Architecture 324

Structures II

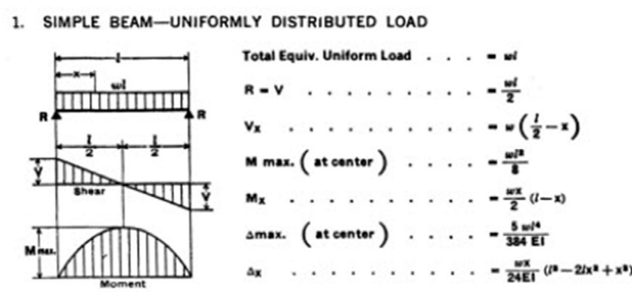
Steel Beam Analysis
Part 2

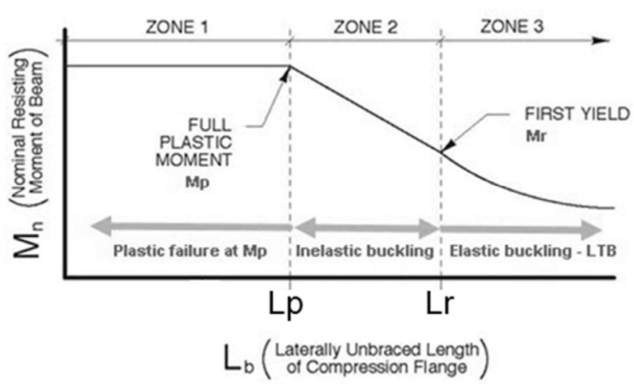
- Steel Codes: ASD vs. LRFD
- Analysis Methods

Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

Given: yield stress, steel section, loading
Find: pass/fail of section

1. Calculate the factored design load w_u
   $w_u = 1.2w_{DL} + 1.6w_{LL}$
2. Determine the design moment Mu. Mu will be the maximum beam moment using the factored loads
3. Insure that Lb < Lp (zone 1)
   $Lp = 1.76\, r_y \sqrt{E/Fy}$
4. Determine the nominal moment, Mn
   Mn = Fy Zx (look up Zx for section)
5. Factor the nominal moment
   øMn = 0.90 Mn
6. Check that Mu < øMn
7. Check shear
8. Check deflection





Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

Example:

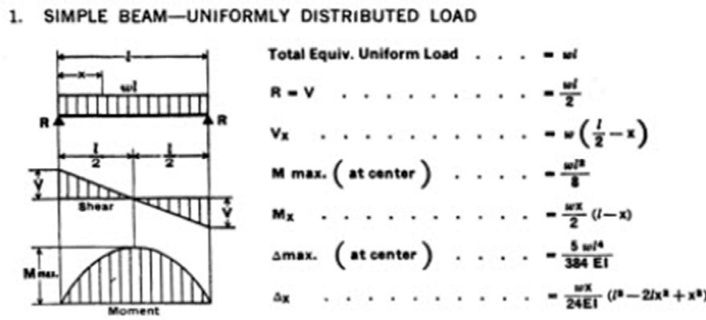
Given: yield stress, steel section,
loading, braced @ 24” o.c.

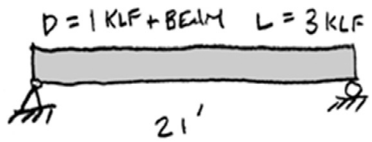
Find: pass/fail of section

1. Calculate the factored design load w_u
   $w_u = 1.2w_{DL} + 1.6w_{LL}$

2. Determine the design moment Mu.
   Mu will be the maximum beam moment using the factored loads.

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD





W21x44
A 992 STEEL
F_y = 50 KSI

FROM TABLE 1-1 AISC $Z_x = 95.4\ in^3$

$w_u = 1.2(1+.044) + 1.6(3) = 6.05$ KLF

$M_u = \frac{w_u l^2}{8} = \frac{6.05\text{ KLF} \times 21'^2}{8} = 333.5$ K-'

Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

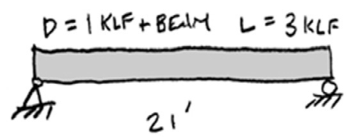
Example:

3. Insure that Lb < Lp (zone 1)
   $Lp = 1.76\ r_y \sqrt{E/Fy}$
   $Lp = 1.76\ (1.26) \sqrt{29000/50}$
   Lp = 53.4 in. = 4.45 ft > 2 ft ok

4. Determine the nominal moment, Mn
   Mn = Mp = Fy Zx (look up Zx for section)

5. Factor the nominal moment
   øMn = 0.90 Mn

6. Check that Mu < øMn



W21x44
A 992 STEEL
F_y = 50 KSI

FROM TABLE 1-1 AISC $Z_x = 95.4\ in^3$

$M_n = F_y Z = 50\text{ ksi } 95.4\ in^3 = 4770$ K-"

$M_n = 4770^{K-''}/12 = 397.5^{K-'}$

$\phi M_n = 0.9\ (397.5) = 357.7^{K-'}$

$M_u = 333.5^{K-'} < 357.7^{K-'} = \phi M_n$

∴ PASS

Analysis of Steel Beam – $L_b < L_p$

W21x44

7. Check shear

CHECK SHEAR:

FROM AISC TABLE 1-1

$h/t_w = 53.6 < 59$ (zone 1)

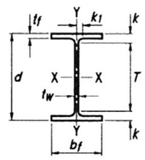


Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $t_w/2$	Flange Width, b_f		Flange Thickness, t_f		Distance k_{des}	Distance k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W21×93	27.3	21.6	21 5/8	0.580	9/16	5/16	8.42	8 3/8	0.930	15/16	1.43	1 5/8	15/16	18 3/8	5 1/2
×83^{c}	24.4	21.4	21 3/8	0.515	1/2	1/4	8.36	8 3/8	0.835	13/16	1.34	1 1/2	7/8	↓	↓
×73^{c}	21.5	21.2	21 1/4	0.455	7/16	1/4	8.30	8 1/4	0.740	3/4	1.24	1 7/16	7/8		
×68^{c}	20.0	21.1	21 1/8	0.430	7/16	1/4	8.27	8 1/4	0.685	11/16	1.19	1 3/8	7/8		
×62^{c}	18.3	21.0	21	0.400	3/8	3/16	8.24	8 1/4	0.615	5/8	1.12	1 5/16	13/16		
×55^{c}	16.2	20.8	20 3/4	0.375	3/8	3/16	8.22	8 1/4	0.522	1/2	1.02	1 3/16	13/16		
×48^{c,f}	14.1	20.6	20 5/8	0.350	3/8	3/16	8.14	8 1/8	0.430	7/16	0.930	1 1/8	13/16	↓	↓
W21×57^{c}	16.7	21.1	21	0.405	3/8	3/16	6.56	6 1/2	0.650	5/8	1.15	1 5/16	13/16	18 3/8	3 1/2
×50^{c}	14.7	20.8	20 7/8	0.380	3/8	3/16	6.53	6 1/2	0.535	9/16	1.04	1 1/4	13/16	↓	↓
×44^{c}	13.0	20.7	20 5/8	0.350	3/8	3/16	6.50	6 1/2	0.450	7/16	0.950	1 1/8	13/16	↓	↓

Table 1-1 (continued)
W-Shapes
Properties

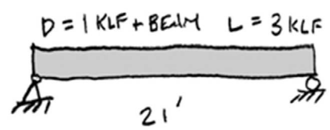
W21-W18

Nominal Wt.	Compact Section Criteria $b_f/2t_f$	Compact Section Criteria h/t_w	Axis X-X I	Axis X-X S	Axis X-X r	Axis X-X Z	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r	Axis Y-Y Z	r_{ts}	h_o	Torsional Properties J	Torsional Properties C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
93	4.53	32.3	2070	192	8.70	221	92.9	22.1	1.84	34.7	2.24	20.7	6.03	9940
83	5.00	36.4	1830	171	8.67	196	81.4	19.5	1.83	30.5	2.21	20.6	4.34	8630
73	5.60	41.2	1600	151	8.64	172	70.6	17.0	1.81	26.6	2.19	20.5	3.02	7410
68	6.04	43.6	1480	140	8.60	160	64.7	15.7	1.80	24.4	2.17	20.4	2.45	6760
62	6.70	46.9	1330	127	8.54	144	57.5	14.0	1.77	21.7	2.15	20.4	1.83	5960
55	7.87	50.0	1140	110	8.40	126	48.4	11.8	1.73	18.4	2.11	20.3	1.24	4980
48	9.47	53.6	959	93.0	8.24	107	38.7	9.52	1.66	14.9	2.05	20.2	0.803	3950
57	5.04	46.3	1170	111	8.36	129	30.6	9.35	1.35	14.8	1.68	20.5	1.77	3190
50	6.10	49.4	984	94.5	8.18	110	24.9	7.64	1.30	12.2	1.64	20.3	1.14	2570
44	7.22	53.6	843	81.6	8.06	95.4	20.7	6.37	1.26	10.2	1.60	20.3	0.770	2110

Pass/Fail Analysis of Steel Beam – $L_b < L_p$

Example cont.:

7. Check shear

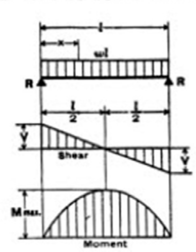


W21×44
A 992 STEEL
$F_y = 50$ KSI

FROM TABLE 1-1 AISC $Z_x = 95.4$ in^3

$w_u = 1.2(1+.044) + 1.6(3) = 6.05$ KLF

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . $= wl$

R = V . . . . $= \frac{wl}{2}$

V_x . . . . $= w\left(\frac{l}{2} - x\right)$

M max. (at center) . . . . . $= \frac{wl^2}{8}$

M_x . . . . $= \frac{wx}{2}(l - x)$

Δmax. (at center) . . . . $= \frac{5wl^4}{384EI}$

Δx . . . . $= \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$

CHECK SHEAR:

$V_u = \frac{w_u l}{2} = \frac{6.05(21)}{2} = 63.5^K$

FROM AISC TABLE 1-1

$h/t_w = 53.6 < 59$ (zone 1)

$V_n = 0.6 F_y A_w = 0.6(50)(20.7 \times 0.35)$ (d, t_w)

$V_n = 217.35^K$

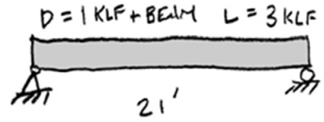
$\phi V_n = 1.0(217.35) = 217.35^K$

$V_u = 63.5^K < 217.3^K = \phi V_n$

Therefore, pass.

Pass/Fail Analysis of Steel Beam – $L_b < L_p$

Example cont.:



W21x44

A 992 STEEL

$F_y = 50$ KSI

FROM TABLE 1-1 AISC $Z_x = 95.4\ in^3$

$w_u = 1.2(1+.044) + 1.6(3) = 6.05$ KLF

8. Check deflection

$$\Delta_{MAX} = \frac{5 w \ell^4}{384 EI} = \frac{5(3000)\,21^4\,(1728)}{384(29000000)(843)}$$

$$= 0.535''$$

$$\frac{\ell}{360} = \frac{21(12)}{360} = 0.7''$$

$$\Delta_{ACTUAL} = 0.535'' < 0.7'' = \Delta_{ALLOWABLE}$$

TABLE 1604.3 DEFLECTION LIMITS[a, b, c, h, i]

CONSTRUCTION	L	S or W[f]	D + L[d, g]
Roof members:[e]			
Supporting plaster or stucco ceiling	l/360	l/360	l/240
Supporting nonplaster ceiling	l/240	l/240	l/180
Not supporting ceiling	l/180	l/180	l/120
Floor members	l/360	—	l/240
Exterior walls:			
With plaster or stucco finishes	—	l/360	—
With other brittle finishes	—	l/240	—
With flexible finishes	—	l/120	—
Interior partitions:[b]			
With plaster or stucco finishes	l/360	—	—
With other brittle finishes	l/240	—	—
With flexible finishes	l/120	—	—
Farm buildings	—	—	l/180
Greenhouses	—	—	l/120

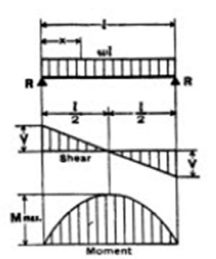
Capacity Analysis of Steel Beam

Given: yield stress, steel section, bracing

Find: moment or load capacity

1. Determine the unbraced length of the compression flange (Lb).
2. Find the Lp and Lr values from the AISC properties table 3-6
3. Compare Lb to Lp and Lr and determine which equation for Mn or Mcr to be used.
4. Determine the beam load equation for maximum moment in the beam. Solve for Mn.
5. Calculate load based on maximum moment. Mu = ϕ_b Mn

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load = wl

R = V = $\frac{wl}{2}$

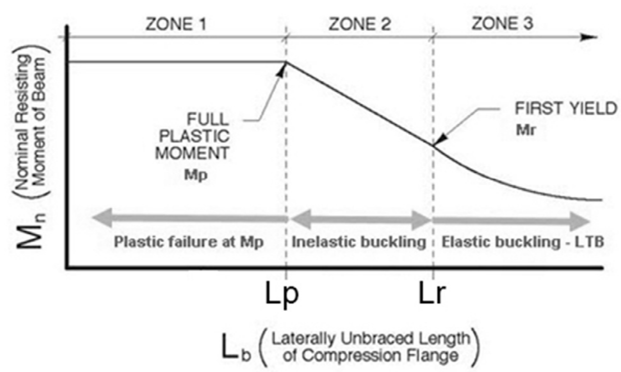
$V_x = w\left(\frac{l}{2} - x\right)$

M max. (at center) = $\frac{wl^2}{8}$

$M_x = \frac{wx}{2}(l - x)$

Δmax. (at center) = $\frac{5wl^4}{384EI}$

$\Delta_x = \frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$



Example – Capacity Analysis of Steel Beam

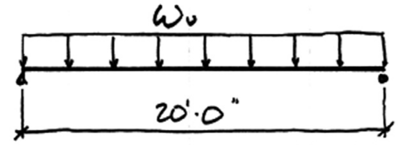
Find applied live load capacity, wLL in KLF
  wu = 1.2wDL + 1.6wLL
  wDL = beam + floor = 44plf + 1500plf

Fy = 50 ksi, Fully Braced

My = Fy * Sx = 50 ksi x 81.6 in^3 = 4080 k-in

1. Find the Plastic Modulus (Zx) and Section Modulus (Sx) for the given section from the AISC table 1-1
2. Determine 1.5 My (limit of Mp)
3. Determine Mp = Fy Zx
4. Compare Mp and 1.5 My, and choose the lesser of the two for Mn
5. Calculate Mu: = ϕ_b Mn
   $\phi_b = 0.90$

GIVEN: $F_y = 50$ ksi
W21x44
Fully Braced



For a W21x44 from Table
$Z_x = 95.4 \text{ in}^3 \quad S_x = 81.6$

$1.5 M_y = 1.5(F_y \cdot S_x) = 6{,}120$ k-in

$M_N = F_y Z_x = 50_{ksi} \cdot 95.4 = 4{,}770$ k-in

$M_N < 1.5 M_y$ (OK)

$M_U = \phi_b \cdot M_N = 0.9 \cdot 4{,}770$ k-in

$M_U = 4{,}293$ k-in $= 357.75$ k-ft

Example – Load Analysis cont.

W21x44

6. Using the maximum moment equation, solve for the factored distributed loading, w_u

$$M_U = \frac{\omega_U \cdot l^2}{8} \Rightarrow \omega_U = \frac{8 M_U}{l^2}$$

$$\omega_U = \frac{8 \cdot 357.75 \text{ kft}}{20_{ft}^2}$$

$$\omega_U = 7.155 \text{ k/ft}$$

7. The applied (unfactored) load
   w = w_u / (γ factors)
   w_u = 1.2wDL + 1.6wLL

$$w_u = 7.155 \text{ KLF} = 1.2(\overset{w_{DL}}{0.044 + 1.5}) + 1.6(w_{LL})$$

$$w_u = 1.853 + 1.6 w_{LL} = 7.155 \text{ KLF}$$

$$w_{LL} = 3.31 \text{ KLF}$$

Steel Beams by LRFD

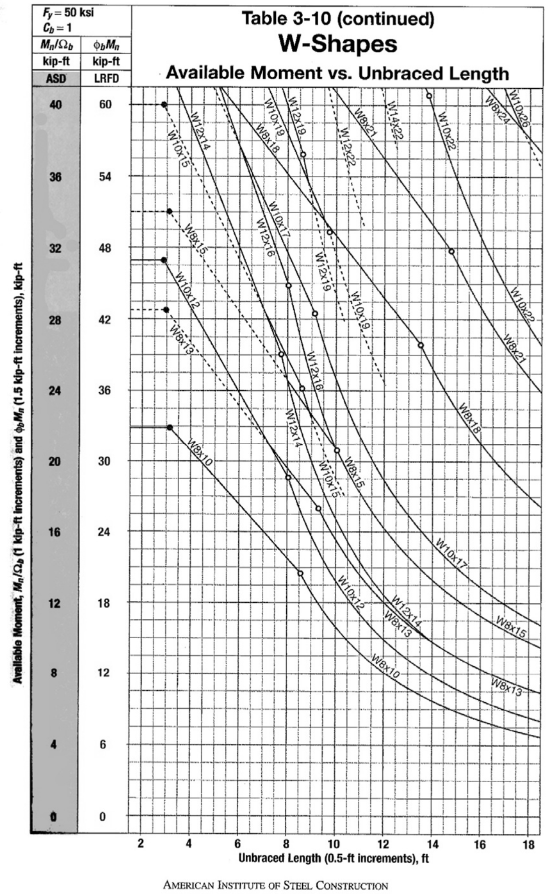
Moment Capacity Graphs

Analysis for Bending

- Plastic Behavior (zone 1)
  Mn = Mp
  Braced against LTB (Lb < Lp)

- Inelastic Buckling “Decreased” (zone 2)
  Mn < Mp
  Lp < Lb < Lr

- Elastic Buckling “Decreased Further” (zone 3)
  Mn = Mcr
  Lb > Lr



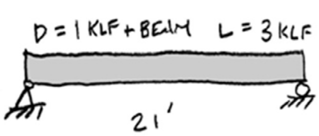
AISC 15th ed.

Pass/Fail Analysis of Steel Beams for Zone 1 $L_b < L_p$

Example:

Given: yield stress, steel section,
loading, braced @ 24” o.c.

Find: pass/fail of section

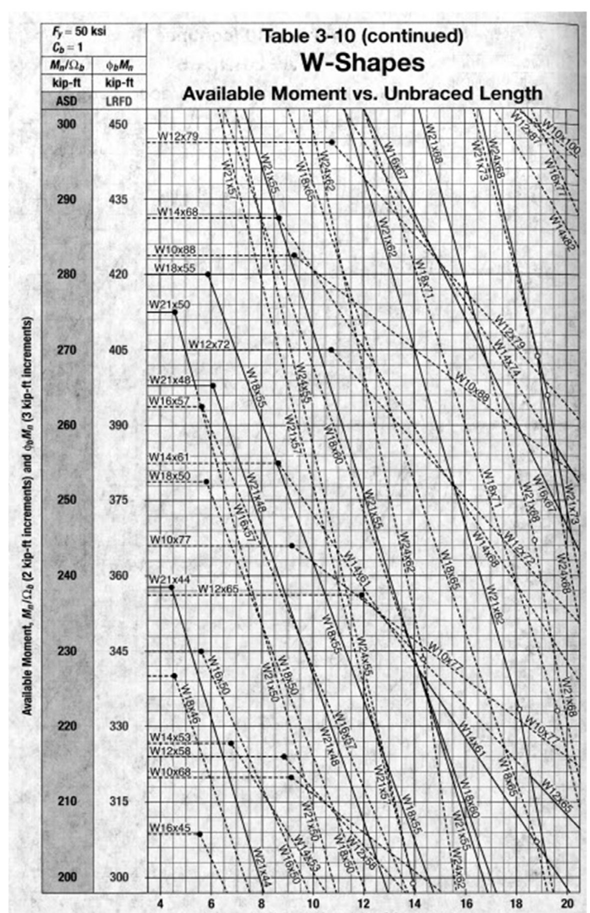


W21 × 44
A 992 STEEL
$F_y = 50$ KSI

FROM TABLE 1-1 AISC $Z_x = 95.4 \text{ in}^3$

$w_u = 1.2(1 + .044) + 1.6(3) = 6.05$ KLF

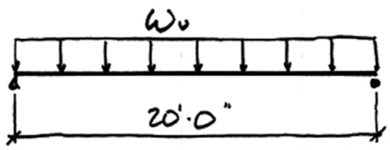
$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21'^2}{8} = 333.5$ K-'



Steel Beams by LRFD

Moment Capacity Graphs

GIVEN: $F_y = 50$ ksi
W21x44
FULLY BRACED



FOR A W21x44 FROM TABLE

$Z_x = 95.4 \text{ in}^3 \quad S_x = 81.6$

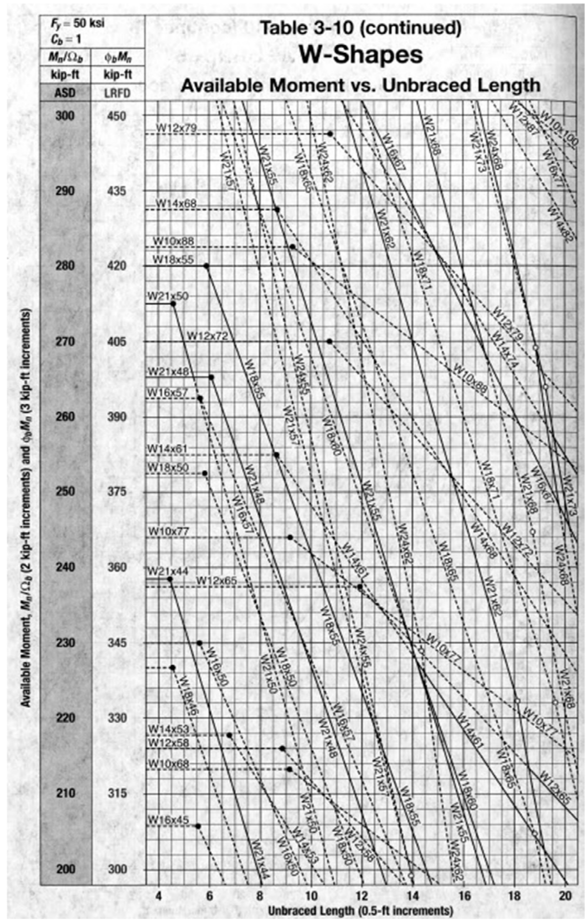
$1.5 M_y = 1.5(F_y \cdot S_x) = 6{,}120 \text{ k-in}$

$M_N = F_y Z_x = 50_{ksi} \cdot 95.4 = 4{,}770 \text{ k-in}$

$M_N < 1.5 M_y$ (OK)

$M_u = \phi_b \cdot M_N = 0.9 \cdot 4{,}770 \text{ k-in}$

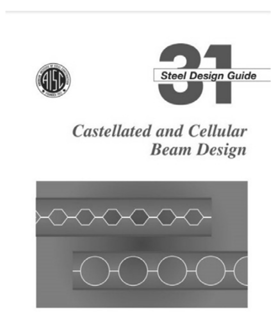
$M_u = 4{,}293 \text{ k-in} = 357.75 \text{ k-ft}$

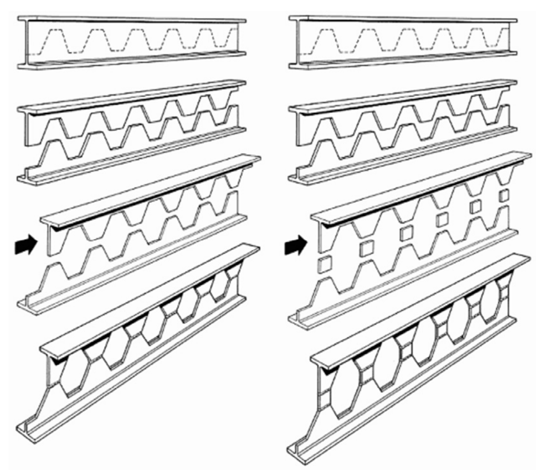


AISC 14th ed.

Modified Sections

- Castellated Sections:
- "Boyd beam"
- round, hexagonal, rectangular, sinusoidal
- extendable (added depth)
- cost-efficient
- lightweight





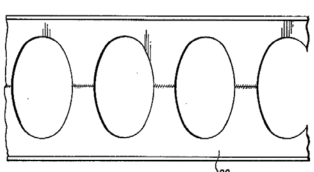


Fig.2A.

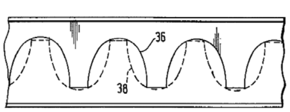


Fig.2B.