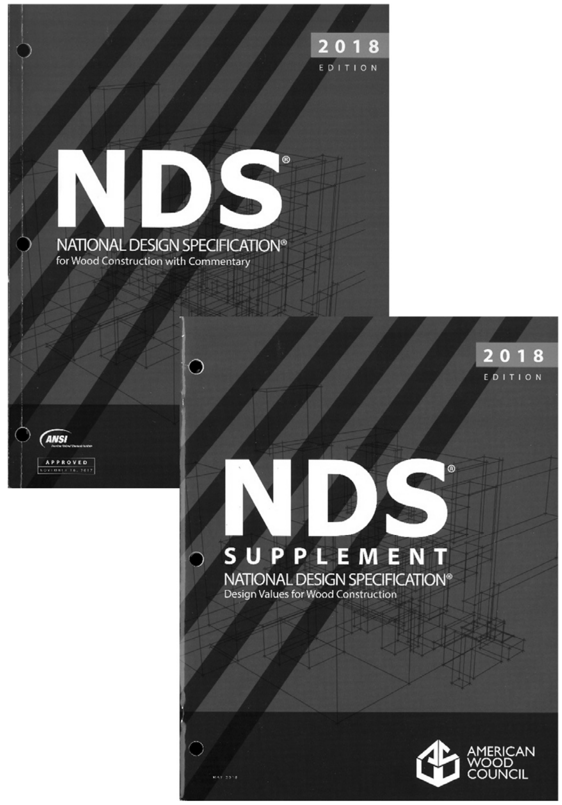
Architecture 324
Structures II

Wood Beam Analysis and Design

- ASD approach
- NDS criteria
- Wood Beam Analysis
- Wood Beam Design



Allowable Stress Design

Allowable Stress ≥ Actual Stress

Fb from the NDS Supplement

DESIGN VALUES FOR WOOD CONSTRUCTION – NDS SUPPLEMENT 35

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

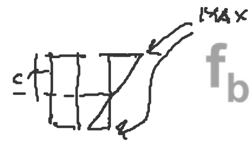
(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity[4] G	Grading Rules Agency
HEM-FIR										
Select Structural	2" & wider	1,400	925	150	405	1,500	1,600,000	580,000	0.43	WCLIB WWPA
No. 1 & Btr		1,100	725	150	405	1,350	1,500,000	550,000		
No. 1		975	625	150	405	1,350	1,500,000	550,000		
No. 2		850	525	150	405	1,300	1,300,000	470,000		
No. 3		500	300	150	405	725	1,200,000	440,000		
Stud	2" & wider	675	400	150	405	800	1,200,000	440,000		
Construction	2" - 4" wide	975	600	150	405	1,550	1,300,000	470,000		
Standard		550	325	150	405	1,300	1,200,000	440,000		
Utility		250	150	150	405	850	1,100,000	400,000		

Allowable Stress Design by NDS
Flexure

F_b' ≥ f_b



Allowable Flexure Stress F_b'

F_b from NDS Supplement tables determined by species and grade

F_b' = F_b (usage factors)

usage factors for flexure:
- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_L Beam Stability Factor
- C_F Size Factor
- C_{fu} Flat Use
- C_i Incising Factor
- C_r Repetitive Member Factor

≥

Actual Flexure Stress f_b

$f_b = Mc/I = M/S$

$S = I/c = bd^2/6$

$c = \frac{d}{2}$

Allowable Stress Design by NDS
Shear

F_v' ≥ f_v

Allowable Shear Stress Fv'

F_v from tables determined by species and grade

F_v' = F_v (usage factors)

usage factors for shear:
- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_i Incising Factor

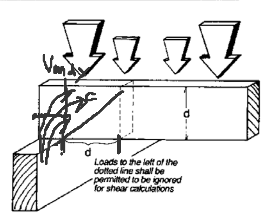
≥

Actual Shear Stress fv

$f_v = VQ / I\,b = 1.5\ V/A$

Can use V at d from support as maximum

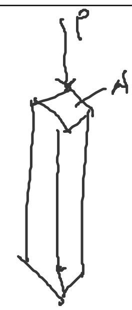
Shear at Supports



Allowable Stress Design by NDS
Compression

$F_c' \geq f_c$

Allowable Compression Stress F_c'	$\geq$	**Actual Compression Stress f_c**
F_c from NDS Supplement tables determined by species and grade $\mathbf{F_c'} = F_c$ (usage factors) usage factors for flexure: C_D Load Duration Factor C_M Moisture Factor C_t Temperature Factor C_F Size Factor C_i Incising Factor C_P Column Stability Factor		$f_c = P/A$



Adjustment Factors

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

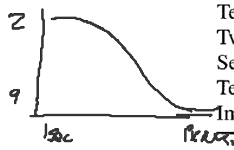
		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	-	-	-	K_F	ϕ_b	λ
$F_t' = F_t$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	-	-	K_F	ϕ_t	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	K_F	ϕ_v	λ
$F_{c\perp}' = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_b	K_F	ϕ_c	λ
$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	-	-	-
$E_{min}' = E_{min}$	x	-	C_M	C_t	-	-	-	C_i	-	-	C_T	-	K_F	ϕ_s	-

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

$F_b' = F_b (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$



Usage factors for flexure:

- **C_D** Load Duration Factor
- **C_t** Temperature Factor

Table 2.3.2 Frequently Used Load Duration Factors, C_D[1]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact[2]	2.0	Impact Load

(1) Actual stress due to (DL) ≤ (0.9) (Design value)
(2) Actual stress due to (DL+LL) ≤ (1.0) (Design value)
(3) Actual stress due to (DL+WL) ≤ (1.6) (Design value)
(4) Actual stress due to (DL+LL+SL) ≤ (1.15) (Design value)
(5) Actual stress due to (DL+LL+WL) ≤ (1.6) (Design value)
(6) Actual stress due to (DL+SL+WL) ≤ (1.6) (Design value)
(7) Actual stress due to (DL+LL+SL+WL) ≤ (1.6) (Design value)

Table 2.3.3 Temperature Factor, C_t

Reference Design Values	In-Service Moisture Conditions[1]	C_t T≤100°F	100°F<T≤125°F	125°F<T≤150°F
F_t, E, E_{min}	Wet or Dry	1.0	0.9	0.9
F_b, F_v, F_c, and $F_{c\perp}$	Dry	1.0	0.8	0.7
	Wet	1.0	0.7	0.5

1. Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joists, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively.

2018 NDS

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from NDS tables

$F_b' = F_b (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$

Usage factors for flexure:

- **C_M** Moisture Factor
- **C_F** Size Factor

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \le 1{,}150$ psi, $C_M = 1.0$
** when $(F_c)(C_F) \le 750$ psi, $C_M = 1.0$

Size Factors, C_F

Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from NDS tables

$F_b' = F_b (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$

Usage factors for flexure:

- **C_{fu}** Flat Use
- **C_r** Repetitive Member Factor

Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be permitted to be multiplied by the following flat use factors:

Flat Use Factors, C_{fu}

Width (depth)	Thickness (breadth) 2" & 3"	4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

Repetitive Member Factor, C_r

Bending design values, F_b, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, C_r = 1.15, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

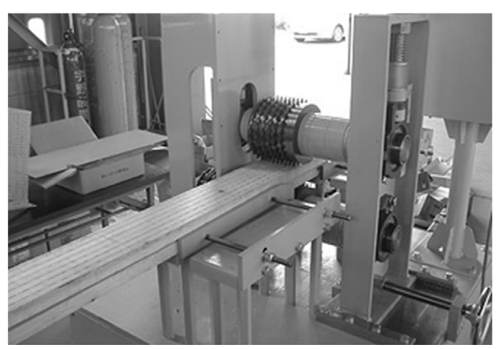
$F_b' = F_b (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$

Usage factors for flexure:

- **C_i** Incising Factor

Table 4.3.8 Incising Factors, C_i

Design Value	C_i
E, E_{min}	0.95
F_b, F_t, F_c, F_v	0.80
$F_{c\perp}$	1.00



Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

$$F_b' = F_b\ (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$$

Usage factors for flexure:

C_L Beam Stability Factor

3.3.3 Beam Stability Factor, C_L

3.3.3.1 When the depth of a bending member does not exceed its breadth, d ≤ b, no lateral support is required and C_L = 1.0.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, C_L = 1.0.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, C_L = 1.0.

2012 NDS

4.4.1 Stability of Bending Members

2x4 (a) d/b ≤ 2; no lateral support shall be required.

2x6-8 (b) 2 < d/b ≤ 4; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

2x10 (c) 4 < d/b ≤ 5; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x12 (d) 5 < d/b ≤ 6; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

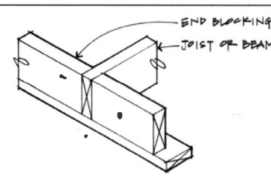
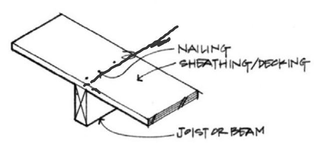
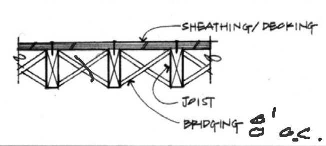
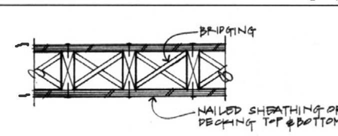
2x14 (e) 6 < d/b ≤ 7; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

C_L

C_L = 1.0
for depth/width ratio in
4.4.1 C_L = 1.0

Otherwise

C_L < 1.0
calculate factor using
section 3.3.3

Beam Depth/ Width Ratio	Type of Lateral Bracing Required	Example
2 to 1	None	
3 to 1 2x6 2x8	The ends of the beam should be held in position	
5 to 1 2x10	Hold compression edge in line (continuously)	
6 to 1 2x12	Diagonal bridging should be used	
7 to 1 2x14	Both edges of the beam should be held in line	

C_L Beam Stability Factor

In the case bracing provisions of 4.4.1 cannot be met, C_L is calculated using equation 3.3-6

The maximum allowable slenderness, R_B is 50

Table 3.3.3 Effective Length, ℓ_e, for Bending Members

Cantilever[1]	when $\ell_u/d < 7$		when $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=1.33 ℓ_u		ℓ_e=0.90 ℓ_u + 3d
Concentrated load at unsupported end	ℓ_e=1.87 ℓ_u		ℓ_e=1.44 ℓ_u + 3d
Single Span Beam[1,2]	when $\ell_u/d < 7$		when $\ell_u/d \geq 7$
Uniformly distributed load	ℓ_e=2.06 ℓ_u		ℓ_e=1.63 ℓ_u + 3d
Concentrated load at center with no intermediate lateral support	ℓ_e=1.80 ℓ_u		ℓ_e=1.37 ℓ_u + 3d
Concentrated load at center with lateral support at center		ℓ_e=1.11 ℓ_u	
Two equal concentrated loads at 1/3 points with lateral support at 1/3 points		ℓ_e=1.68 ℓ_u	
Three equal concentrated loads at 1/4 points with lateral support at 1/4 points		ℓ_e=1.54 ℓ_u	
Four equal concentrated loads at 1/5 points with lateral support at 1/5 points		ℓ_e=1.68 ℓ_u	
Five equal concentrated loads at 1/6 points with lateral support at 1/6 points		ℓ_e=1.73 ℓ_u	
Six equal concentrated loads at 1/7 points with lateral support at 1/7 points		ℓ_e=1.78 ℓ_u	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application		ℓ_e=1.84 ℓ_u	
Equal end moments		ℓ_e=1.84 ℓ_u	

1. For single span or cantilever bending members with loading conditions not specified in Table 3.3.3:
   $\ell_e = 2.06\,\ell_u$ when $\ell_u/d < 7$
   $\ell_e = 1.63\,\ell_u + 3d$ when $7 \leq \ell_u/d \leq 14.3$
   $\ell_e = 1.84\,\ell_u$ when $\ell_u/d > 14.3$
2. Multiple span applications shall be based on table values or engineering analysis.

3.3.3.6 The slenderness ratio, R_B, for bending members shall be calculated as follows:

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \qquad (3.3\text{-}5)$$

3.3.3.7 The slenderness ratio for bending members, R_B, shall not exceed 50.

3.3.3.8 The beam stability factor shall be calculated as follows:

$$C_L = \frac{1+(F_{bE}/F_b^*)}{1.9} - \sqrt{\left[\frac{1+(F_{bE}/F_b^*)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}} \qquad (3.3\text{-}6)$$

where:

F_b^* = reference bending design value multiplied by all applicable adjustment factors except C_{fu}, C_V (when $C_V \leq 1.0$), and C_L (see 2.3), psi

$$F_{bE} = \frac{1.20\,E_{min}'}{R_B^2}$$

Adjustment Factors for Shear

Allowable Flexure Stress F_v'

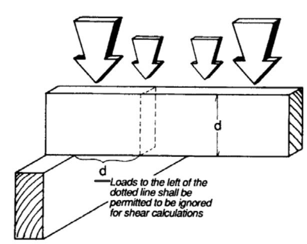
F_v from tables determined by species and grade

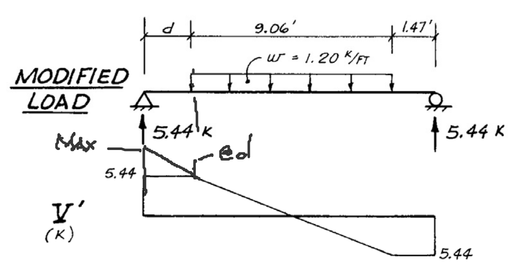
F_v' = F_v (usage factors)

Usage factors for shear:

- C_D Load Duration Factor
- C_M Moisture Factor
- C_t Temperature Factor
- C_i Incising Factor

Shear at Supports





Modified shear V' used to compute reduced shear f_v'

Analysis Procedure

Given: loading, member size, material and span.

Req'd: Safe or Unsafe (Pass/Fail)

1. Find Max Shear & Moment
   - Simple case – equations
   - Complex case - diagrams
2. Determine actual stresses
   - $f_b = M/S$
   - $f_v = 1.5\ V/A$
3. Determine allowable stresses
   - F_b and F_v (from NDS)
   - $F_b' = F_b$ (usage factors)
   - $F_v' = F_v$ (usage factors)
4. Check that actual ≤ allowable
   - $f_b \le F'_b$
   - $f_v \le F'_v$
5. Check deflection
6. Check bearing ($F_{c\perp} \ge$ Reaction/$A_{bearing}$)

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.²	X-X AXIS Section Modulus S_{xx} in.³	X-X AXIS Moment of Inertia I_{xx} in.⁴	Y-Y AXIS Section Modulus S_{yy} in.³	Y-Y AXIS Moment of Inertia I_{yy} in.⁴
Boards[1]						
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)						
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49

from *NDS 2012*

Analysis Example

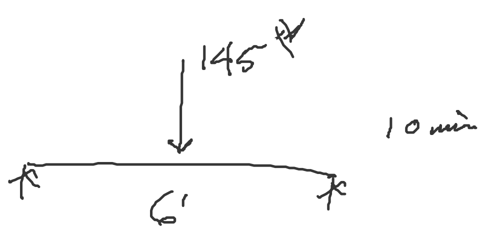
Example – Pass / Fail

Given: loading, member size, material and span.

Req'd: Safe or Unsafe?

GIVEN: LOAD = 145#
SPAN = 6'
SECTION = 2x4 (1.5 x 3.5)
Fb = 875 Fv = 135 psi

REQ'D: PASS OR FAIL



10 min

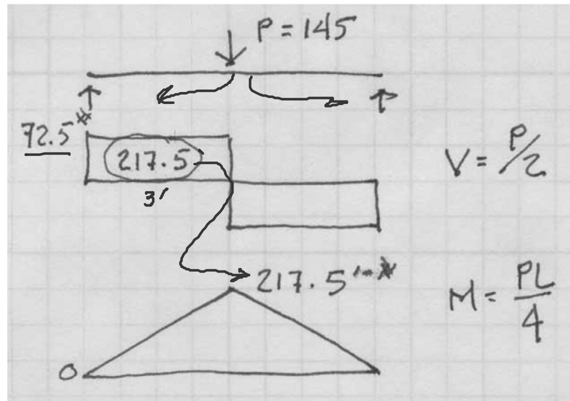
Analysis Example

GIVEN: LOAD = 145#
SPAN = 6'
SECTION = 2x4 (1.5 x 3.5)
F_b = 875 F_v = 135 psi

REQ'D: PASS OR FAIL

1. **Find Max Shear & Moment**
   - Simple cases – equations
   - Complex cases - diagrams

By Diagrams:



By equations:

$$V = P/2 = 145/2 = 72.5^{\#}$$

$$M_{\text{℄}} = \frac{PL}{4} = \frac{145(6')}{4}$$

$$M_{\text{℄}} = 217.5'^{\#} = 2610''^{-\#}$$

Analysis Example

2. **Determine actual stresses**
   - f_b = M/S
   - f_v = 1.5 V/A

$$f_b = M/S_x$$

$$S_x = \frac{bd^2}{6} = \frac{1.5(3.5)^2}{6} = 3.063\ in^3 \quad 2\times4$$

$$f_b = \frac{2610''^{-\#}}{3.063\ in^3} = 852\ psi$$

$$f_v = \frac{3}{2}\frac{V}{A} = 1.5\,(72.5^{\#}/5.25\ in^2) \quad 2\times4$$

$$f_v = 20.71\ psi$$

Species and Grade

S-P-F No.2

F_b = 875 psi
F_v = 135 psi

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific Gravity[4]	Grading Rules Agency
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity			
S-P-F		F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}	G	
SPRUCE-PINE-FIR										
Select Structural	2" & wider	1,250	700	135	425	1,400	1,500,000	550,000	0.42	NLGA
No. 1/ No. 2	2" & wider	875	450	135	425	1,150	1,400,000	510,000	0.42	NLGA
No. 3	2" & wider	500	250	135	425	650	1,200,000	440,000	0.42	NLGA
Stud	2" & wider	675	350	135	425	725	1,200,000	440,000	0.42	NLGA
Construction	2" - 4" wide	1,000	500	135	425	1,400	1,300,000	470,000	0.42	NLGA
Standard	2" - 4" wide	550	275	135	425	1,150	1,200,000	440,000	0.42	NLGA
Utility	2" - 4" wide	275	125	135	425	750	1,100,000	400,000	0.42	NLGA

Analysis Example

3. Determine allowable stresses

- F_b = 875 psi
- F_v = 135 psi

Determine factors:

- **CD = ?** 10 min
- CM = 1 DRY
- Ct = 1 < 100F
- CL = 1 2x4
- **CF = ?**
- Cfu = 1 NO
- Ci = 1 NO
- Cr = 1 NO

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	-	-	-	K_F	ϕ_b	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	K_F	ϕ_v	λ

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

$F_b' = F_b (C_D\, C_M\, C_t\, C_L\, C_F\, C_{fu}\, C_i\, C_r)$

Usage factors for flexure:

C_L Beam Stability Factor

3.3.3 Beam Stability Factor, C_L

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \le b$, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

2012 NDS

4.4.1 Stability of Bending Members

2x4 (a) $d/b \le 2$; no lateral support shall be required.

2x6-8 (b) $2 < d/b \le 4$; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

2x10 (c) $4 < d/b \le 5$; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x12 (d) $5 < d/b \le 6$; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x14 (e) $6 < d/b \le 7$; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

Analysis Example

C_D Load duration factor

Use 1.6 (10 minutes)

Table 2.3.2 Frequently Used Load Duration Factors, C_D[1]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact[2]	2.0	Impact Load

C_F Size factor

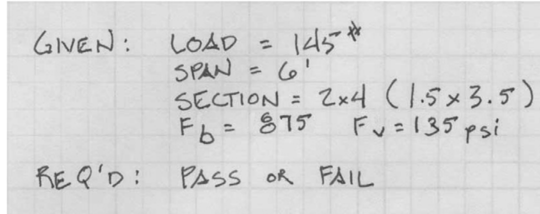
2 x 4
use 1.5

Size Factors, C_F

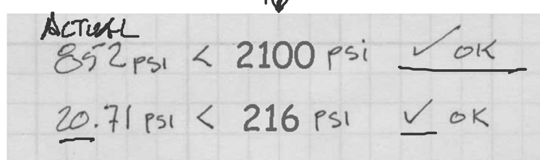
Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b Thickness (breadth) 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Analysis Example

3. **Determine allowable stresses**
   - $F_b' = F_b (C_D)(C_F)$
   - $F_b' = 875 (1.6)(1.5) = 2100$ psi
   - $F_v' = F_v (C_D)$
   - $F_v' = 135 (1.6) = 216$ psi



4. **Check that actual ≤ allowable**
   - $f_b < F'_b$
   - $f_v < F'_v$



5. **Check deflection**
6. **Check bearing** ($F_{c\perp} \geq R/A_b$)

Analysis Procedure

Given: member size, material and span.
Req'd: Max. Safe Load (**capacity**)

1. Assume f = F
   - Maximum actual = allowable stress
2. Solve stress equations for force
   - $M = F_b S$
   - $V = 0.66 F_v A$
3. Use maximum moment to find loads
   - Back calculate a load from moment
   - Assumes moment controls
4. Check Shear
   - Use load found is step 3 to check shear stress.
   - If it fails (fv > F'v), then find load based on shear.
5. Check deflection
6. Check bearing

Table 4A (Cont.) **Reference Design Values for Visual (2" - 4" thick)[1,2,3]**
(All species except Southern Pine—see duration and dry service conditions. See NDS adjustment factors.)

USE WITH TABLE 4A AD

Species and commercial grade	Size classification	Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v
SPRUCE-PINE-FIR				
Select Structural	2" & wider	1,250	700	135
No. 1/ No. 2	2" & wider	875	450	135
No. 3	2" & wider	500	250	135
Stud	2" & wider	675	350	135
Construction	2" - 4" wide	1,000	500	135
Standard	2" - 4" wide	550	275	135
Utility	2" - 4" wide	275	125	135

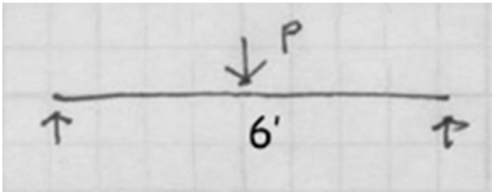
from *NDS 2012*

Analysis Example

Given: member size, material and span.
Req'd: Max. Safe Load (capacity)

GIVEN : SPAN = 6' P @ ℄
SECTION = 2x4 (1.5 x 3.5)
F_b = 875 psi F_v = 135 psi

REQ'D : MAXIMUM LOAD P



1. **Assume f = F'**
   - Maximum actual = allowable stress

$$f_b = F'_b = \underset{F_b}{875}\,(\underset{C_D}{1.6})(\underset{C_F}{1.5})$$
$$F'_b = 2100 \text{ psi}$$

$$S_x = 3.063 \text{ in}^3$$

2. **Solve stress equation for moment**
   - M = F'$_b$ S (i.e. moment capacity)

$$M_{℄} = F'_b S_x = 2100(3.063)$$
$$= 6432.3 \text{ "-\#}$$
$$= 536 \text{ '-\#}$$

Analysis Example (cont.)

3. **Use maximum forces to find loads**
   - Back calculate a maximum load from moment capacity

$$M_{℄} = PL/4$$
$$P = M_{℄}\, 4/L$$
$$P = 536(4)/6$$
$$P = 357\#$$

4. **Check shear**
   - Check shear for load capacity from step 3.
   - Use P from moment to find Vmax
   - Check that fv < Fv'

$$V_{max} = P/2 = 357/2 = 178.6\#$$
$$f_v = \frac{3}{2}\frac{V}{A} = 1.5\,\frac{178.6}{5.25} = 51 \text{ psi}$$
$$51 \text{ psi} < 180 \quad \checkmark \text{ OK}$$

4. **Check deflection** (serviceability)
5. **Check bearing** (serviceability)

Analysis Example

Given: loading, <u>member size</u>, material and span.

Req'd: Safe or Unsafe

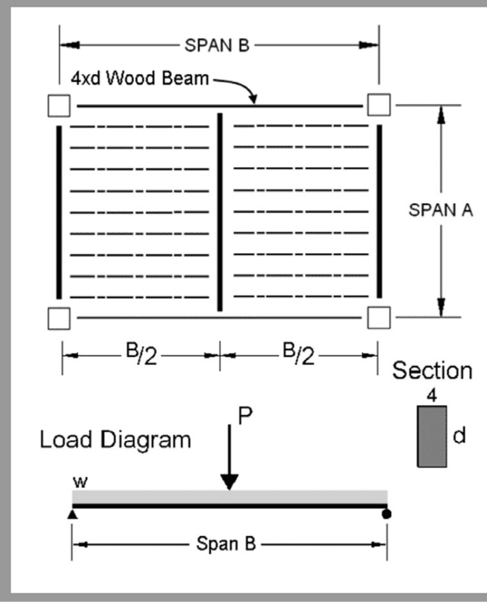
Logged in as: PvB

3. Wood Beam Analysis

Analyze the given 4x dimensioned lumber beam to determine if it passes or fails the NDS code criteria. The beam carries both dead and live floor load plus its own selfweight(CD = 1.0). Check the actual shear and bending stresses against the factored allowable stresses including all applicable factors from the NDS. Load duration is based on the live load. Assume normal temperature, and no incising (Ct = Ci = 1.0). Find the beam selfweight including the given moisture content. The beam is braced at the ends and the C.L. (meets criteria in 4.4.1) so CL = 1.0.

DATASET: 1 -2- -3-

Wood Species	HEM-FIR
Wood Grade	Select Structural
Span A	16 FT
Span B	12 FT
Nominal Depth of Beam, d	12 IN
Moisture Content, m.c.	15 %
Floor DL	7 PSF
Floor LL	35 PSF



Analysis Example

Find Specific Gravity for Hem-Fir

- (from NDS)

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific Gravity[4]	Grading Rules Agency
		Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	E_{min}	G	
HEM-FIR										
Select Structural	2" & wider	1,400	925	150	405	1,500	1,600,000	580,000	0.43	WCLIB WWPA
No. 1 & Btr		1,100	725	150	405	1,350	1,500,000	550,000		
No. 1		975	625	150	405	1,350	1,500,000	550,000		
No. 2		850	525	150	405	1,300	1,300,000	470,000		
No. 3		500	300	150	405	725	1,200,000	440,000		
Stud	2" & wider	675	400	150	405	800	1,200,000	440,000		
Construction	2" - 4" wide	975	600	150	405	1,550	1,300,000	470,000		
Standard		550	325	150	405	1,300	1,200,000	440,000		
Utility		250	150	150	405	850	1,100,000	400,000		

Analysis Example

Section Properties:

4 x 12 (3.5” x 11.25”)

Area = 39.38 in^2

Sx = 73.83 in^3

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}
Boards[1]						
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)						
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49

Analysis Example

Determine Loading

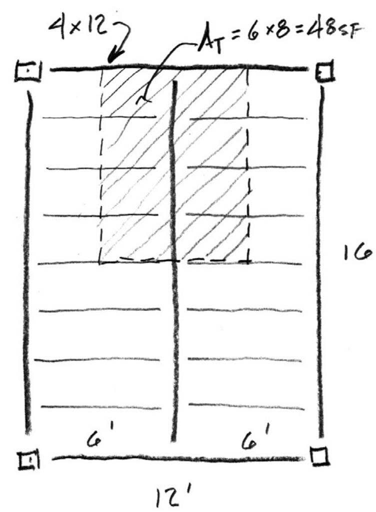
- Find Tributary area, A_T
  6’ x 8’ = 48 SF

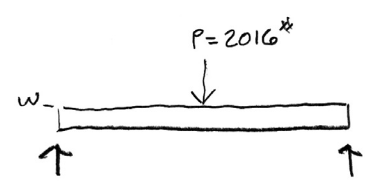
- Determine member selfweight (w)

DL = 7 PSF
LL = 35 PSF

TOTAL = 42 PSF

P = A_T × PSF
= 48 × 42 = 2016#





Analysis Example

Selfweight of member:

Density at 0 m.c. = 62.4 x G (dry)
62.4 x 0.43 = 26.8 PCF

m.c. = 15% (given)

To include m.c. use NDS formula.

w (PLF) = D (PCF) x Area (IN^2)/144

The following formula shall be used to determine the density in lbs/ft³ of wood:

$$\text{density} = 62.4\left[\frac{G}{1+G(0.009)(m.c.)}\right]\left[1+\frac{m.c.}{100}\right]$$

where:

G = specific gravity of wood

m.c. = moisture content of wood, %

$$D = 62.4\left[\frac{0.43}{1+0.43(0.009)(15)}\right]\left[1+\frac{15}{100}\right]$$

$$25.35 \times 1.15 = 29.16 \text{ PCF}$$

$$w = PLF = D\,\frac{AREA}{144} = 29.16\,\frac{39.38}{144}$$

$$w = 7.975 \text{ PLF}$$

Analysis Example

Determine Beam Forces

by superposition equations or by diagrams

$$R = \frac{w\ell}{2} + \frac{P}{2}$$

$$\frac{7.975(12)}{2} + \frac{2016}{2}$$

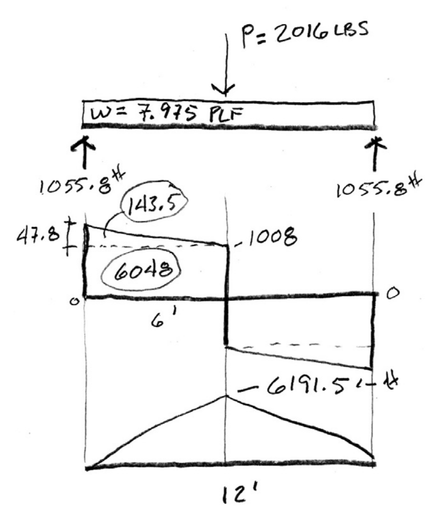
$$47.85 + 1008 = 1055.8^{\#}$$

$$V_{max} = R$$

$$M_{\mathbb{C}} = \frac{w\ell^2}{8} + \frac{PL}{4}$$

$$\frac{7.975(12)^2}{8} + \frac{2016(12)}{4}$$

$$143.5 + 6048 = 6191.5^{\,'-\#}$$



Analysis Example

Determine actual stresses

- $f_b = M/S$
- $f_v = 1.5\ V/A$

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.²	X-X AXIS Section Modulus S_{xx} in.³	X-X AXIS Moment of Inertia I_{xx} in.⁴	Y-Y AXIS Section Modulus S_{yy} in.³	Y-Y AXIS Moment of Inertia I_{yy} in.⁴
Boards[1]						
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)						
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49

ACTUAL STRESS:

$$f_b = M/S_x = \frac{6191.5(12)}{73.83} = 1006.3 \text{ PSI}$$

$$f_v = \frac{3}{2}\frac{V}{A} = 1.5\ \frac{1055.8}{39.38} = 40.22 \text{ PSI}$$

Analysis Example

Determine allowable stresses

- F_b and F_v (from NDS)

DESIGN VALUES FOR WOOD CONSTRUCTION – NDS SUPPLEMENT 35

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)**[1,2,3]

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity[4] G	Grading Rules Agency
HEM-FIR										
Select Structural	2" & wider	1,400	925	150	405	1,500	1,600,000	580,000	0.43	WCLIB WWPA
No. 1 & Btr		1,100	725	150	405	1,350	1,500,000	550,000		
No. 1		975	625	150	405	1,350	1,500,000	550,000		
No. 2		850	525	150	405	1,300	1,300,000	470,000		
No. 3		500	300	150	405	725	1,200,000	440,000		
Stud	2" & wider	675	400	150	405	800	1,200,000	440,000		
Construction	2" - 4" wide	975	600	150	405	1,550	1,300,000	470,000		
Standard		550	325	150	405	1,300	1,200,000	440,000		
Utility		250	150	150	405	850	1,100,000	400,000		

Analysis Example

3. Determine allowable stresses

- F_b = 1400 psi
- F_v = 150 psi

Determine factors:

CD =
CM =
Ct =
CL =
CF =
Cfu =
Ci =
Cr =

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	-	-	-	K_F	ϕ_b	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	K_F	ϕ_v	λ

Analysis Example

Determine allowable stresses
M.C. = 15% size: 4x12

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}
Boards[1]						
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)						
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547
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2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04
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3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \le 1{,}150$ psi, $C_M = 1.0$
** when $(F_c)(C_F) \le 750$ psi, $C_M = 1.0$

Size Factors, C_F

		F_b	
		Thickness (breadth)	
Grades	Width (depth)	2" & 3"	4"
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5
	5"	1.4	1.4
	6"	1.3	1.3
	8"	1.2	1.3
	10"	1.1	1.2
	12"	1.0	1.1
	14" & wider	0.9	1.0
Stud	2", 3", & 4"	1.1	1.1
	5" & 6"	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design v	
Construction, Standard	2", 3", & 4"	1.0	1.0
Utility	4"	1.0	1.0
	2" & 3"	0.4	—

Adjustment Factors

Allowable Flexure Stress F_b'

F_b from tables determined by species and grade

$F_b' = F_b\ (C_D\ C_M\ C_t\ C_L\ C_F\ C_{fu}\ C_i\ C_r)$

b/d = 3.5 / 11.25 = 3.11 (case b)

Assuming ends are braced, CL = 1.0

3.3.3 Beam Stability Factor, C_L

3.3.3.1 When the depth of a bending member does not exceed its breadth, d ≤ b, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

4.4.1 Stability of Bending Members

2x4 (a) d/b ≤ 2; no lateral support shall be required.

2x6-8 (b) 2 < d/b ≤ 4; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

2x10 (c) 4 < d/b ≤ 5; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x12 (d) 5 < d/b ≤ 6; bridging, full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

2x14 (e) 6 < d/b ≤ 7; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

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Analysis Example

3. Determine allowable stresses
    - $F_b' = F_b$ (usage factors)

F_b

$C_D = 1.0$ (LIVE LOAD)

$C_{M_b} = 1.0$ 15% < 19% (NDS SUP. P. 32)

$C_t = 1.0$ TEMP < 100°

$C_L = 1.0$ BRACED PER NDS 4.4.1

$C_F = 1.1$ FOR 4×12 (NDS SUP. P 32)

$C_{fu} = 1.0$ (NOT) (NDS SUP P32)

$C_i = 1.0$ (NOT) (NDS P29-30)

$C_r = 1.0$ (NOT) (NDS SUP P32)

$F'_b = 1400(1.1) = 1540$ PSI

ACTUAL STRESS:

$$f_b = M/S_x = \frac{6191.5(12)}{73.83} = 1006.3 \text{ PSI}$$

Analysis Example

3. Determine allowable stresses
   - $F_v' = F_v$ (usage factors)

F_v

$C_D = 1.0$

$C_{M_v} = 1.0$

$C_t = 1.0$

$C_i = 1.0$

$F'_v = 150(1.0) = 150$ PSI

$f_v = \frac{3}{2}\frac{V}{A} = 1.5\,\frac{1055.8}{39.38} = 40.22$ PSI

Analysis Example

Check that actual ≤ allowable
- $f_b \le F'_b$
- $f_v \le F'_v$

$f_b < F'_b$

1006.3 PSI < 1540 ∴ ✓OK

$f_v < F'_v$

40.22 PSI < 150 ∴ ✓OK

Check deflection

Check bearing ($F_{c\perp} \ge \text{Reaction}/A_{bearing}$)

Design Procedure

Given: load, wood, span
Req'd: member size

1. **Find Max Shear & Moment**
   - Simple case – equations
   - Complex case - diagrams
2. **Determine allowable stresses**
3. **Solve $S=M/F_b'$**
4. **Choose a section from Table 1B**
   - Revise DL and F_b'
5. **Check shear stress**
   - First for V max (easier)
   - If that fails try V at d distance from support.
   - If the section still fails, choose a new section with $A=1.5V/F_v'$
6. **Check deflection**
7. **Check bearing**

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}
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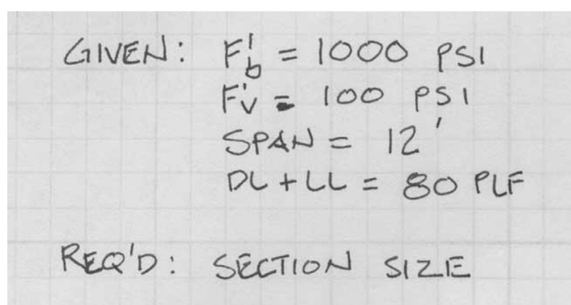
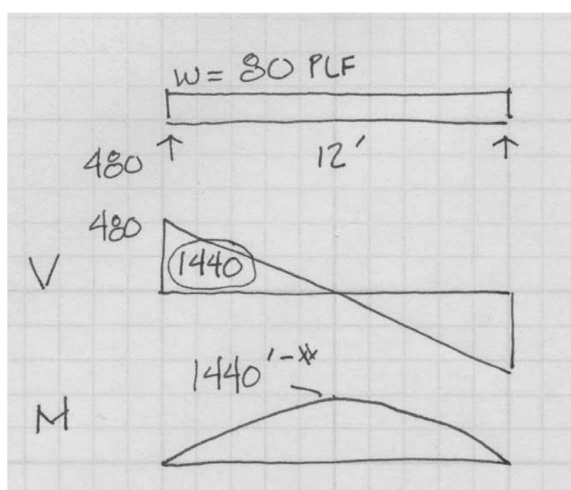
Design Example

Given: load, wood, span
Req'd: member size



1. **Find Max Shear & Moment**
   - Simple case – equations
   - Complex case - diagrams



Design Example

2. **Determine allowable stresses**
   (given in this example)
   F'_b = 1000 psi
   F'_v = 100 psi

$$F'_b = M/S_x \qquad S_x = M/F'_b$$

$$S_x = \frac{1440(12)}{1000} = 17.28\ in^3$$

3. **Solve $S=M/F_b'$**

4. **Choose a section from S table**
   - Revise DL and F_b'

2 x 10 $\quad S_x = 21.39 > 17.28$ ✓
$A = 13.88\ in^2$

5. **Check shear stress**
   - First for V max (easier)
   - If that fails try V at d distance (remove load d from support)
   - If the section still fails, choose a new section with $A=1.5V/F_v'$

$$f_v = \frac{3}{2}\frac{V}{A} = \frac{1.5(480^{\#})}{13.88\ in^2} = 51.87$$

51.87 psi < 100 psi ✓ OK

6. **Check deflection**
7. **Check bearing**

Design Example

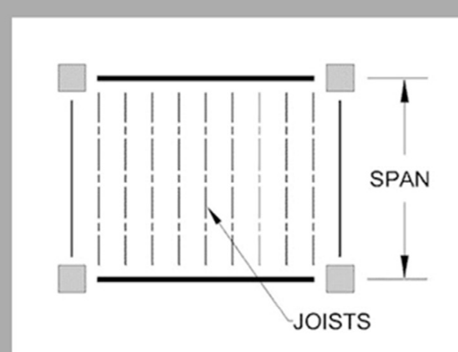
Given: load, wood, span
Req'd: member size

4. Wood Beam Design

Design a 2x dimensioned lumber floor joist to carry the given dead + live floor load. Assume the floor meets conditions of 4.4.1 so CL=1.0. Also Ct, Cfu, and Ci = 1.0. Find the short term deflection of your chosen beam under live load only (100% LL is sustained). Compare your LL deflection with the code limit of L/360.

DATASET: 1 -2- -3-

Wood Species	HEM-FIR
Wood Grade	No.1
Span	20 FT
Joist Spacing, o.c.	12 IN
Moisture Content, m.c.	15 %
Floor DL	7 PSF
Floor LL	35 PSF



Design Example

Determine allowable stresses

- F_b and F_v (from NDS)

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Design Example

Determine allowable stresses

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4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49

Table 4A Adjustment Factors

Repetitive Member Factor, C_r

Bending design values, F_b, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r = 1.15$, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \leq 1{,}150$ psi, $C_M = 1.0$
** when $(F_c)(C_F) \leq 750$ psi, $C_M = 1.0$

Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be multiplied by the following flat use factors:

Flat Use Factors, C_{fu}

Width (depth)	Thickness (breadth) 2" & 3"	Thickness (breadth) 4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

NOTE

To facilitate the use of Table 4A, shading has been employed to distinguish design values based on a 4" nominal width (Construction, Standard, and Utility grades) or a 6" nominal width (Stud grade) from design values based on a 12" nominal width (Select Structural, No.1 & Btr, No.1, No.2, and No.3 grades).

Size Factor, C_F

Tabulated bending, tension, and compression parallel to grain design values for dimension lumber 2" to 4" thick shall be multiplied by the following size factors:

Size Factors, C_F

Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b Thickness (breadth) 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Design Example

Determine allowable stresses.

Since the size is not known you have to skip C_F (or make a guess).

$$F_b' = F_b(\text{FACTORS})$$

$$= 975(1.0 \times 1.15 \times 1.0 \times C_F?) \approx 1121 \text{ psi}$$

(C_D, C_r, C_M)

$$F_v' = F_v(C_D, C_M, C_t, C_i)$$

$$= 150(1.0 \times 1.0 \times 1.0 \times 1.0) = 150 \text{ psi}$$

Design Example

Determine Moment from Loading

First find the uniform beam load, w, from the floor loading.

$$w = (\text{PSF}) \frac{\text{O.C.}}{12} = \text{PLF}$$

$$(7+35)\frac{12}{12} = 42 \text{ PLF}$$

With the beam loading, calculate the maximum moment.

$$M = \frac{w l^2}{8} = \frac{42(20')^2}{8} = 2100 \text{ 1-\#}$$

Design Example

Estimate the Required Section Modulus.

$$S_x = \frac{M}{F'_b} = \frac{2100(12)}{1121} = 22.47\ in^3$$

Compare this required Sx to the actual Sx of available sections in NDS Table 1B. Remember CF will be multiplied which may make some pass which at first fail.

FROM TABLE 1B (NDS)

	S_x	
2×10	21.39	($C_F = 1.1$) MIGHT WORK
2×12	31.64	($C_F = 1.0$)

Design Example

Choose a section and test it (by analysis with all factors including C_F)

TRY 2×10 $C_F = 1.1$

$$F'_b = 975(1.15\ \ 1.1) = 1233.3\ psi$$

$$f_b = M/S_x = \frac{2100\ (12)}{21.39} = 1178\ psi < 1233\ psi \ \checkmark OK$$

$$f_v = \frac{3}{2}\frac{V}{A} = \frac{1.5\ (420)}{13.88} = 45.39\ psi < 160\ psi \ \checkmark OK$$

∴ USE 2×10

Design Example

Check Deflection

In this case LL only against code limit of L/360

LL = 35 PSF = 35 PLF

$$\Delta_{LL} = \frac{5 w \ell^4}{384 EI} = \frac{5(35)(20)^4(1728)}{384(1500000)(98.93)} = 0.849''$$

$$\Delta_{LIMIT} \quad \frac{L}{360} = \frac{20'(12)}{360} = 0.667''$$

$0.849 > 0.667 \therefore$ FAILS