

# Steel Beam Analysis

## Part 1

- Steel Properties
- Steel Profiles
- Steel Codes: ASD vs. LRFD
- Analysis Method



## Nomenclature of steel shapes

Standard section shapes:

**W** – wide flange

**S** – American standard beam

**C** – American standard channel

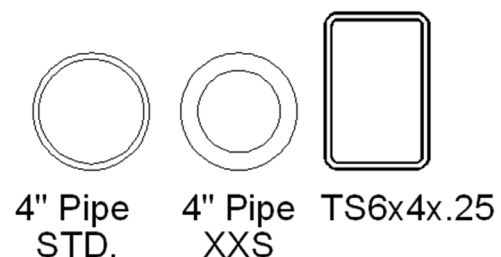
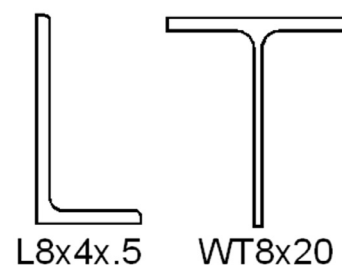
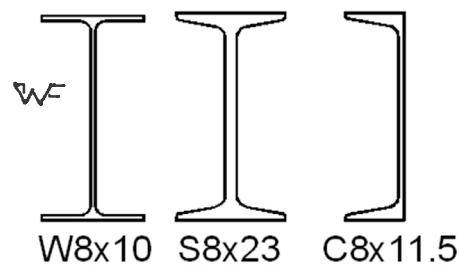
**L** – angle

**WT** or **ST** – structural T

**STD**, **XS** or **XXS** – Pipe

**HSS** – Hollow Structural Sections  
Rectangular, Square, Round

**LLBB** , **SLBB** - Double Angles



# Steel Grades – Rolled Sections

Different sections are made with different grades of steel.

Most structural shapes are:

A36 Carbon Steel  $F_y = 36\text{ksi}$

A992 High Strength  $F_y = 50\text{ ksi}$

Steel Type	ASTM Designation	$F_y$ Min. Yield Stress (ksi)	$F_u$ Tensile Stress <sup>a</sup> (ksi)	Applicable Shape Series								HSS		Pipe		
				W	M	S	HP	C	MC	L	Rect.	Round				
Carbon	A36	36	58–80 <sup>b</sup>													
	A53 Gr. B	35	60													
	A500	42	58													
		46	58													
		46	62													
		50	62													
	A501	Gr. A	36	58												
		Gr. B	50	70												
	A529 <sup>c</sup>	Gr. 50	50	65–100												
		Gr. 55	55	70–100												
High-Strength Low-Alloy	A572	Gr. 42	42	60												
		Gr. 50	50	65 <sup>d</sup>												
		Gr. 55	55	55												
		Gr. 60 <sup>e</sup>	60	60												
		Gr. 65 <sup>e</sup>	65	65												
	A618	Gr. I & II	50 <sup>f</sup>	70 <sup>f</sup>												
		Gr. III	50	50												
	A913	50	50 <sup>h</sup>	60 <sup>h</sup>												
		60	60	75												
		65	65	80												
Corrosion Resistant High-Strength Low-Alloy		70	70	90												
	A992	50	65 <sup>i</sup>													
	A242	42 <sup>j</sup>	63 <sup>j</sup>													
		46 <sup>k</sup>	67 <sup>k</sup>													
		50 <sup>l</sup>	70 <sup>l</sup>													
	A588	50	70													
A847	50	70														

■ = Preferred material specification

▒ = Other applicable material specification, the availability of which should be confirmed prior to specification

□ = Material specification does not apply

<sup>a</sup> Minimum unless a range is shown.

<sup>b</sup> For shapes over 426 lb/ft, only the minimum of 58 ksi applies.

<sup>c</sup> For shapes with a flange thickness less than or equal to 1½ in., only. To improve weldability, a maximum carbon equivalent can be specified (per ASTM Supplementary Requirement S78). If desired, maximum tensile stress of 90 ksi can be specified (per ASTM Supplementary Requirement S79).

<sup>d</sup> If desired, maximum tensile stress of 70 ksi can be specified (per ASTM Supplementary Requirement S91).

<sup>e</sup> For shapes with a flange thickness less than or equal to 2 in., only.

<sup>f</sup> ASTM A618 can also be specified as corrosion-resistant; see ASTM A618.

<sup>g</sup> Minimum applies for walls nominally ¾-in. thick and under. For wall thicknesses over ¾ in.,  $F_y = 46$  ksi and  $F_u = 67$  ksi.

<sup>h</sup> If desired, maximum yield stress of 65 ksi and maximum yield-to-tensile strength ratio of 0.85 can be specified (per ASTM Supplementary Requirement S75).

<sup>i</sup> A maximum yield-to-tensile strength ratio of 0.85 and carbon equivalent formula are included as mandatory in ASTM A992.

<sup>j</sup> For shapes with a flange thickness greater than 2 in., only.

<sup>k</sup> For shapes with a flange thickness greater than 1½ in., and less than or equal to 2 in., only.

<sup>l</sup> For shapes with a flange thickness less than or equal to 1½ in., only.

■ = Preferred material specification  
 ■ = Other applicable material specification, the availability of which should be confirmed prior to specification  
 □ = Material specification does not apply

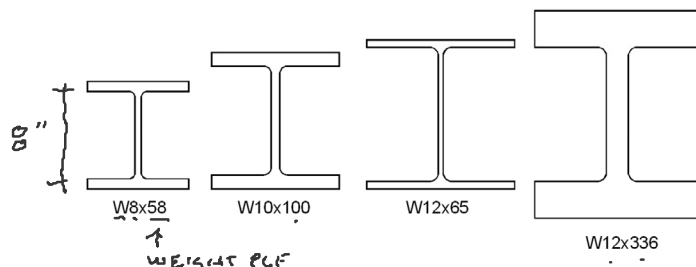
<sup>a</sup> Minimum unless a range is shown.  
<sup>b</sup> For shapes over 426 lb/ft, only the minimum of 58 ksi applies.  
<sup>c</sup> For shapes with a flange thickness less than or equal to 1½ in. only. To improve weldability, a maximum carbon equivalent can be specified (per ASTM Supplementary Requirement S78). If desired, maximum tensile stress of 90 ksi can be specified (per ASTM Supplementary Requirement S79).  
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<sup>k</sup> For shapes with a flange thickness greater than 1½ in. and less than or equal to 2 in. only.  
<sup>l</sup> For shapes with a flange thickness less than or equal to 1½ in. only.

Source: AISC Manual, Table 2-4, p. 2-48, 14th ed, 2011. Copyright © American Institute of Steel Construction. Reprinted with permission. All rights reserved.

## Steel W-sections for beams and columns

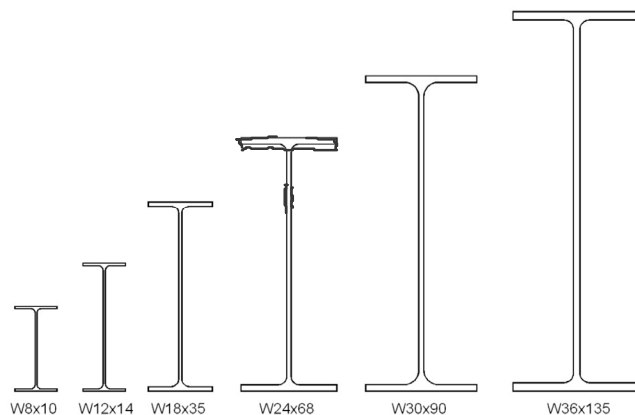
### Columns:

Closer to square  
Thicker web & flange



### Beams:

Deeper sections  
Flange thicker than web



# Steel W-sections for beams and columns

## Columns:

Closer to square  
Thicker web & flange

## Beams:

Deeper sections  
Flange thicker than web



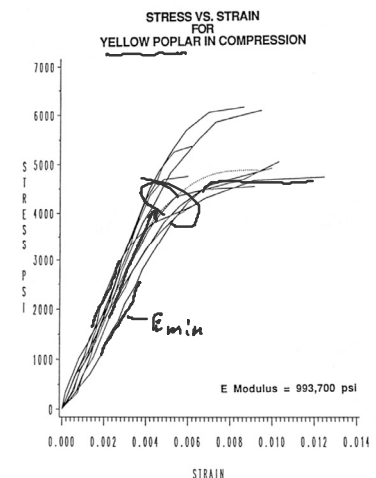
Photo by Gregor Y.

## Young's Modulus

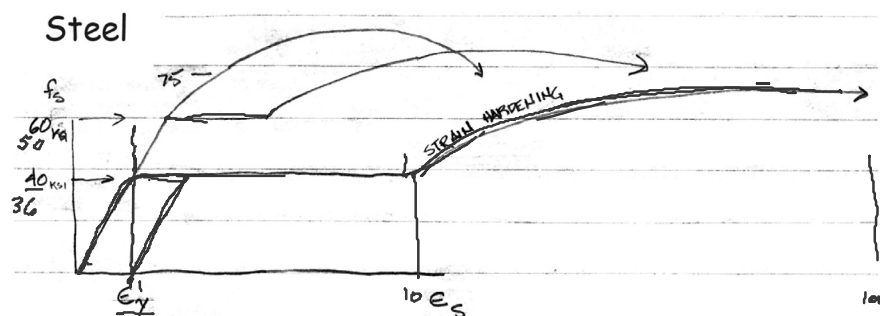
Young's Modulus or the Modulus of Elasticity, is obtained by dividing the stress by the strain present in the material. (Thomas Young, 1807)

$$E = \frac{P/A}{D/L} = \frac{\sigma}{\epsilon}$$

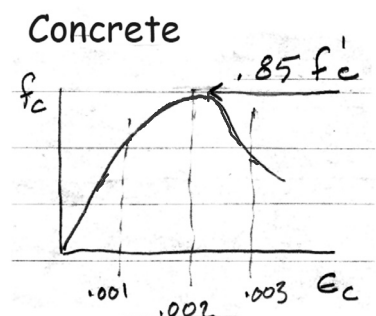
It thus represents a measure of the stiffness of the material.



$$E = 1000 \text{ ksi}$$

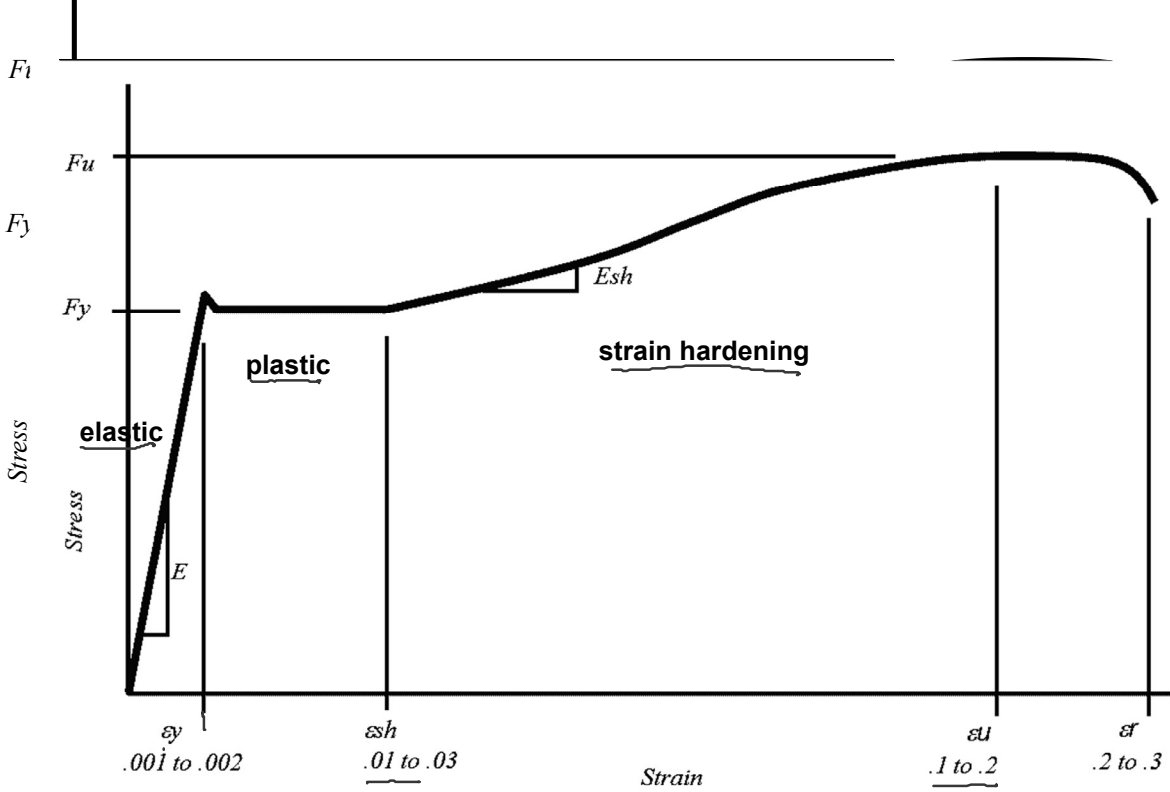


$$E = 29000 \text{ ksi}$$



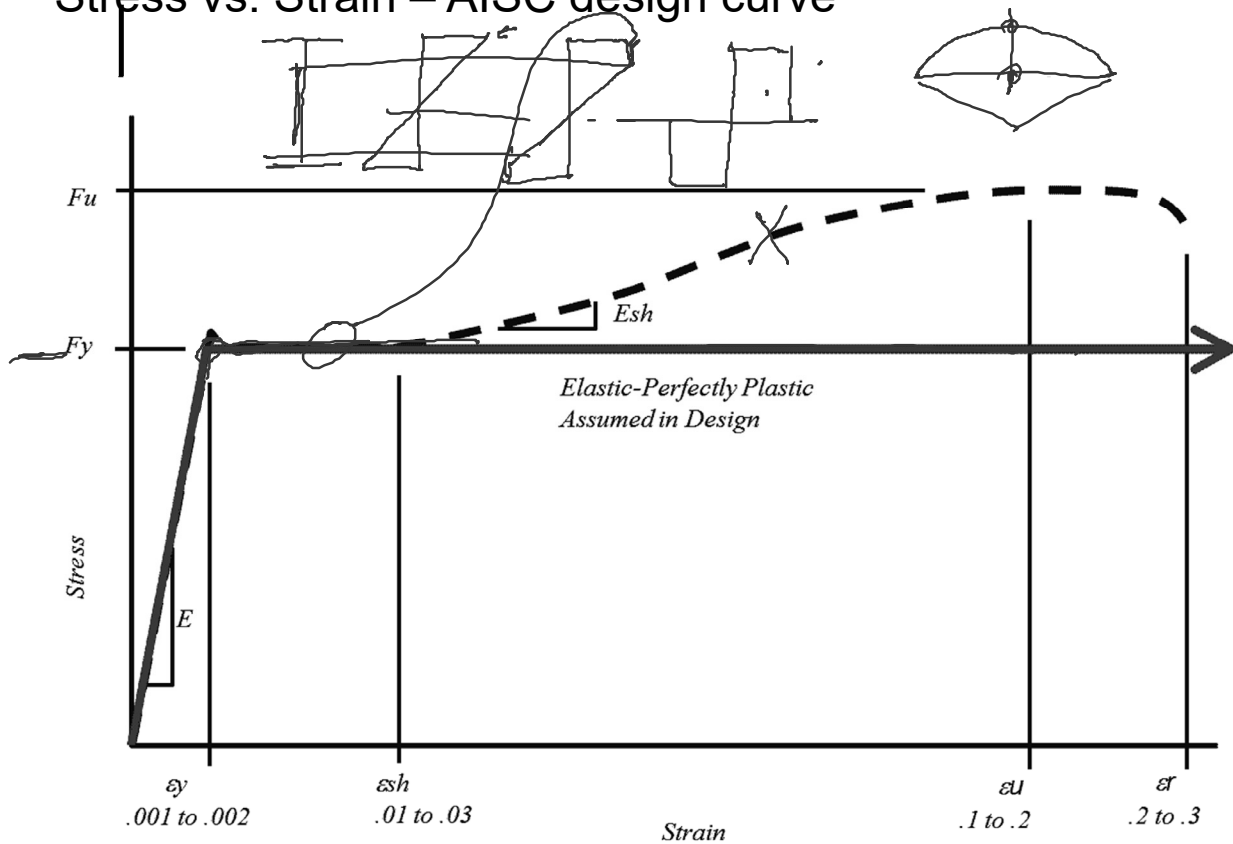
$$E = 3500 \text{ ksi}$$

## Stress vs. Strain – mild steel



Developed by Scott Civan  
University of Massachusetts, Amherst

## Stress vs. Strain – AISC design curve



# Stress Analysis – Two Methods

## Allowable Stress Design (ASD)

- use design loads (no F.S. on loads)
- reduce stress by a Factor of Safety F.S.

$$f_{actual} = \frac{P}{A}$$

$$f_{actual} \leq F_{allowable}$$

$$F_{allowable} = \underline{F.S.} \cdot f_{yield}$$

## Load & Resistance Factored Design (LRFD)

- Use loads with safety factor  $\gamma$
- Use factor on ultimate strength  $\phi$

$$P_{load} = \gamma \cdot P_{applied\_load}$$

$$P_{load} \leq P_{resisting}$$

$$P_{resisting} = \phi \cdot P_{material\_strength}$$

## LRFD Analysis

### Load & Resistance Factored Design (LRFD)

- Use loads with safety factor  $\gamma$
- Use forces with strength factor  $\phi$

$$P_{load} = \gamma \cdot P_{applied}$$

$$P_{load} \leq P_{resisting}$$

$$P_{resisting} = \phi \cdot P_{material}$$

Design Strength

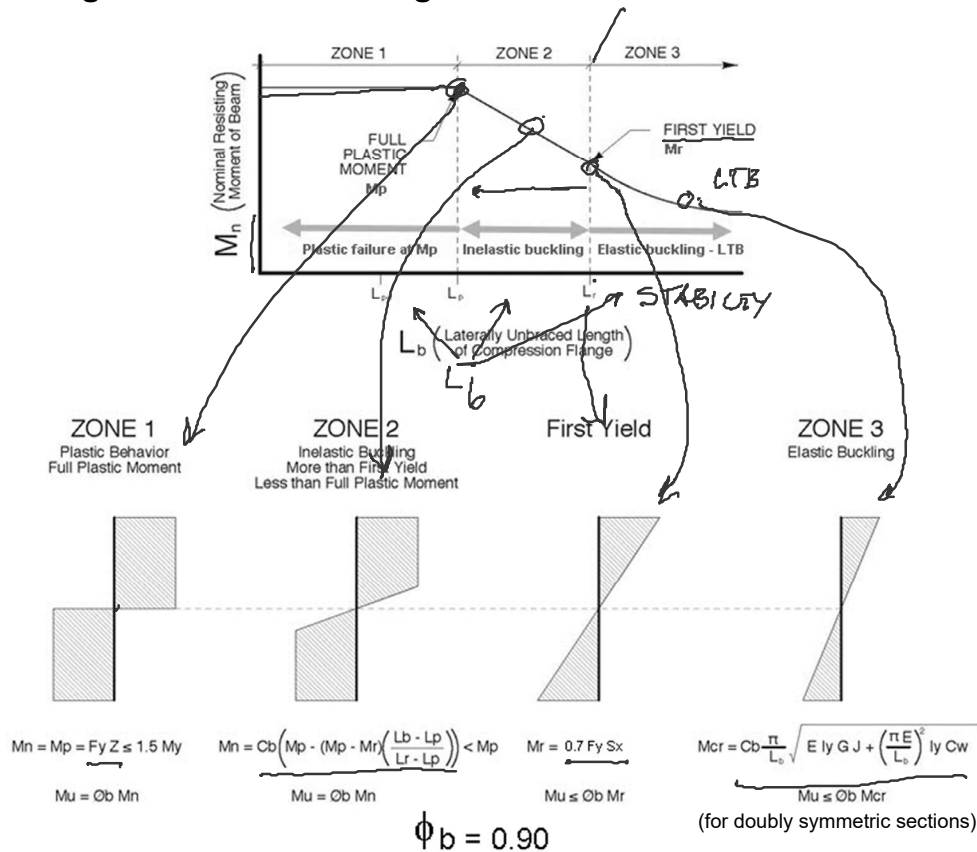
$$P_u \leq \phi P_n$$

Required (Nominal) Strength

## 2.3 LOAD COMBINATIONS FOR STRENGTH DESIGN

1.  $1.4D$
2.  $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3.  $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
4.  $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
5.  $0.9D + 1.0W$

## Beam Strength vs Unbraced Length



## Steel Beams by LRFD

### Yield Stress Values

- A36 Carbon Steel  $F_y = 36$  ksi
- A992 High Strength  $F_y = 50$  ksi

### Elastic Analysis for Bending

#### Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z < 1.5 M_y$$

- Braced against LTB ( $L_b < L_p$ )

#### Inelastic Buckling "Decreased" (zone 2)

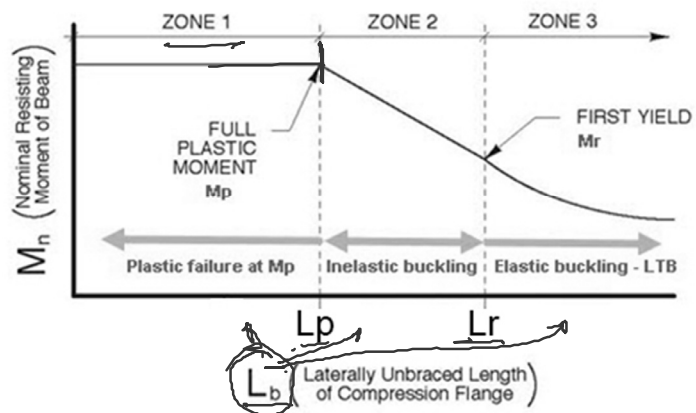
$$M_n = C_b \left( M_p - (M_p - M_r) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right) \leq M_p$$

- $L_p < L_b < L_r$

#### Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b \frac{\pi^2 E I_y G J}{L_b^2} + \left( \frac{\pi E}{L_b} \right)^2 I_y C_w$$

- $L_b > L_r$



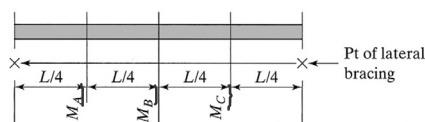
$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$M_p = F_y Z_x$$

$$M_r = 0.7 F_y S_x$$

$C_b$  is LTB modification factor

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$



# Steel Beams by LRFD

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DESIGN OF FLEXURAL MEMBERS

## Yield Stress Values

A36 Carbon Steel  $F_y = 36$  ksi

A992 High Strength  $F_y = 50$  ksi

## Analysis for Bending

- Plastic Behavior (zone 1)
  - $M_n = M_p = F_y Z_x < 1.5 M_y$ 
    - Braced against LTB ( $L_b < L_p$ )
- Inelastic Buckling "Decreased" (zone 2)
  - $M_n = C_b(M_p - (M_p - M_r)[(L_b - L_p)/(L_r - L_p)]) < M_p$ 
    - $L_p < L_b < L_r$
- Elastic Buckling "Decreased Further" (zone 3)
  - $M_{cr} = C_b * \pi^2 I_y / L_b^2 \sqrt{E I_y G^* J + (\pi^2 E I_y / L_b^2 * I_y C_w)}$ 
    - $L_b > L_r$

Table 3-2 (continued)  
W-Shapes  
Selection by  $Z_x$   
 $F_y = 50$  ksi

$Z_x$  ASD LRFD

Shape	$Z_x$ in. <sup>3</sup>	$M_n/\Omega_b$		$M_n/\phi_b$		$\phi_b M_n$		$\phi_b M_n$		$\phi_b M_n$		$\phi_b M_n$		$\phi_b M_n$		$\phi_b M_n$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217					
W16x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186					
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195					
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154					
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132					
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	40.6	394	97.8	147					
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167					
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169					
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141					
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125					
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129					
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146					
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	80.3	135					
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154					
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125					
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112					
W18x35	66.5	166	249	101	151	8.14	12.3	4.31	12.3	510	106	159					
W12x45	64.2	160	241	101	151	3.80	5.80	6.89	22.4	348	81.1	122					
W16x36	64.0	160	240	98.7	148	6.24	9.36	5.37	15.2	448	93.8	141					
W14x38	61.5	153	231	95.4	143	5.37	8.20	5.47	16.2	385	87.4	131					
W10x49	60.4	151	227	95.4	143	2.46	3.71	8.97	31.6	272	68.0	102					
W8x58	59.8	149	224	90.8	137	1.70	2.55	7.42	41.6	228	89.3	134					
W12x40	57.0	142	214	89.9	135	3.66	5.54	6.85	21.1	307	70.2	105					
W10x45	54.9	137	206	85.8	129	2.59	3.89	7.10	26.9	248	70.7	106					
W14x34	54.6	136	205	84.9	128	5.01	7.55	5.40	15.6	340	79.8	120					
W16x31	54.0	135	203	82.4	124	6.86	10.3	4.13	11.8	375	87.5	131					
W12x35	51.2	128	192	79.6	120	4.34	6.45	5.44	16.6	285	75.0	113					
W8x48	49.0	122	184	75.4	113	1.67	2.55	7.35	35.2	184	68.0	102					
W14x30	47.3	118	177	73.4	110	4.63	6.95	5.26	14.9	291	74.5	112					
W10x39	46.8	117	176	73.5	111	2.53	3.78	6.99	24.2	209	62.5	93.7					
W16x26	44.2	110	166	67.1	101	5.93	8.98	3.96	11.2	301	70.5	106					
W12x30	43.1	108	162	67.4	101	3.97	5.96	5.37	15.6	238	64.0	95.9					

ASD LRFD

$\Omega_b = 1.67$   
 $\phi_b = 0.90$   
 $\Omega_b = 1.50$   
 $\phi_b = 1.00$

\* Shape does not meet the  $h/t_w$  limit for shear in AISC Specification Section G2.1(a) with  $F_y = 50$  ksi; therefore,  $\phi_v = 0.90$  and  $\Omega_v = 1.67$ .

AISC 15<sup>th</sup> ed.

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Structures II

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## Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

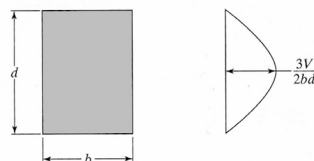
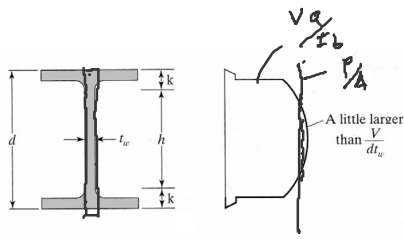
$$F_v = V / A_w$$

$$A_w = d * t_w$$

To adjust the stress a reduction factor of 0.6 is applied to  $F_y$

$$F_v = 0.6 F_y$$

$$\text{so, } V_n = 0.6 F_y A_w \quad (\text{Zone 1})$$



The equations for the 3 stress zones:  
( $\phi$  in all cases = 1.0)

### Zone 1:

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \left[ \frac{h}{t_w} \right] \leq 2.45 \sqrt{E/F_y} = 59 \quad (\text{for } 50 \text{ ksi steel})$$

$$\text{then: } V_n = 0.6 F_y A_w$$

### Zone 2:

INELASTIC WEB BUCKLING

$$\text{if } 2.45 \sqrt{E/F_y} < \frac{h}{t_w} \leq 3.07 \sqrt{E/F_y} = 74 \quad (\text{for } 50 \text{ ksi steel})$$

$$\text{then: } V_n = 0.6 F_y A_w (2.45 \sqrt{E/F_y}) / \frac{h}{t_w}$$

### Zone 3:

ELASTIC WEB BUCKLING

$$\text{if } 3.07 \sqrt{E/F_y} < \left[ \frac{h}{t_w} \right] \leq 260$$

$$\text{then: } V_n = A_w \left[ \frac{4.25 E}{\left( \frac{h}{t_w} \right)^2} \right]$$

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Structures II

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# Design for Shear

## Steel

