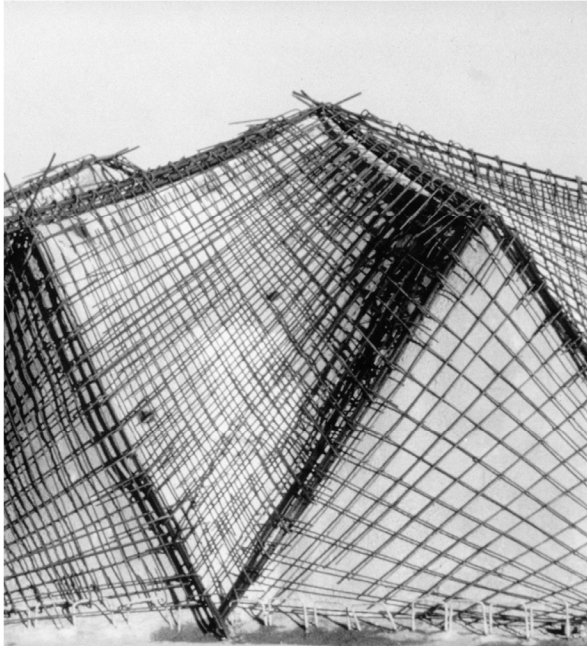


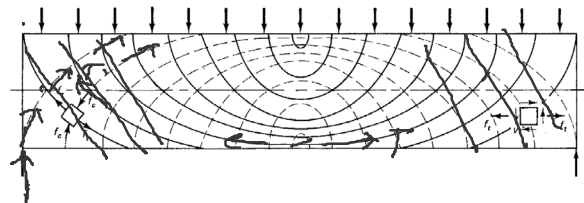
# Reinforced Concrete Beams Ultimate Strength Design (ACI 318 - 2019)



- Flexure in Concrete
- Ultimate Strength Design (LRFD)
- Failure Modes
- Flexure Equations
- Analysis of Rectangular Beams
- Design of Rectangular Beams
- Analysis of Non-rectangular Beams

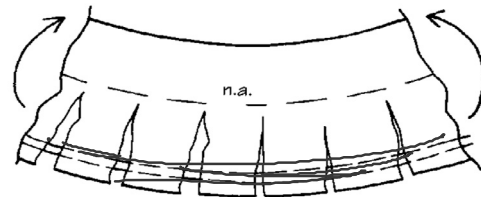
## Flexure

The stress trajectories in this simple beam, show principle tension as solid lines.

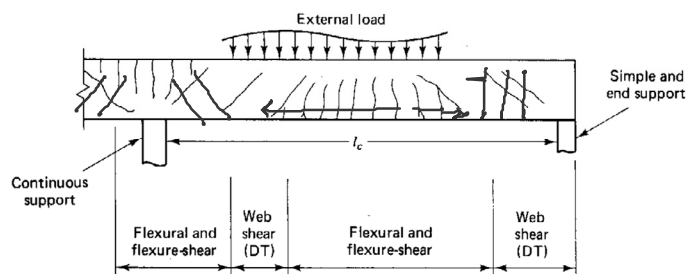
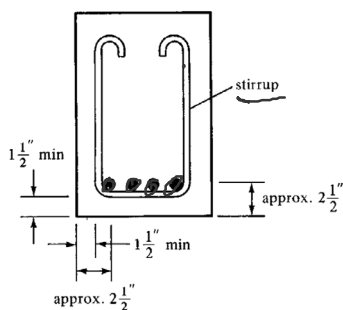


Reinforcement must be placed to resist these tensile forces

In beams continuous over supports, the stress reverses (negative moment). In such areas, tensile steel is on top.



Shear reinforcement is provided by vertical or sloping stirrups.



# Ultimate Strength – (LRFD)

Nominal Strength  $\geq$  Design Strength  
(strength of member  $\geq$  required by loads)

LRFD uses 2 safety factors:  $\gamma$  and  $\phi$   
 $\phi$  nominal strength  $\geq \gamma$  required strength

$\gamma$  increases the required strength of the member and is placed on the loads

$\phi$  reduces the member strength capacity and is placed on the calculated force

Loads increased:

$\gamma$  Factors: DL=1.2 LL=1.6  
U is the required strength  
U=1.2DL+1.6LL  
(factors from ASCE 7)

Strength reduced:

$\phi$  Factors: e.g. flexure = 0.9  
in tension-controlled beams

**Table 21.2.1—Strength reduction factors  $\phi$**

Action or structural element	$\phi$	Exceptions
(a) Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pre-tensioned members where strands are not fully developed, $\phi$ shall be in accordance with 21.2.3.
(b) Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c) Torsion	0.75	—
(d) Bearing	0.65	—
(e) Post-tensioned anchorage zones	0.85	—
(f) Brackets and corbels	0.75	—
(g) Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h) Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i) Plain concrete elements	0.60	—
(j) Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

# Ultimate Strength – (ACI 318)

Reduced Nominal Strength  $\geq$  Factored Load Effects

$$\phi S_n \geq U$$

$\gamma$  Factored Loads (see ACSE 7)

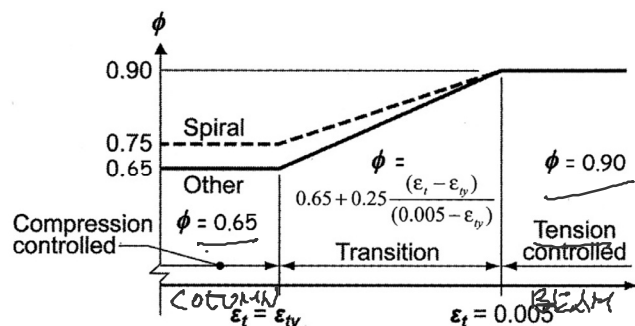
- 1) 1.4D
- 2) 1.2D + 1.6L + 0.5(Lr or S or R)
- 3) 1.2D + 1.6(Lr or S or R) + (1.0L or 0.5W)
- 4) 1.2D + 1.0W + 1.0L + 0.5(Lr or S or R)
- 5) 1.2D + 1.0E + 1.0L + 0.2S
- 6) 0.9D + 1.0W
- 7) 0.9D + 1.0E

- D = service dead loads
- L = service live load
- Lr = service roof live load
- S = snow loads
- W = wind loads
- R = rainwater loads
- E = earthquake loads

## Strength Reduction Factors, $\phi$

Mn	Flexural ( $\epsilon > 0.005$ )	0.90
Vn	Shear	0.75
Pn	Compression (spiral)	0.75
Pn	Compression (other)	0.65
Bn	Bearing	0.65
Tn	Torsion	0.75
Nn	Tension	0.90
Combined stress		0.65 to 0.90

ACI 318 21.2.2



# Strength Measurement

## • Compressive strength

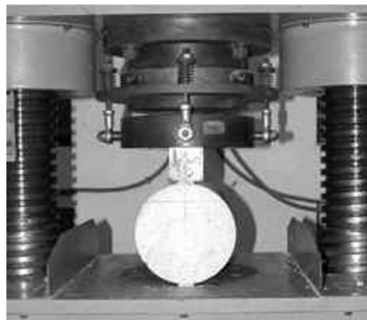
- 12" x 6" cylinder
- 28 day moist cure
- Ultimate (failure) strength
- Usable strain  $\epsilon_{cu} = 0.003$  (ACI 318)

$$f'_c$$

## • Tensile strength ASTM C496

- 12" x 6" cylinder
- 28 day moist cure
- Ultimate (failure) strength
- Split cylinder test
- ca. 10% of  $f_c$
- Neglected in flexure analysis

$$f'_t$$



# Failure Modes Based on $A_s$

## • No Reinforcing

- ~~o Less than  $A_{s, min}$~~
- ~~o Brittle failure~~

$$\rho = \frac{A_s}{bd}$$

$A_{s, min}$ :  
greater of a and b

## • Reinforcing < balance (use this)

- o Steel yields before concrete fails
- o ductile failure
- o  $(\sim A_{s, min}) 0.06 \geq \epsilon_t \geq 0.004$  ( $A_{s, max}$ )
- o  $\epsilon_t \geq 0.005$  for tension controlled

$$(a) \frac{3\sqrt{f'_c}}{f_y} b_w d$$

$$(b) \frac{200}{f_y} b_w d$$

## • Reinforcing = balance

- ~~o Concrete fails just as steel yields~~
- ~~o  $\epsilon_t$  at balance = 0.00207 for Gr 60~~

$A_{s, max}$  when  $\epsilon_t = 0.004$

$$\rho_{bal} = \left( \frac{0.85\beta_1 f'_c}{f_y} \right) \left( \frac{87000}{87000 + f_y} \right)$$

## • Reinforcing > balance

- ~~o Concrete fails before steel yields~~
- ~~o Low ductility~~
- ~~o Sudden failure~~

$A_s > A_{s, max}$  SuddenDeath!!

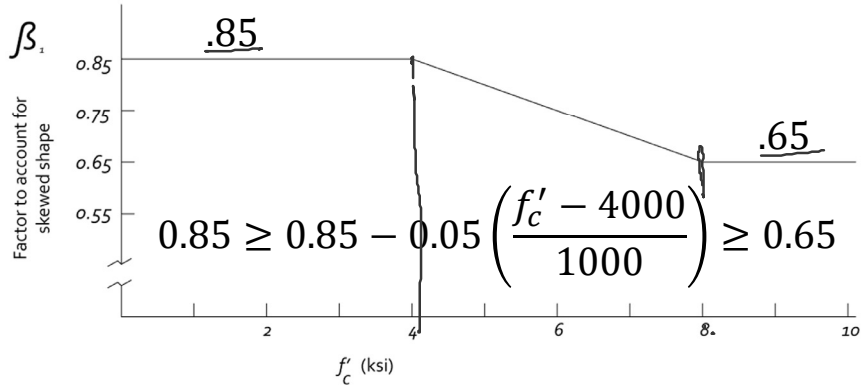
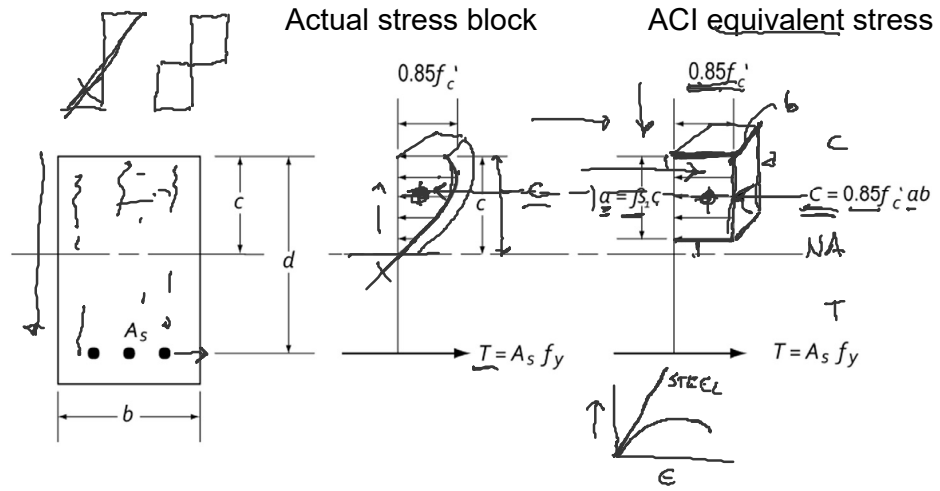
# ACI Stress Block

$\beta_1$  is a factor to account for the non-linear shape of the compression stress block.

$$a = \beta_1 c$$

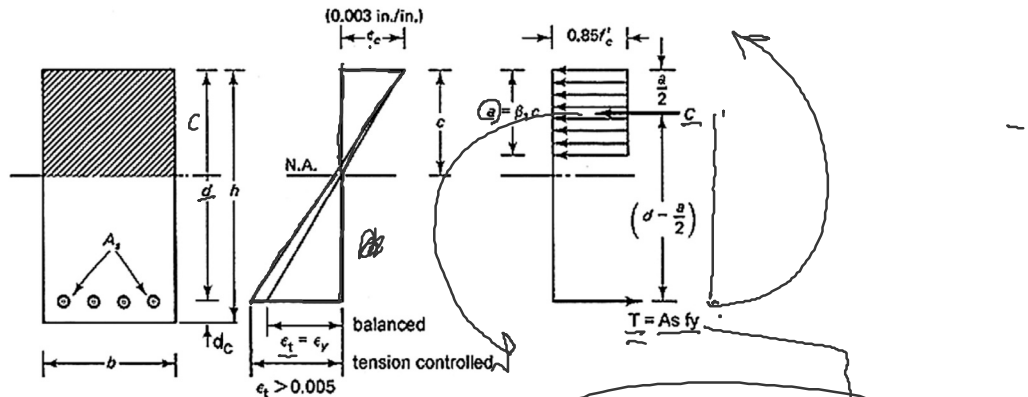
psi

$f'_c$	$\beta_1$
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



# Flexure Equations

strain      ACI equivalent stress block



$$C = T$$

$$0.85f'_c ab = A_s f_y$$

solving for  $a$ ,

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{\rho f_y d}{0.85f'_c}$$

$$M_n = T \left( d - \frac{a}{2} \right) = A_s f_y \left( d - \frac{a}{2} \right)$$

Load  $\rightarrow M_u = \phi M_n \leftarrow$  STRENGTH

$$M_u = \phi M_n = \phi A_s f_y \left( d - \frac{a}{2} \right)$$

$$\epsilon_t = \frac{d - c}{c} (0.003)$$

$$\rho = \frac{A_s}{bd} \quad \frac{\epsilon_c}{c} = \frac{\epsilon_t}{d - c}$$

$$M_u = \phi A_s f_y d \left( 1 - 0.59 \frac{\rho f_y}{f'_c} \right)$$



# Balance Condition

From similar triangles at balance condition:

$$\frac{c}{d} = \frac{0.003 \cdot 0.003}{0.003 + (f_y/E_s)} = \frac{0.003}{0.003 + (f_y/29 \times 10^6)}$$

$$c = \frac{87,000}{87,000 + f_y} d$$

Use equation for a. Substitute into  $c = a / \beta_1$

$$a = \frac{\rho f_y d}{0.85 f'_c}$$

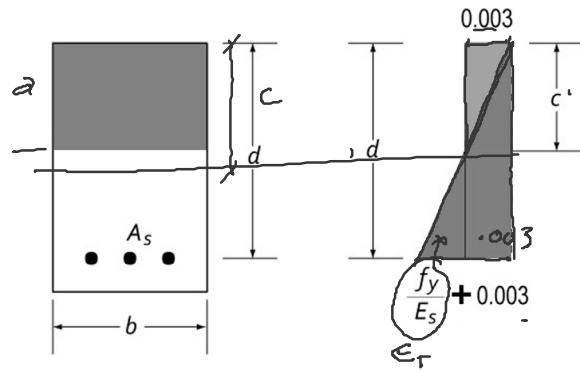
$$\rho = \frac{A_s}{bd}$$

$$c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f'_c}$$

Equate expressions for c:

$$\frac{\rho f_y d}{0.85 \beta_1 f'_c} = \frac{87,000}{87,000 + f_y} d$$

$$\rho_b = \left( \frac{0.85 \beta_1 f'_c}{f_y} \right) \left( \frac{87,000}{87,000 + f_y} \right)$$



Strain diagram for balanced condition.

Table A.8 Balanced Ratio of Reinforcement  $\rho_b$  for Rectangular Sections with Tension Reinforcement Only

$f_y$	$f'_c$	2,500 psi	3,000 psi	4,000 psi	5,000 psi	6,000 psi
		(17.2 MPa)	(20.7 MPa)	(27.6 MPa)	(34.5 MPa)	(41.4 MPa)
		$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.80$	$\beta_1 = 0.75$
Grade 40 40,000 psi (275.8 MPa)	$\rho_b$	0.0309	0.0371	0.0495	0.0582	0.0655
	$0.75 \rho_b$	0.0232	0.0278	0.0371	0.0437	0.0492
	$0.50 \rho_b$	0.0155	0.0186	0.0247	0.0291	0.0328
Grade 50 50,000 psi (344.8 MPa)	$\rho_b$	0.0229	0.0275	0.0367	0.0432	0.0486
	$0.75 \rho_b$	0.0172	0.0206	0.0275	0.0324	0.0365
	$0.50 \rho_b$	0.0115	0.0138	0.0184	0.0216	0.0243
Grade 60 60,000 psi (413.7 MPa)	$\rho_b$	0.0178	0.0214	0.0285	0.0335	0.0377
	$0.75 \rho_b$	0.0134	0.0161	0.0214	0.0252	0.0283
	$0.50 \rho_b$	0.0089	0.0107	0.0143	0.0168	0.0189
Grade 75 75,000 psi (517.1 MPa)	$\rho_b$	0.0129	0.0155	0.0207	0.0243	0.0274
	$0.75 \rho_b$	0.0097	0.0116	0.0155	0.0182	0.0205
	$0.50 \rho_b$	0.0065	0.0078	0.0104	0.0122	0.0137

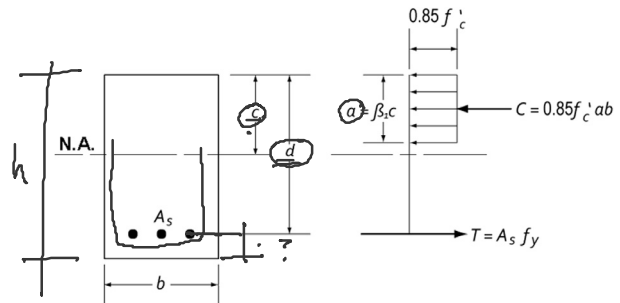
# Rectangular Beam Analysis

Data:

- Section dimensions –  $b, h, d$ , (span)
- Steel area -  $A_s$
- Material properties –  $f'_c, f_y$

Required:

- Nominal Strength (of beam) Moment -  $M_n$
- Required (by load) Design Moment –  $M_u$
- Load capacity



$A_{s,min}$  greater of a and b.

$$(a) \frac{3\sqrt{f'_c}}{f_y} b_w d$$

$$(b) \frac{200}{f_y} b_w d$$

$$c = \frac{a}{\beta_1}$$

$$\epsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right)$$

$$\phi M_n \geq M_u$$

$$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$$

$$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$$

- Calculate  $d$
- Check  $A_{s,min}$
- Calculate  $a$
- Determine  $c$
- Check that  $\epsilon_t \geq 0.005$  (tension controlled)
- Find nominal moment,  $M_n$
- Calculate required moment,  $\phi M_n \geq M_u$   
(if  $\epsilon_t \geq 0.005$  then  $\phi = 0.9$ )
- Determine max. loading (or span)

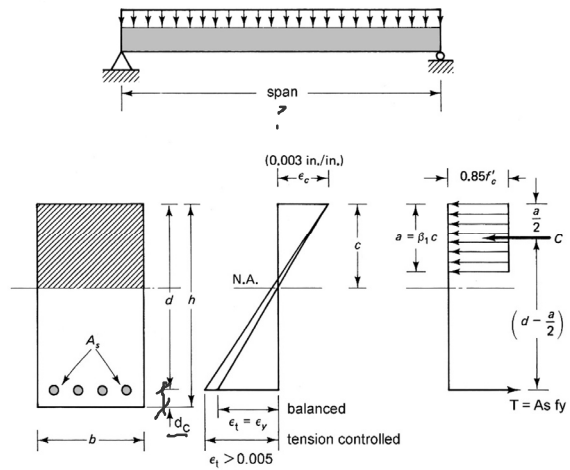
# Rectangular Beam Analysis

## Data:

- dimensions –  $12" \times 23"$
- Steel –  $4 \times \#6$   $f_y = 60\text{ksi}$
- Concrete  $f'_c = 6000\text{ psi}$
- Stirrup  $\#3$  cover  $1.5"$  Agg  $\frac{3}{4}"$

## Required:

- Required Moment –  $\phi M_n = M_u$  (capacity)

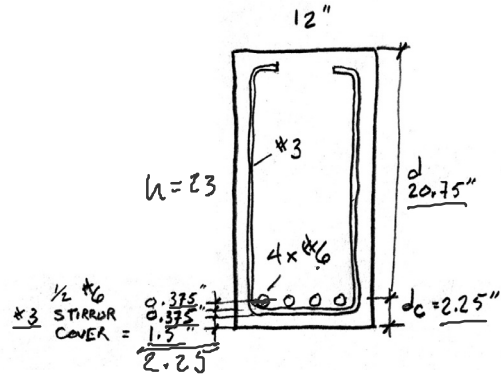


1. Calculate  $d$

$$d_c = \text{COVER} + \#3 + \frac{1}{2}(\#6)$$

$$= 1.5 + 0.375 + \frac{0.75}{2} = 2.25"$$

$$d = h - d_c = 23" - 2.25" = 20.75" \checkmark$$



# Rectangular Beam Analysis cont.

## Data:

dimensions –  $12" \times 23"$

Steel –  $4 \times \#6$  –  $A_s = 1.76 \text{ in}^2$   
 $f'_c = 6000 \text{ psi}$   $f_y = 60\text{ksi}$

$$4 \times 0.44 =$$

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Customary Units			SI Units		
	Diameter (in.)	Cross-sectional Area (in. <sup>2</sup> )	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm <sup>2</sup> )	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

2. Check  $A_{s, \min}$

$$A_{s, \min}$$

$$\textcircled{1} \frac{3\sqrt{f'_c}}{f_y} b d = \frac{3\sqrt{6000}}{60000} (12 \times 20.75) = 0.964 \text{ in}^2 < 1.76 \checkmark$$

controls

$$\textcircled{2} \frac{200 b d}{f_y} = \frac{200 (12) (20.75)}{60000} = 0.83 \text{ in}^2$$

$$\therefore A_{s, \min} = 0.964 \text{ in}^2$$

$$A_s = A_b (\text{N.B.}) = 0.44 (4) = 1.76 \text{ in}^2 \checkmark > 0.964 \text{ in}^2$$

# Rectangular Beam Analysis cont.

Data:

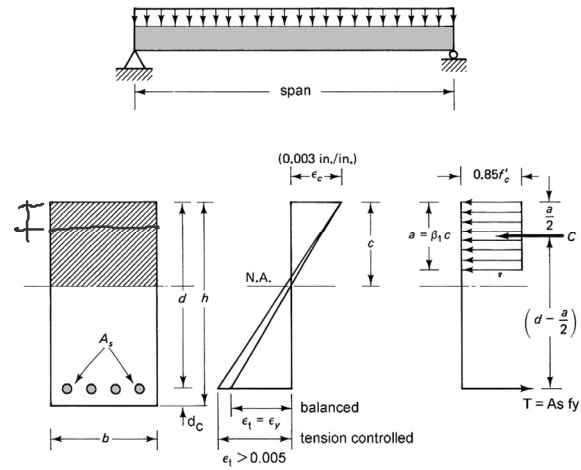
dimensions – 12"x23"

Steel – 4 x # 6 –  $A_s = 1.76 \text{ in}^2$

$f'_c = 6000 \text{ psi}$   $f_y = 60 \text{ ksi}$

6 ksi

$f'_c$	$\beta_1$
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



3. Find a ✓

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1.76)(60)}{0.85(6)(12)} = 1.725''$$

4. Find c ✓

$$\beta_1 = 0.85 - 0.05 \frac{f'_c - 4000}{1000} = 0.85 - 0.1 = 0.75$$

$$c = \frac{a}{\beta_1} = \frac{1.725}{0.75} = 2.300''$$

# Rectangular Beam Analysis cont.

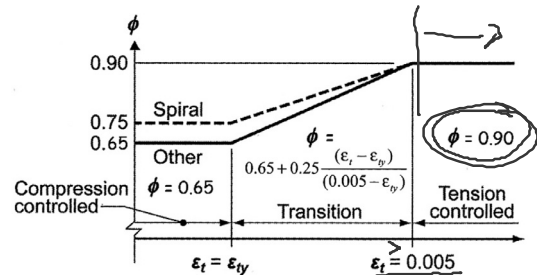
5. Check that  $A_s$  is  $< A_{s \text{ max}}$

$$\epsilon_t \geq 0.004 \quad \checkmark$$

6. Check that  $\epsilon_t \geq 0.005$

(for tension controlled section)

$$\phi = 0.9$$



$$\epsilon_t = \frac{d-c}{c} 0.003 = \frac{20.75-2.3}{2.3} 0.003$$

$$\epsilon_t = 0.02406 > 0.004 \quad \therefore \text{OK } \checkmark$$

$$= 0.02406 > 0.005 \quad \therefore \text{tension controlled}$$

7. Find nominal moment, Mn

$$T = A_s f_y = 1.76 (60 \text{ ksi}) = 105.6 \text{ K}$$

$$M_n = T \left( d - \frac{a}{2} \right) = 105.6 \left( 20.75 - \frac{1.725}{2} \right)$$

$$M_n = 2100 \text{ K}\cdot\text{ft}$$

8. Calculate required moment

$$\phi M_n \geq M_u$$

$$\phi M_n = 0.9 (2100) = 1890 \text{ K}\cdot\text{ft}$$

$$\frac{\gamma_w l^2}{8} \rightarrow M_u = \phi M_n = 1890 / 12 = 157.5 \text{ K}\cdot\text{ft}$$

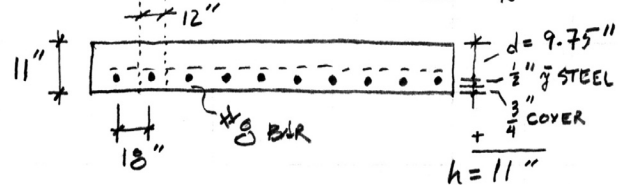
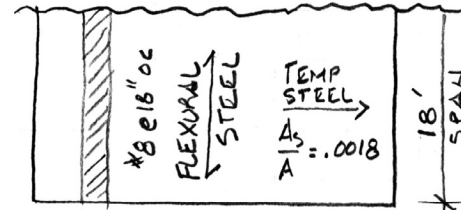
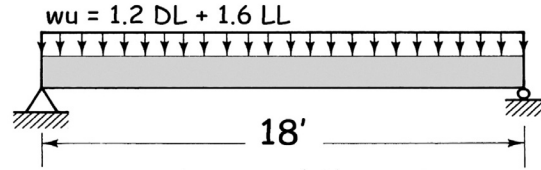
# Slab Analysis

Data:

- Span = 18 ft
- h = 11"
  - take b = 12"
- Steel #8 @ 18" o.c.
- f'c = 3000 psi
- fy = 60 ksi

Required:

- Design moment capacity – Mu
- Maximum LL in PSF



$$d = 11 - \frac{1}{2} - \frac{3}{4} = 9.75''$$

- Find d
- Find As

$$A_s = \frac{12''}{18''} (0.79 \text{ in}^2) = 0.5267 \text{ in}^2/\text{FT}$$

Check As,min

$$A_g = 11'' \times 12'' = 132 \text{ in}^2$$

$$[0.0018(60)/60] 132 = 0.237 \text{ in}^2$$

$$0.0014 (132) = 0.1848 \text{ in}^2$$

$$0.527 > 0.237 \text{ ok}$$

Table 7.6.1.1—As,min for nonprestressed one-way slabs

Reinforcement type	fy, psi	As,min
Deformed bars	< 60,000	0.0020Ag
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of: $\frac{0.0018 \times 60,000}{f_y} A_g$
		0.0014Ag

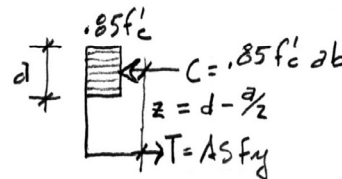
ACI 318-14

# Slab Analysis

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

- Find a

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.5267 (60)}{.85 (3) (12)} = 1.033''$$



- Find c = β1 a

$$c = \frac{a}{\beta_1} = \frac{1.033''}{0.85} = 1.215''$$

- Check failure mode

$$\epsilon_t \geq 0.005 \text{ for tension controlled}$$

$$\epsilon_t = \frac{0.003 d}{c} - 0.003$$

$$\epsilon_t = \frac{0.003 (9.75'')}{1.215''} - 0.003 = 0.021 \%$$

- Find force T

$$\epsilon_t = 0.021 > 0.005 \therefore \text{TENSION CONTROLLED}$$

- Find moment arm z

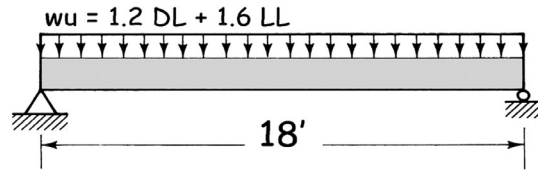
$$T = A_s f_y = 0.5267 (60) = 31.6 \text{ K}$$

$$z = d - \frac{a}{2} = 9.75 - \frac{1.033}{2} = 9.23''$$

- Find nominal strength moment, Mn

$$M_n = T z = 31.6 (9.23) = 291.8 \text{ K-in}$$

# Slab Analysis



9. Find required moment,  $M_u$

$$M_u = \phi M_n = 0.9 (291.8) \frac{1000}{12} = 21885 \text{ l-ft}$$

10. Find slab DL

$$w_{DL} = 2 \frac{h}{12} = 150 \frac{11}{12} = 137.5 \text{ PSF}$$

11. Determine max. loading

$$M_u = 21885 \text{ l-ft} = \frac{(1.2 w_{DL} + 1.6 w_{LL}) l^2}{8}$$

$$\frac{21885 (8)}{(18')^2} = 1.2 (137.5) + 1.6 (w_{LL})$$

$$540.37 = 165 + 1.6 (w_{LL})$$

$$w_{LL} = 234.6 \text{ PSF}$$

## Details of Reinforcement

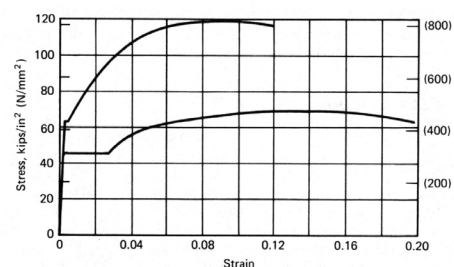
### Size

- Nominal 1/8" increments

### Grade

- 40 40 ksi
- 60 60 ksi
- 75 75 ksi

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257



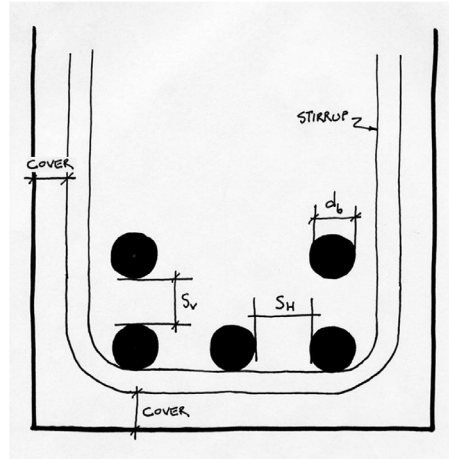
# Details of Reinforcement

## ACI 318 Chapter 25.2 Placement of Reinforcement

- cover (ACI 20.6.1)
- horizontal spacing in beams (ACI 25.2.1)  
1 inch  
 $d_b$   
 $4/3$  max aggregate
- vertical spacing in beams (ACI 25.2.2)  
1 inch

**Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members**

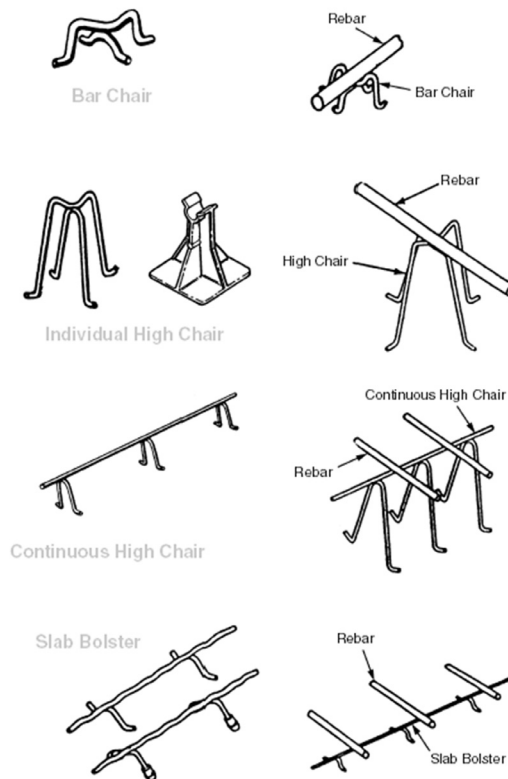
Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2



# Details of Reinforcement

## ACI 318 Chapter 25 Placement of Reinforcement

- Chairs
- Bolsters



# Details of Reinforcement

## ACI 318 Chapter 25

### Minimum bend diameter

- factor x  $d_b$

### Hooks for bars in tension

- ACI Table 25.3.1
- Inside diameter

### Bends for stirrups

- ACI Table 25.3.2

**Table 25.3.1—Standard hook geometry for development of deformed bars in tension**

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension <sup>(1)</sup> $\ell_{ext}$ , in.	Type of standard hook
90-degree hook	No. 3 through No. 8	$6d_b$	$12d_b$	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		
180-degree hook	No. 3 through No. 8	$6d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		

<sup>(1)</sup>A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

**Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops**

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension <sup>(1)</sup> $\ell_{ext}$ , in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

<sup>(1)</sup>A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

# Details of Reinforcement

## ACI 318 Chapter 25

### Development length of bars

- 12" minimum
- based on table 25.4.2.2

**Table 25.4.2.4—Modification factors for development of deformed bars and deformed wires in tension**

Modification factor	Condition	Value of factor
Lightweight $\lambda$	Lightweight concrete	0.75
	Lightweight concrete, where $f_{cr}$ is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy <sup>(1)</sup> $\psi_e$	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size $\psi_s$	No. 7 and larger bars	1.0
	No. 6 and smaller bars and deformed wires	0.8
Casting position <sup>(1)</sup> $\psi_t$	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

<sup>(1)</sup>The product  $\psi_s\psi_e$  need not exceed 1.7.

**Table 25.4.2.2—Development length for deformed bars and deformed wires in tension**

Spacing and cover	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than $d_b$ , clear cover at least $d_b$ , and stirrups or ties throughout $\ell_d$ not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least $d_b$	$\left( \frac{f_y \psi_s \psi_e}{25 \lambda \sqrt{f'_c}} \right) d_b$	$\left( \frac{f_y \psi_s \psi_e}{20 \lambda \sqrt{f'_c}} \right) d_b$
Other cases	$\left( \frac{3 f_y \psi_s \psi_e}{50 \lambda \sqrt{f'_c}} \right) d_b$	$\left( \frac{3 f_y \psi_s \psi_e}{40 \lambda \sqrt{f'_c}} \right) d_b$

# Other Useful Tables:

**Table A.1** Values of Modulus of Elasticity for Normal-Weight Concrete

Customary Units		SI Units	
$f'_c$ (psi)	$E_c$ (psi)	$f'_c$ (MPa)	$E_c$ (MPa)
3,000	3,140,000	20.7	21 650
3,500	3,390,000	24.1	23 373
4,000	3,620,000	27.6	24 959
4,500	3,850,000	31.0	26 545
5,000	4,050,000	34.5	27 924

Jack C McCormac, 1978  
Design of Reinforced Concrete,

**Table A.2** Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Customary Units			SI Units		
	Diameter (in.)	Cross-sectional Area (in. <sup>2</sup> )	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm <sup>2</sup> )	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

**Table A.4** Areas of Groups of Standard Bars (in.<sup>2</sup>)

Bar No.	Number of Bars												
	2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

## Rectangular Beam Design

Two approaches:

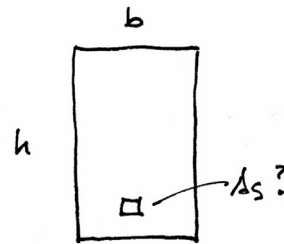
### Method 1:

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions:  $h$  and  $b$

Required:

- Steel area -  $A_s$



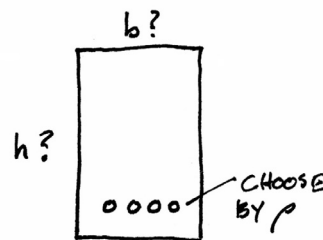
### Method 2:

Data:

- Load and Span
- Some section dimensions –  $h$  or  $b$
- Material properties –  $f'_c$ ,  $f_y$
- $\rho$

Required:

- Steel area -  $A_s$
- Beam dimensions –  $b$  and  $h$





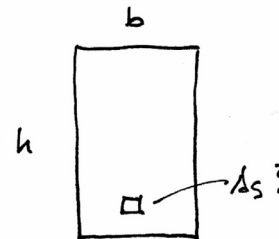
# Rectangular Beam Design – method 1

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions –  $b$  and  $h$

Required:

- Steel area -  $A_s$



1. Calculate the factored load and find factored required moment,  $M_u$
2. Find  $d = h - \text{cover} - \text{stirrup} - d_b/2$  (one layer)
3. Estimate moment arm  $z = jd$ . For beams  $j \approx 0.9$  for slabs  $j \approx 0.95$
4. Estimate  $A_s$  based on estimate of  $jd$ .
5. Use  $A_s$  to find  $a$
6. Use  $a$  to find  $A_s$  (repeat...until 2% accuracy)
7. Choose bars for  $A_s$  and check  $A_s$  max & min
8. Check that  $\epsilon_t \geq 0.005$
9. Check  $M_u \leq \phi M_n$  (final condition)

$$M_u = \frac{(\gamma w_{DL} + \gamma w_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi f_y \left( d - \frac{a}{2} \right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

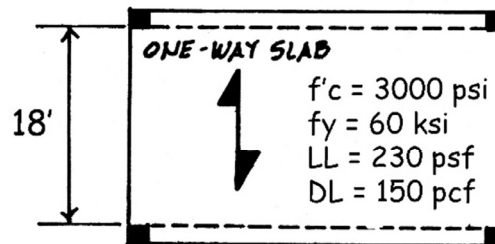
10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, steel development length.

$$M_n = A_s f_y \left( d - \frac{a}{2} \right)$$

# Rectangular Slab Design

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions:
- $h$  (based on deflection limit)
- $b = \text{typical } 12'' \text{ width}$



Required:

- Steel area -  $A_s$

First estimate the slab thickness,  $h$ .

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

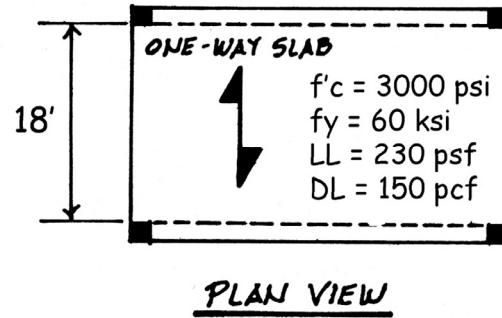
**Table 7.3.1—Minimum thickness of solid nonprestressed one-way slabs**

Support condition	Minimum $h^{(1)}$
Simply supported	$l/20$
One end continuous	$l/24$
Both ends continuous	$l/28$
Cantilever	$l/10$

THICKNESS,  $h$ , BASED ON DEFLECTION

$$h = \frac{l}{20} = \frac{18 \times 12}{20} = 10.8'' \text{ USE } 11''$$

## Rectangular Slab Design



1. Calculate the dead load and find required  $M_u$

FACTOR LOADS

$$DL = \frac{11''}{12} (150) = 137.5 \text{ PSF}$$

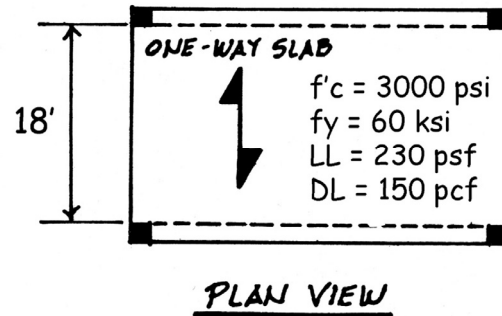
$$LL \text{ (GIVEN)} = 230 \text{ PSF}$$

$$w_D = 1.2(137.5) + 1.6(230) = 533$$

$$M_u = \frac{w_D l^2}{8} = \frac{533 \text{ PLF} (18')^2}{8} = 21587 \text{ ft-k}$$

$$= 259 \text{ ''-k}$$

## Rectangular Slab Design



2. Estimate moment arm  
 $z \approx 0.95 d$

FOR  $j \approx 0.95$ ,  $d = h - \text{COVER} - \frac{1}{2} \text{ BAR}$

$$d = 11'' - \frac{3}{4}'' - \frac{1}{2} (\frac{1}{2}'')$$

$$d = 11'' - 1'' = 10''$$

$$z \approx j d \approx 0.95 (10'') = 9.5''$$

# Rectangular Slab Design

- Estimate  $A_s$  based on estimate of  $j_d$ .
- Use  $A_s$  to find  $a$
- Use  $a$  to find  $A_s$  (repeat...)

TRIAL 1

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{259 \text{ ft-k}}{0.9(60 \text{ ksi})(9.5 \text{ ft})} = 0.505 \text{ in}^2$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.505(60)}{.85(3)(12)} = 0.99 \text{ in}$$

TRIAL 2

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9(60)(10 - \frac{.99}{2})}$$

$$A_s = 0.5046 \text{ in}^2 \quad \text{WITHIN 2\%}$$

# Rectangular Slab Design

- Choose bars for  $A_s$  required:  
either  
**choose bars** and calculate spacing  
or  
**choose spacing** and find bar size

If the bar size changes, re-calculate to find new  $d$ . Then re-calculate  $A_s$ ...

- Check  $A_{s,min}$   
(for slabs  $A_{s,min}$  from ACI Table 7.6.1.1)

Table 7.6.1.1— $A_{s,min}$  for nonprestressed one-way slabs

Reinforcement type	$f_y$ , psi		$A_{s,min}$
Deformed bars	< 60,000		$0.0020A_g$
Deformed bars or welded wire reinforcement	$\geq 60,000$	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			$0.0014A_g$

CHOOSE BARS

USING #4

$$\frac{0.505}{12 \text{ in}} ; \frac{0.2}{s} \quad s = 4.75 \text{ in}$$

$\therefore$  USE #4 @ 4" o.c. (always round down)

$$A_s = 0.60 \text{ in}^2/\text{FT} > 0.505 \checkmark$$

ALTERNATE FOR MAX. S = 18"

$$\frac{0.505}{12 \text{ in}} ; \frac{A_b}{18 \text{ in}} \quad A_b = 0.75 \text{ in}^2$$

$$\#8 = 0.79$$

$\therefore$  USE #8 @ 18" o.c.

$$A_s = 0.526 \text{ in}^2/\text{FT} > 0.505 \checkmark$$

Check  $A_{s,min}$

$$A_{s,min} = 0.0018 bh = 0.0018(12)(11 \text{ in})$$

$$= 0.24 \text{ in}^2 < 0.526 \text{ in}^2 \checkmark \text{ OK}$$

## Rectangular Slab Design

8. Check that  $\epsilon_t \geq 0.005$

$$\text{RE-CALC 2 FOR } A_s = 0.6 \text{ m}^2/\text{ft}$$
$$a = \frac{A_s f_y}{0.85 F_c' b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d-c}{c} 0.003 =$$
$$= \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$$0.01759 > 0.005$$

∴ TENSION CONTROLLED ✓

## Rectangular Slab Design

9. Check  $M_u \leq \phi M_n$   
(final condition)  
 $A_s = A_{s, \text{used}}$

$$M_n = T z$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right)$$

$$M_n = 0.6(60) \left( 9.5'' - \frac{1.176}{2} \right)$$

$$M_n = 36(8.911'') = 320.8 \text{ k-in}$$

$$\phi M_n = 0.9(320.8) = 288.7 \text{ k-in}$$

10. Check deflection, crack control, steel development length.

$$M_u = 259 \text{ k-in} < 288.7 \text{ k-in}$$

$$M_u < \phi M_n \quad \checkmark \text{ OK}$$

## Rectangular Beam Design – method 2

### Data:

- Load and Span
- Some section dimensions – b or h
- Material properties –  $f'_c$ ,  $f_y$

$$M_u = \frac{(\gamma w_{DL} + \gamma w_{LL})l^2}{8}$$

### Required:

- Steel area -  $A_s$
  - Beam dimensions – b and h
1. Estimate the dead load ( $h \approx L/12$ ) and find  $M_u$
  2. Choose  $\rho$  (equation assumes  $\epsilon_t = 0.0075$ )
  3. Calculate  $bd^2$
  4. Choose b and solve for d (or d and solve b)  
b is based on form size – matches column size  
h is between  $L/12$  to  $L/18$  and  $b:h \approx 1:2$  to  $2:3$
  5. Estimate h and correct weight and  $M_u$
  6. Find  $A_s = \rho b d$
  7. Choose bars for  $A_s$  and determine spacing and cover. Recheck h and weight.
  8. Check that  $\epsilon_t \geq 0.005$  (if not, increase h and reduce  $A_s$ )
  9. Design shear reinforcement (stirrups)
  10. Check deflection, crack control, steel development length.

$$\rho = \frac{\beta_1 f'_c}{4 f_y}$$

$$b d^2 = \frac{M_u}{\phi \rho f_y (1 - 0.59 \rho (f_y / f'_c))}$$

$$A_s = \rho b d$$

$$a = \frac{\rho f_y d}{0.85 f'_c}$$

## Rectangular Beam Design

### Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$

### Required:

- Steel area -  $A_s$
- Beam dimensions – b and d

Estimate b and h to get beam selfweight.

1. Estimate the dead load ( $h \approx L/12$ ) and find  $M_u$ .

Table 9.3.1.1—Minimum depth of nonprestressed beams

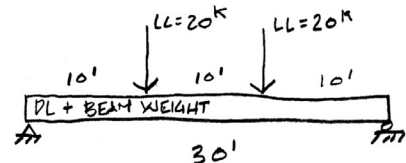
Support condition	Minimum $h^{(1)}$
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

<sup>(1)</sup>Expressions applicable for normalweight concrete and  $f_y = 60,000$  psi. For other cases, minimum  $h$  shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

2. Choose  $\rho$  (equation assumes  $\epsilon_t = 0.0075$ )

$f'_c = 3000$  psi  
 $f_y = 60$  ksi

DL = 2 klf + beam  
LL = 2 x 20 k



$$\text{ASSUME } h \approx \frac{L}{12} = \frac{360''}{12} = 30''$$

$$\text{ASSUME } b:h \approx 1:2 \therefore b \approx 15''$$

$$\text{BEAM DL} = 150 \frac{15 \times 30}{144} = 469 \text{ PLF}$$

ESTIMATE  $M_u$

$$\begin{aligned} M_u &= P_2 + \frac{w l^2}{8} \\ &= 1.6(20)(10') + \frac{1.2(2.469 \text{ KLF})(30')^2}{8} \\ &= 320 + 333.3 = 653.3 \text{ K-}' \end{aligned}$$

CHOOSE  $\rho$

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

## Rectangular Beam Design cont.

3. Calculate  $bd^2$

$$bd^2 = \frac{M_u}{\phi \rho F_y (1 - 0.59 \rho (f_y / f_c))}$$

$$bd^2 = \frac{653.3 (12)}{0.01(0.9)60 [1 - 0.59(0.01)(\frac{60}{3})]}$$

$$bd^2 = \frac{7840}{0.573(0.882)} = 15492 \text{ in}^3$$

4. Choose  $b$  and solve for  $d$

(or  $d$  and solve for  $b$ )

$b$  is based on form size – matches column size

$h$  is between  $L/12$  to  $L/18$  and  $b:h \approx 1:2$  to  $2:3$

TRY

$b$	$d$	$h \approx 1.12 d$	$A$
14"	33.27"	38"	532
15"	32.14"	36"	540
16"	31.11"	35"	560

5. Estimate  $h$  and correct weight and  $M_u$

CHOOSE 15 x 36

## Rectangular Beam Design cont.

6. Choose  $b$  and solve for  $d$

(or  $d$  and solve for  $b$ )

$b$  is based on form size – matches column size

$h$  is between  $L/12$  to  $L/18$  and  $b:h \approx 1:2$  t

USE 15 x 36

$$\text{REVISE } PL = 150 \frac{540}{144} = 563 \text{ PLF}$$

CHECK  $M_u$

$$M_u = 320 + \frac{1.2(2,563)30^2}{8} = 666 \text{ K-ft}$$

REVISE  $bd$

$$bd^2 = \frac{666(12)}{0.505} = 15814 \text{ in}^3$$

$$\text{FOR } b = 15" \quad d = 32.5"$$

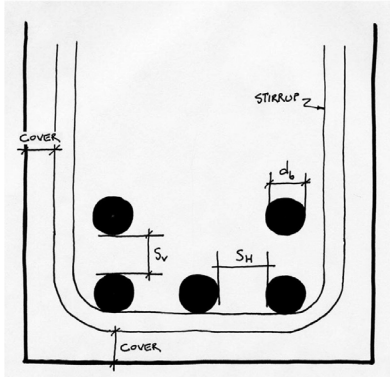
8. Find  $A_s = \rho bd$

$$A_s = \rho bd = (0.01)(15")(32.5")$$

$$A_s = 4.87 \text{ in}^2$$

# Rectangular Beam Design

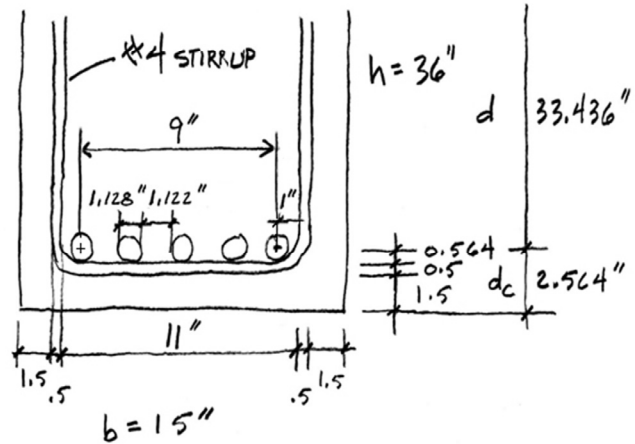
9. Choose bars for  $A_s$  and determine spacing and cover. Recheck  $h$  and weight.



If bars do not fit in one layer,  $d$  is measured to the centroid of the pattern.

CHOOSE BARS (SEE TABLE A.4)

TRY 5 x #9 BARS  $A_s = 5.0 \text{ in}^2$



$$\bar{x} = \frac{\sum A \times d_x}{\sum A}$$

Table A.4 Areas of Groups of Standard Bars (in.<sup>2</sup>)

Bar No.	Number of Bars													
	2	3	4	5	6	7	8	9	10	11	12	13	14	
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75	
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30	
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19	
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42	
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00	
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72	
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87	
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50	
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	

Jack C McCormac, 1978  
Design of Reinforced Concrete,

# Rectangular Beam Design

7. Choose bars for  $A_s$  and determine spacing and cover. Recheck  $h$  and weight.

$$d = 33.436 \text{ inches}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5(60)}{0.85(3)15} = 7.843 \text{ inches}$$

Make final check of  $M_n$  using final  $d$  and Check that  $M_u \leq \phi M_n$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) = 5(60) \left( 33.436 - \frac{7.843}{2} \right)$$

$$M_n = 8854 \text{ K-in} = 737.8 \text{ K-ft}$$

$$\phi M_n = 0.9(737.8) = 664 \text{ K-ft}$$

$$M_u = 653.3 < 664 \text{ } \checkmark \text{ OK}$$

8. Check that  $\epsilon_t \geq 0.005$  (if not, increase  $h$  and reduce  $A_s$ )

$$c = \frac{a}{\beta_1} = \frac{7.843}{0.85} = 9.227 \text{ inches}$$

$$\epsilon_t = \frac{d - c}{c} (0.003)$$

9. Design shear reinforcement (stirrups)

10. Check deflection, crack control, steel development length.

$$\epsilon_t = \frac{33.436 - 9.227}{9.227} (0.003)$$

$$\epsilon_t = 0.00787 > 0.005 \text{ } \checkmark \text{ OK}$$

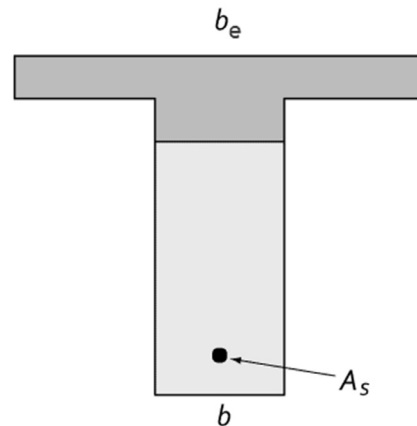
# Non-Rectangular Beam Analysis

## Data:

- Section dimensions –  $b$ ,  $b_e$ ,  $h$ , (span)
- Steel area -  $A_s$
- Material properties –  $f'_c$ ,  $f_y$

## Required:

- Required Moment –  $M_u$  (or load, or span)
1. Find  $T = A_s f_y$  and  $C = 0.85 f'_c A_c$
  2. Set  $T = C$  and solve for  $A_c$
  3. Draw and label diagrams for section and stress
    1. Determine  $b$  effective (for T-beams)
    2. Locate  $T$  and  $C$  (or  $C_1$  and  $C_2$ )
  4. Determine the location of  $a$ .  
Working from the top down,  
add up area to make  $A_c$
  5. Find moment arms ( $z$ ) for each block of area
  6. Find  $M_n = \sum C z$
  7. Find  $M_u = \phi M_n$
  8. Check  $A_{s \min} < A_s < A_{s \max}$
  9. Check that  $\epsilon_t \geq 0.005$

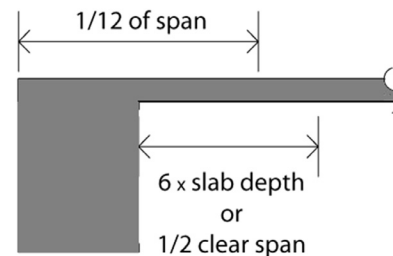


## Effective Flange Width, $b_e$

### Slab on one side:

$b_e$  least of either (total width) or (overhang + stem)

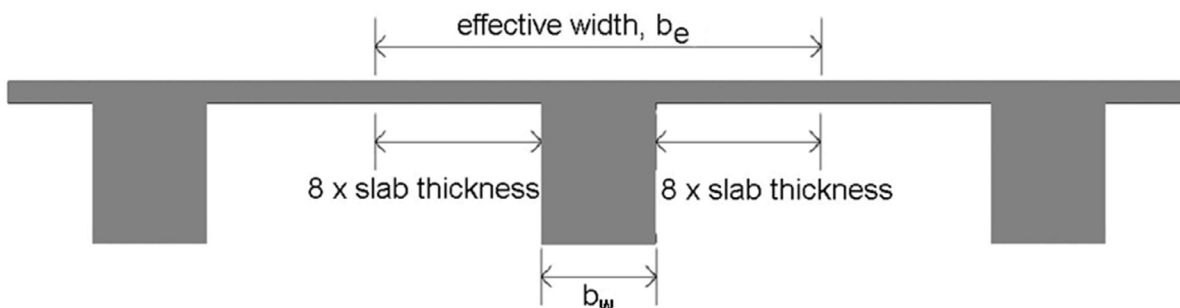
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: 1/2 the clear distance to next beam



### Slab on both sides:

$b_e$  least of either (total width) or (2 x overhang + stem)

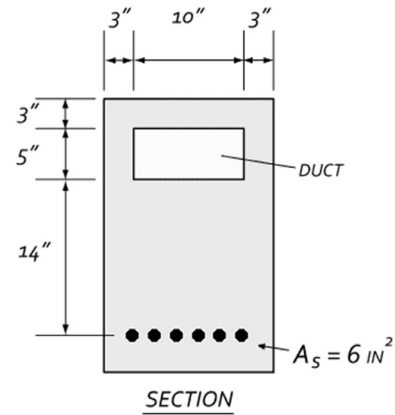
- Total width: 1/4 of the beam span
- Overhang: 8 x slab thickness
- Overhang: 1/2 the clear distance to next beam (i.e. the web on center spacing)





## Non-rectangular shape - example

Given:  $f'_c = 3000$  psi  
 $f_y = 60$  ksi  
 $A_s = 6 \times \#9 = 6 \text{ in}^2$   
 Req'd: Capacity,  $M_u$



- 1a. Find  $T$
- 1b. Find  $C$  in terms of  $A_c$
2. Set  $T = C$  and solve for  $A_c$

$$T = A_s f_y = 6 \text{ in}^2 (60,000 \text{ psi})$$

$$T = 360,000 \text{ lb} = 360 \text{ k}$$

$$C = 0.85 f'_c A_c = 0.85 (3000 \text{ psi}) A_c \text{ in}^2$$

$$C = (2550 A_c) \text{ lb} = (2.55 A_c) \text{ k}$$

$$T = C$$

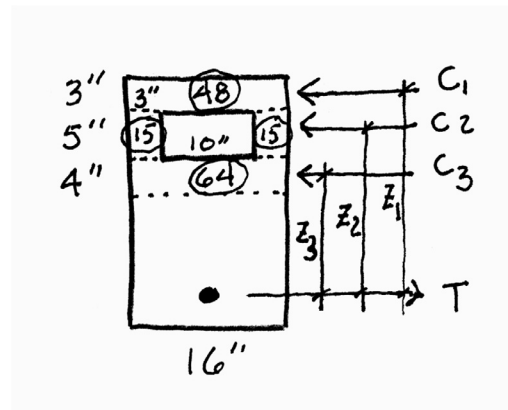
$$360 \text{ k} = 2.55 A_c \text{ k}$$

$$A_c = 142 \text{ in}^2$$

## Non-rectangular shape (cont.)

3. Draw section and determine areas to make  $A_c$
4. Find the location of  $a$ .  
 $a = 3'' + 5'' + 4''$

$$C = 0.85 f'_c A_c$$



$f'_c = 3$  ksi  
 $A_{c1} = 48 \text{ in}^2$   
 $A_{c2} = 30 \text{ in}^2$   
 $A_{c3} = 64 \text{ in}^2$

$$A_c = 142 \text{ in}^2 = A_{c1} + A_{c2} + A_{c3}$$

$$142 = 48 + 30 + A_{c3}$$

$$A_{c3} = 64 \text{ in}^2$$

$$C_1 = 48 (2.55) = 122.4 \text{ k}$$

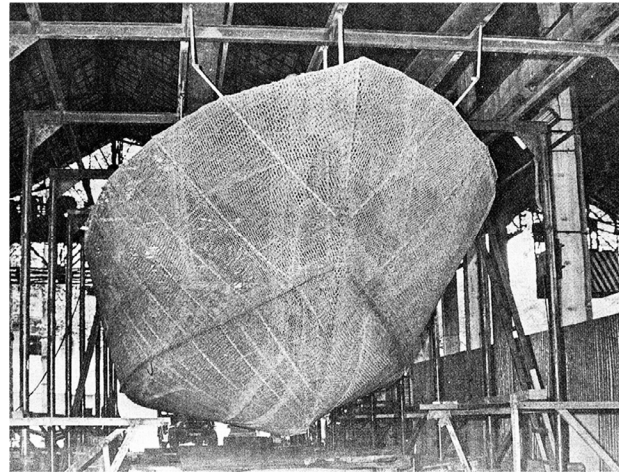
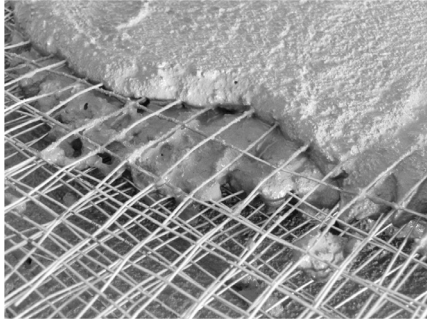
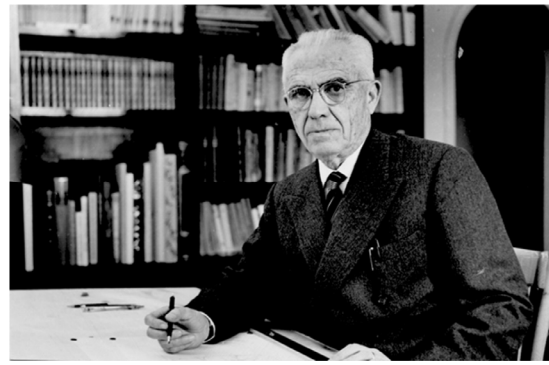
$$C_2 = 30 (2.55) = 76.5 \text{ k}$$

$$C_3 = 64 (2.55) = 163.2 \text{ k}$$



## Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking

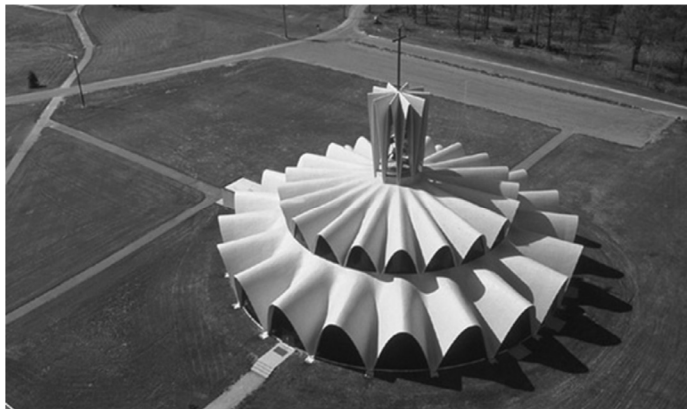


## Ferrocement

- Pioneered by Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking
- Low-tech applications



Palazzetto dello Sport, Rome, 1957. P.L. Nervi



Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

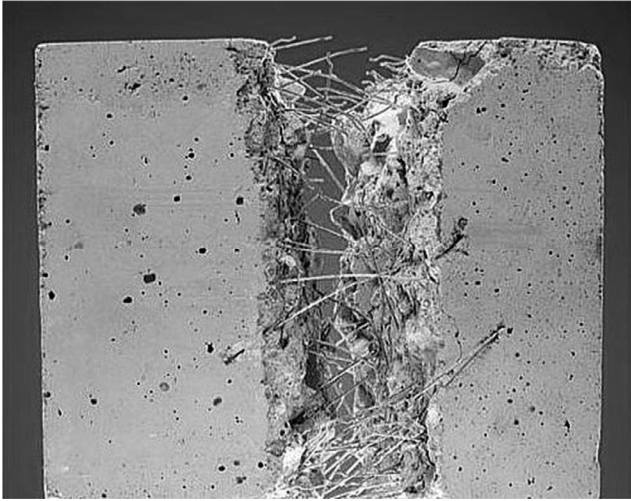


# Fiber Reinforced Concrete

Several different fiber types:

- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

Better crack control  
Secondary reinforcement



Single

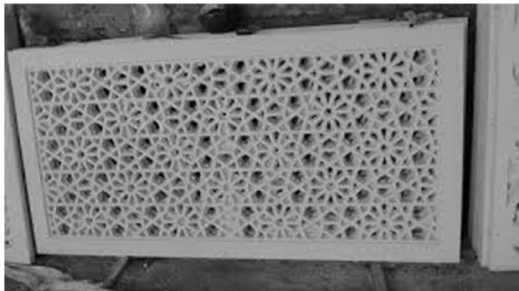


Wave



Bundle

# Glass Fiber Reinforced Concrete - GFRC



## Carbon Fiber



University of Michigan, TCAUP

## Bamboo

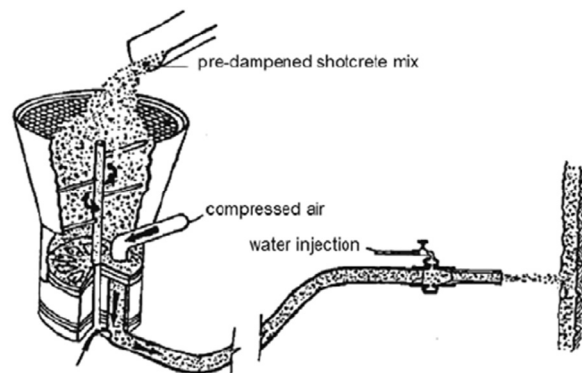


Structures II

Slide 49 of 51

## Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels



University of Michigan, TCAUP

Structures II

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Textile Reinforced Concrete (TRC)

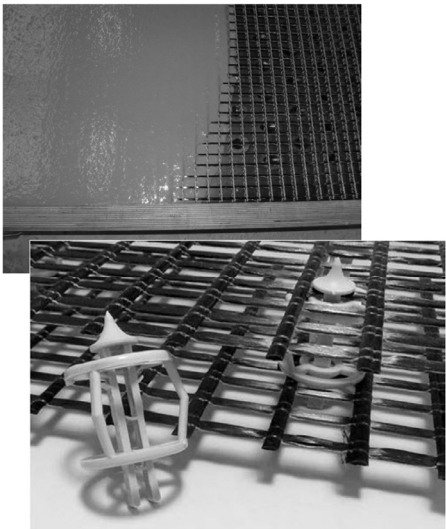


Figure 12: distTEX: special spacers for textile grids [photo: Frank Schladitz, TU Dresden]

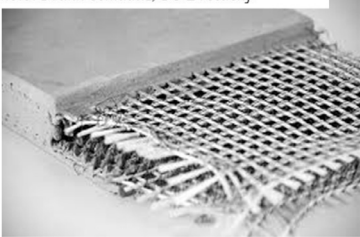


Figure 13: Manufacturing of the TRC hyper-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]

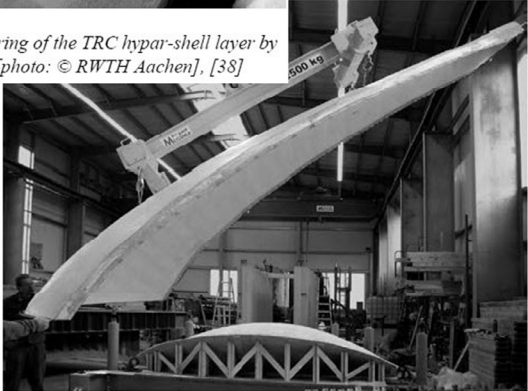


Figure 10: Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]