Architecture 324 Structures II

Wood Columns

- Failure Modes
- Euler Equation
- · End Conditions and Lateral Bracing
- · Analysis of Wood Columns
- Design of Wood Columns



Solemar, Bad Dürrheim Klaus Linkwitz, 1987

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Failure Modes

FLEXURE

AXIAL

Strength

$$f_b = \frac{Mc}{I} \qquad f_v = \frac{VQ}{Ib}$$

$$f_c = \frac{P}{A}$$

Stability

$$C_{L} = \frac{1 + \left(F_{bE}/F_{b}^{*}\right)}{1.9} - \sqrt{\left[\frac{1 + \left(F_{bE}/F_{b}^{*}\right)}{1.9}\right]^{2} - \frac{F_{bE}/F_{b}^{*}}{0.95}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}} \\ C_{P} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}{c}}$$

$$C_{p} = \frac{1 + \left(F_{cE} \middle/ F_{c}^{*}\right)}{2c} - \sqrt{\left\lceil \frac{1 + \left(F_{cE} \middle/ F_{c}^{*}\right)}{2c} \right\rceil^{2} - \frac{F_{cE} \middle/ F_{c}^{*}}{c}}$$

Serviceability

Deflection

Bearing (crushing limit)

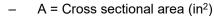
Leonhard Euler (1707 - 1783)

Euler Buckling (elastic buckling)

$$P_{Cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

$$I = Ar^2$$



E = Modulus of elasticity of the material (lb/in²)

K = Stiffness (curvature mode) factor

L = Column length between pinned ends (in.)

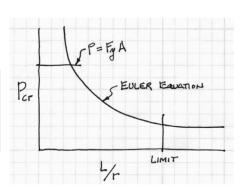
r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$
 $F_{cE} = \frac{0.822 \text{ E'}_{\min}}{\left(\frac{le}{d}\right)^2}$



portrait by Emanuel Handmann, 1753



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Failure Mode - Strength

Short Columns – fail by crushing

$$f_c = \frac{P}{A} \le F_c \qquad A = \frac{P}{F_c}$$

$$A = \frac{P}{F_c}$$

f_c = Actual compressive stress

A = Cross-sectional area of column (in²)

P = Load on the column

F_c = Allowable compressive stress per codes



Failure Modes - Stability

Long Columns - fail by buckling

Traditional Euler

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

- E = Modulus of elasticity of the column material (psi)
- K = Stiffness (curvature mode) factor
- L = Column length between ends (inches)
- r = radius of gyration = $\sqrt{I/A}$ (inches)

NDS Equation



– le = Ke l, (inches)

 $F_{cE} = \frac{0.822E'_{min}}{\left(\frac{l_e}{d}\right)^2}$

- d (inches)
- 0.822 = $\pi^2/12$

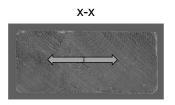
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Slenderness Ratios:

The larger ratio will govern. Try to balance for efficiency.

Slenderness Ratio le/d

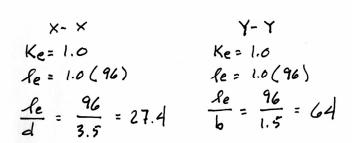
Slenderness Limited to < 50

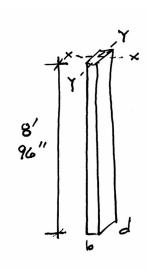


$$d = 3.5$$



$$b = 1.5$$





End Support Conditions

K_e is a constant based on the end conditions

 ℓ is the actual length

 $\ell_{\rm e}$ is the effective length (curved part)

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Allowable Flexure Stress F_c'

Actual Flexure Stress f_b

 ${\sf F}_{\sf c}$ from tables determined by species and grade

 $f_c = P/A$

 F_c ' = F_c (adjustment factors)

 $F_c' \geq f_c$

Table 4A Base Design Values for Visually Graded Dimension Lumber (2"-4" thick)^{1,2} (Cont.) (All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load)

(All species except Southern Pine — see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

			Design values in pounds per square inch (psi)					
Species and commercial grade	Size classification	Bending F _b	Tension parallel to grain F _t	Shear parallel to grain F _v	Compression perpendicular to grain F _{c⊥}	Compression parallel to grain F _c	Modulus of Elasticity E	Grading Rules Agency
EASTERN HEMLOCK-BA	LSAM FIR							
Select Structural		1250	575	140	335	1200	1,200,000	
No.1		775	350	140	335	1000	1,100,000	
No.2	2" & wider	575	275	140	335	825	1,100,000	
No.3		350	150	140	335	475	900,000	NELMA
Stud Exist of an incorporate	2" & wider	450	200	140	335	525	900,000	NSLB
Construction		675	300	140	335	1050	1,000,000	
Standard	2"-4" wide	375	175	140	335	850	900,000	
Utility		175	75	140	335	550	800,000	

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		ASD only				AS	SD an	d LRI	FD				LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Suffness Factor	Bearing Area Factor	4 Format Conversion Factor	- Resistance Factor	Time Effect Factor
$F_b' = F_b$	Х	C_{D}	$C_{\mathbf{M}}$	C_{t}	C_{L}	C_{F}	C_{fu}	C_i	C_{r}	-	-	-	2.54	0.85	λ
$F_t' = F_t$	х	C_{D}	$C_{\mathbf{M}}$	C_t	-	C_{F}	-	C_i	-	-	-	-	2.70	0.80	λ
$F_{v} = F_{v}$	х	C_{D}	C _M	C_t	-	-	-	C_{i}	-	-	-	-	2.88	0.75	λ
$F_c' = F_c$	х	C_{D}	C_{M}	Ct	-	C_{F}	-	C_i	-	Ср	-	-	2.40	0.90	λ
$F_{c\perp} = F_{c\perp}$	Х	-	C_{M}	C_{t}	-	-	-	$C_{\mathbf{i}}$	-	-	-	Сь	1.67	0.90	-
E' = E	Х	-	C_{M}	C_{t}	-	-	-	C_{i}	-	-	-	-	-	-	-
$E_{\min} = E_{\min}$	Х	-	C_{M}	C_{t}	-	-	-	C_{i}	-	-	C_{T}	-	1.76	0.85	-

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Allowable Flexure Stress F_c'

 $\boldsymbol{F}_{\mathrm{c}}$ from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression:

C_D Load Duration Factor

C_t Temperature Factor

Table 2.3.2 Frequently Used Load Duration Factors, C_D¹

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact ²	2.0	Impact Load

Table 2.3.3	Temperature	Factor,	Cŧ
-------------	-------------	---------	----

Reference Design Values	In-Service Moisture		Ct	
v andes	Conditions ¹	T≤100°F	100°F <t≤125°f< th=""><th>125°F<t≤150°f< th=""></t≤150°f<></th></t≤125°f<>	125°F <t≤150°f< th=""></t≤150°f<>
F _t , E, E _{min}	Wet or Dry	1.0	0.9	0.9
F F F 1F	Dry	1.0	0.8	0.7
F_b , F_v , F_c , and $F_{c\perp}$	Wet	1.0	0.7	0.5

 Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joists, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively.

(1) Actual stress due	
to (DL)	\leq (0.9) (Design value)
(2) Actual stress due	
to (DL+LL)	\leq (1.0) (Design value)
(3) Actual stress due	
to (DL+WL)	\leq (1.6) (Design value)
(4) Actual stress due	
to (DL+LL+SL)	\leq (1.15) (Design value)
(5) Actual stress due	
to (DL+LL+WL)	\leq (1.6) (Design value)
(6) Actual stress due	
to (DL+SL+WL)	\leq (1.6) (Design value)
(7) Actual stress due	
to (DL+LL+SL+WL)	\leq (1.6) (Design value)

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Allowable Flexure Stress F_c' (For Dimensioned Lumber)

F_c from tables determined by species and grade

 $F_c' = F_c (C_D C_M C_t C_F C_i C_P)$

Adjustment factors for compression:

C_M Moisture Factor C_F Size Factor

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

	F_{b}	\mathbf{F}_{t}	$F_{\rm v}$	$F_{c\perp} \\$	F_c	\boldsymbol{E} and $\boldsymbol{E}_{\text{min}}$
0	.85*	1.0	0.97	0.67	0.8**	0.9

^{*} when $(F_b)(C_F) \le 1,150 \text{ psi}, C_M = 1.0$

Wet Service Factor, C_M

Size Factors, C_F

		Size Pactors,	∪ _F	1	
		F	b	F _t	F _c
		Thickness	(breadth)		
Grades	Width (depth)	2" & 3"	4"		
	2", 3", & 4"	1.5	1.5	1.5	1.15
Select	5"	1.4	1.4	1.4	1.1
Structural,	6"	1.3	1.3	1.3	1.1
No.1 & Btr,	8"	1.2	1.3	1.2	1.05
No.1, No.2,	10"	1.1	1.2	1.1	1.0
No.3	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
Stud	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade	tabulated design	values and size fac	ors
Construction,	2", 3", & 4"	1.0	1.0	1.0	1.0
Standard					
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	_	0.4	0.6

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Allowable Flexure Stress F_c' (For Timbers)

F_c from tables determined by species and grade

 $F_c' = F_c (C_D C_M C_t C_F C_i C_P)$

Adjustment factors for compression:

C_M Moisture Factor C_F Size Factor

Size Factor, C_F

When visually graded timbers are subjected to loads applied to the narrow face, tabulated design values shall be multiplied by the following size factors:

Size Factors, CF

Depth	F _b	F_t	Fe
d > 12"	$(12/d)^{1/9}$	1.0	1.0
$d \leq 12\text{"}$	1.0	1.0	1.0

Wet Service Factor, C_M

When timbers are used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table (for Southern Pine and Mixed Southern Pine, use tabulated design values without further adjustment):

Wet Service Factors, C_M

F_b	F _t	$F_{\rm v}$	$F_{c\perp}$	F_c	E and E _{min}	
1.00	1.00	1.00	0.67	0.91	1.00	

^{**} when $(F_c)(C_F) \le 750 \text{ psi}, C_M = 1.0$

Allowable Flexure Stress F_c'

F_c from tables determined by species and grade

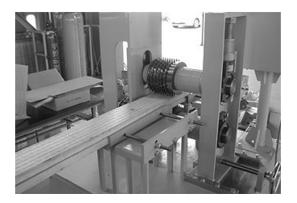
$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

 $\begin{array}{c} \text{Adjustment factors for compression}: \\ \text{C_{i} Incising Factor} \end{array}$

Table 4.3.8 Incising Factors, C,

Design Value	C_{i}	
E, E _{min}	0.95	
F_b, F_t, F_c, F_v	0.80	
$F_{c\perp}$	1.00	





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Allowable Flexure Stress F_c'

F_c from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

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3.7 Solid Columns

3.7.1 Column Stability Factor, C,

- 3.7.1.1 When a compression member is supported throughout its length to prevent lateral displacement in all directions, $C_P = 1.0$.
- 3.7.1.2 The effective column length, ℓ_e , for a solid column shall be determined in accordance with principles of engineering mechanics. One method for determining effective column length, when end-fixity conditions are known, is to multiply actual column length by the appropriate effective length factor specified in Appendix G, $\ell_e = (K_e)(\ell)$.
- 3.7.1.3 For solid columns with rectangular cross section, the slenderness ratio, ℓ_e/d , shall be taken as the larger of the ratios ℓ_{e1}/d_1 or ℓ_{e2}/d_2 (see Figure 3F) where each ratio has been adjusted by the appropriate buckling length coefficient, K_e , from Appendix G.
- 3.7.1.4 The slenderness ratio for solid columns, ℓ_e /d, shall not exceed 50, except that during construction ℓ_e /d shall not exceed 75.
- 3.7.1.5 The column stability factor shall be calculated as follows:

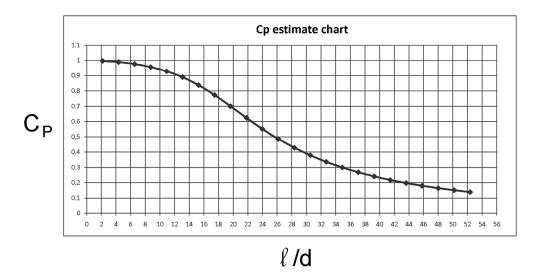
$$C_{p} = \frac{1 + (F_{cE}/F_{c}^{*})}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_{c}^{*})}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}$$
(3.7-1)

where:

 F_c = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_p (see 2.3), psi

$$F_{cE} = \frac{0.822 \ E_{min}'}{(\ell_e / d)^2}$$

- c = 0.8 for sawn lumber
- c = 0.85 for round timber poles and piles
- c = 0.9 for structural glued laminated timber or structural composite lumber



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Analysis of Wood Columns

Data:

- Column <u>size</u>, length
- Support conditions
- Material properties F_c , E
- Load

Required:

- Pass/Fail or margin of safety
- 1. Calculate slenderness ratio l_e/d largest ratio governs. Must be < 50
- 2. Find adjustment factors $C_D C_M C_t C_F C_i$
- 3. Calculate C_P
- 4. Determine F'c by multiplying the tabulated Fc by all the above factors
- 5. Calculate the actual stress: fc = P/A
- Compare Allowable and Actual stress.F'c > fc passes



Analysis Example:

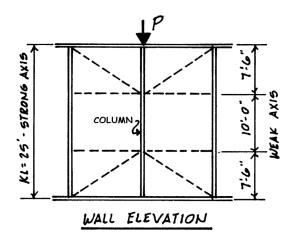
Data: section 4x8 (nominal) Douglas Fir-Larch No1 M.C. 15% P = 7000 LBS (Snow Load)

Find: Pass/Fail

From NDS Supplement Table 4A Fc = 1500 psi Emin = 620000 psi

$$C_D = 1.15 \text{ (snow)}$$

 $C_M = 1.0$
 $C_t = 1.0$
 $C_F = 1.05 \text{ (4x8)}$
 $C_i = 1.0$
 $C_P = ?$



	Size Factors, C _F	
		F _c
Grades	Width (depth)	
	2", 3", & 4"	1.15
Select	5"	1.1
Structural,	6"	1.1
No.1 & Btr,	8"	1.05
No.1, No.2,	10"	1.0
No.3	12"	1.0
	14" & wider	0.9
	2", 3", & 4"	1.05
Stud	5" & 6"	1.0
	8" & wider	
Construction,	2", 3", & 4"	1.0
Standard		
Utility	4"	1.0
	2" & 3"	0.6

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Allowable Flexure Stress F_c'

F_c from tables determined by species and grade

 $F_c' = F_c (C_D C_M C_t C_F C_i C_P)$

Adjustment factors for compression:

C_M Moisture Factor C_F Size Factor

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_{t}	$F_{\rm v}$	$F_{c\perp}$	F_c	\boldsymbol{E} and $\boldsymbol{E}_{\text{min}}$
0.85*	1.0	0.97	0.67	0.8**	0.9

^{*} when $(F_b)(C_F) \le 1,150 \text{ psi}, C_M = 1.0$

Size Factors, C.

		Size Factors,	CF		
		F	b	Ft	F _c
		Thickness (breadth)			
Grades	Width (depth)	2" & 3"	4"		
	2", 3", & 4"	1.5	1.5	1.5	1.15
Select	5"	1.4	1.4	1.4	1.1
Structural,	6"	1.3	1.3	1.3	1.1
No.1 & Btr,	8"	1.2	1.3	1.2	1.05
No.1, No.2,	10"	1.1	1.2	1.1	1.0
No.3	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
Stud	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade	tabulated design	values and size fac	ors
Construction,	Construction, 2", 3", & 4"		1.0	1.0	1.0
Standard					
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	_	0.4	0.6

^{**} when $(F_c)(C_F) \le 750 \text{ psi}, C_M = 1.0$

Analysis Example:

Calculate C_P

$$C_{p} = \frac{1 + (F_{cE}/F_{c}^{*})}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_{c}^{*})}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}}$$
(3.7-1)

where:

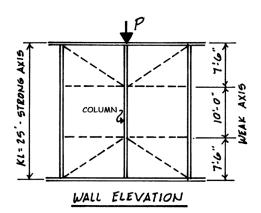
F_c = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_p (see 2.3), psi

$$F_{cE} = \frac{0.822 E_{min}'}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber



$$x-x$$
 $le_{x}=25'=300''$
 $le_{y}=10'=120''$
 $le_{x}=\frac{300''}{7.25''}$
 $le_{y}/d_{z}=\frac{120''}{3.5''}$
 $le_{y}/d_{z}=\frac{120''}{3.5''}$
 $le_{y}/d_{z}=\frac{120''}{3.5''}$
 $le_{y}/d_{z}=\frac{120''}{3.5''}$

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Analysis Example:

Calculate C_P

$$C_{p} = \frac{1 + \left(F_{cE} / F_{c}^{\star}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE} / F_{c}^{\star}\right)}{2c}\right]^{2} - \frac{F_{cE} / F_{c}^{\star}}{c}} \quad (3.7-1)$$

where:

 F_c = reference compression design value parallel to grain multiplied by all applicable adjustment factors except $C_{\mbox{\tiny p}}$ (see 2.3), psi

$$F_{cE} = \frac{0.822 E_{min}'}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

$$F_{CE} = \frac{0.822 \text{ Emin}}{(4e/d)^2}$$

$$= \frac{0.822(620000)}{(41.4)^2}$$

$$= 297.6 \text{ ps}$$

Analysis Example:

Calculate C_P

$$C_{\text{p}} = \frac{1 + \left(F_{\text{cE}}/F_{\text{c}}^{\star}\right)}{2c} - \sqrt{\left\lceil\frac{1 + \left(F_{\text{cE}}/F_{\text{c}}^{\star}\right)}{2c}\right\rceil^{2} - \frac{F_{\text{cE}}/F_{\text{c}}^{\star}}{c}}{c}} \quad (3.7 - 1)$$

where:

 F_c^{\cdot} = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_p (see 2.3), psi

$$F_{cE} = \frac{0.822 E_{min}'}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Compare Allowable and Actual stress F'c > fc passes

$$C_{p} = \frac{1 + 0.164}{2(0.8)} - \sqrt{\left[\frac{1 + 0.164}{2(0.8)}\right]^{2} - \frac{0.164}{.8}}$$

$$F_c^1 = 1500(1.15 \cdot 1.05 \cdot 0.1584)$$

= 286.9 PS1

$$f_c = \frac{P}{A} = \frac{7000^{\frac{1}{4}}}{25.38 \, \text{m}^2} = 275.8 \, \text{ps}$$

$$F_c > f_c \qquad \text{OK}$$

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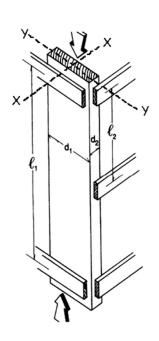
Capacity Analysis of Columns

Data:

- Column size, length
- Support conditions
- Material properties F_c, E

Required:

- Maximum Load Capacity, Pmax
- Calculate slenderness ratio l_e/d largest ratio governs. Must be < 50
- Find adjustment factors
 C_D C_M C_t C_F C_i
- 3. Calculate C_P
- 4. Determine F'c by multiplying the tabulated Fc by all the above factors
- 5. Set actual stress = allowable, fc = F'c
- Find the maximum allowable load Pmax = F'c A



Capacity Example

Data:

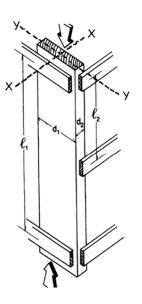
- 4x10
- Hem Fir, No 2 M.C. = 20%
- Wind Load
- L1 = 8' L2 = 4' $K_e = 1.0$

Required:

Maximum Load Capacity, Pmax

From NDS Supplement Table 4A Fc = 1300 psi Emin = 470000 psi

$$\begin{split} &C_{D} = 1.6 \quad \text{(Table 2.3.2.)} \\ &C_{Mc} = 0.8 \quad C_{ME} = 0.9 \\ &C_{t} = 1.0 \\ &C_{F} = 1.0 \quad \text{(chart in Supplement)} \\ &C_{i} = 1.0 \\ &C_{P} = ? \end{split}$$



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Allowable Flexure Stress F_c'

4 x 10

F_c from tables determined by species and grade

$$F_c' = F_c (C_D C_M C_t C_F C_i C_P)$$

Adjustment factors for compression: C_M Moisture Factor C_F Size Factor

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

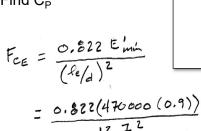
F _b	F _t	F_{v}	$F_{c\perp}$	F_c	\boldsymbol{E} and \boldsymbol{E}_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

- * when $(F_b)(C_F) \le 1,150 \text{ psi}, C_M = 1.0$
- ** when $(F_c)(C_F) \le 750 \text{ psi}, C_M = 1.0$

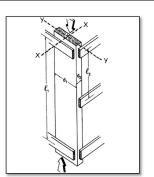
		Size Factors,	C_F		
		F	,	F_t	F _c
		Thickness (breadth)			
Grades	Width (depth)	2" & 3"	4"		
	2", 3", & 4"	1.5	1.5	1.5	1.15
Select	5"	1.4	1.4	1.4	1.1
Structural,	6"	1.3	1.3	1.3	1.1
No.1 & Btr,	8"	1.2	1.3	1.2	1.05
No.1, No.2,	10"	1.1	1.2	1.1	1.0
No.3	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
Stud	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade	ors		
Construction,	2", 3", & 4"	1.0	1.0	1.0	1.0
Standard					
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	_	0.4	0.6

Capacity Example

Find C_P



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$$C_{p} = \frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c} - \sqrt{\left[\frac{1 + \left(F_{cE}/F_{c}^{*}\right)}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}} \quad (3.7-1)$$

where:

F_c = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_o (see 2.3), psi

$$F_{cE} = \frac{0.822 \ E_{min}'}{(\ell_e / d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Find the maximum load, Pmax

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Stud Wall Example

Data:

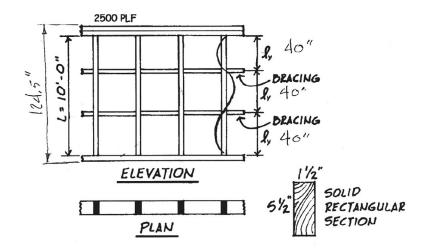
- 2x6
- S-P-F, Stud M.C. = 12%
- D+L Load = 2500 PLF
- Braced as shown K_e=1.0

Required:

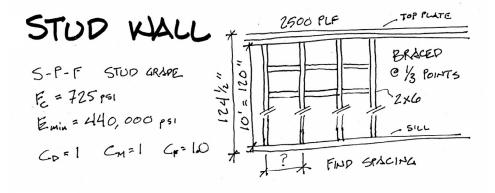
o.c. spacing

From NDS Supplement Table 4A Fc = 725 psi Emin = 440000 psi

$$C_D = 1.0 \text{ (LL)}$$
 $C_{Mc} = 1.0 \quad C_{ME} = 1.0$
 $C_t = 1.0$
 $C_F = 1.0 \text{ (stud)}$
 $C_i = 1.0$
 $C_P = ?$



Stud Wall Example



$$k_{\text{ex}} = 124.5$$
"
 $k_{\text{ex}} = 124.5$ "
 $k_{\text{ex}} = 40$ "
 k_{\text

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Stud Wall Example

CP

$$k_{\text{ex}} = 124.5$$
"
 $k_{\text{ex}} = 124.5$ "
 $k_{\text{ex}} = 40$ "
 k_{\text

$$F_{CE}^{*} = \frac{0.822 \text{ Emin}}{(18/d)^{2}} = \frac{0.822 (440000)}{24.7^{2}} = 508.6$$

$$F_{C}^{*} = 725 (1 \times 1 \times 1) = 725 \text{ psi}$$

$$F_{C}^{*} = \frac{508.6}{725} = 0.702$$

$$NDS \text{ eq. } 3.7-1 \rightarrow C_{p} = 0.559$$

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Stud Wall Example

Find max allowable stress, F'c

Calculate max load per stud

Determine max stud spacing

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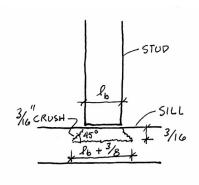
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Stud Wall Example

Check bearing on sill plate



3.10.4 Bearing Area Factor, Cb

Reference compression design values perpendicular to grain, $F_{c\perp}$, apply to bearings of any length at the ends of a member, and to all bearings 6" or more in length at any other location. For bearings less than 6" in length and not nearer than 3" to the end of a member, the reference compression design value perpendicular to grain, $F_{c\perp}$, shall be permitted to be multiplied by the following bearing area factor, C_b :

$$C_{b} = \frac{\ell_{b} + 0.375}{\ell_{b}}$$
 (3.10-2)

where:

 $\ell_{\rm b}$ = bearing length measured parallel to grain, in.

Equation 3.10-2 gives the following bearing area factors, C_b , for the indicated bearing length on such small areas as plates and washers:

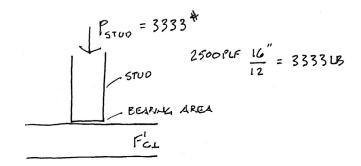
Table 3.10.4			Bearing Area Factors, C _b				
ℓ_{b}	0.5"	1"	1.5"	2"	3"	4"	6" or more
C_{b}	1.75	1.38	1.25	1.19	1.13	1.10	1.00

For round bearing areas such as washers, the bearing length, ℓ_b , shall be equal to the diameter.

Stud Wall Example

Check bearing on sill plate

- · determine Cb
- calculate F'_c_
- calculate f_{c⊥}
- · check stress



$$f_{CL} = \frac{P}{A} = \frac{3333^{4}}{8.25 \text{m}^2} = 404 \text{ ps}$$

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