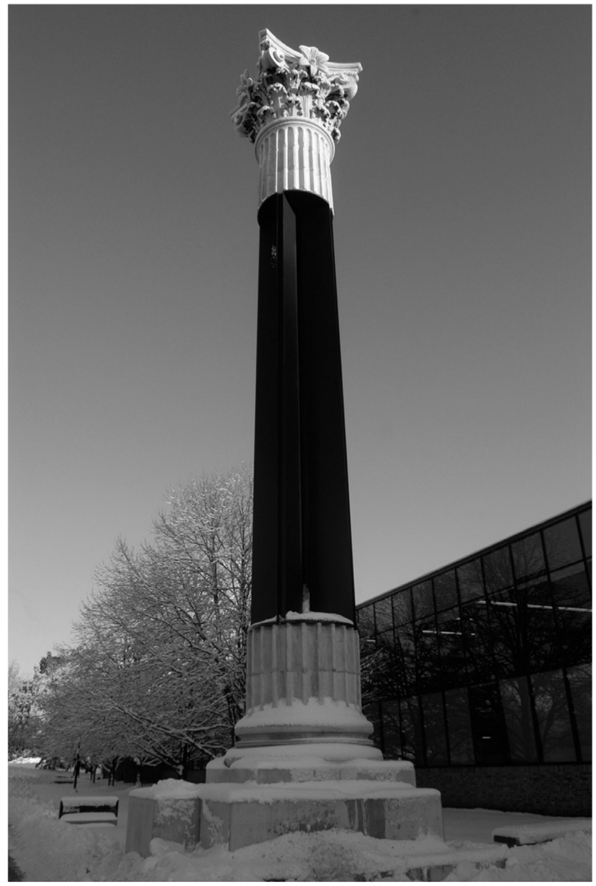
Architecture 324
Structures II

Steel Column Analysis

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns



Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

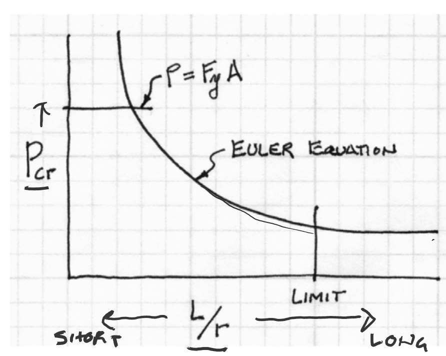
$$r = \sqrt{\frac{I}{A}}$$

$$I = Ar^2$$

portrait by Emanuel Handmann,1753

- A = Cross sectional area (in^2)
- E = Modulus of elasticity of the material (lb/in^2)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

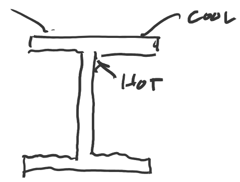
$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \le F_{cr}$$



Analysis of Steel Columns

Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions

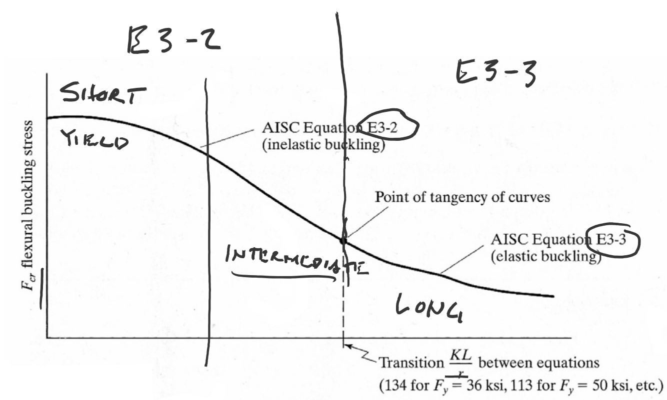


Analysis of Steel Columns

Short columns
Fail by material crushing
Plastic behavior

Intermediate columns
Crush partially and then buckle
Inelastic behavior
Local buckling – flange or web
Flexural torsional buckling - twisting

Long columns
Fail in Euler buckling
Elastic behavior



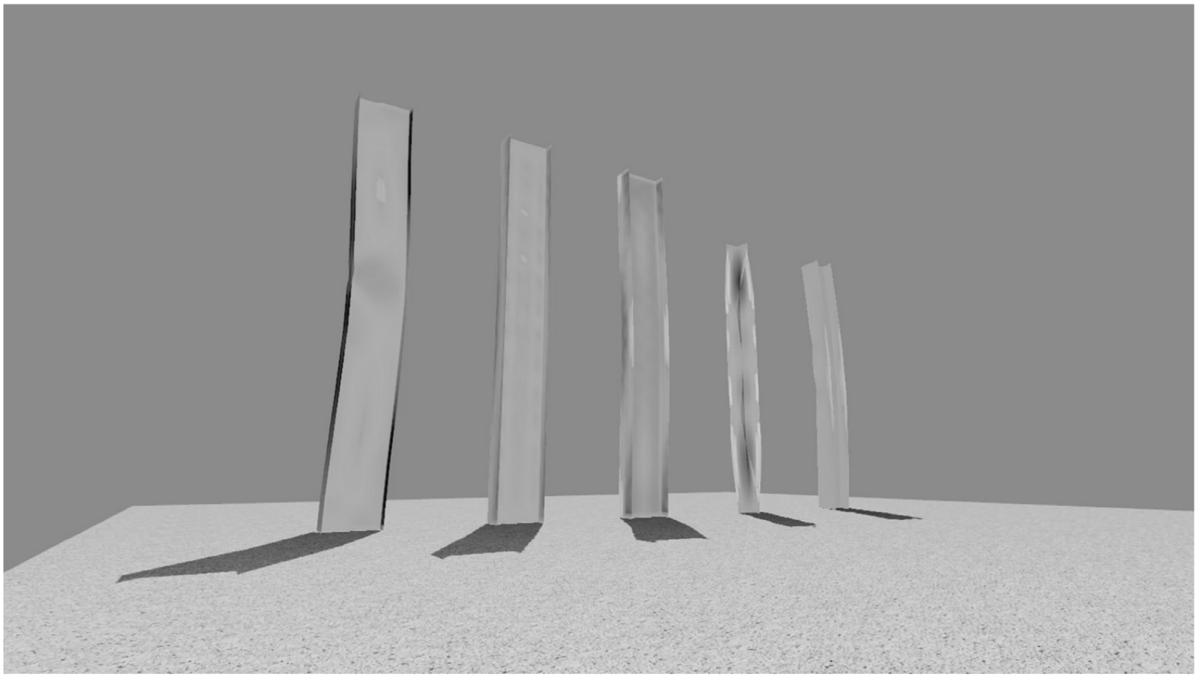
$$slenderness = \frac{KL}{r}$$

short intermediate long

Failure Modes

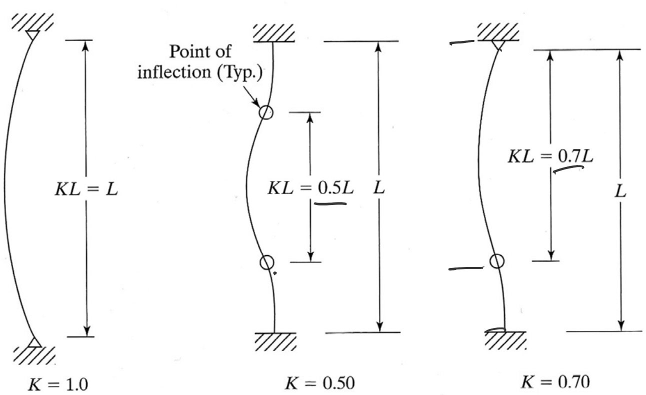
Column 1: Strong axis flexural buckling
Column 2: Web local buckling
Column 3: Weak axis flexural buckling
Column 4: Torsional buckling
Column 5: Flange local buckling

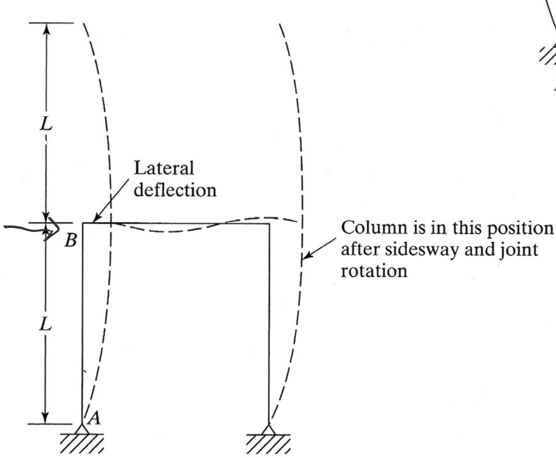
"Dancing Columns"
Sherif El-Tawil

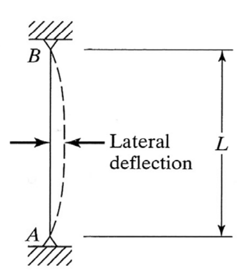


Analysis of Steel Columns

Estimate of effective length factor, K





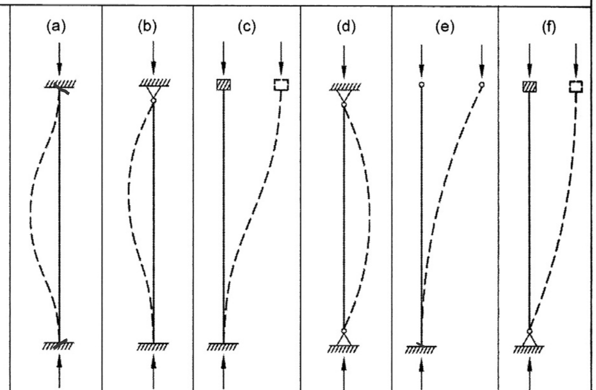


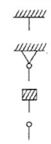
Analysis of Steel Columns

Estimate of K:

TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code						





Determining K factors
by Alignment Charts

Sidesway Inhibited:
- Braced frame
- 1.0 > K > 0.5

Sidesway Uninhibited:
- Un-braced frame
- unstable > K > 1.0

More Pinned:
If I_c/L_c is large
and I_g/L_g is small
The connection is more pinned

More Fixed:
If I_c/L_c is small
and I_g/L_g is large
The connection is more fixed

Sidesway inhibited

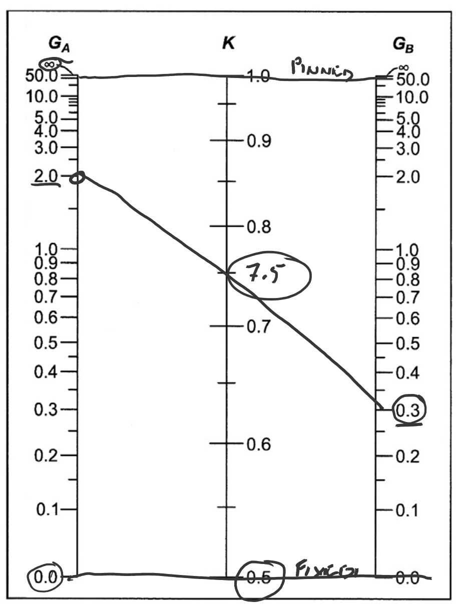
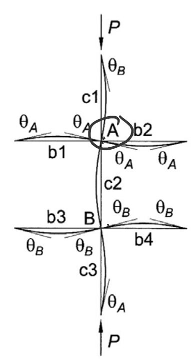


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).



$$G = \frac{\Sigma\left(\frac{EI}{L}\right)_{column}}{\Sigma\left(\frac{EI}{L}\right)_{beam}}$$

STIFFNESS

0 / 100 100 / 0

Determining K factors
by Alignment Charts

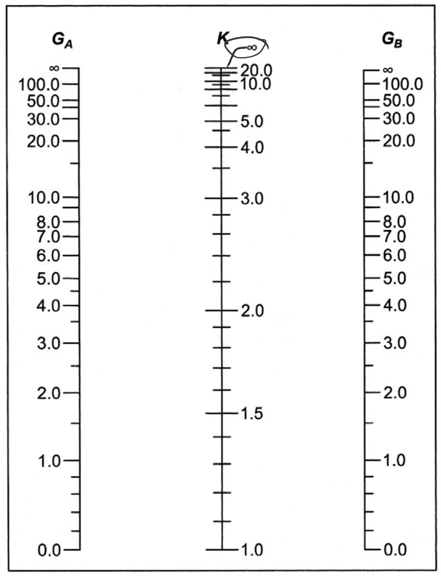
Sidesway uninhibited

Sidesway Inhibited:
- Braced frame
- 1.0 > K > 0.5

Sidesway Uninhibited:
- Un-braced frame
- unstable > K > 1.0

More Pinned:
If Ic/Lc is large
and Ig/Lg is small
The connection is more pinned

More Fixed:
If Ic/Lc is small
and Ig/Lg is large
The connection is more fixed



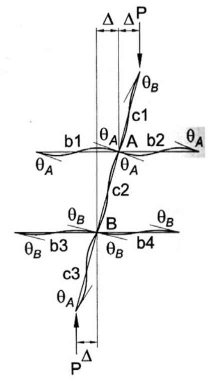


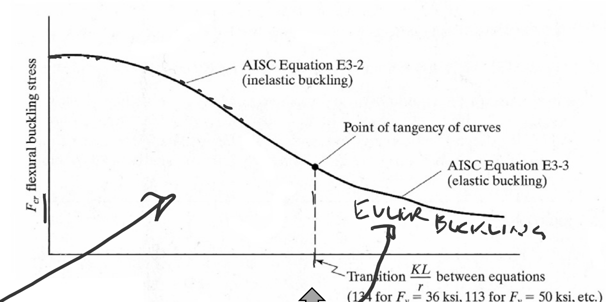
Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\Sigma\left(\frac{EI}{L}\right)_{column}}{\Sigma\left(\frac{EI}{L}\right)_{beam}}$$

Analysis of Steel Columns - LRFD

Euler equation:

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$



short long

Short & Intermediate Columns:

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$

Equation E3-2

Transition Slenderness $4.71\sqrt{\frac{E}{F_y}}$

Long Columns:

$$F_{cr} = 0.877 F_e$$

Equation E3-3

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

Analysis of Steel Columns

pass / fail by LRFD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Factored load – P_u

Required:

- $P_u \leqq ø P_n$ (pass)

1. Calculate slenderness ratios: L_c/r_x and L_c/r_y (L_c = KL)
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$
   and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine $øP_n$ and compare to P_u
   $P_n = F_{cr}\ A_g \qquad ø = 0.9$
6. If $P_u \leqq ø P_n$, then OK

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad \text{Short}$$

$$F_{cr} = 0.877 F_e \qquad \text{Long}$$

Example - Analysis of Steel Columns

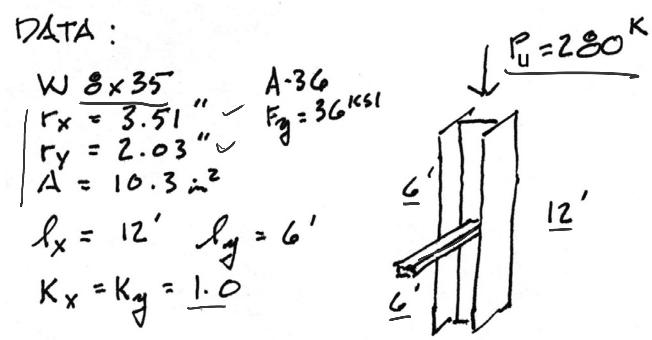
pass / fail by ASD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Factored Load – P_u

Required:

- $P_u \leqq ø P_n$ (pass)



1. Calculate slenderness ratios: L_c/r_x and L_c/r_y (L_c = KL)
   The largest ratio governs.

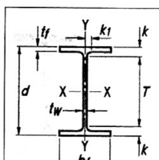


Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	k k_{det}	k_1	T	Workable Gage
	in.²	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W8×67	19.7	9.00	9	0.570	9/16	5/16	8.28	8 1/4	0.935	15/16	1.33	1 5/8	15/16	5 3/4	5 1/2
×58	17.1	8.75	8 3/4	0.510	1/2	1/4	8.22	8 1/4	0.810	13/16	1.20	1 1/2	7/8	↓	↓
×48	14.1	8.50	8 1/2	0.400	3/8	3/16	8.11	8 1/8	0.685	11/16	1.08	1 3/8	13/16		
×40	11.7	8.25	8 1/4	0.360	3/8	3/16	8.07	8 1/8	0.560	9/16	0.954	1 1/4	13/16		
×35	10.3	8.12	8 1/8	0.310	5/16	3/16	8.02	8	0.495	1/2	0.889	1 3/16	13/16		
×31[f]	9.13	8.00	8	0.285	5/16	3/16	8.00	8	0.435	7/16	0.829	1 1/8	3/4		

Table 1-1 (continued)
W-Shapes
Properties
W8-W4

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	Axis X-X I	S	r	Z	Axis Y-Y I	S	r	Z	r_{ts}	h_o	Torsional Properties J	C_w
lb/ft			in.⁴	in.³	in.	in.³	in.⁴	in.³	in.	in.³	in.	in.	in.⁴	in.⁶
67	4.43	11.1	272	60.4	3.72	70.1	88.6	21.4	2.12	32.7	2.43	8.07	5.05	1440
58	5.07	12.4	228	52.0	3.65	59.8	75.1	18.3	2.10	27.9	2.39	7.94	3.33	1180
48	5.92	15.9	184	43.2	3.61	49.0	60.9	15.0	2.08	22.9	2.35	7.82	1.96	931
40	7.21	17.6	146	35.5	3.53	39.8	49.1	12.2	2.04	18.5	2.31	7.69	1.12	726
35	8.10	20.5	127	31.2	3.51	34.7	42.6	10.6	2.03	16.1	2.28	7.63	0.769	619
31	9.19	22.3	110	27.5	3.47	30.4	37.1	9.27	2.02	14.1	2.26	7.57	0.536	530

Example - Analysis of Steel Columns

pass / fail by ASD

Data:

- Column – <u>size</u>, length
- Support conditions
- Material properties – F_y
- Factored Load – P_u

Required:

- $P_u \leqq ø P_n$ (pass)

1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)

DATA:

W 8x35 A-36

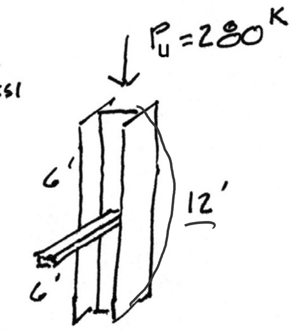
$r_x = 3.51''$ $F_y = 36^{KSI}$

$r_y = 2.03''$

$A = 10.3\ in^2$

$l_x = 12'$ $l_y = 6'$

$K_x = K_y = 1.0$



X - X AXIS

$$\frac{K_x l_x}{r_x} = \frac{144''}{3.51''}$$

$41.03 < 200$

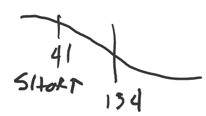
Y-Y AXIS

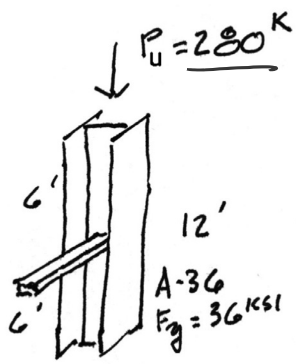
$$\frac{K_y l_y}{r_y} = \frac{72''}{2.03''}$$

35.47

Example - Analysis of Steel Columns

pass / fail by ASD





W8x35

3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine $øP_n$ and compare to P_u
6. If $P_u \leqq ø P_n$, then OK

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$41 < 134$ ∴ SHORT

Euler Equation

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2\ 29000^{KSI}}{41^2} = 170.2^{KSI}$$

Short Column Equation

$$F_{cr} = \left[.658^{\frac{F_y}{F_e}}\right] F_y = 0.9153\,(36) = 32.95\ KSI$$

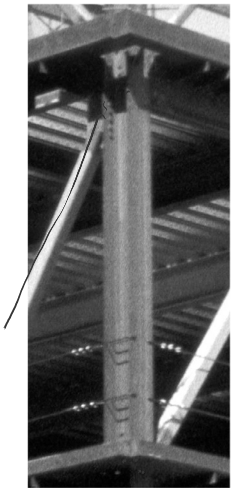
Column Strength

$$P_n = F_{cr} A_g = 32.95\ KSI \times 10.3\ in^2 = 339.39^K$$

$$\phi P_n = 0.9\ P_n = 0.9\,(339.39) = 305.4^K$$

$$P_u = 280^K < 305.4^K = \phi P_n$$ ✓OK

Analysis of Steel Columns
capacity by LRFD



Data:
- Column – size, length
- Support conditions
- Material properties – F_y

Required:
- Max load capacity

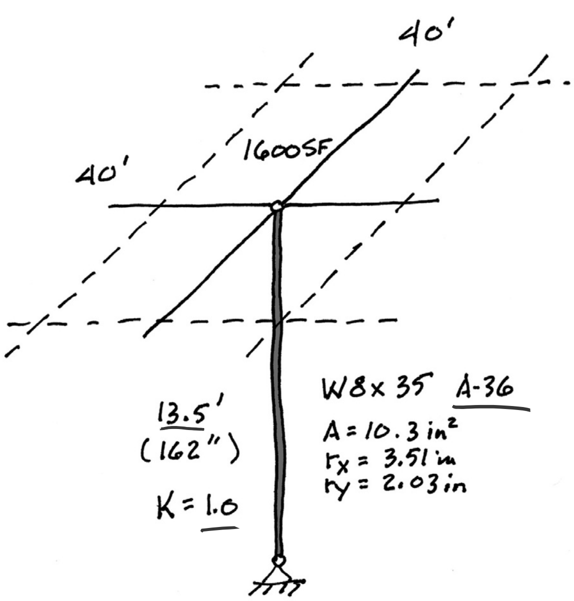
1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness
5. Determine øPn and Compute allowable capacity: $P_n = F_{cr} A_g$ $P_u = øP_n$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$ Short

$$F_{cr} = 0.877 F_e$$ Long

Capacity Example 1

Free standing column
Third floor studio space
Supports roof load = 20 psf DL + SL
snow ≈ 15lbs / FT depth



Capacity Example 1

1. Calculate slenderness ratios. The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness $4.71\sqrt{E/Fy}$ and determine column type (short or long)
4. Calculate F_{cr} based on slenderness

y-y AXIS (CONTROLS)

$$\frac{K_y l_y}{r_y} = \frac{1.(162'')}{2.03''} = 79.8 < 200 \checkmark$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$79.8 < 134$ ∴ SHORT

Euler Buckling

$$F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2\, 29000}{79.8^2} = 44.94 \text{ KSI}$$

Short Column Equation

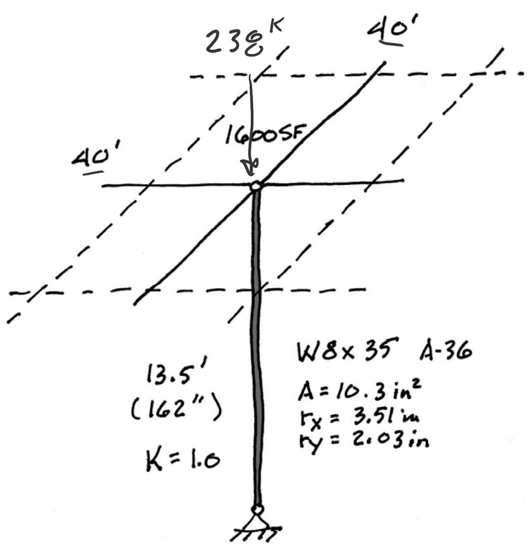
$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y = \left[0.7151\right] 36 = 25.74 \text{ KSI}$$

Capacity Example 1

5. Determine $øP_n$ and Compute allowable capacity: $P_u = øP_n$

DL = 20 psf

20 psf (1600 sf) = 32k on column



Column nominal strength

$$P_n = F_{cr} A_g = 25.74 \text{ KSI } 10.3 \text{ in}^2 = 265.1^K$$

$$\phi P_n = 0.9(265) = 238.6^K = P_U$$

Load capacity DL ?

$$P_U = 1.2(32) + 1.6(SL) = 238.6^K$$

$$SL = 125.1^K$$

$$\text{FOR } A_T = 40 \times 40 = 1600 \text{ SF}$$

$$SL = \frac{125100^{\#}}{1600 \text{ SF}} = 78.2 \text{ PSF}$$

78.2 lbs / 15 lbs/ft = 5.21 ft

Capacity Example 2
long column – using equations

Find the capacity for the 25 ft. column shown.

r_x = 3.51 in.
r_y = 2.03 in.

W8 x 35
F_y = 50 KSI
E = 29000 KSI
L = 25' (NO BRACING)

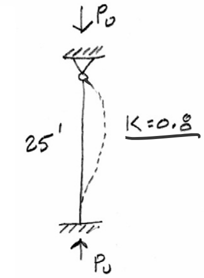
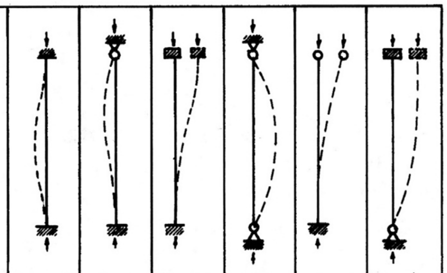


Table G1 Buckling Length Coefficients, K_e

Buckling modes						
Theoretical K_e value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design K_e when ideal conditions approximated	0.65	0.80	1.2	1.0	2.10	2.4
End condition code		Rotation fixed, translation fixed Rotation free, translation fixed Rotation fixed, translation free Rotation free, translation free				

Slenderness y-y

$$\frac{K\ell_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$$

$$4.71\sqrt{\frac{E}{F_y}} = 11.3 < 118.2 \quad \therefore \text{LONG}$$

(F_y = 50)

Euler Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{K\ell}{r}\right)^2} = \frac{\pi^2\, 29000}{118.2^2} = 20.47 \text{ KSI}$$

Long Column Equation

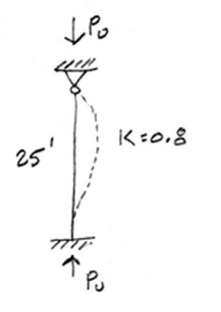
$$F_{cr} = 0.877(20.47) = 17.95 \text{ KSI}$$

Column strength

$$\phi P_n = \phi\, F_{cr}\, A_g = 0.9(17.95 \text{ KSI})(10.3 \text{ in}^2) = 166.4^{K}$$

Capacity Example 2
long column – using table

W8 x 35
F_y = 50 KSI
E = 29000 KSI
L = 25' (NO BRACING)



r_y CONTROLS

KL = 0.8 (25') = 20'

Table 4-1a (continued)
Available Strength in Axial Compression, kips
W-Shapes

W8 — F_y = 50 ksi

Shape	W8×											
lb/ft	67		58		48		40		35		31	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, L_c (ft), with respect to least radius of gyration, r_y: 0	590	886	512	769	422	634	350	526	308	463	273	411
6	542	815	470	706	387	581	320	481	281	423	249	374
7	526	790	455	685	375	563	309	465	272	409	241	362
8	508	763	439	660	361	543	298	448	262	394	232	348
9	488	733	422	634	347	521	285	429	251	377	222	333
10	467	701	403	606	331	497	272	409	239	359	211	317
11	444	668	384	576	314	473	258	388	226	340	200	301
12	421	633	363	546	297	447	243	366	213	321	189	283
13	397	597	342	514	280	421	228	343	200	301	177	266
14	373	560	321	482	262	394	213	321	187	281	165	248
15	348	523	299	450	244	367	198	298	174	261	153	230
16	324	487	278	418	226	340	183	275	160	241	141	212
17	300	450	257	386	209	314	169	253	147	221	130	195
18	276	415	236	355	192	288	154	232	135	203	118	178
19	253	381	216	325	175	264	141	211	123	184	108	162
20	231	347	197	296	159	239	127	191	111	166	97.2	146
22	191	287	163	244	132	198	105	158	91.5	138	80.3	121
24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101
26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5
28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5
30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9
32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1
34	79.9	120	68.1	102	55.1	82.8	44.0	66.1				
Properties												
P_{wo}, kips	126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.1
P_{wi}, kip/in.	19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.3
P_{wb}, kips	507	761	363	546	174	262	127	192	81.1	122	63.0	94.7
P_{fb}, kips	164	246	123	185	87.8	132	58.7	88.2	45.9	68.9	35.4	53.2
L_p, ft	7.49		7.42		7.35		7.21		7.17		7.18	
L_r, ft	47.6		41.6		35.2		29.9		27.0		24.8	
A_g, in.^{2}	19.7		17.1		14.1		11.7		10.3		9.13	
I_x, in.^{4}	272		228		184		146		127		110	
I_y, in.^{4}	88.6		75.1		60.9		49.1		42.6		37.1	
r_y, in.	2.12		2.10		2.08		2.04		2.03		2.02	
r_x/r_y	1.75		1.74		1.74		1.73		1.78		1.72	
$P_{ex}L_c^2/10^4$, k-in.^{2}	7790		6530		5270		4180		3630		3150	
$P_{ey}L_c^2/10^4$, k-in.^{2}	2540		2150		1740		1410		1220		1060	

ASD	LRFD	Note: Heavy line indicates L_c/r_y equal to or greater than 200.
$\Omega_c = 1.67$	$\phi_c = 0.90$	