

## Pre- and Post-Tensioning

- Cable Trusses
- Concrete Beams
- Stressed Membranes



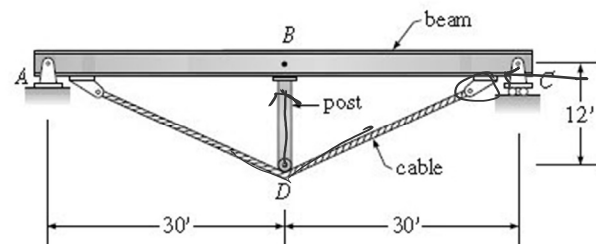
## Cable Trusses

- Reduce flexure stress
- Reduce deflection
- Produces stiffer section with less material
- Lighter weight
- Longer spans possible
- Analysis by combined stress



DETROIT

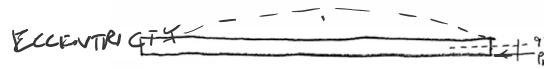
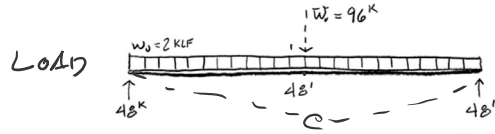
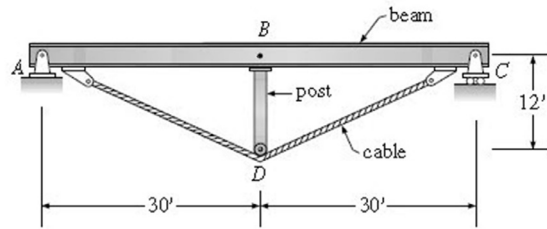
$$f = -\frac{P}{A} \pm \frac{M}{S} \pm \left[ \frac{Pe}{S} \right]$$



# Cable Truss – stress analysis

determine cable prestress

- Break beam load into 3 FBDs.
  - applied load
  - cable + strut
  - eccentric load (if any)
- Solve moment for beam at the center line (C.L.) for applied load
- Solve C.L. moments for other 2 FBDs in terms of strut force,  $P_s$
- Equate the moments from the three moment equations to cancel at the CL
- Solve for the strut and cable forces.
- Construct moment diagram for the beam with all loadings combined: applied load + cable at ends + struts.
- Solve combined stress in beam using interaction equation.



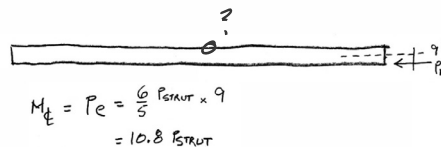
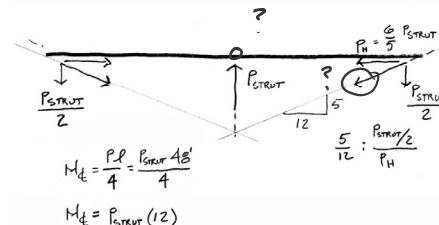
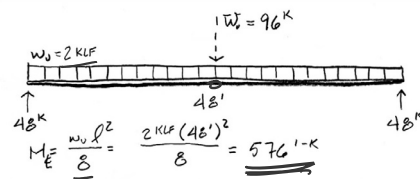
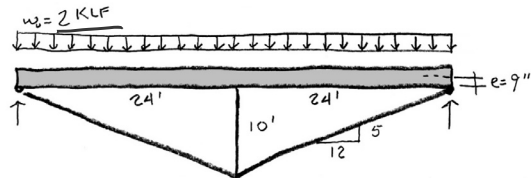
# Cable Truss Analysis

## Example

Given: truss configuration with applied load

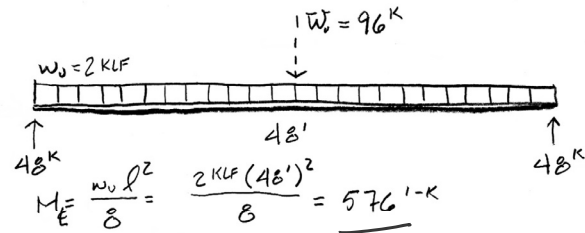
Required: force in the cable which will result in zero moment at the center line, C.L.

- Divide the truss into 3 Free Body Diagrams:
  - applied load
  - cable + strut
  - eccentric load (if any)

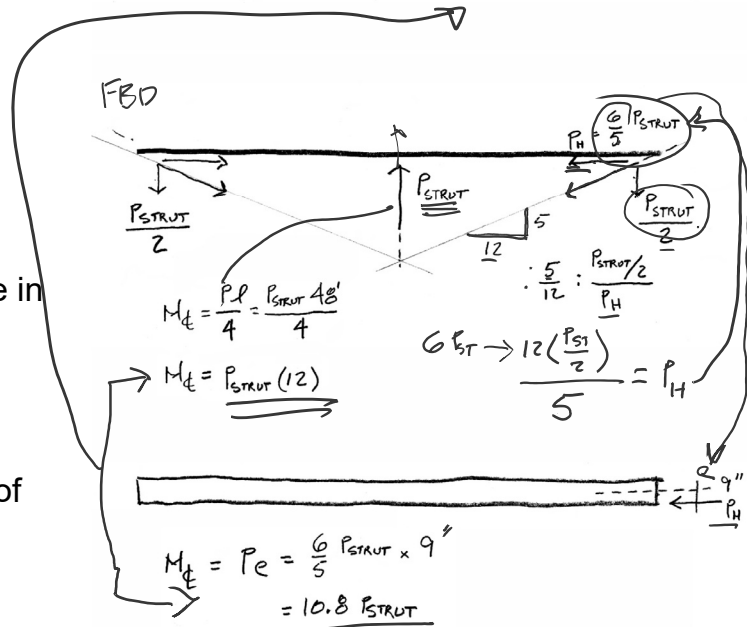


# Cable Truss Analysis

- Find the C.L. moment based on applied load alone.

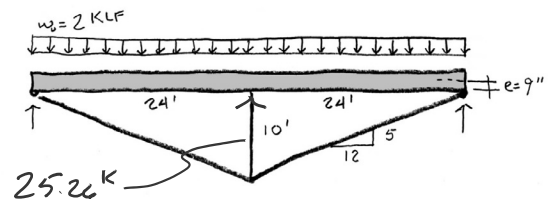


- Find the C.L. moment for the cable and strut in terms of the strut force,  $P_{\text{strut}}$ . Write the components of the cable force in terms of  $P_{\text{strut}}$



- Find the C.L. moment for the eccentric cable load in terms of  $P_{\text{strut}}$

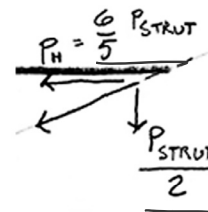
# Cable Truss Analysis



- Set the sum of the C.L. moments equal to zero and solve for the strut force,  $P_{\text{strut}}$

LOAD    STRUT    Ecc.  
 $+ M_U - M_{\text{STRUT}} - M_e = 0$   
 $+ 576 - 12 P_{\text{STRUT}} - 10.8 P_{\text{STRUT}} = 0$   
 $22.8 P_{\text{STRUT}} = -576 \text{ K-FT}$   
 $P_{\text{STRUT}} = 25.26 \text{ K}$

- Sum the cable components to find the total cable force.

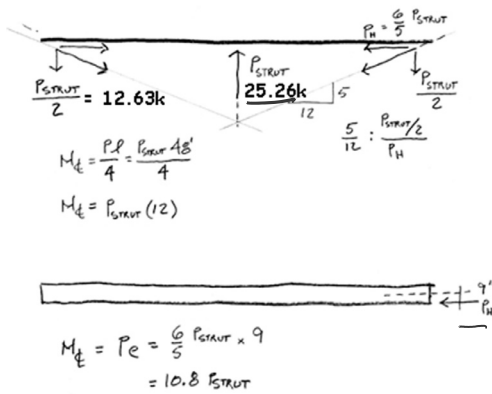
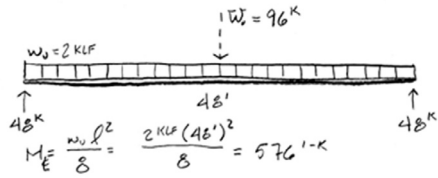
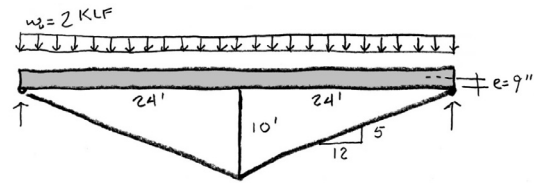


CABLE FORCE

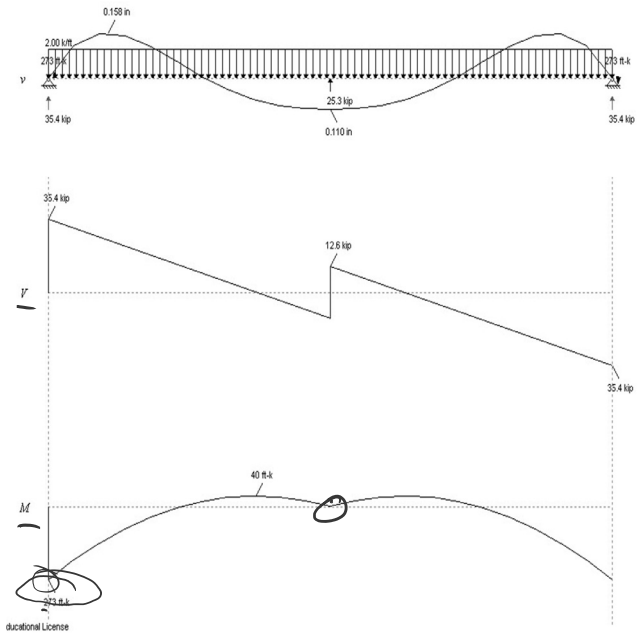
$$\sqrt{\left(\frac{6}{5} P_{\text{STRUT}}\right)^2 + \left(\frac{P_{\text{STRUT}}}{2}\right)^2} = 32.84 \text{ K}$$

# Cable Truss Analysis

7. find end reactions and calculate shear & moment

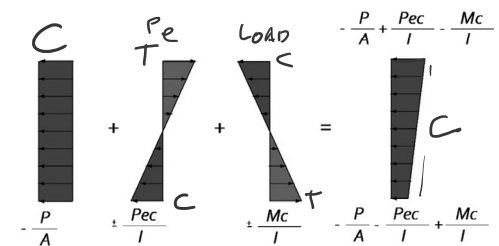
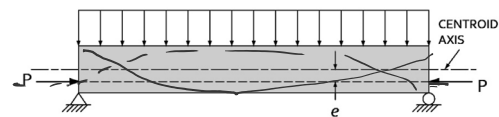


DR BEAM

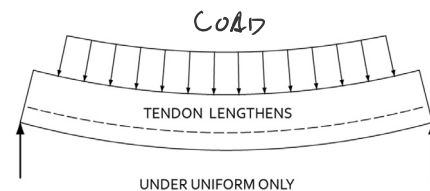
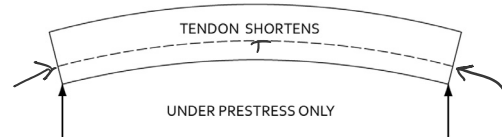


# Pre-stressed Concrete

- More concrete active in resisting moment
- Produces stiffer section with less material
- Lighter weight
- Longer spans possible
- Analysis by combined stress



$$f = -\frac{P}{A} \pm \frac{Pec}{I} \pm \frac{Mc}{I}$$



# Pre-stressed Concrete

## Steel:

high strength wires 250 or 270 ksi  
 wire diameter 0.105 – 0.276  
 used in strands of bundled wire  
 most common is 7 wire strand



## Concrete:

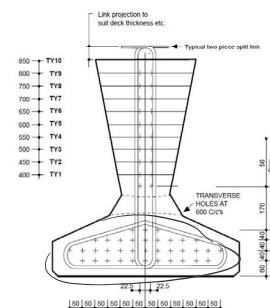
higher strength 5 – 10 ksi  
 to reduce creep and strain  
 reduced cracking  
 stiffer sections

Photo by Angelo Marasco

# Pre-stressed Concrete



Photo by MACRETE



Schlaich Bergermann & Partners

Neckarsulm, 1989



University of Michigan, TCAUP

Structures II

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Schlaich Bergermann & Partners

History of Hamburg Museum



University of Michigan, TCAUP

Structures II

Slide 15 of 20

# Stressed Membrane

Renaissance Center  
Entrance Pavilion  
Detroit 2004  
SOM

- Point supported glass
- “fish belly” cable truss bacing



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Structures II

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# Stressed Membrane

Renaissance Center  
Entrance Pavilion  
Detroit 2004  
SOM



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Structures II

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# Expo '67, Montreal

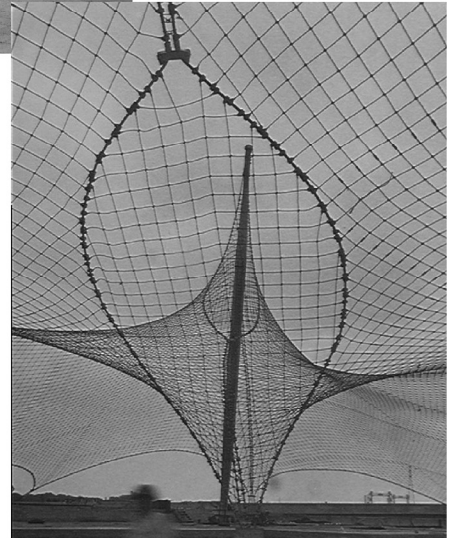
Frei Otto  
German Pavilion



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Structures II



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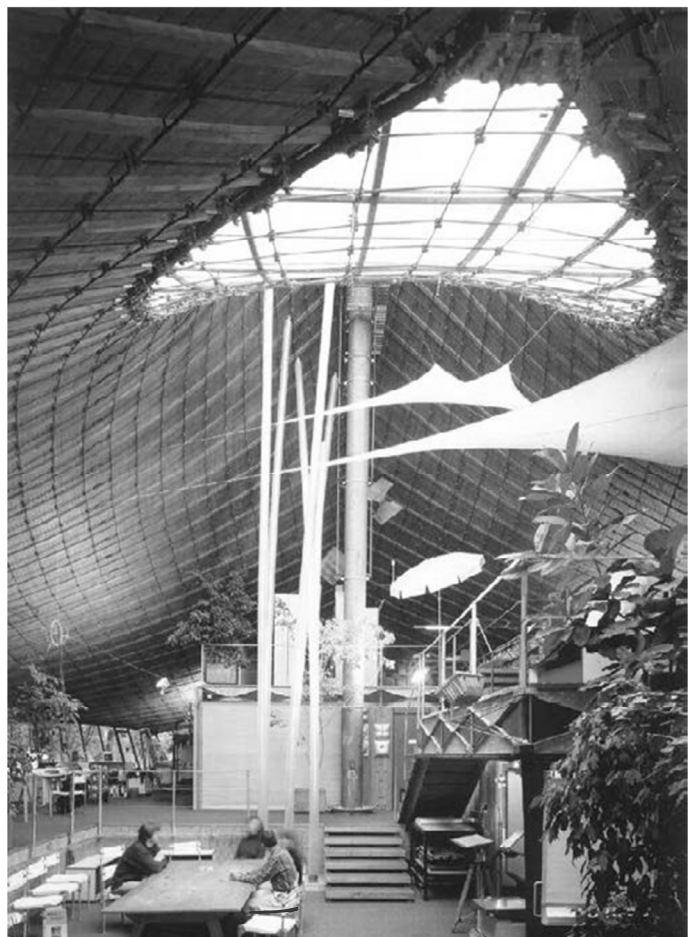
# Institute for Lightweight Structures – IL (now ILEK)

University of Stuttgart



Frei Otto, IL building, University of Stuttgart

University of Michigan, TCAUP



Structures II

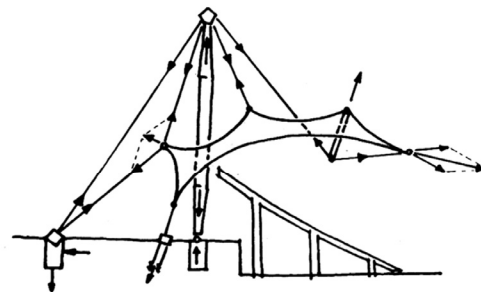
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# Stressed Membrane

Olympic Buildings, Munich 1972  
Eng. Otto, Leonhardt, Schlaich  
Arch: Behnisch

- Opposing curvature
- Stressed by anchors and masts



Frei Otto, Munich Soccer Stadium (from back)

# Stressed Membrane Olympic Stadium, Munich 1972



# Bundesgartenschau Köln Frei Otto

