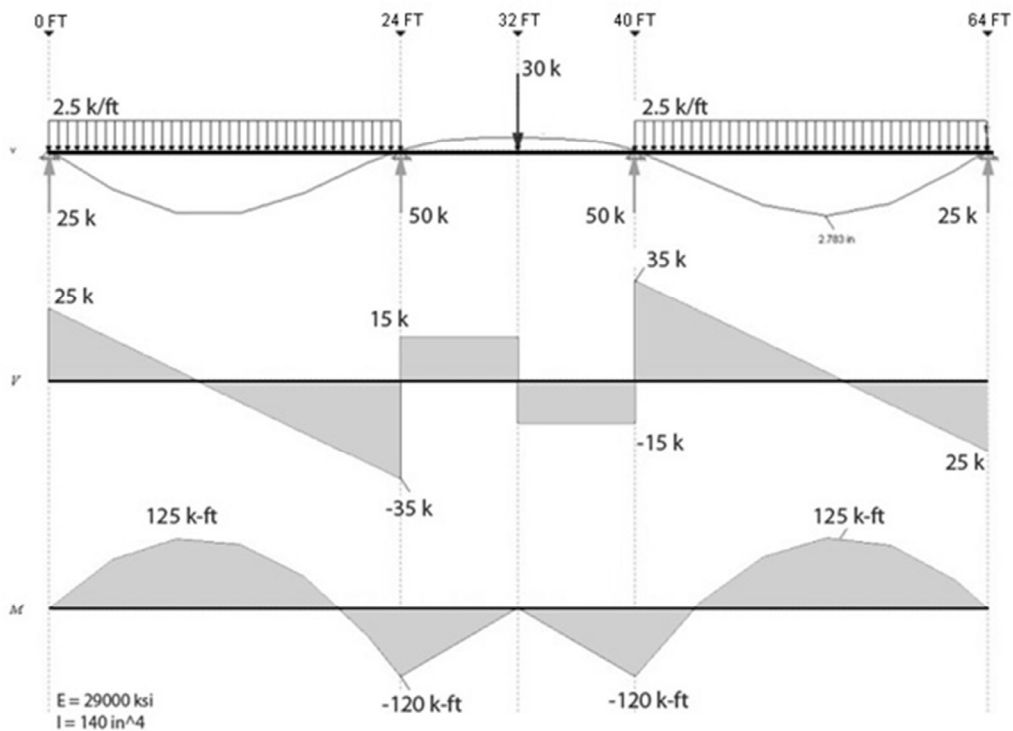


Three-Moment Theorem – 2 Spans



$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Three-Moment Theorem – 3 Spans

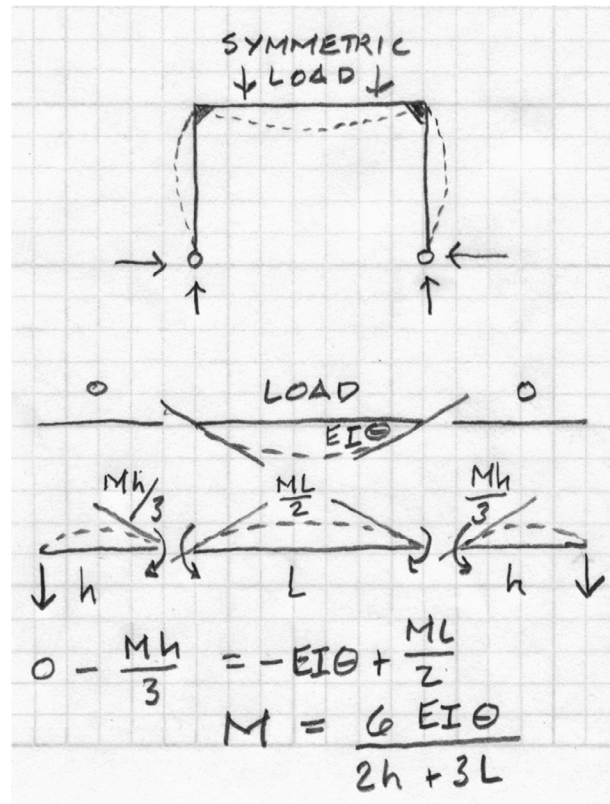


$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

2-Hinge Frame

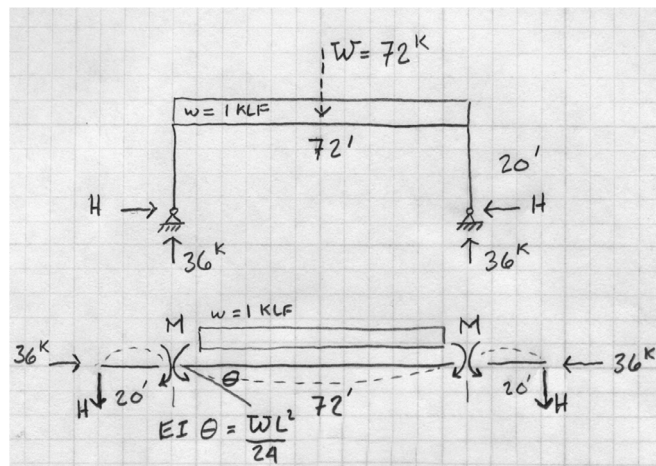
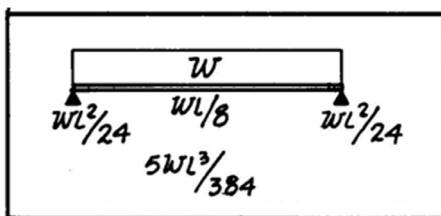
- Statically indeterminate
- Find negative moment at knee
- Symmetric case solution

$$M = \frac{6 E I \theta}{2h + 3L}$$



2-Hinge Frame example

- Symmetric case solution
- Vertical reactions by symmetry
- Find moment at knee
- With FBD of one leg find H

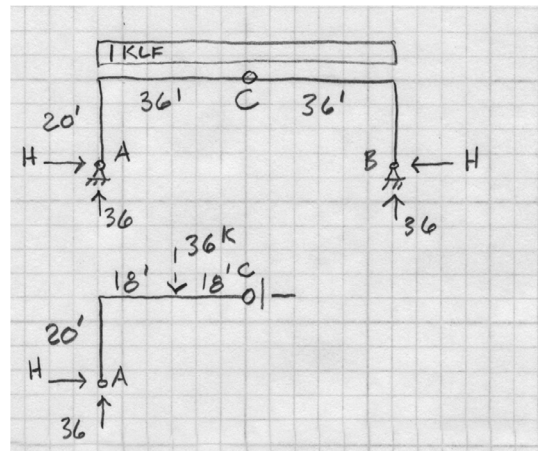


$$M = \frac{6 E I \theta}{2h + 3L} = \frac{6 \frac{72(72)^2}{24}}{2(20) + 3(72)} = \underline{364.5^K}$$

$$M = H(20), \quad H = \frac{M}{20} = \frac{364.5}{20} = \underline{18.2^K}$$

3-Hinge Frame comparison

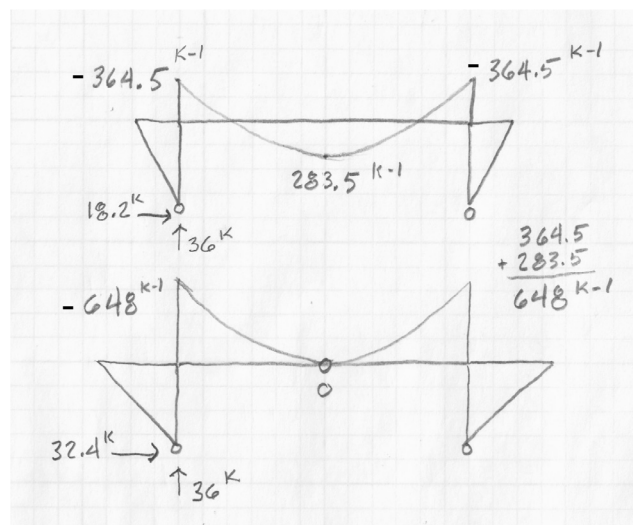
- Statically determinate
- Solve with statics
- FBD of half from hinge
- Solve for H
- Use FBD of leg to solve M



$$\begin{aligned} \sum M_C = 0 &= -36(18) + 36(36) - H(20) \\ H &= 32.4^k \\ M &= H(20) = 32.4(20) = 648^{k-1} \end{aligned}$$

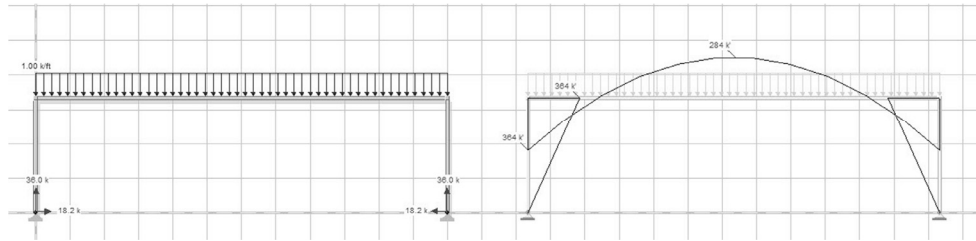
Comparison of moments

- 2-hinge frame
- 3-hinge frame



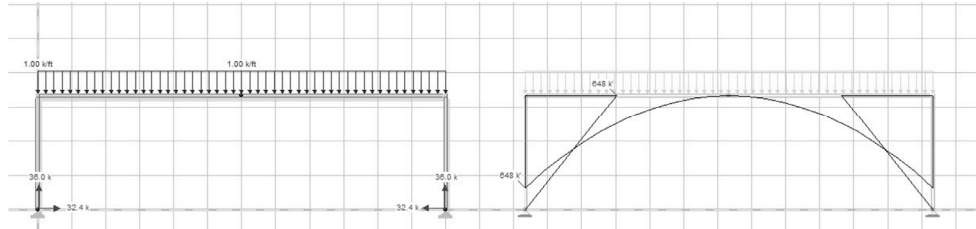
The effect of shape and hinges

Moment:
 knee: -364 ft-lbs
 center: +284 ft-lbs
 horz. react. = 18.2 k



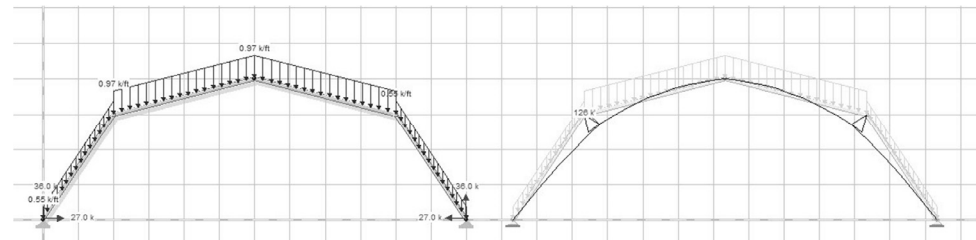
Continuous Beam

Moment:
 knee: -648 ft-lbs
 center: 0 ft-lbs
 horz. react. = 32.4 k



Beam with center hinge

Moment:
 knee: -126 ft-lbs
 center: 0 ft-lbs
 horz. react. = 27.0 k



3 Hinged Arch