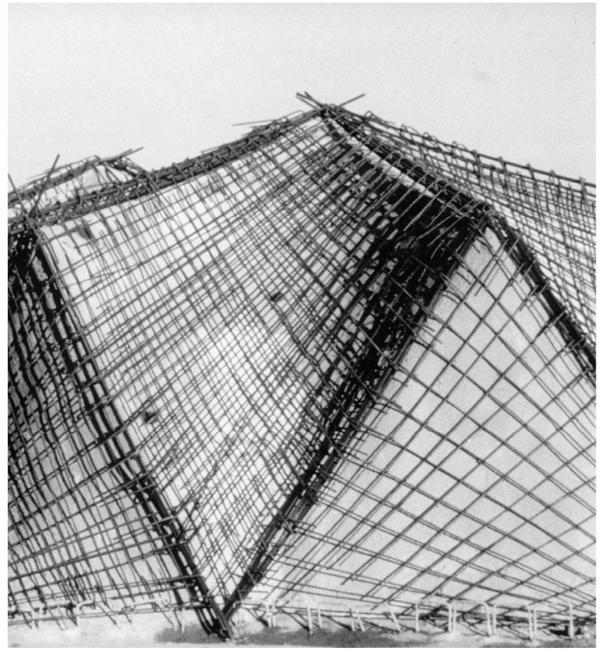
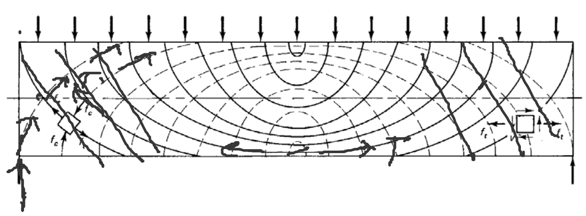
Architecture 324
Structures II

Reinforced Concrete Beams Ultimate Strength Design
(ACI 318 - 2019)

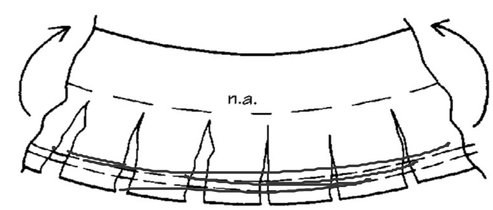


- Flexure in Concrete
- Ultimate Strength Design (LRFD)
- Failure Modes
- Flexure Equations
- Analysis of Rectangular Beams
- Design of Rectangular Beams
- Analysis of Non-rectangular Beams

Flexure



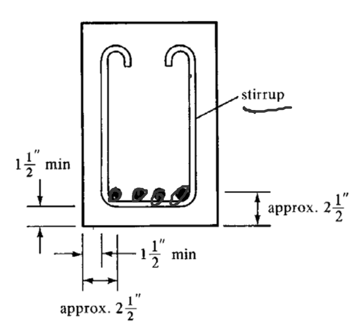
The stress trajectories in this simple beam, show principle tension as solid lines.

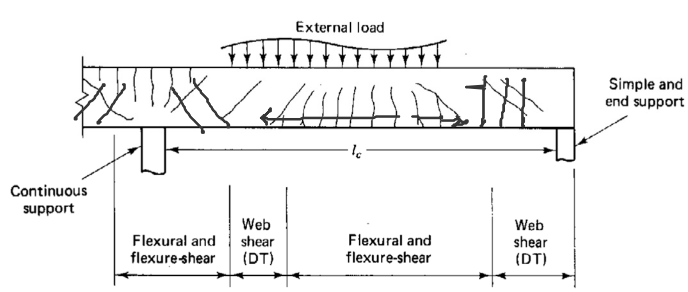


Reinforcement must be placed to resist these tensile forces

In beams continuous over supports, the stress reverses (negative moment).
In such areas, tensile steel is on top.

Shear reinforcement is provided by vertical or sloping stirrups.





Ultimate Strength – (LRFD)

Nominal Strength ≥ Design Strength
(strength of member ≥ required by loads)

LRFD uses 2 safety factors: γ and ϕ
ϕ nominal strength ≥ γ required strength

γ increases the required strength of the member and is placed on the loads

ϕ reduces the member strength capacity and is placed on the calculated force

Loads increased:
γ Factors: DL=1.2 LL=1.6
U is the required strength
U=1.2DL+1.6LL
(factors from ASCE 7)

Strength reduced:
ϕ Factors: e.g. flexure = 0.9 in tension-controlled beams

Table 21.2.1—Strength reduction factors ϕ

	Action or structural element	ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

Ultimate Strength – (ACI 318)

Reduced Nominal Strength ≥ Factored Load Effects

$$\Phi Sn \geq U$$

γ **Factored Loads** (see ACSE 7)

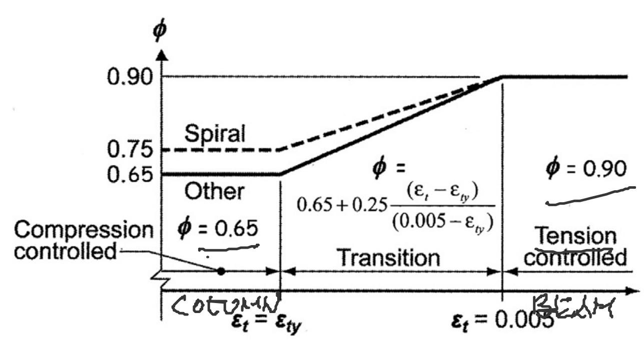
1) 1.4D
2) 1.2D + 1.6L + 0.5(Lr or S or R)
3) 1.2D + 1.6(Lr or S or R) + (1.0L or 0.5W)
4) 1.2D + 1.0W + 1.0L + 0.5(Lr or S or R)
5) 1.2D + 1.0E + 1.0L + 0.2S
6) 0.9D + 1.0W
7) 0.9D + 1.0E

D = service dead loads
L = service live load
Lr = service roof live load
S = snow loads
W = wind loads
R = rainwater loads
E = earthquake loads

Strength Reduction Factors, Φ

Mn	Flexural (ϵ > 0.005)	0.90
Vn	Shear	0.75
Pn	Compression (spiral)	0.75
Pn	Compression (other)	0.65
Bn	Bearing	0.65
Tn	Torsion	0.75
Nn	Tension	0.90
Combined stress		0.65 to 0.90

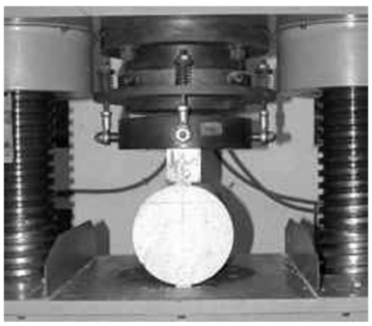
ACI 318 21.2.2

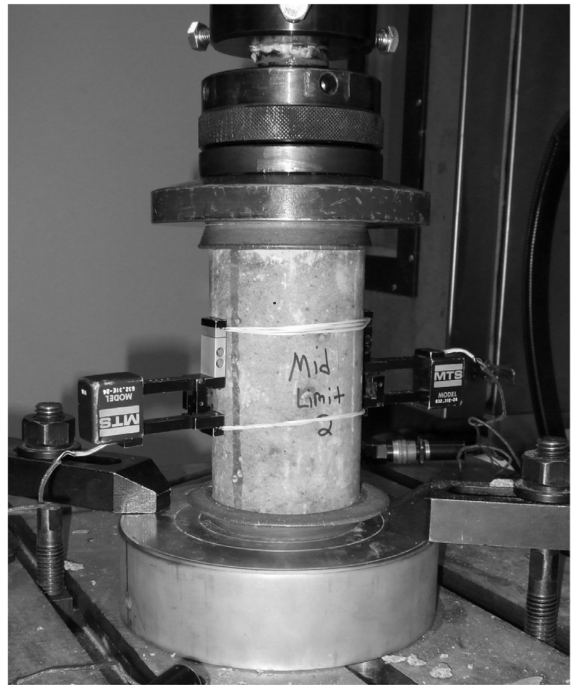


Strength Measurement

- Compressive strength $f_c^{'}$
  - 12” x 6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength
  - Usable strain ϵ_{cu} = 0.003 (ACI 318)

- Tensile strength ASTM C496 $f_t^{'}$
  - 12” x 6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength
  - Split cylinder test
  - ca. 10% of f’c
  - Neglected in flexure analysis





Failure Modes Based on As

$$\rho = \frac{A_s}{bd}$$

- **No Reinforcing**
  - Less than As min
  - Brittle failure

- **Reinforcing < balance (use this)**
  - Steel yields before concrete fails
  - ductile failure
  - (~As min) 0.06 ≥ ϵ_t ≥ 0.004 (As max)
  - ϵ_t ≥ 0.005 for tension controlled

As,min

3 BAR

As,min:
greater of a and b

(a) $\dfrac{3\sqrt{f_c'}}{f_y} b_w d$

(b) $\dfrac{200}{f_y} b_w d$

$As_{max}\ when\ \varepsilon_t = 0.004$

- **Reinforcing = balance**
  - Concrete fails just as steel yields
  - ϵ_t at balance = 0.00207 for Gr 60

C

4 BAR

T

$$\rho_{bal} = \left(\frac{0.85\beta_1 f_c^{'}}{f_y}\right)\left(\frac{87000}{87000 + f_y}\right)$$

- **Reinforcing > balance**
  - Concrete fails before steel yields
  - Low ductility
  - Sudden failure

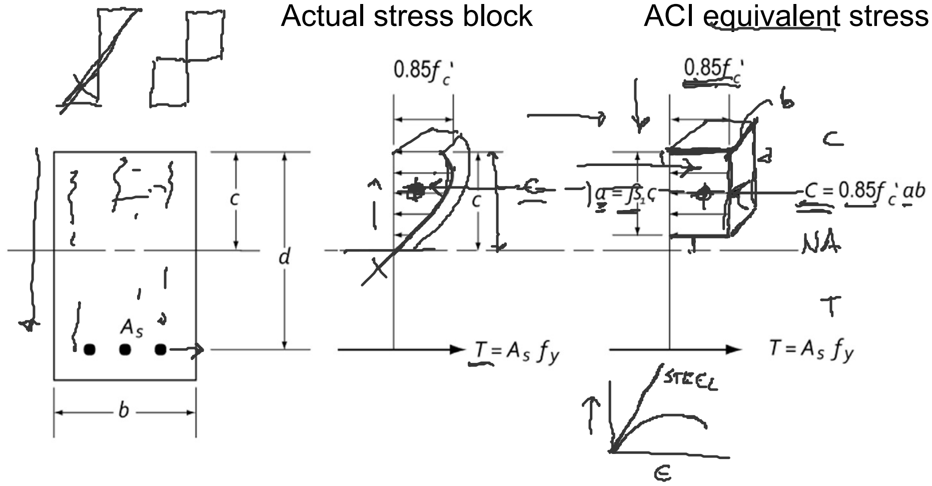
5 BAR

$As > As_{max}$ SuddenDeath!!

ACI Stress Block

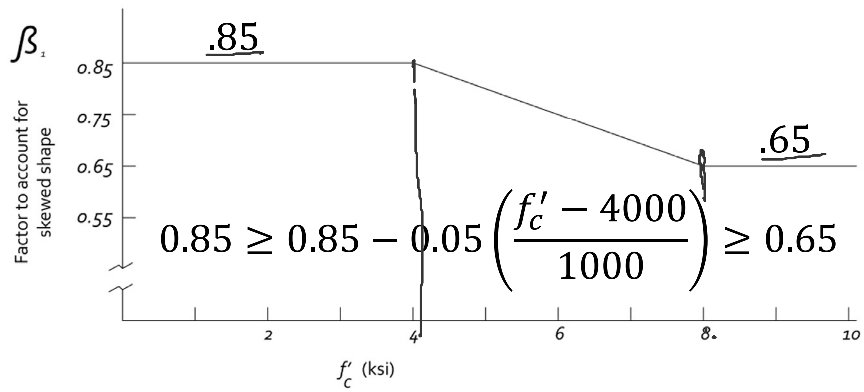
β_1 is a factor to account for the non-linear shape of the compression stress block.

$$a = \beta_1 c$$



psi

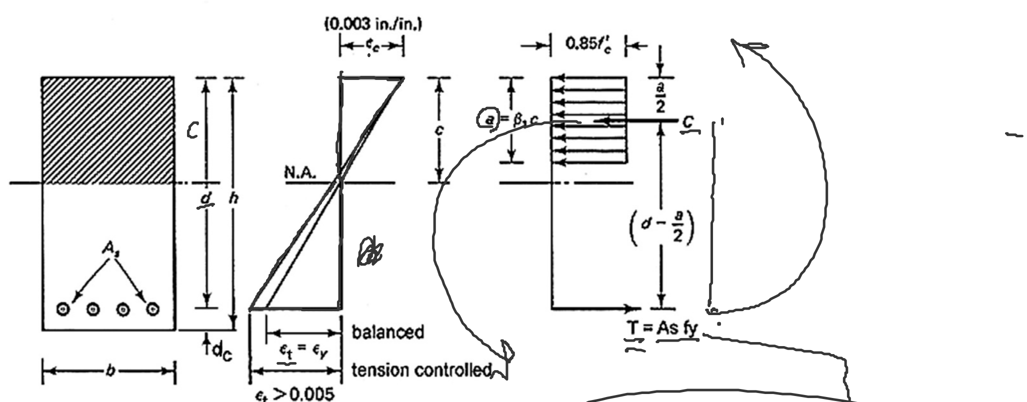
f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



$$0.85 \geq 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) \geq 0.65$$

Flexure Equations

strain ACI equivalent stress block



$$C = T$$

$$0.85 f_c' ab = A_s f_y$$

solving for a,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{\rho f_y d}{0.85 f_c'}$$

F d

$$M_n = T\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

LOADS → $M_u = \phi M_n$ ← STRENGTH

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right)$$

$$\boxed{\epsilon_t = \frac{d-c}{c}(0.003)}$$

$$\rho = \frac{A_s}{bd}$$

$\frac{A_s}{bd}$

$\frac{\epsilon_c}{c} = \frac{\epsilon_t}{d-c}$

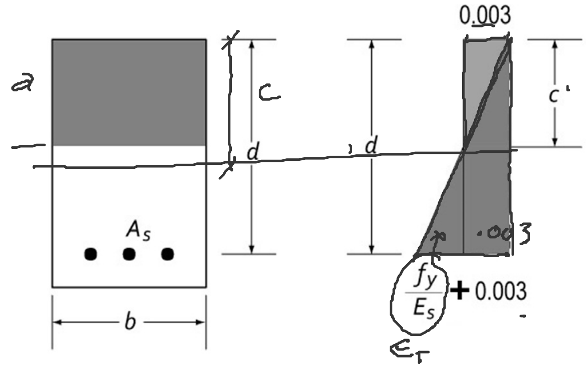
$$M_u = \phi A_s f_y d\left(1 - 0.59\frac{\rho f_y}{f_c'}\right)$$

Balance Condition

From similar triangles at balance condition:

$$\frac{c}{d} = \frac{0.003}{0.003 + (f_y/E_s)} = \frac{0.003}{0.003 + (f_y/29 \times 10^6)}$$

$$c = \frac{87{,}000}{87{,}000 + f_y} d$$



Strain diagram for balanced condition.

Use equation for a. Substitute into c = a / β_1

$$a = \frac{\rho f_y d}{0.85 f_c'} \qquad \rho = \frac{A_s}{bd}$$

$$c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f_c'}$$

Equate expressions for c:

$$\frac{\rho f_y d}{0.85 \beta_1 f_c'} = \frac{87{,}000}{87{,}000 + f_y} d$$

$$\rho_b = \left(\frac{0.85 \beta_1 f_c'}{f_y}\right)\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

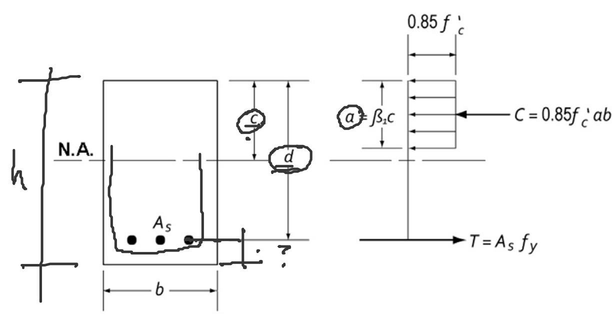
Table A.8 Balanced Ratio of Reinforcement ρ_b for Rectangular Sections with Tension Reinforcement Only

f_y \ f_c'		2,500 psi (17.2 MPa) $\beta_1 = 0.85$	3,000 psi (20.7 MPa) $\beta_1 = 0.85$	4,000 psi (27.6 MPa) $\beta_1 = 0.85$	5,000 psi (34.5 MPa) $\beta_1 = 0.80$	6,000 psi (41.4 MPa) $\beta_1 = 0.75$
Grade 40	ρ_b	0.0309	0.0371	0.0495	0.0582	0.0655
40,000 psi	$0.75\rho_b$	0.0232	0.0278	0.0371	0.0437	0.0492
(275.8 MPa)	$0.50\rho_b$	0.0155	0.0186	0.0247	0.0291	0.0328
Grade 50	ρ_b	0.0229	0.0275	0.0367	0.0432	0.0486
50,000 psi	$0.75\rho_b$	0.0172	0.0206	0.0275	0.0324	0.0365
(344.8 MPa)	$0.50\rho_b$	0.0115	0.0138	0.0184	0.0216	0.0243
Grade 60	ρ_b	0.0178	0.0214	0.0285	0.0335	0.0377
60,000 psi	$0.75\rho_b$	0.0134	0.0161	0.0214	0.0252	0.0283
(413.7 MPa)	$0.50\rho_b$	0.0089	0.0107	0.0143	0.0168	0.0189
Grade 75	ρ_b	0.0129	0.0155	0.0207	0.0243	0.0274
75,000 psi	$0.75\rho_b$	0.0097	0.0116	0.0155	0.0182	0.0205
(517.1 MPa)	$0.50\rho_b$	0.0065	0.0078	0.0104	0.0122	0.0137

Rectangular Beam Analysis

Data:
- Section dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity



As,min
greater of a and b.

(a) $\dfrac{3\sqrt{f_c'}}{f_y} b_w d$

(b) $\dfrac{200}{f_y} b_w d$

1. Calculate d
2. Check As min
3. Calculate a
4. Determine c
5. Check that $\epsilon_t \geq$ 0.005 (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, ϕMn ≥ Mu

   (if $\epsilon_t \geq$ 0.005 then phi = 0.9)
8. Determine max. loading (or span)

$$c = \frac{a}{\beta_1} \qquad \varepsilon_t = \frac{d-c}{c} 0.003 \geq 0.005$$

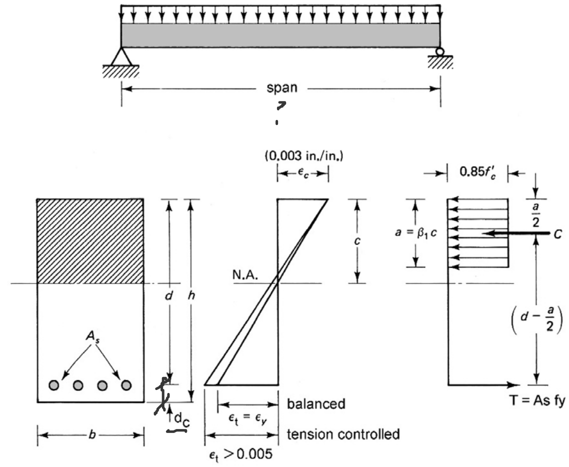
$$a = \frac{A_s f_y}{0.85 f_c' b} \qquad M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\phi\, M_n \geq M_u$$

$$M_u = \frac{(1.2 w_{DL} + 1.6 w_{LL}) l^2}{8}$$

$$1.6 w_{LL} = \frac{M_u 8}{l^2} - 1.2 w_{DL}$$

Rectangular Beam Analysis



Data: b h

- dimensions – 12”x23”
- Steel – 4 x # 6 fy = 60ksi
- Concrete f’c = 6000 psi
- Stirrup # 3 cover 1.5” Agg ¾”

Required:

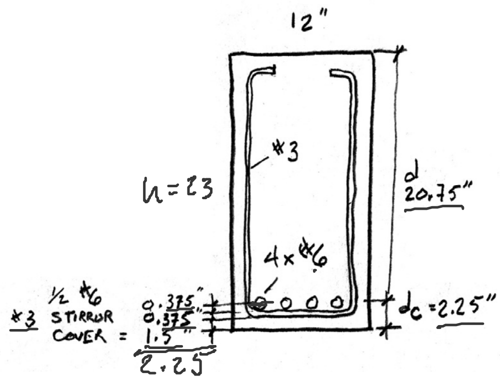
- Required Moment – ϕMn = Mu (capacity)

1. Calculate d

d_c = COVER + #3 + ½(#6)

$= 1.5 + 0.375 + \frac{0.75}{2} = 2.25''$

$d = h - d_c = 23'' - 2.25 = 20.75''$ ✓



Rectangular Beam Analysis cont.

Data:

dimensions – 12”x23”

Steel – 4 x # 6 – As = 1.76 in^2

f’c = 6000 psi fy = 60ksi

$4 \times 0.44 =$

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

	Customary Units			SI Units		
Bar No.	Diameter (in.)	Cross-sectional Area (in.^{2})	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm^2)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	$\boxed{0.44}$	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

2. Check As,min

As, min

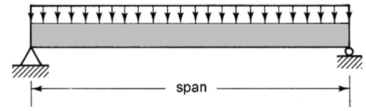
① $\frac{3\sqrt{f'_c}}{f_y} b d = \frac{3\sqrt{6000}}{60\,000}(12 \times 20.75) = 0.964 \text{ in}^2$ ← CONTROLS < 1.76 ✓

② $\frac{200\, b\, d}{f_y} = \frac{200(12)(20.75)}{60\,000} = 0.83 \text{ in}^2$

$\therefore A_{s\,min} = 0.964 \text{ in}^2$

$A_s = A_b\,(\text{N.B.}) = 0.44(4) = 1.76 \text{ in}^2$ ✓ $> 0.964 \text{ in}^2$

Rectangular Beam Analysis cont.



Data:

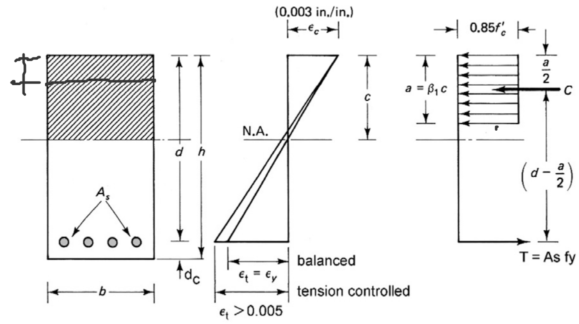
dimensions – 12"x23"

Steel – 4 x # 6 – As = 1.76 in^2

f'c = 6000 psi fy = 60ksi

6 ksi

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



3. Find a

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1.76)(60)}{.85(6)(12)''} = 1.725''$$

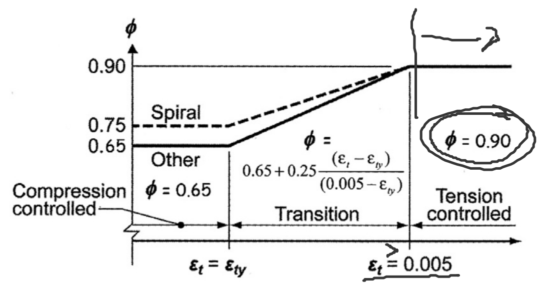
in² ksi, ksi

4. Find c

$$\beta_1 = 0.85 - 0.05\,\frac{f'_c - 4000}{1000} = 0.85 - 0.1 = 0.75$$

$$c = a/\beta_1 = \frac{1.725}{0.75} = 2.300''$$

Rectangular Beam Analysis cont.



5. Check that As is < As max

   $\epsilon_t \geq 0.004$

6. Check that $\epsilon_t \geq 0.005$

   (for tension controlled section)

   $\phi = 0.9$

$$\epsilon_t = \frac{d-c}{c}\,0.003 = \frac{20.75-2.3}{2.3}\,0.003$$

$$\epsilon_t = \boxed{0.02406} > 0.004 \therefore \text{ok}$$

$$= 0.02406 > 0.005 \therefore \text{tension controlled}$$

7. Find nominal moment, Mn

$$T = A_s f_y = 1.76^{in^2}(60^{ksi}) = 105.6\text{ K}$$

$$M_n = T\left(d - \frac{a}{2}\right) = 105.6\left(20.75 - \frac{1.725''}{2}\right)$$

$$M_n = 2100\text{ K-''}$$

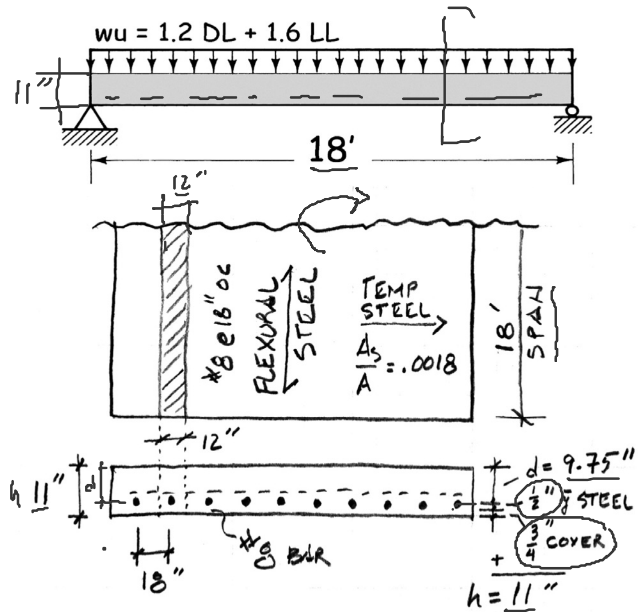
8. Calculate required moment

   ϕMn ≥ Mu

$$\phi M_n = 0.9(2100) = 1890\text{ K-''}$$

$$\frac{w l^2}{8} \longrightarrow M_u = \phi M_n = 1890/12 = 157.5\text{ K-'}$$

Slab Analysis



Data:

- Span = 18 ft
- h = 11" dia. = $\frac{8}{8}$ = 1"
  - take b = 12"
- Steel #8 @ 18" o.c.
- f'c = 3000 psi
- fy = 60 ksi

Required:

- Design moment capacity – Mu LOAD
- Maximum LL in PSF

$d = 11" - \frac{1}{2} - \frac{3}{4} = 9.75"$

1. Find d
2. Find As

$A_s = \left(\frac{12"}{18"}\right)(0.79 in^2)$ #8

OC, = 0.5267 in^2/FT

Check As,min

Ag = 11" x 12" = 132 in^2

[0.0018(60)/60] 132 = 0.237 in^2

0.0014 (132) = 0.1848 in^2

0.527 > 0.237 ok

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi		$A_{s,min}$
Deformed bars	< 60,000		$0.0020A_g$
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60{,}000}{f_y \; 60} A_g$
			$0.0014A_g$

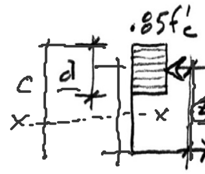
ACI 318-14

Slab Analysis

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

3. Find a

$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.5267(60)}{.85(3)(12")} = 1.033"$ (ksi)



$C = .85 f'_c a b$

$z = d - a/2$ $Tz = M_n$

$T = A_s f_y$

4. Find c = β_1 a

$c = \frac{a}{\beta_1} = \frac{1.033"}{0.85} = 1.215"$

5. Check failure mode

$\epsilon_t \geq$ 0.005 for tension controlled

$\epsilon_t = \frac{0.003\,d}{c} - 0.003$

$\epsilon_t = \frac{0.003(9.75")}{1.215"} - 0.003 = 0.021$ "/"

$\epsilon_t = 0.021 > 0.005$ ∴ TENSION CONTROLLED

6. Find force T

7. Find moment arm z

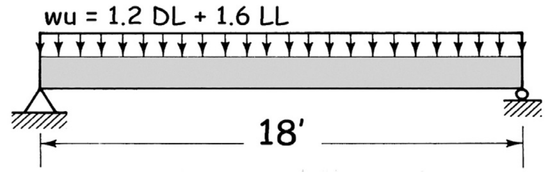
8. Find nominal strength moment, Mn

$T = A_s f_y = 0.5267(60) = 31.6^K$

$z = d - \frac{a}{2} = 9.75" - \frac{1.033"}{2} = 9.23"$

$M_n = Tz = 31.6(9.23) = 291.8^{K-"}$

Slab Analysis



9. Find required moment, Mu

$$M_U = \phi M_n = 0.9(\overset{k''}{291.8})\frac{1000}{12} = 21885 \text{ l-\#}$$

10. Find slab DL

$$w_{DL} = \gamma_c \frac{h}{12}\times 1' = \overset{PCF}{150}\ \frac{11''}{12} = 137.5 \text{ PSF}$$

11. Determine max. loading

$$M_U = 21885 \text{ l-\#} = \frac{(1.2 w_{DL} + 1.6 w_{LL})\, l^2}{8}$$

$$\frac{21885(8)}{(18')^2} = 1.2(137.5) + 1.6(w_{LL})$$

$$540.37 = 165 + 1.6(w_{LL})$$

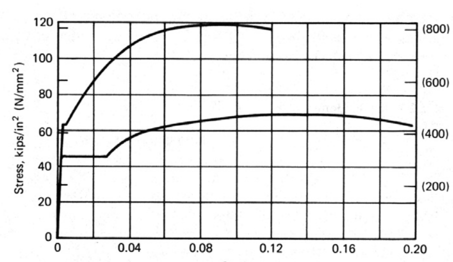
$$w_{LL} = 234.6 \text{ PSF}$$

Details of Reinforcement

Size
- Nominal 1/8" increments

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257

Grade
- 40 40 ksi
- 60 60 ksi
- 75 75 ksi



Details of Reinforcement

ACI 318 Chapter 25.2
Placement of Reinforcement

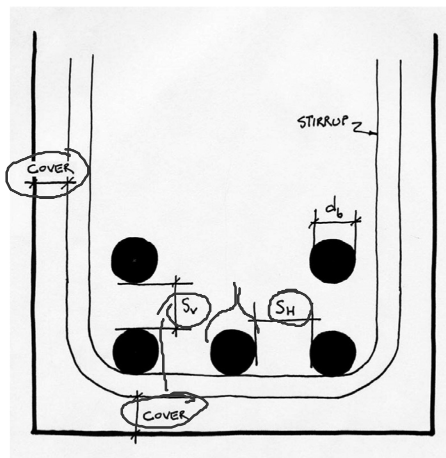
- cover
  (ACI 20.6.1)

- horizontal spacing in beams
  (ACI 25.2.1)
  1 inch
  d_b
  4/3 max aggregate $\frac{3}{4}\,\frac{4}{3} = 1''$

- vertical spacing in beams
  (ACI 25.2.2)
  1 inch

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

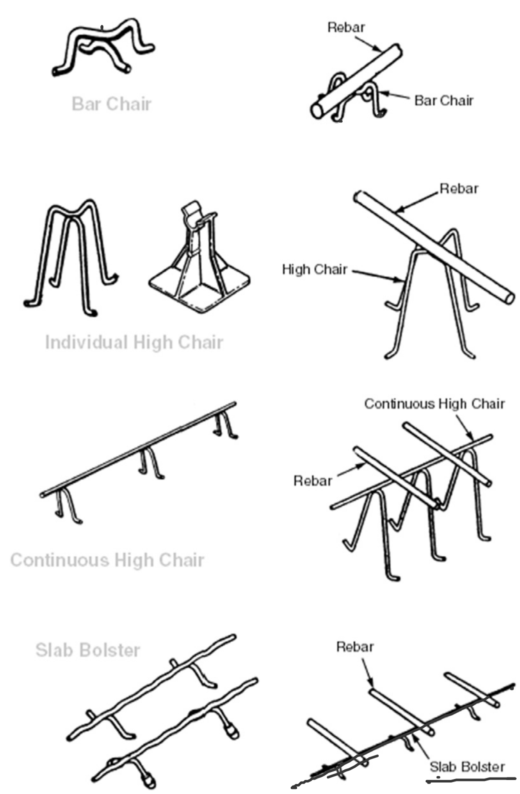
Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2



Details of Reinforcement

ACI 318 Chapter 25
Placement of Reinforcement

- Chairs
- Bolsters



Details of Reinforcement

ACI 318 Chapter 25

Minimum bend diameter
- factor x d_b

Hooks for bars in tension
- ACI Table 25.3.1
- Inside diameter

Bends for stirrups
- ACI Table 25.3.2

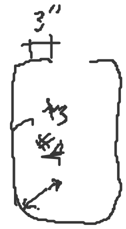
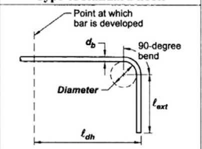
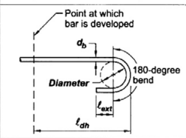
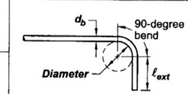
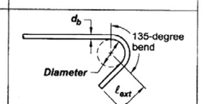
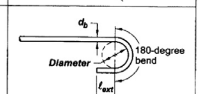


Table 25.3.1—Standard hook geometry for development of deformed bars in tension BEAMS

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 8	$6d_b$	$12d_b$	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		
180-degree hook	No. 3 through No. 8	$6d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Details of Reinforcement

ACI 318 Chapter 25

Development length of bars
- 12" minimum
- based on table 25.4.2.2

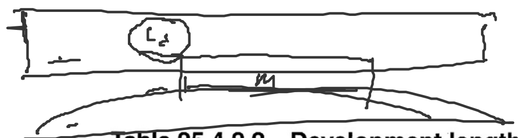


Table 25.4.2.2—Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\psi_t\psi_e}{25\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{20\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\frac{3f_y\psi_t\psi_e}{50\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{3f_y\psi_t\psi_e}{40\lambda\sqrt{f'_c}}\right)d_b$

Table 25.4.2.4—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 7 and larger bars	1.0
	No. 6 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

Other Useful Tables:

Table A.1 Values of Modulus of Elasticity for Normal-Weight Concrete

Customary Units		SI Units	
f'_c (psi)	E_c (psi)	f'_c (MPa)	E_c (MPa)
3,000	3,140,000	20.7	21 650
3,500	3,390,000	24.1	23 373
4,000	3,620,000	27.6	24 959
4,500	3,850,000	31.0	26 545
5,000	4,050,000	34.5	27 924

Jack C McCormac, 1978
Design of Reinforced Concrete,

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

	Customary Units			SI Units		
Bar No.	Diameter (in.)	Cross-sectional Area (in.^{2})	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm^2)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

Table A.4 Areas of Groups of StandardBars (in.^{2})

SIZE	Number of Bars												
Bar No.	2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	~~4.71~~	$\boxed{5.50}$	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

Rectangular Beam Design

Two approaches:

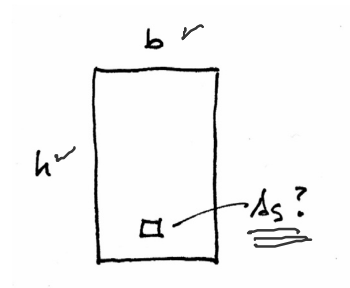
Method 1:

Data:

- Load and Span
- Material properties – f'c, fy
- All section dimensions: h and b

Required:

- Steel area - As



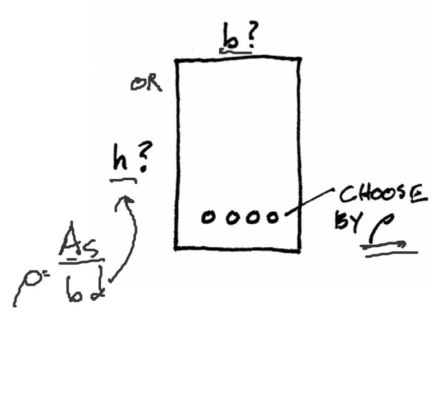
Method 2:

Data:

- Load and Span
- Some section dimensions – h or b
- Material properties – f'c, fy
- ρ

Required:

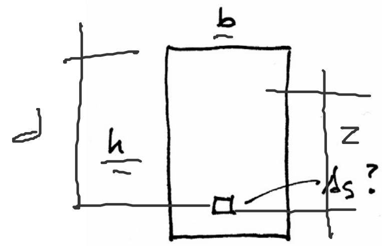
- Steel area - As
- Beam dimensions – b and h



Rectangular Beam Design – **method 1**

Data:
- Load and Span
- Material properties – f'c, fy
- All section dimensions – b and h

Required:
- Steel area - As



1. Calculate the factored load and find factored required moment, Mu
2. Find d = h – cover – stirrup – $d_b/2$ (one layer)
3. Estimate moment arm z = jd. For beams j ≈ 0.9 for slabs j ≈ 0.95
4. Estimate As based on estimate of jd.
5. Use As to find a
6. Use a to find As (repeat…until 2% accuracy)
7. Choose bars for As and check As max & min
8. Check that $\epsilon_t \geq 0.005$
9. Check Mu ≤ ϕ Mn (final condition)
10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, steel development length.

$$M_u = \frac{(\gamma\, w_{DL} + \gamma\, w_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi\, f_y\left(d - \frac{a}{2}\right)}$$

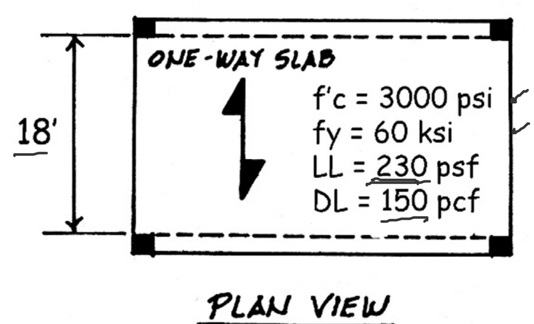
$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

Rectangular Slab Design

Data:
- Load and Span
- Material properties – f'c, fy
- All section dimensions:
- h (based on deflection limit)
- b = typical 12" width

Required:
- Steel area - As



First estimate the slab thickness, h.

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

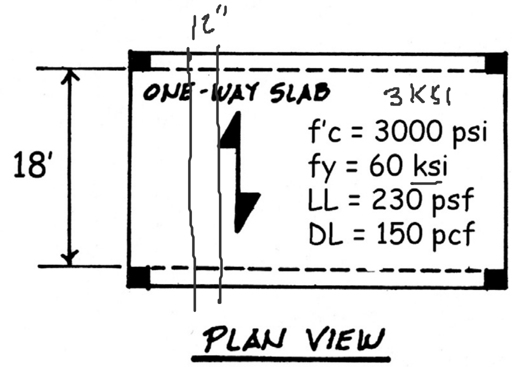
Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	ℓ/20
One end continuous	ℓ/24
Both ends continuous	ℓ/28
Cantilever	ℓ/10

THICKNESS, h, BASED ON DEFLECTION

$h = \ell/20 = \frac{18 \times 12}{20} = 10.8"$ USE 11"

Rectangular Slab Design



1. Calculate the dead load and find required Mu

FACTOR LOADS

$DL = \frac{11''}{12}(150 \text{ PCF}) = 137.5 \text{ PSF}$

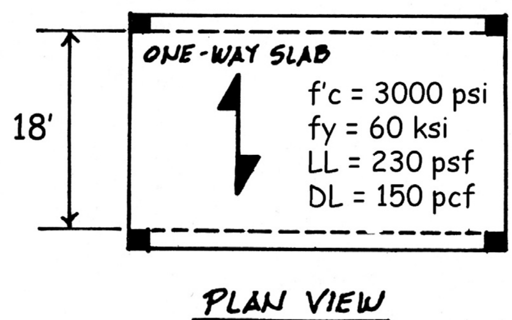
LL (GIVEN) = 230 PSF

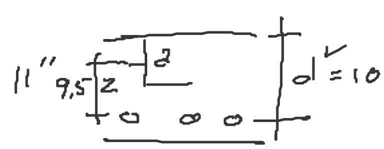
$w_u = 1.2(137.5) + 1.6(230) = 533$

$M_u = \frac{w_u \ell^2}{8} = \frac{533 \text{ PLF} (18')^2}{8} = 21\,587$ '-#

$\phi M_n = 259$ "-K

Rectangular Slab Design





2. Estimate moment arm
   $z \approx 0.95\ d$

FOR $j \approx 0.95$, $d = h - \text{COVER} - \frac{1}{2} \text{BAR}$

$d = 11'' - \frac{3}{4}'' - \frac{1}{2}(\frac{1}{2}'')$ 8#

$d = 11'' - 1'' = 10''$

$z \approx jd \approx 0.95(10'') = 9.5''$

Rectangular Slab Design

3. Estimate As based on estimate of jd.
4. Use As to find a
5. Use a to find As (repeat…)

$(d - \frac{a}{2})$

TRIAL 1

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{259''^{-k}}{0.9(60\text{ KSI})(9.5)''} = 0.505 \text{ in}^2$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.505(60)}{.85(3)(12)} = 0.99''$$

TRIAL 2

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9(60)(10 - \frac{.99}{2})}$$

$A_s = 0.5046 \text{ in}^2$ WITHIN 2%

Rectangular Slab Design

6. Choose bars for As required:
   either
   choose bars and calculate spacing
   or
   choose spacing and find bar size

   If the bar size changes, re-calculate to find new d. Then re-calculate As…

7. Check As,min
   (for slabs As,min from ACI Table 7.6.1.1)

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi	$A_{s,min}$	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			$0.0014A_g$

CHOOSE BARS

USING #4 — AREA of #4

$\frac{0.505}{12''} : \frac{0.2}{s''}$ $s = 4.75''$ $\frac{.2}{4}$ $\frac{.6}{12}$

∴ USE 4'' o.c. (always round down)

$A_s = 0.60 \text{ in}^2/\text{FT} > 0.505$ ✓

ALTERNATE FOR MAX. S = 18''

$\frac{0.505}{12''} : \frac{A_b}{18''}$ $A_b = 0.75 \text{ in}^2$

#8 = 0.79

∴ USE #8 @ 18'' o.c.

$A_s = 0.526 \text{ in}^2/\text{FT} > 0.505$ ✓

Check As,min ✓

$$A_{s\,min} = 0.0018\,bh = 0.0018(12)(11'')$$
$$= 0.24 \text{ in}^2 < 0.526 \text{ in}^2 \checkmark \text{ OK}$$

.6

Rectangular Slab Design

8. Check that $\epsilon_t \geq 0.005$

#4 @ 4" o.c.

RE-CALC a FOR $A_s = 0.6 \text{ in}^2/\text{FT}$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d - c}{c} 0.003 =$$

$$= \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$0.01759 > 0.005$

∴ TENSION CONTROLLED ✓✓

Rectangular Slab Design

9. Check Mu ≤ ϕ Mn
   (final condition)
   As = As,used

   Mn = T z

10. Check deflection, crack control, steel development length.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 0.6(60)\left(9.5'' - \frac{\boxed{1.176}}{2}\right)$$

$$M_n = 36(8.911'') = 320.8^{\text{K-''}}$$

$$\phi M_n = 0.9(320.8) = 288.7^{\text{K-''}}$$

LOAD STRENGTH

$$M_u = 259^{\text{K-''}} < 288.7^{\text{K''}}$$

$M_u < \phi M_n$ ✓ OK ✓

Rectangular Beam Design – **method 2**

Data:

- Load and Span
- Some section dimensions – b or h
- Material properties – f'c, fy

Required:

- Steel area - As
- Beam dimensions – b and h

1. Estimate the dead load (h ≈ L/12) and find Mu
2. Choose ρ (equation assumes ϵ_t = 0.0075)
3. Calculate bd^2
4. Choose b and solve for d (or d and solve b)
   b is based on form size – matches column size
   h is between L/12 to L/18 and b:h ≈ 1:2 to 2:3
5. Estimate h and correct weight and Mu
6. Find As = ρ b d
7. Choose bars for As and determine spacing and cover. Recheck h and weight.
8. Check that $\epsilon_t \geq$ 0.005 (if not, increase h and reduce As)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length.

$$M_u = \frac{(\gamma\ w_{DL} + \gamma\ w_{LL})l^2}{8}$$

$$\rho = \frac{\beta_1 f'_c}{4 f_y}$$

$$b\,d^2 = \frac{M_u}{\phi\,\rho\,f_y\left(1 - 0.59\,\rho\left(fy / f'_c\right)\right)}$$

$$A_s = \rho\,b\,d$$

$$a = \frac{\rho\,f_y\,d}{0.85 f'_c}$$

Rectangular Beam Design

Data:

- Load and Span
- Material properties – f'c, fy

Required:

- Steel area - As
- Beam dimensions – b and d

Estimate b and h to get beam selfweight.

1. Estimate the dead load (h ≈ L/12) and find Mu.

Table 9.3.1.1—Minimum depth of nonprestressed beams

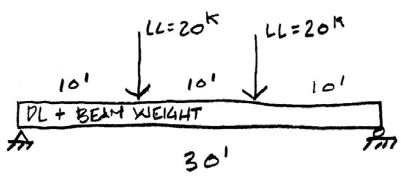
Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and f_y = 60,000 psi. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

2. Choose ρ (equation assumes ϵ_t = 0.0075)

f'c = 3000 psi
fy = 60 ksi

DL = 2 klf + beam
LL = 2 x 20 k



ASSUME $h \approx \frac{L}{12} = \frac{360''}{12} = 30''$

ASSUME $b:h \approx 1:2 \therefore b \approx 15''$

BEAM DL = $150 \frac{15 \times 30}{144} = 469$ PLF

ESTIMATE M_U

$$M_U = Pa + \frac{w l^2}{8}$$

$$= 1.6(20^k)(10') + \frac{1.2(2.469 \text{ KLF})(30')^2}{8}$$

$$= 320 + 333.3 = 653.3 \text{ K-'}$$

CHOOSE ρ

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

Rectangular Beam Design cont.

3. Calculate bd^2

$$bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - 0.59 \rho \left(f_y / f'_c\right)\right)}$$

$$bd^2 = \frac{653.3\,(12)}{0.01(0.9)60\left[1 - 0.59(0.01)\left(\frac{60}{3}\right)\right]}$$

$$bd^2 = \frac{7840}{0.573\,(0.882)} = 15492 \text{ in}^3$$

4. Choose b and solve for d
   (or d and solve for b)
   b is based on form size – matches column size
   h is between L/12 to L/18 and b:h ≈ 1:2 to 2:3

TRY

b	d	$h \approx 1.12d$	A
14"	33.27"	38"	532
15"	32.14"	36"	540
16"	31.11"	35"	560

5. Estimate h and correct weight and Mu

CHOOSE 15 × 36

Rectangular Beam Design cont.

6. Choose b and solve for d
   (or d and solve for b)
   b is based on form size – matches column size
   h is between L/12 to L/18 and b:h ≈ 1:2 t

USE 15 × 36

REVISE DL $= 150 \frac{540}{144} = 563$ PLF

7. Estimate h and correct weight and Mu

CHECK M_u

$$M_u = 320 + \frac{1.2(2.563)\,30^2}{8} = 666^{\text{K-'}}$$

REVISE bd

$$bd^2 = \frac{666\,(12)}{0.505} = 15814 \text{ in}^3$$

FOR $b = 15''$ $d = 32.5''$

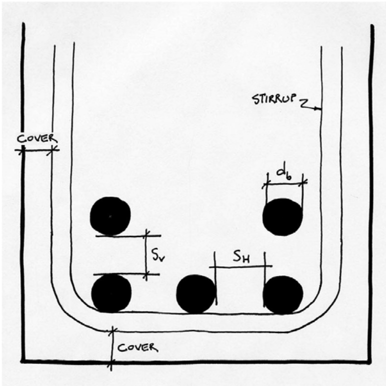
8. Find $As = \rho\, bd$

$$A_s = \rho\, bd = (0.01)(15'')(32.5'')$$

$$A_s = 4.87 \text{ in}^2$$

Rectangular Beam Design

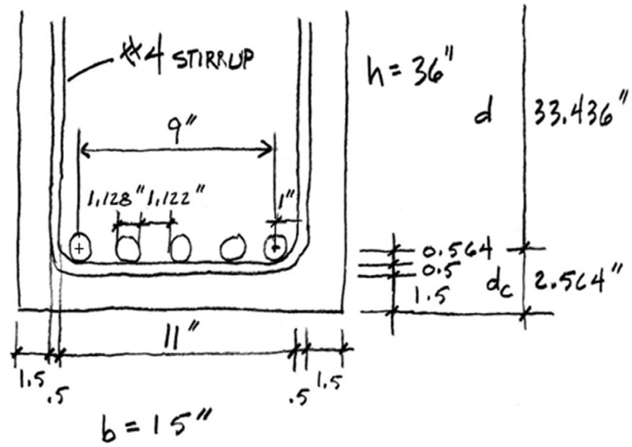
9. Choose bars for As and determine spacing and cover. Recheck h and weight.



If bars do not fit in one layer, d is measured to the centroid of the pattern.

CHOOSE BARS (SEE TABLE A.4)

TRY 5 x #9 BARS $A_s = 5.0 \text{ in}^2$



$$\bar{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}}$$

Table A.4 Areas of Groups of StandardBars (in.^{2})

Bar No.	Number of Bars 2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

Jack C McCormac, 1978
Design of Reinforced Concrete,

Rectangular Beam Design

7. Choose bars for As and determine spacing and cover. Recheck h and weight.

$d = 33.436''$

$a = \dfrac{A_s f_y}{.85 f'_c\, b} = \dfrac{5(60)}{.85(3)15} = 7.843''$

Make final check of Mn using final d and Check that Mu ≤ ø Mn

$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5(60)\left(33.436 - \frac{7.843}{2}\right)$

$M_n = 8854 \text{ K-''} = 737.8 \text{ K-'}$

$\phi M_n = 0.9(737.8) = 664 \text{ K-'}$

$M_u = 653.3 < 664$ ✓ ok

$c = \dfrac{a}{\beta_1} = \dfrac{7.843''}{0.85} = 9.227''$

8. Check that $\varepsilon_t \geq 0.005$ (if not, increase h and reduce As)

$\varepsilon_t = \dfrac{d-c}{c}(0.003)$

$\varepsilon_t = \dfrac{33.436 - 9.227}{9.227}(0.003)$

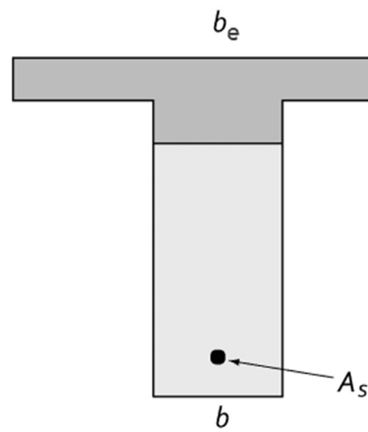
$\varepsilon_t = 0.00787 > 0.005$ ✓ok

9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length.

Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, be, h, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Required Moment – Mu (or load, or span)

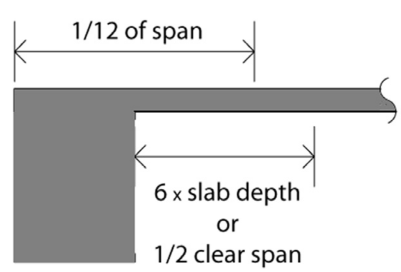


1. Find T=As fy and C= 0.85 f'c Ac
2. Set T = C and solve for Ac
3. Draw and label diagrams for section and stress
    1. Determine b effective (for T-beams)
    2. Locate T and C (or C_1 and C_2)
4. Determine the location of a.
    Working from the top down,
    add up area to make Ac
5. Find moment arms (z) for each block of area
6. Find Mn = Σ C z
7. Find Mu = ϕ Mn
8. Check As min < As < As max
9. Check that $\epsilon_t \geq 0.005$

Effective Flange Width, b_e

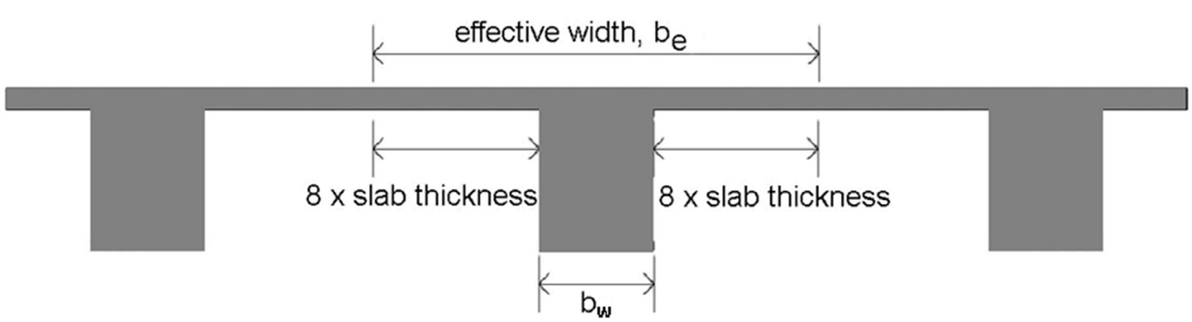
Slab on one side:

b_e least of either (**total width**) or (**overhang + stem**)
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam

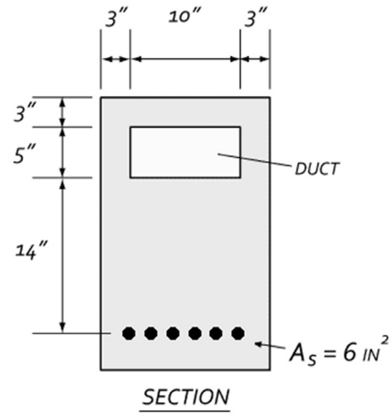


Slab on both sides:

b_e least of either (**total width**) or (**2 x overhang + stem**)
- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)



Non-rectangular shape - example



Given: f'c = 3000 psi
fy = 60 ksi
As = 6 x #9 = 6 in^2

Req'd: Capacity, Mu

1a. Find T

1b. Find C in terms of Ac

2. Set T = C and solve for Ac

$T = A_s f_y = 6\ in^2\ (60000\ psi)$

$T = 360\ 000\ \# = 360^k$

$C = 0.85 f'_c A_c = 0.85(3000 psi)\ A_c\ in^2$

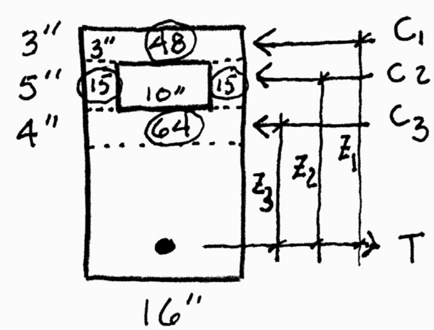
$C = (2550\ A_c)^\# = (2.55\ A_c)^k$

$T = C$

$360^k = 2.55\ A_c\ ^k$

$A_c = 142\ in^2$

Non-rectangular shape (cont.)



3. Draw section and determine areas to make Ac

4. Find the location of a.
a = 3"+5"+4"

C = 0.85 f'c Ac

f'c = 3 ksi
Ac_1= 48 in^2
Ac_2= 30 in^2
Ac_3= 64 in^2

$A_c = 142\ in^2 = A_{c1} + A_{c2} + A_{c3}$

$142 = 48 + 30 + A_{c3}$

$A_{c3} = 64\ in^2$

$C_1 = 48(2.55) = 122.4^k$

$C_2 = 30(2.55) = 76.5^k$

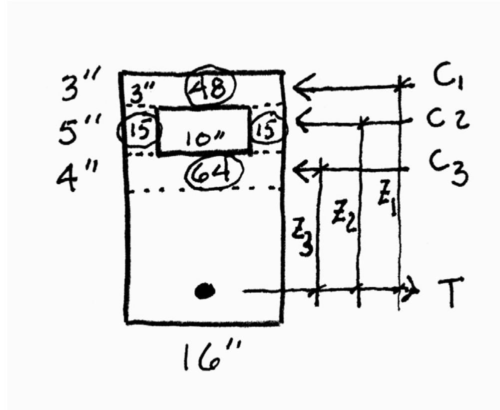
$C_3 = 64(2.55) = 163.2^k$

Non-rectangular shape (cont.)

5. Determine moment arms to areas, z. (d = 22”)

6. Calculate Mn by summing the Cz moments.

7. Find Mu = ϕMn



$z_1 = 22 - 1.5 = 20.5''$

$z_2 = 22 - (3 + 2.5) = 16.5''$

$z_3 = 22 - (8 + 2) = 12.0''$

$M_n = \Sigma C z$

$M_n = (C_1 z_1) + (C_2 z_2) + (C_3 z_3)$

$M_n = 2509 + 1262 + 1959$

$M_n = 5730$

$M_u = \phi M_n = 0.9(5730) = 5157^{K\text{-}'}$

Non-rectangular shape (cont.)

8. Check As,min

$3 \frac{\sqrt{f'c}}{fy}$ bw d =

$3 \left(\frac{\sqrt{3000}}{60000}\right)$ 22 (16) =0.964 in^2

$(200/f_y)\ b_w d$ =

(200/60000) (16x22) = 1.17 in^2

As,min is the greater

(a) $\frac{3\sqrt{f'_c}}{f_y} b_w d$

(b) $\frac{200}{f_y} b_w d$

9. Check $\epsilon_t \geq 0.005$

Find $c = a/\beta_1$

Check that $\epsilon_t \geq 0.005$ (tension controlled)

And $\epsilon_t \geq 0.004$ (As max)

When As > As,max, ϵ_t must be increased:

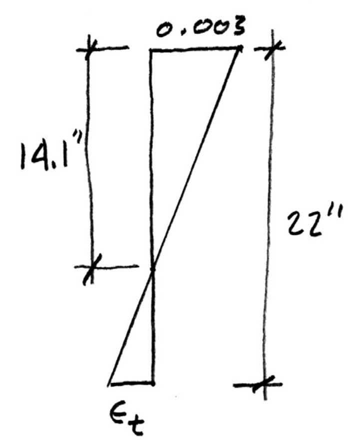
Reduce As (but would also reduce Mn)

Add compression steel

Increase h

Increase f’c

$c = \frac{a}{\beta_1} = \frac{12''}{0.85} = 14.1176''$



$\epsilon_t = \frac{d - c}{c} 0.003 = \frac{22 - 14.1}{14.1}$

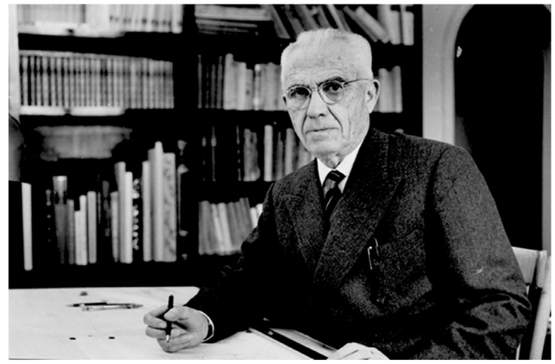
$\epsilon_t = 0.00168$

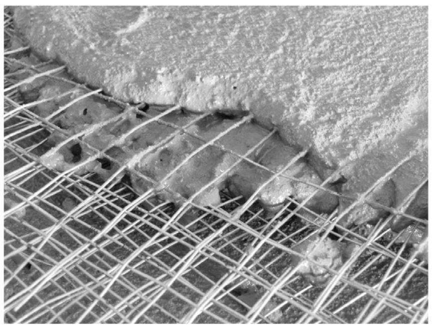
$0.00168 < 0.004$

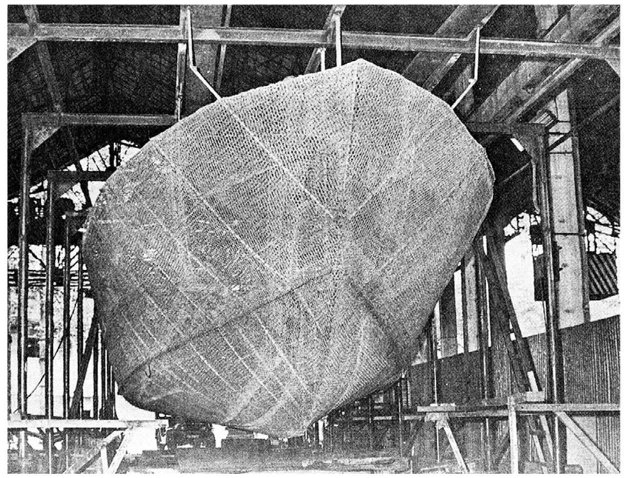
$\therefore$ NG!

Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking







Ferrocement

- Pioneered by Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking
- Low-tech applications

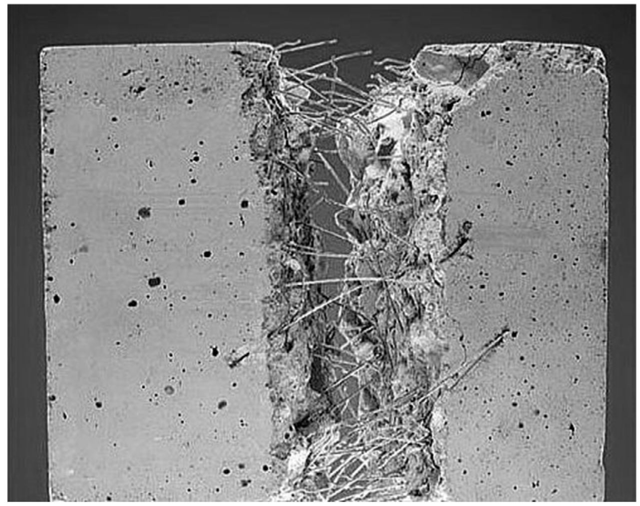
Palazetto dello Sport, Rome, 1957. P.L. Nervi

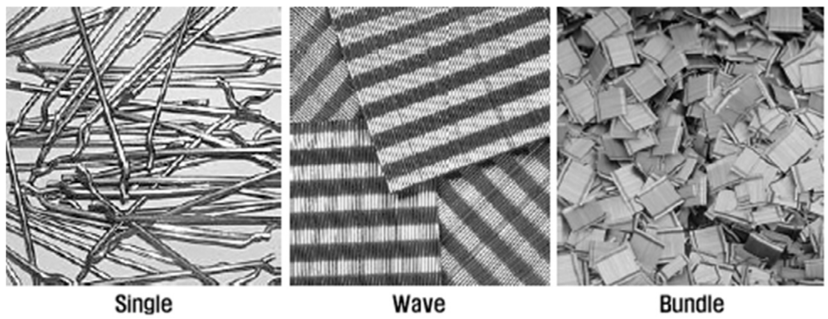
Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

Fiber Reinforced Concrete

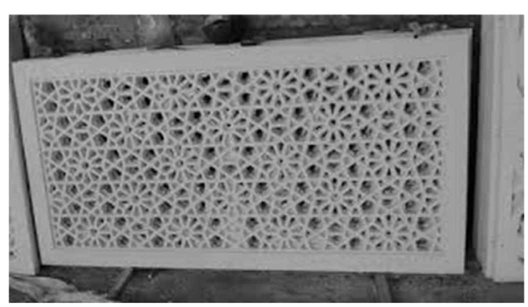
Several different fiber types:
- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

Better crack control
Secondary reinforcement





Glass Fiber Reinforced Concrete - GFRC

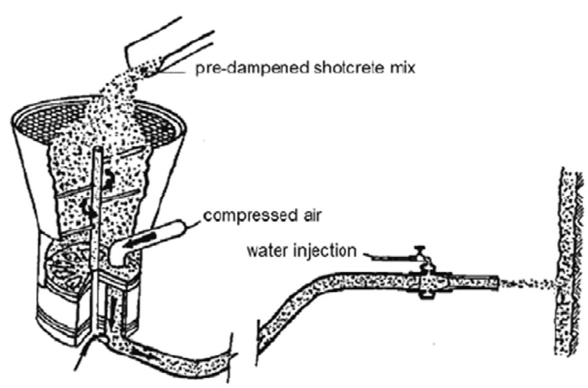


Carbon Fiber

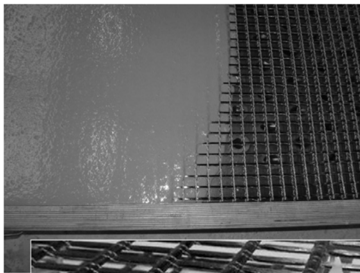
Bamboo

Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels



Textile Reinforced Concrete (TRC)



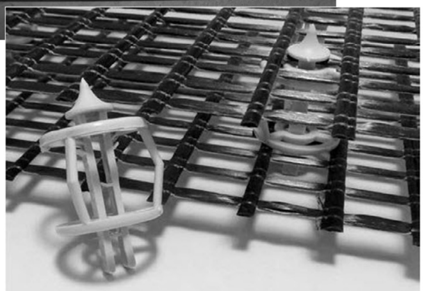


Figure 12: distTEX: special spacers for textile grids [photo: Frank Schladitz, TU Dresden]

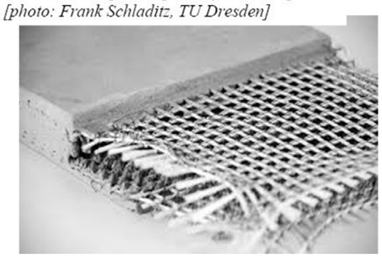


Figure 13: Manufacturing of the TRC hypar-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]

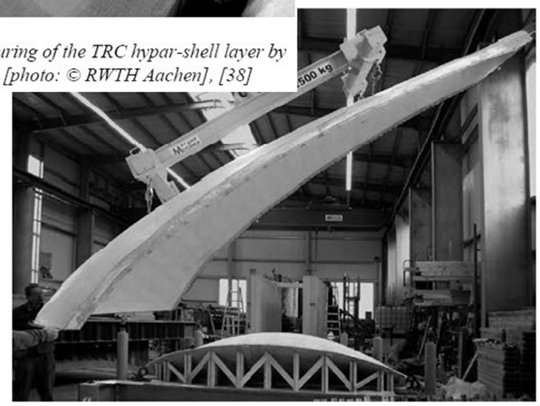


Figure 10: Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]