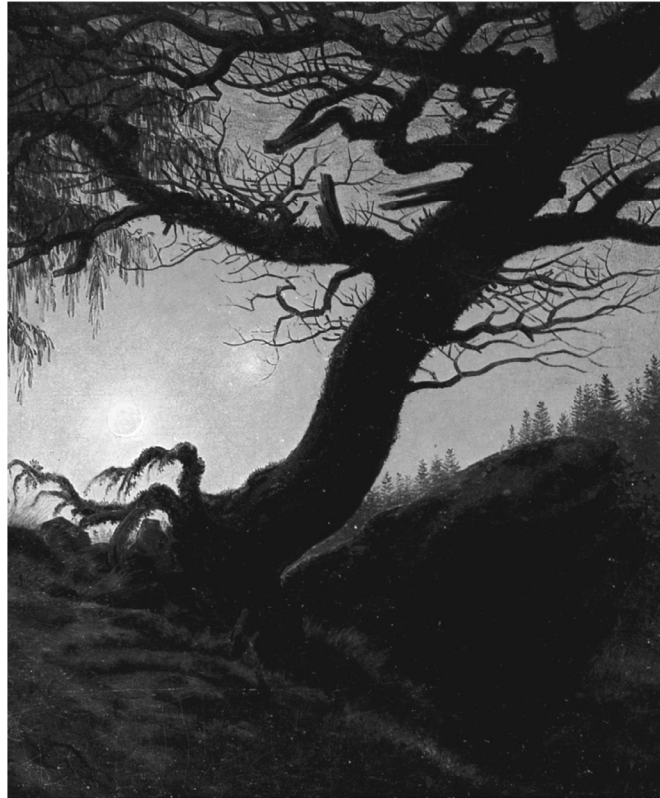


Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas



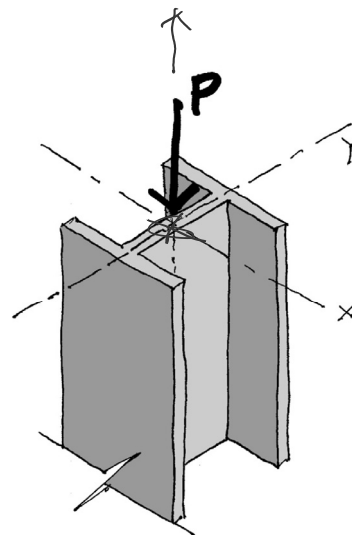
from "Man und Frau den Mond betrachtend"
1830-35 by Caspar David Friedrich
Alte Nationalgalerie, Berlin

Axial Stress

- Loads pass through the centroid of the section, i.e. axially loaded
- Member is straight
- Load less than buckling load

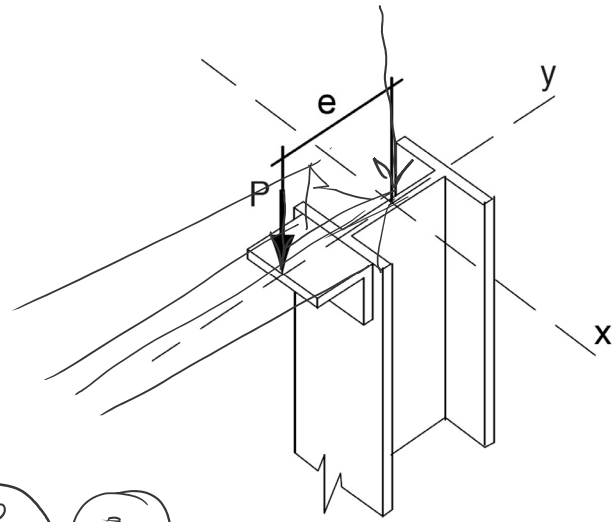
Then:

$$\underline{f_a = \frac{P}{A}}$$



Eccentric Loads

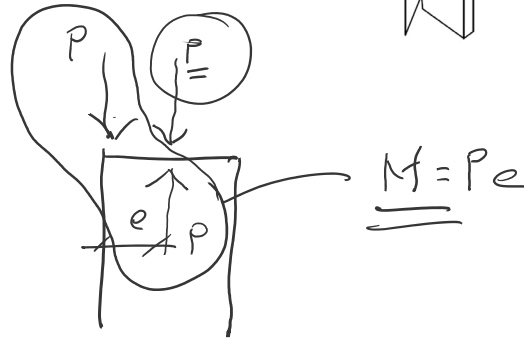
- Load is offset from centroid
- Bending Moment = $P e$
- Total load = $P + M$



Interaction formula

AXIAL MOMENT

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$

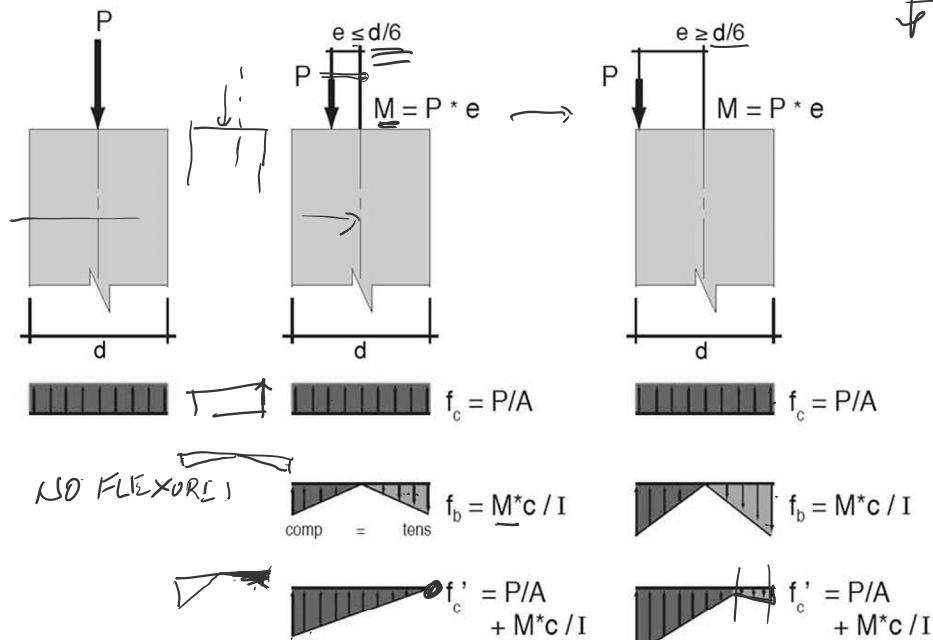
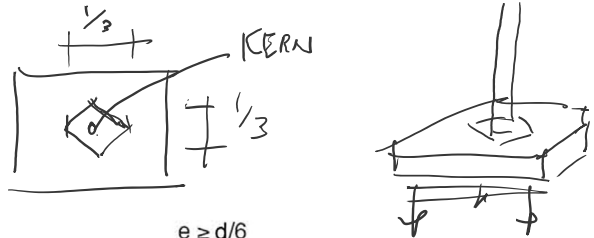


% AXIAL % BENDING < 100%

$$\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0$$

Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign



axial loaded - uniform compressive stress.

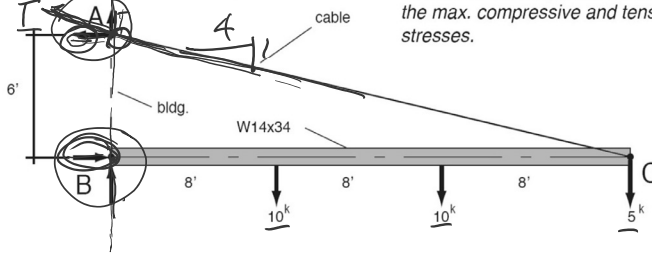
small eccentricity - linearly varying stress.

large eccentricity - tensile stress on part of cross section.

Example

CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.
The supporting cable is pin-connected on the centroidal axis of the steel beam.

Reactions at face of building.



FOR THE W14x34:
Determine the magnitude and location of the max. compressive and tensile unit stresses.

1. Determine external reactions

$$\begin{aligned} \sum M_A = 0 &= -B_H(6') + 10^k(8') + 10^k(16') + 5^k(24') \\ B_H &= 60^k \rightarrow \\ \sum M_B = 0 &= -A_H(6') + 10^k(8') + 10^k(16') + 5^k(24') \\ A_H &= 60^k \leftarrow \\ \text{CHECK } \sum F_H = 0 &= 60^k - 60^k \checkmark \end{aligned}$$

FBD of A

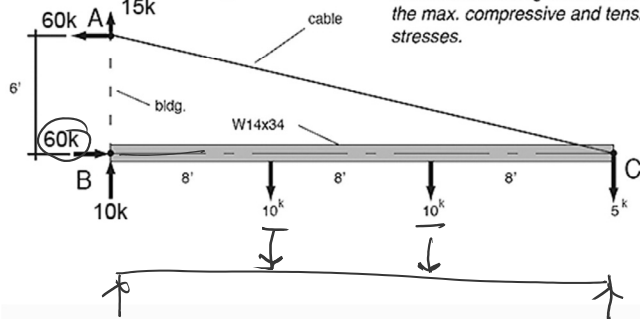
$$\frac{60}{4} = \frac{A_V}{1} \Rightarrow A_V = 15^k$$

$$\sum F_V = 0 = 15^k - 10^k - 10^k - 5^k + B_V \Rightarrow B_V = 10^k$$

Example

CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.
The supporting cable is pin-connected on the centroidal axis of the steel beam.

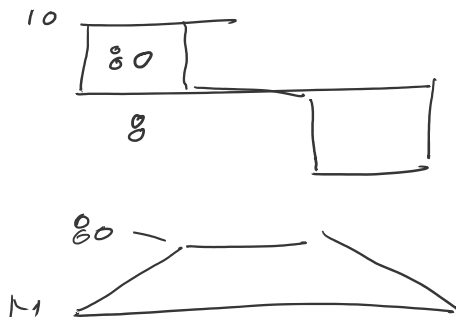
Reactions at face of building.



FOR THE W14x34:
Determine the magnitude and location of the max. compressive and tensile unit stresses.

2. Determine internal member forces: Axial and Flexural

3. Determine axial and flexural stresses



W14x34

$$A = 10.0 \text{ in}^2$$

$$S_x = 48.4 \text{ in}^3$$

FORCE:

$$\text{AXIAL} = 60^k$$

$$\text{FLEXURAL} = M = \frac{PL}{3} = 10^k(8') = 80 \text{ k-ft}$$

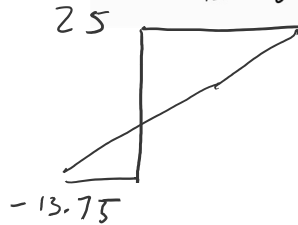
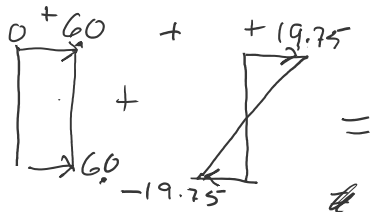
STRESS:

$$\text{AXIAL} = f_a = \frac{P}{A} = \frac{60^k}{10 \text{ in}^2} = 6.0 \text{ KSI}$$

$$\text{FLEXURAL} = f_b = \frac{M}{S} = \frac{80 \text{ k-ft}(12)}{48.4 \text{ in}^3} = 19.75 \text{ KSI}$$

Example

2. Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)



COMBINED STRESS

TOP SIDE:

$$f_t + f_b = \frac{6.0 + 19.75}{25} = 25.75 \text{ KSI (COMP)}$$

BOTTOM SIDE:

$$f_t - f_b = \frac{6.0 - 19.75}{25} = -13.75 \text{ KSI (TENS)}$$

$$F \approx 0.6 F_y = 30 \text{ KSI}$$

$$F_y = 50 \text{ KSI}$$

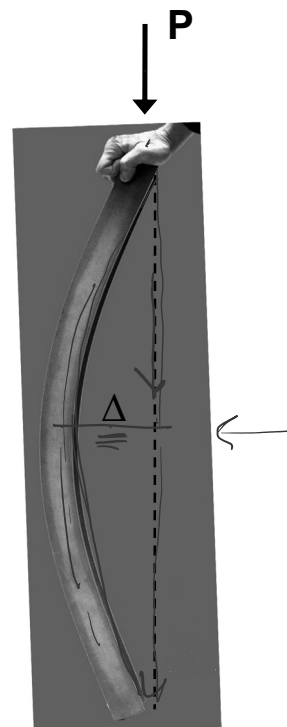
$$\frac{f_t}{F_t} + \frac{f_b}{F_b} = \frac{6.0}{30} + \frac{19.75}{30} =$$

$$0.2 + 0.66 = 0.86 < 1.0 \checkmark$$

Second Order Stress "P Delta Effect"

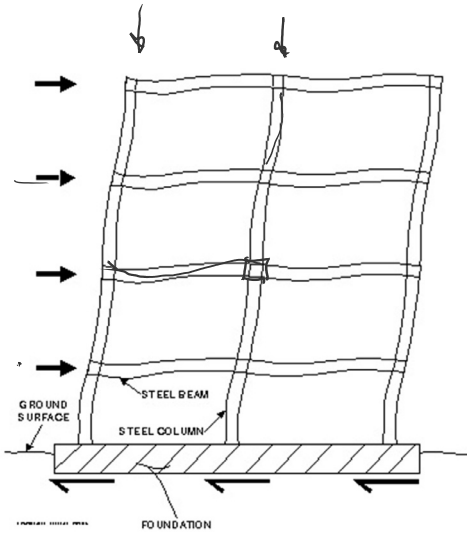
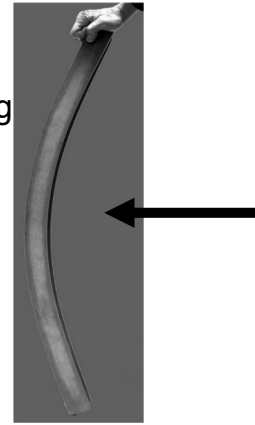
With larger deflections this can become significant.

1. Eccentric load causes bending moment
2. Bending moment causes deflection, Δ
3. $P \times \Delta$ causes additional moment

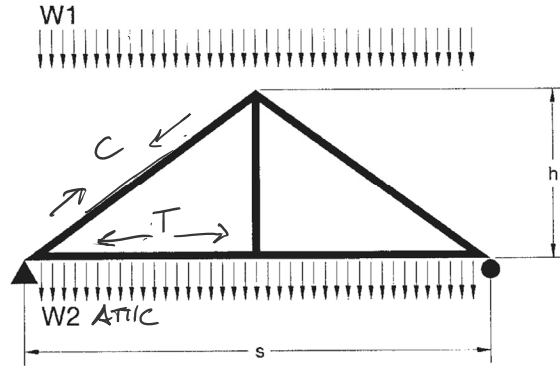


Other Examples of Combined Stress

Columns with side loading

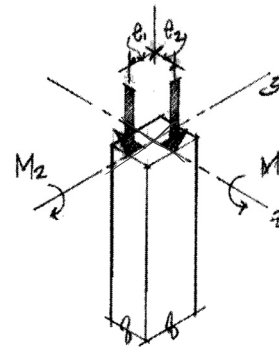
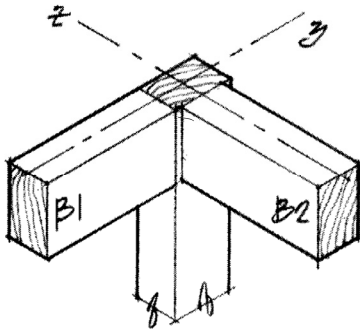


Moment frames



Trusses loaded on members

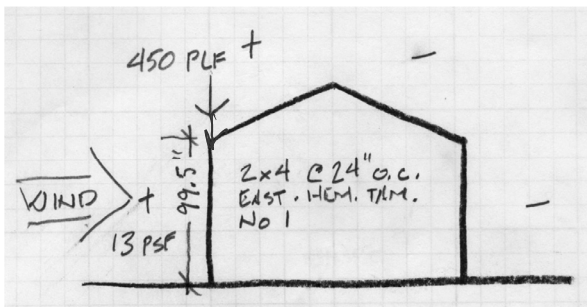
Other Examples of Combined Stress



$$M_1 = P_1 \times e_1 \text{ (ABOUT THE } z\text{-axis)}$$

$$M_2 = P_2 \times e_2 \text{ (ABOUT THE } y\text{-axis)}$$

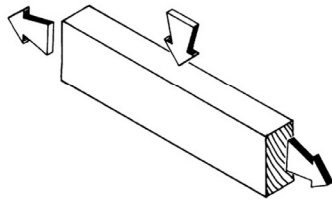
Eccentrically loaded columns



Wind load on walls

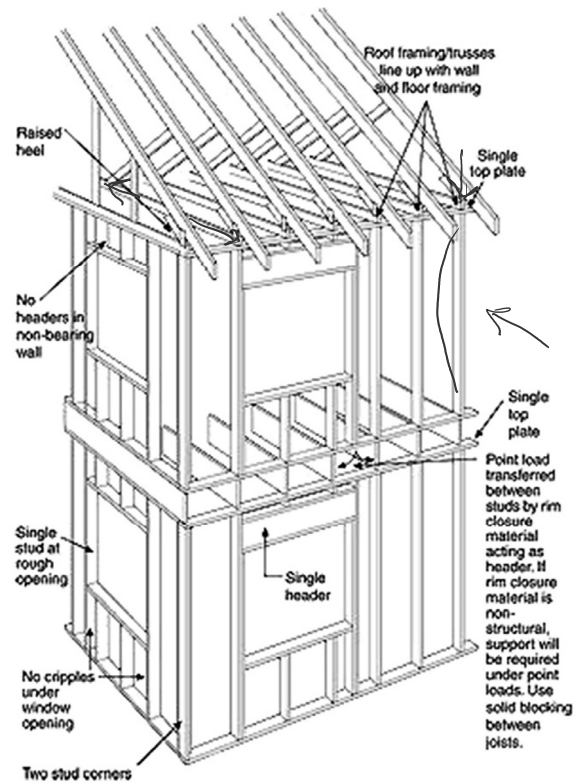
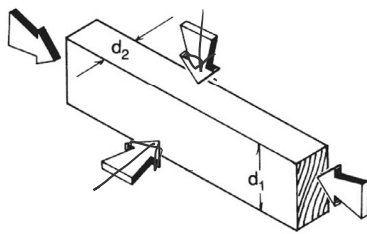
Combined Stress in NDS

Figure 3G Combined Bending and Axial Tension



3.9.2 Bending and Axial Compression

Figure 3H Combined Bending and Axial Compression



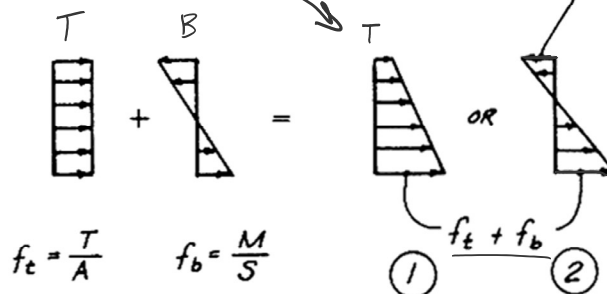
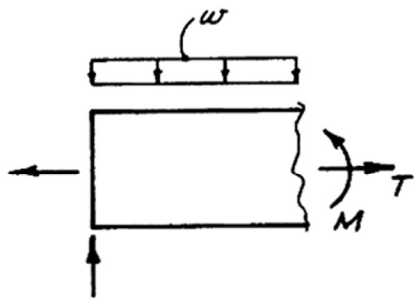
Tension + Flexure NDS Equations

CASE 1. Tension is critical. Eq. 3.9-1
* no C_L

CASE 2. Flexure is critical. Eq. 3.9-2
** no C_V C_L

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0$$

$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0$$



TENSION + BENDING = COMBINED STRESSES

Tension + Flexure

3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$

and

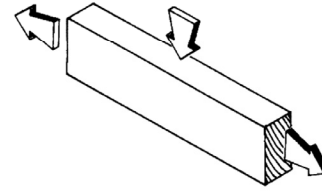
$$\frac{f_b - f_t}{F_b''} \leq 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2)$$

where:

F_b' = reference bending design value multiplied by all applicable adjustment factors except C_L

F_b'' = reference bending design value multiplied by all applicable adjustment factors except C_v

Figure 3G Combined Bending and Axial Tension



Example Problem

Given: Queen Post truss

Hem-Fir No.1 & Better

$F_b = 1100$ psi

$F_t = 725$ psi

$F_c = 1350$ psi

$E_{min} = 550000$ psi

$C_L C_P$

span = 30 ft. spaced 48" o.c.

D + S Load = 44 psf (projected)

D (attic + ceiling) = 8 psf

bottom chord: 2x8

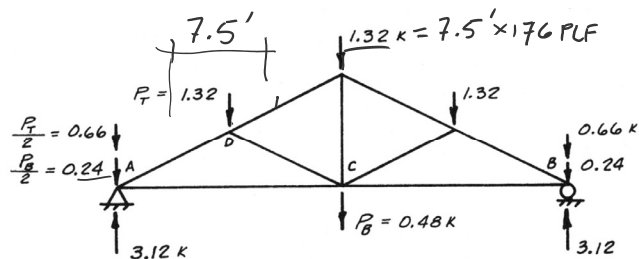
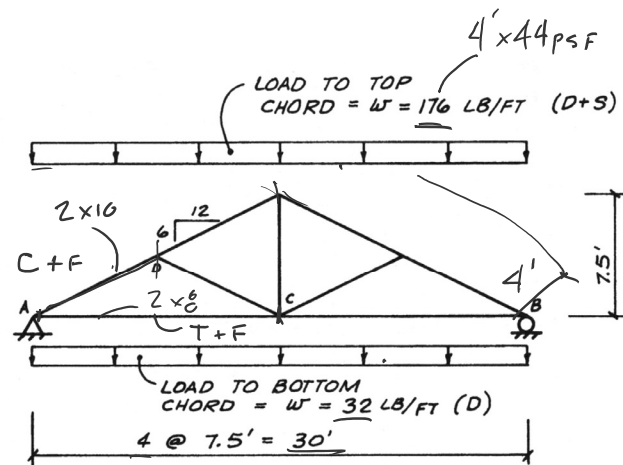
top chord: 2x10

Find: pass/fail

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0$$

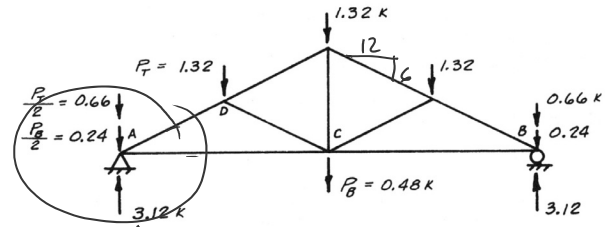
$$\frac{f_b - f_t}{F_b''} \leq 1.0$$

1. Determine truss joint loading



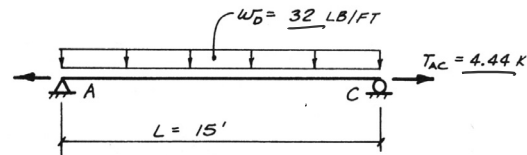
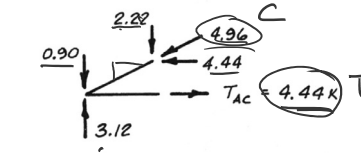
Example (cont.)

- Determine the external **end reactions** of the whole truss. The geometry and loads are symmetric, so each reaction is $\frac{1}{2}$ of the total load.



- Use an FBD of the reaction joint to find the **chord forces**. Sum the forces horizontal and vertical to find the components.

Top chord = 4.96 k compression
Bottom chord = 4.44 k tension



Example bottom chord 2x8

- Calculate the **actual** axial and flexural stress.

$$f_t = 408.3 \text{ psi}$$

$$f_b = 821.9 \text{ psi}$$

- Determine **allowable** stresses using applicable factors:

(tension: D+S)

$$F_t' = F_t (C_D C_F) \quad \checkmark$$

$$F_t' = \frac{725}{1.05} (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3$$

(flexure: D+S)

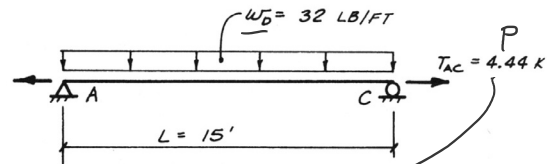
$$F_b' = F_b (C_D C_L C_F)$$

$$F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi}$$

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0$$

and

$$\frac{f_b - f_t}{F_b'} \leq 1.0$$



$$f_t = \frac{P}{A} = \frac{4440 \text{ lbs}}{10.875 \text{ in}^2} = 408.3 \text{ psi}$$

2x8

$$f_b = \frac{M}{S_x} = \frac{900 (12)''}{13.14 \text{ in}^3} = 821.9 \text{ psi}$$

$$M = \frac{w l^2}{8} = \frac{32 (15')^2}{8} = 900 \text{ ft} \cdot \text{lb}$$

$$S_x = 13.14 \text{ in}^3$$

C_L is 1.0 BY 4.4.1
 $d/b = 4$, ENDS ARE HELD

Example

bottom chord 2x8

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0$$

and

$$\frac{f_b - f_t}{F_b''} \leq 1.0$$

5. Determine **allowable** stresses using applicable factors:

(tension: D+S)

$$F_t' = F_t (C_D C_F)$$

$$F_t' = 725 (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3$$

(flexure: D+S)

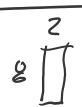
$$F_b' = F_b (C_D C_L C_F)$$

$$F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi}$$

Size Factors, C_F

Grades	Width (depth)	Thickness (breadth)		F_t'	F_c
		2" & 3"	4"		
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
Stud	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
Construction Standard	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3			
Utility	2", 3", & 4"	1.0	1.0	1.0	1.0
	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

C_L is 1.0 by 4.4.1
 $d/b = 4$, ENDS ARE HELD



Example

bottom chord 2x8

3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$

and

$$\frac{f_b - f_t}{F_b''} \leq 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2)$$

where:

F_b' = reference bending design value multiplied by all applicable adjustment factors except C_L

F_b'' = reference bending design value multiplied by all applicable adjustment factors except C_v

(3.9-1)

$$\frac{408.3}{1000} + \frac{821.9}{1518}$$

$$0.4083 + 0.5414 = 0.95$$

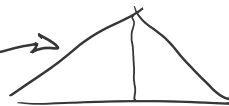
$0.95 < 1.0$ ✓ PASS

(3.9-2)

$$\frac{821.9 - 408.3}{1518} = 0.2724$$

$$0.27 < 1.0 \quad \checkmark \text{ PASS}$$

Bending + Axial Compression



3.9.2 Bending and Axial Compression

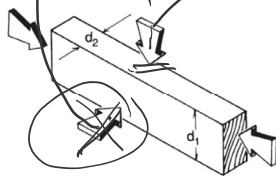
where:

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

$$\left[\frac{f_c}{F_c'} \right]^2 + \frac{F_{b1}' \left[1 - (f_c / F_{cE1}) \right]}{F_{b2}' \left[1 - (f_c / F_{cE2}) - (f_{b1} / F_{bE})^2 \right]} \leq 1.0 \quad (3.9-3)$$

and

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}} \right)^2 < 1.0 \quad (3.9-4)$$



$$f_c < F_{cE1} = \frac{0.822 E_{min}'}{(l_{e1} / d_1)^2} \quad \text{for either uniaxial edgewise bending or biaxial bending}$$

and

$$f_c < F_{cE2} = \frac{0.822 E_{min}'}{(l_{e2} / d_2)^2} \quad \text{for uniaxial flatwise bending or biaxial bending}$$

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}'}{(R_B)^2} \quad \text{for biaxial bending}$$

f_{b1} = actual edgewise bending stress (bending load applied to narrow face of member), psi

f_{b2} = actual flatwise bending stress (bending load applied to wide face of member), psi

d_1 = wide face dimension (see Figure 3H), in.

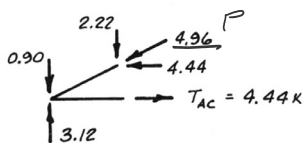
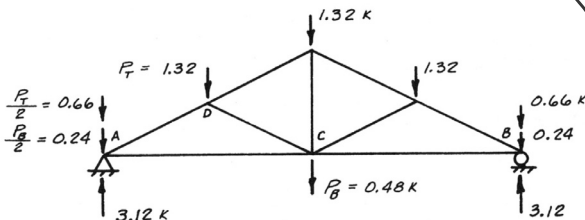
d_2 = narrow face dimension (see Figure 3H), in.

Example top chord 2x10

4. Calculate the **actual** axial and flexural stress.

$$f_c = 357.5 \text{ psi}$$

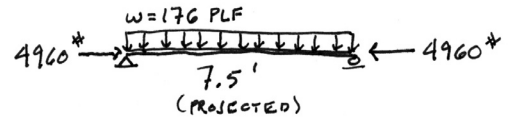
$$f_b = 694.2 \text{ psi}$$



$$\left[\frac{f_c}{F_c'} \right]^2 + \frac{f_{b1}}{F_{b1}' \left[1 - (f_c / F_{cE1}) \right]}$$

and

$$\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}} \right)^2$$



$$f_c = \frac{P}{A} = \frac{4960^*}{1.5 \times 9.25} = 357.5 \text{ psi}$$

$$f_b = \frac{M}{S_x} = \frac{1237.5 (12)^*}{21.39} = 694.2 \text{ psi}$$

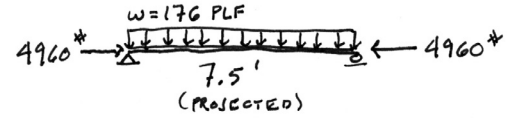
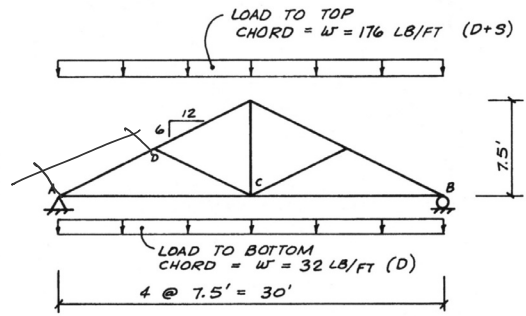
$$M = \frac{w l^2}{8} = \frac{176 \text{ PLF} (7.5')^2}{8} = 1237.5 \text{ ft-lb}$$

$$S_x = 21.39 \text{ in}^3$$

Example

top chord 2x10

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c / F_{cE1}) \right]}$$



5. Determine **allowable** stresses using applicable factors:

(compression: D+S)

$$F'_c = F_c (C_D C_F C_P)$$

$$F'_c = 1350 (1.15 \cdot 1.0 \cdot 0.897) = 1392.7 \text{ psi} > 357.5$$

(flexure: D+S)

$$F'_b = F_b (C_D C_L C_F)$$

$$F'_b = 1100 (1.15 \cdot 1.0 \cdot 1.1) = 1392 \text{ psi} > 694.2$$

$$C_p$$

$$l_e = 8.385' \quad d = 9.25" \quad 2 \times 10$$

$$l_e/d = \frac{8.385(12)}{9.25} = 10.88$$

$$F_{cE} = \frac{0.822 E_{min}}{(l_e/d)^2} = \frac{0.822(550000)}{10.88^2} = 3820 \text{ psi}$$

$$F'_c = 1350 (1.15 \cdot 1.0) = 1552.5 \text{ psi}$$

$$F_{cE}/F'_c = \frac{3820}{1552.5} = 2.46 \quad d = 0.8 \text{ DINK LUM}$$

$$C_p = 0.897$$

Example

top chord 2x10

Eq. 3.9-3

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c / F_{cE1}) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

where:

$$f_c < F_{cE1} = \frac{0.822 E_{min}}{(l_{e1}/d_1)^2} \quad \text{EULER 1}$$

for either uniaxial edge-wise bending or biaxial bending

and

$$f_c < F_{cE2} = \frac{0.822 E_{min}}{(l_{e2}/d_2)^2} \quad \text{EULER 2}$$

for uniaxial flatwise bending or biaxial bending

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}}{(R_B)^2} \quad \text{LTB}$$

for biaxial bending

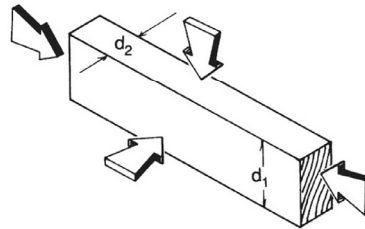
f_{b1} = actual edgewise bending stress (bending load applied to narrow face of member)

f_{b2} = actual flatwise bending stress (bending load applied to wide face of member)

d_1 = wide face dimension (see Figure 3H)

d_2 = narrow face dimension (see Figure 3H)

Figure 3H Combined Bending and Axial Compression



COMPRESSION:

$$\left[\frac{f_c}{F'_c} \right]^2 = \left[\frac{357.5}{1392.7} \right]^2 = 0.0659$$

Example

top chord 2x10

Figure 3H Combined Bending and Axial Compression

Eq. 3.9-3

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c / F_{cE1}) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

where:

$$f_c < F_{cE1} = \frac{0.822 E_{min}}{(\ell_{e1} / d_1)^2}$$

EULER 1 for either uniaxial edge-wise bending or biaxial bending

and

$$f_c < F_{cE2} = \frac{0.822 E_{min}}{(\ell_{e2} / d_2)^2}$$

EULER 2 for uniaxial flatwise bending or biaxial bending

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}}{(R_B)^2}$$

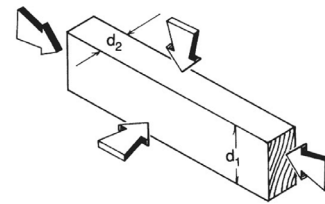
LTB for biaxial bending

f_{b1} = actual edgewise bending stress (bending load applied to narrow face of member)

f_{b2} = actual flatwise bending stress (bending load applied to wide face of member)

d_1 = wide face dimension (see Figure 3H)

d_2 = narrow face dimension (see Figure 3H)



FLEXURE:

$$\frac{f_{b1}}{F'_{b1}} = \frac{694.2}{1392} = 0.4987$$

AMPLIFICATION FACTOR:

$$\frac{1}{1 - (357.5 / 3820)} = 1.103$$

$$0.4987 (1.103) = 0.550$$

COMBINATION:

$$0.0659 + 0.550 = 0.616$$

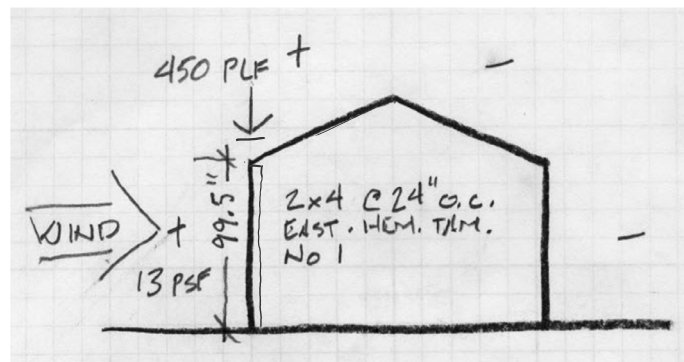
$$0.616 < 1.0 \quad \checkmark \text{ PASS}$$

$f_c = 357.5 \text{ psi}$
 $E_{min} = 550,000 \text{ psi}$
 $\ell_{e1} = 8.385'$
 $d_1 = 9.25''$
 $\ell_{e1} / d_1 = \frac{8.385(12)}{9.25} = 10.88$
 $F_{cE1} = \frac{0.822(550,000)}{10.88^2} = 3820 \text{ psi}$

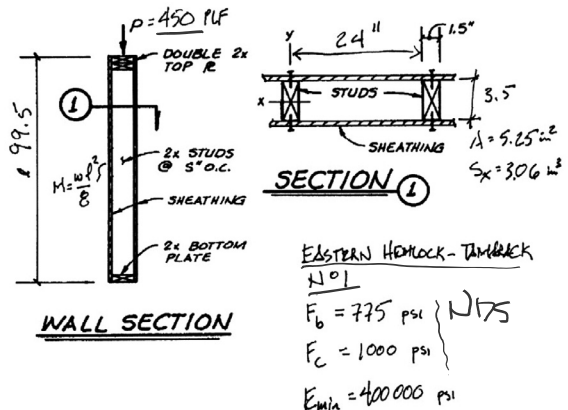
Combined Stress in NDS procedure

Exterior stud wall under bending + axial compression

1. Determine load per stud
2. Use axial load and moment to find actual stresses f_c and f_b
3. Determine load factors
4. Calculate factored stresses
5. Check NDS equations



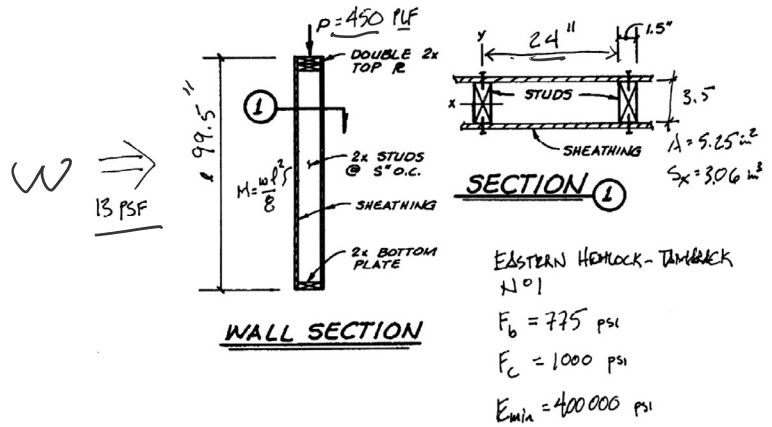
$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c / F_{cE1}) \right]} \leq 1.0 \quad (3.9-3)$$



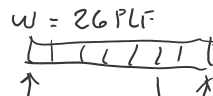
Combined Stress in NDS

example

Exterior stud wall under bending + axial compression



1. Determine load per stud



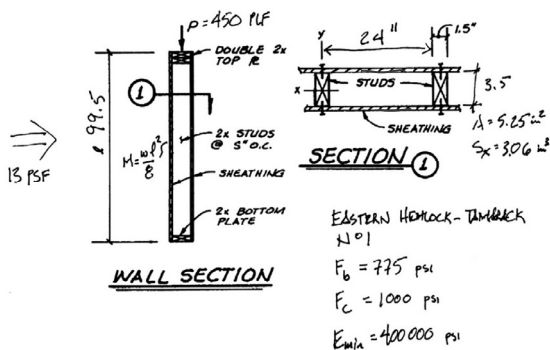
2. Use axial load and moment to find actual stresses f_c and f_b

$P = \text{Load/STUD}$
 $P = 450 \text{ PLF} \frac{\text{o.c.}}{12} = 450 \frac{24}{12} = \boxed{900 \text{ LBS}}$
 $w = 13 \text{ PSF} \frac{\text{o.c.}}{12} = 13 \frac{24}{12} = \underline{26 \text{ PLF/STUD}}$
 $M_x = \frac{w l^2}{8} = \frac{26 (99.5/12)^2}{8} = \boxed{223.4 \text{ ft-lb}}$
 $f_c = \frac{P}{A} = \frac{900}{5.25 \text{ in}^2} = \boxed{171.43 \text{ psi}}$
 $f_b = \frac{M}{S_x} = \frac{223.4 (1.2)^3}{3.06 \text{ in}^3} = \boxed{875.5 \text{ psi}}$

Combined Stress in NDS

example

Exterior stud wall under bending + axial compression



Size Factors, C_F

Grades	Width (depth)	Thickness (breadth)		F_t	F_c
		2" & 3"	4"		
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
Stud	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
Construction Standard	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3			
Utility	2", 3", & 4"	1.0	1.0	1.0	1.0
	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

$F_b = 775 \text{ psi}$ $F_c = 1000 \text{ psi}$ $E_{min} = 400,000 \text{ psi}$

3. Determine load factors (bending)

FACTORS :
 $C_D = 1.6$ (WIND)
 $C_F = 1.5$ (FOR F_b) 1.15 (FOR F_c)
 $C_L = 1.0$ (BRACED BY SHEATHING)
 $C_r = 1.15$ ($\leq 24" \text{ o.c.}$)

Combined Stress in NDS

example

Exterior stud wall under
bending + axial compression

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F_{b1}' \left[1 - (f_c / F_{cE1}) \right]}$$

$$F_b = 775 \text{ psi}$$

$$C_D = 1.6$$

$$C_F = 1.5$$

$$C_M = 1.0$$

$$C_{FV} = 1.0$$

$$C_t = 1.0$$

$$C_i = 1.0$$

$$C_L = 1.0$$

$$C_r = 1.15$$

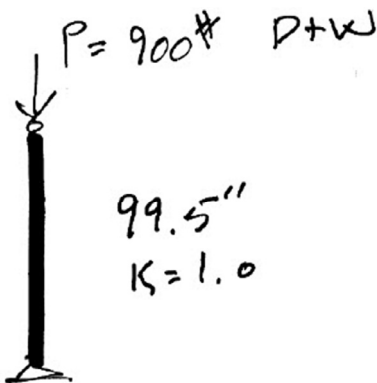
4. Calculate factored stresses
(bending stress)

$$F'_b = 775 (1.6)(1.5)(1.15) \\ = 2139 \text{ psi}$$

Combined Stress in NDS

example

Exterior stud wall under
bending + axial compression



$$C_p = \frac{1 + (F_{CE} / F_c^*)}{2c} - \sqrt{\left[\frac{1 + (F_{CE} / F_c^*)}{2c} \right]^2 - \frac{F_{CE} / F_c^*}{c}}$$

C_p

$$F_c^* = 1000 (1.6)(1.15) = 1840$$

$$F_{cE} = \frac{0.822 (400,000)}{(99.5 / 3.5)^2} = 406.8$$

$$c = 0.8 \text{ DIM LWM}$$

$$C_p = 0.21$$

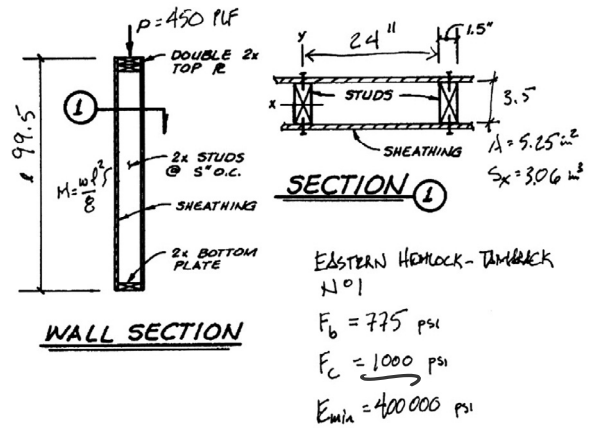
3. Determine load factors
(compression)

Combined Stress in NDS

example

Exterior stud wall under bending + axial compression

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c/F_{cE1}) \right]}$$



Actual Stress

4. Calculate stresses (compression stress)

$$f_c = \frac{P}{A} = \frac{900}{5.25} = 171.4 \text{ psi}$$

Factored Allowable Stress

$$F'_c = 1000 (1.6) (1.15) (0.21) = 386.4 \text{ psi}$$

Combined Stress in NDS

example

Exterior stud wall under bending + axial compression

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c/F_{cE1}) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

5. Combined Stress Calculation (eq. 3.9-3)

$$\left[\frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[1 - (f_c/F_{cE1}) \right]} \leq 1.0$$

AMPLIFICATION

$$\left[\frac{171.4}{386.4} \right]^2 + \frac{876}{2139} \frac{1}{1 - (171.4/406.8)} \leq 1.0$$

$$0.1967 + (0.4095) (1.728) =$$

$$0.1967 + 0.7077 = \underline{\underline{0.9045}} \leq 1.0 \quad \checkmark \text{OK}$$