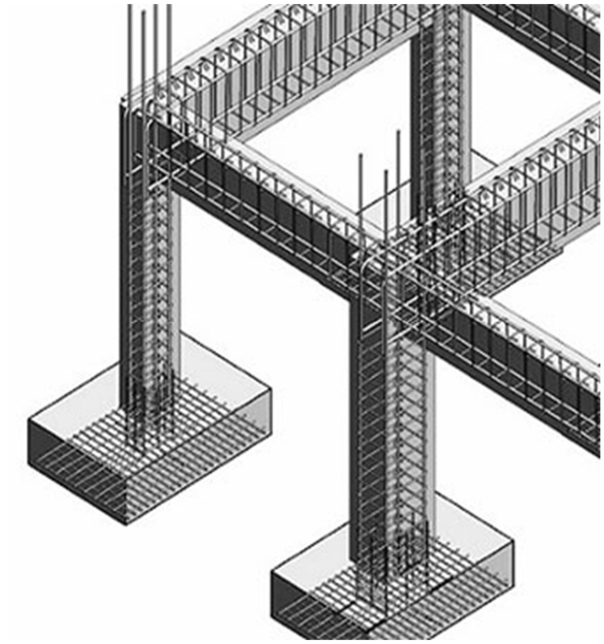
# Reinforced Concrete Beams Ultimate Strength Design
(ACI 318-14) – PART III



- Rectangular Beam Design – Method 2
- Non-Rectangular Beam Analysis
- Reinforced Concrete Examples
- 3D-Print Evolution (Video)

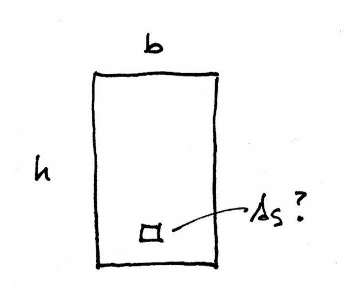


## Rectangular Beam Design
Two approaches:

**Method 1:**

Data:
- Load and Span
- Material properties – $f'_c$, $f_y$
- All section dimensions: h and b

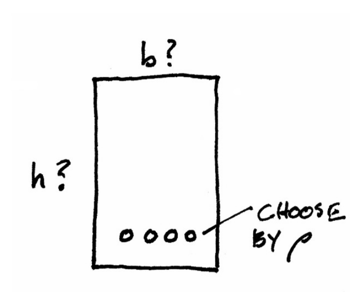
Required:
- Steel area – $A_s$




**Method 2:**

Data:
- Load and Span
- Some section dimensions – h or b
- Material properties – $f'_c$, $f_y$
- Choose $\rho$

Required:
- Steel area – $A_s$
- Beam dimensions – b and h



# Rectangular Beam Design – **Method 2**

**Data:**
- Load and Span
- Some section dimensions – b or h
- Material properties – $f'_c$, $f_y$

**Required:**
- Steel area - $A_s$
- Beam dimensions – b and h

1. Estimate the dead load (estimate h and b) ($L/21 \le h \le L/8$, $h \approx L/12$ and $b:h \approx 1:2$ to $2:3$), find $M_u$
2. Choose $\rho$ (equation assumes $\varepsilon_t = 0.0075$)
3. Calculate $bd^2$
4. Choose b and solve for d (or d and solve b)
5. Revise h, weight, $M_u$, and $bd^2$
6. Find $A_s = \rho bd$
7. Choose bars for $A_s$, determine spacing and cover, and revise d
8. Check that $\varepsilon_t \ge 0.005$ (if not, increase h and reduce $A_s$)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

$$\rho = \frac{\beta_1 f'_c}{4 f_y}$$

$$bd^2 = \frac{M_u}{\varphi \rho f_y \left(1 - 0.59\rho (fy/f'_c)\right)}$$

$$A_s = \rho bd$$

$$a = \frac{\rho f_y \, d}{0.85 f'_c}$$



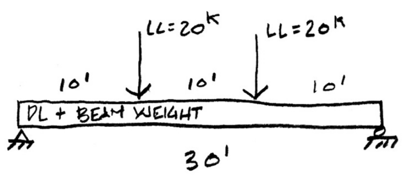
# Rectangular Beam Design

**Data:**
- Load and Span
- Material properties – $f'_c$, $f_y$

**Required:**
- Steel area - $A_s$
- Beam dimensions – b and d

f'c = 3000 psi
fy = 60 ksi

DL = 2 klf + beam
LL = 2 x 20 k




1. Estimate the dead load (self-weight), and find $M_u$ ($h \approx L/12$ and $b:h \approx 1:2$ to $2:3$)

ASSUME $h \approx \frac{L}{12} = \frac{360''}{12} = 30''$

ASSUME $b:h \approx 1:2 \therefore b \approx 15''$

BEAM DL = $150 \frac{15 \times 30}{144} = 469$ PLF

**Table 9.3.1.1—Minimum depth of nonprestressed beams**

| Support condition | Minimum $h$[1] |
|---|---|
| Simply supported | $\ell/16$ |
| One end continuous | $\ell/18.5$ |
| Both ends continuous | $\ell/21$ |
| Cantilever | $\ell/8$ |

[1]Expressions applicable for normalweight concrete and $f_y$ = 60,000 psi. For other cases, minimum $h$ shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

ESTIMATE $M_U$

$$M_U = Pa + \frac{w l^2}{8}$$

$$= 1.6(20^k)(10') + \frac{1.2(2.469 \text{ KLF})(30')^2}{8}$$

$$= 320 + 333.3 = 653.3 \text{ K-'}$$

2. Choose $\rho$ (equation assumes $\varepsilon_t = 0.0075$)

CHOOSE $\rho$

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

# Rectangular Beam Design cont.

3. Calculate $bd^2$

$$bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - 0.59\rho\left(f_y/f'_c\right)\right)}$$

$$bd^2 = \frac{653.3\,(12)}{0.01(0.9)60\left[1 - 0.59(0.01)\left(\frac{60}{3}\right)\right]}$$

$$bd^2 = \frac{7840}{0.573\,(0.882)} = 15492 \text{ in}^3$$

4. Choose b and solve for d
   (or d and solve for b)
   b is based on form size – matches column size
   $h \approx L/12$, $b{:}h \approx 1{:}2$ to $2{:}3$

TRY

| b | d | $h \approx 1.12d$ | A |
|---|---|---|---|
| 14" | 33.27" | 38" | 532 |
| 15" | 32.14" | 36" | 540 |
| 16" | 31.11" | 35" | 560 |

5. Revise h, weight, $M_u$, and $bd^2$

CHOOSE 15 × 36



# Rectangular Beam Design cont.

5. Revise h, weight, $M_u$, and $bd^2$

USE 15 × 36

REVISE DL $= 150\,\frac{540}{144} = 563$ PLF

CHECK $M_u$

$$M_u = 320 + \frac{1.2\,(2{,}563)\,30^2}{8} = 666^{\text{k-'}}$$

REVISE bd

$$bd^2 = \frac{666\,(12)}{0.505} = 15814 \text{ in}^3$$

FOR $b = 15''$ $d = 32.5''$

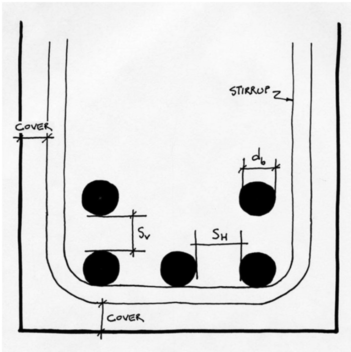
6. Find $A_s = \rho bd$

$$A_s = \rho bd = (0.01)(15'')(32.5'')$$

$$A_s = 4.87 \text{ in}^2$$

# Rectangular Beam Design

7. Choose bars for $A_s$, determine spacing and cover, and revise d




If bars do not fit in one layer, d is measured to the centroid of the pattern.

$$\overline{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}}$$

CHOOSE BARS (SEE TABLE A.4)

TRY 5 x #9 BARS $A_s = 5.0$ in$^2$

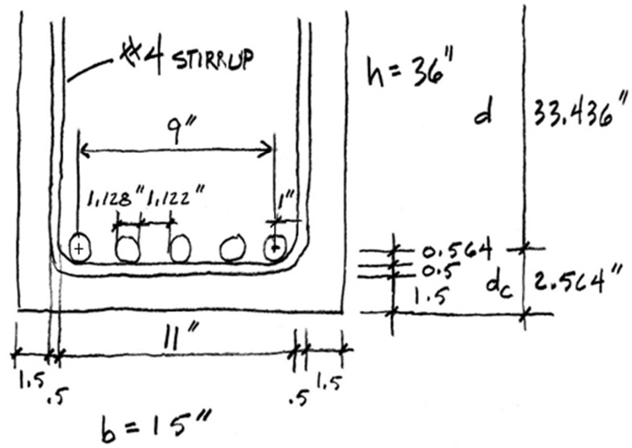



**Table A.4** Areas of Groups of StandardBars (in.$^2$)

| Bar No. | Number of Bars | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 4 | 0.39 | 0.58 | 0.78 | 0.98 | 1.18 | 1.37 | 1.57 | 1.77 | 1.96 | 2.16 | 2.36 | 2.55 | 2.75 |
| 5 | 0.61 | 0.91 | 1.23 | 1.53 | 1.84 | 2.15 | 2.45 | 2.76 | 3.07 | 3.37 | 3.68 | 3.99 | 4.30 |
| 6 | 0.88 | 1.32 | 1.77 | 2.21 | 2.65 | 3.09 | 3.53 | 3.98 | 4.42 | 4.86 | 5.30 | 5.74 | 6.19 |
| 7 | 1.20 | 1.80 | 2.41 | 3.01 | 3.61 | 4.21 | 4.81 | 5.41 | 6.01 | 6.61 | 7.22 | 7.82 | 8.42 |
| 8 | 1.57 | 2.35 | 3.14 | 3.93 | 4.71 | 5.50 | 6.28 | 7.07 | 7.85 | 8.64 | 9.43 | 10.21 | 11.00 |
| 9 | 2.00 | 3.00 | 4.00 | 5.00 | 6.00 | 7.00 | 8.00 | 9.00 | 10.00 | 11.00 | 12.00 | 13.00 | 14.00 |
| 10 | 2.53 | 3.79 | 5.06 | 6.33 | 7.59 | 8.86 | 10.12 | 11.39 | 12.66 | 13.92 | 15.19 | 16.45 | 17.72 |
| 11 | 3.12 | 4.68 | 6.25 | 7.81 | 9.37 | 10.94 | 12.50 | 14.06 | 15.62 | 17.19 | 18.75 | 20.31 | 21.87 |
| 14 | 4.50 | 6.75 | 9.00 | 11.25 | 13.50 | 15.75 | 18.00 | 20.25 | 22.50 | 24.75 | 27.00 | 29.25 | 31.50 |
| 18 | 8.00 | 12.00 | 16.00 | 20.00 | 24.00 | 28.00 | 32.00 | 36.00 | 40.00 | 44.00 | 48.00 | 52.00 | 56.00 |

Jack C McCormac, 1978
*Design of Reinforced Concrete,*



# Rectangular Beam Design

7. Choose bars for $A_s$ and determine spacing and cover, recheck h and weight

$d = 33.436"$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{5(60)}{.85(3)15} = 7.843"$$

Make final check of $M_n$ using final d, and check that $M_u \leq \phi M_n$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5(60)\left(33.436 - \frac{7.843}{2}\right)$$

$M_n = 8854$ K-" $= 737.8$ K-'

$\phi M_n = 0.9(737.8) = 664$ K-'

$M_u = 653.3 < 664$ ✓ ok

8. Check that $\varepsilon_t \geq 0.005$ (if not, increase h and reduce $A_s$)

$$c = \frac{a}{\beta_1} = \frac{7.843"}{0.85} = 9.227"$$

$$\epsilon_t = \frac{d-c}{c}(0.003)$$

$$\epsilon_t = \frac{33.436 - 9.227}{9.227}(0.003)$$

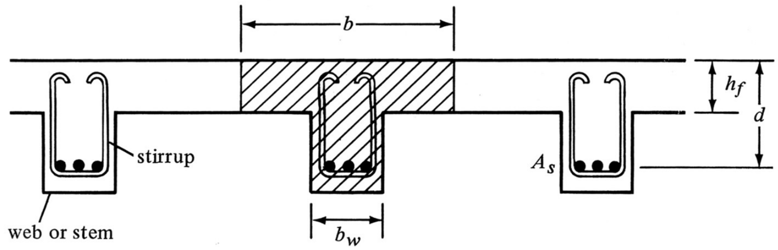
$\epsilon_t = 0.00787 > 0.005$ ✓ok

9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

# T Beams

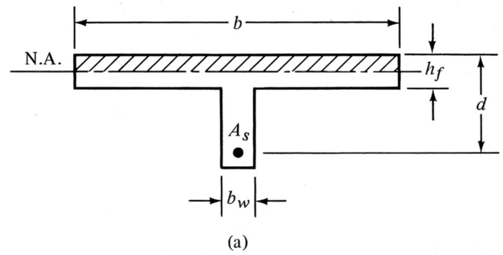
Dimensional limits

Nomenclature

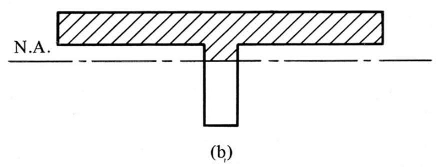



Possible N.A. locations:

Within flange – rectangular




Within stem – non-rectangular



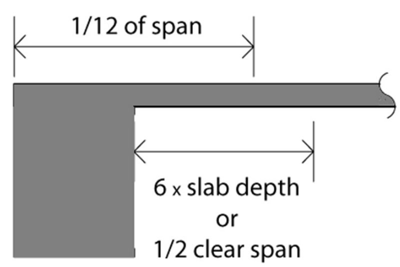



# T Beams - Effective Flange Width, $b_e$

**Slab on one side:**

$b_e$ least of either (total width) or (overhang + stem)

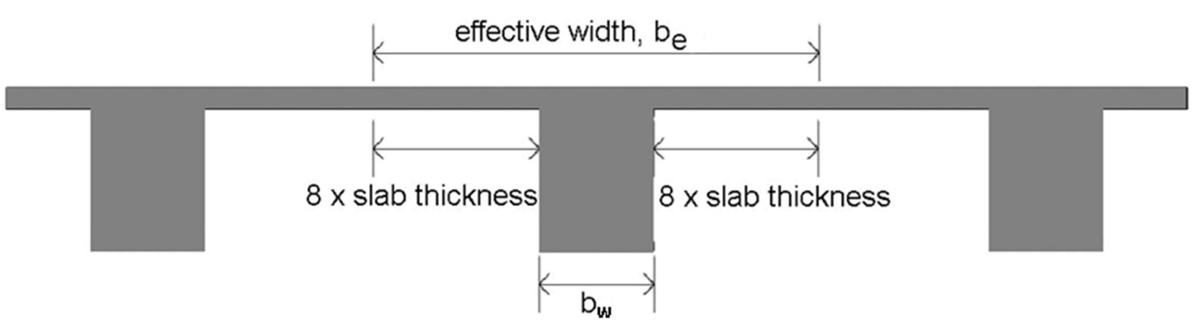
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam




**Slab on both sides:**

$b_e$ least of either (total width) or (2 x overhang + stem)

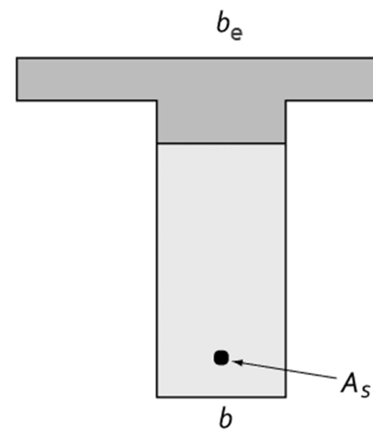
- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)



# Non-Rectangular Beam Analysis

**Data:**
- Section dimensions – b, $b_e$, h, (span)
- Steel area - $A_s$
- Material properties – $f'_c$, $f_y$

**Required:**
- Required Moment – $M_u$ (or load, or span)



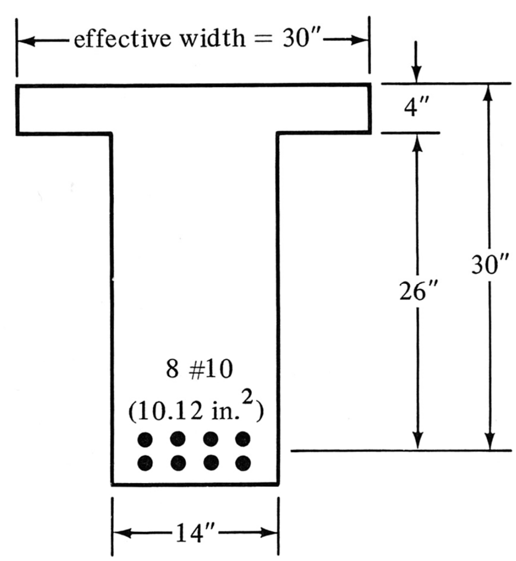

1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set T = C and solve for $A_c = T/(0.85\ f'c)$
3. Draw and label diagrams for section and stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or $C_1$ and $C_2$)
4. Determine the location of a
   Working from the top down,
   add up area to make $A_c$
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$
8. Check $A_{s,min} < A_s < A_{s,max}$
9. Check that $\varepsilon_t \geq 0.005$



# T Beam Analysis

Given: $f'_c$ = 3000 psi
$f_y$ = 50 ksi
dimensions. Use $b_{eff}$ = 30”

Req’d: Moment capacity, Mu



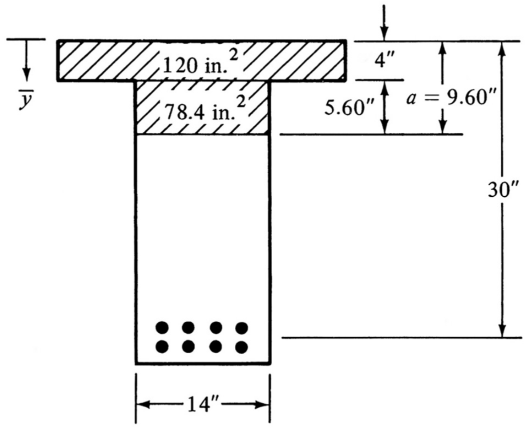

1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$

$$T = A_s f_y = 10.12\ in^2\ 50^{ksi} = 506^k$$

2. Set T = C and solve for $A_c = T/(0.85\ f'c)$

$$A_c = \frac{T}{0.85 f'_c} = \frac{506^k}{0.85\ 3^{ksi}} = 198.4\ in^2$$

# T Beam Analysis (cont.)




FLANGE = 30″ × 4″ = 120 < 198.4 ∴ NA IN WEB

$198.4 - 120 = 78.4 \text{ in}^2 = 14'' \times 5.60''$

$a = 4'' + 5.60'' = 9.60''$

BY PARTS (FOR EACH AREA)

$Z_1 = 30'' - 4''/2 = 28''$

$Z_2 = 30'' - 4'' - 5.6/2'' = 23.2''$

$C_1 = A_{c1}\, 0.85 f'_c = 120(0.85)(3) = 306^K$

$C_2 = A_{c2}\, 0.85 f'_c = 78.4(0.85)(3) = 199.9^K$

$M_n = \sum C_i z_i = 306(28) + 199.9(23.2) = 8568 + 4638 = 13206^{K\text{-}IN} = 1101^{K\text{-}FT}$

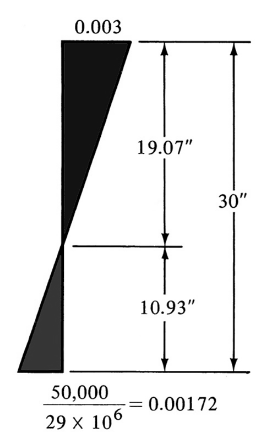
$M_u = \phi M_n = 0.9(1101) = 991^{K\text{-}FT}$

3. Draw and label diagrams for section & stress
    1. Determine b effective (for T-beams)
    2. Locate T and C (or $C_1$ and $C_2$)
4. Determine the location of a
    Working from the top down, add up area to make $A_c$
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$



# T Beam Analysis (cont.)

$\rho_{bal}$




$\rho_{max} = 0.75\, \rho_{bal}$

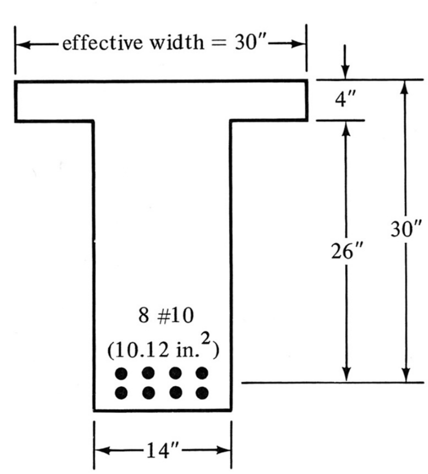
$a_{bal}$ = beta $c_{bal}$ = 0.85 (19.07”) = 16.21”

Ac = (4”) (30”) + (12.21”) (14”) = 291 in$^2$

$C_{bal}$ = (0.85) (3) (291) = 742$^k$

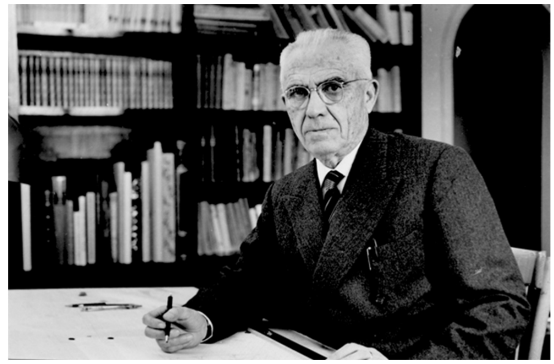
$T_{max}$ = 0.75 $C_{bal}$ = (0.75) (742) = 556$^k$

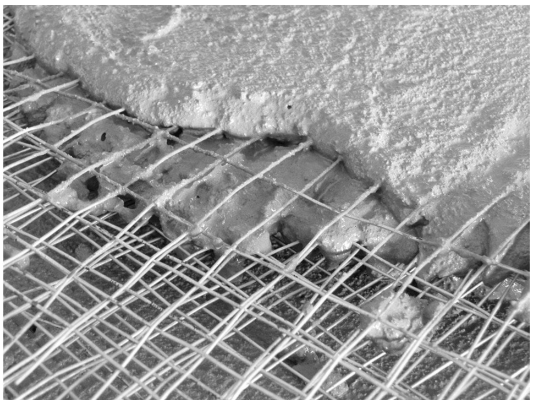
$T_{used} = A_s f_y$ = (10.12) (50) = 506$^k$ < 556$^k$ ok

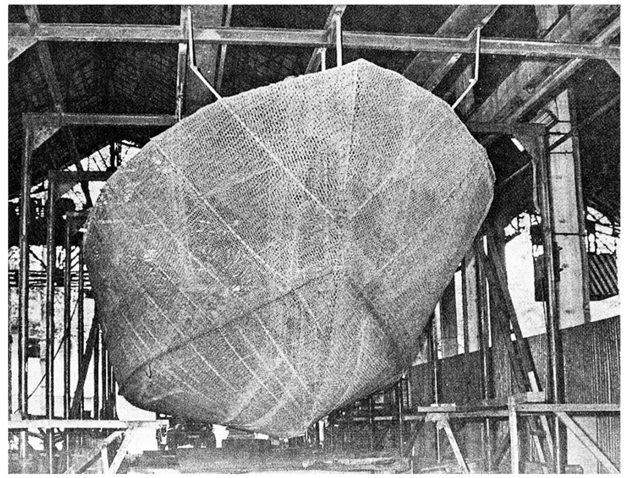


# Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking









# Ferrocement

- Pioneered by Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking
- Low-tech applications

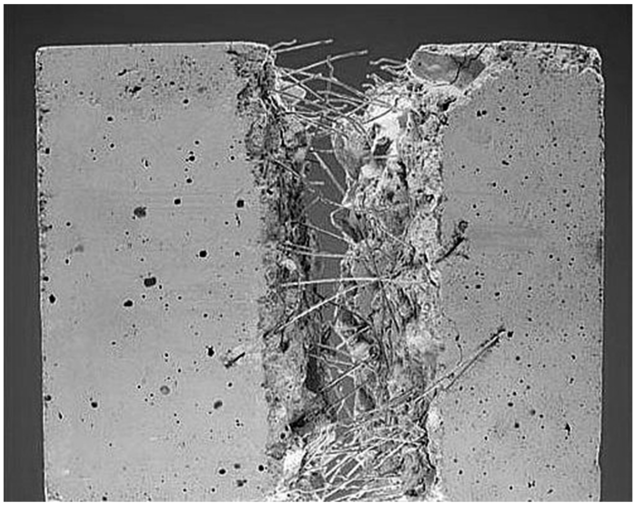
Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

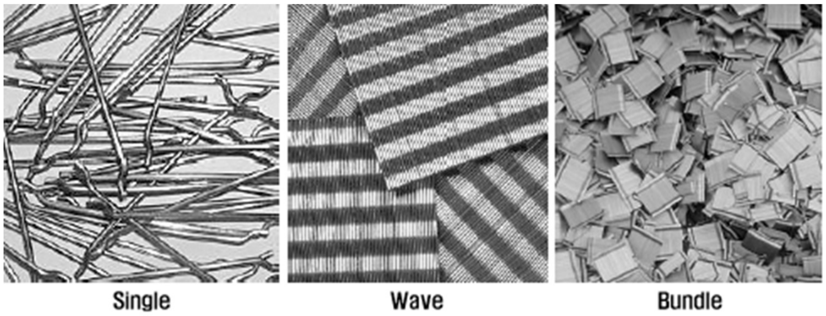
Palazetto dello Sport, Rome, 1957. P.L. Nervi

# Fiber Reinforced Concrete

Several different fiber types:
- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

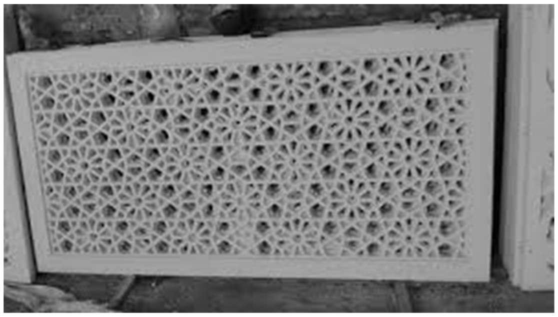
Better crack control
Secondary reinforcement








# Glass Fiber Reinforced Concrete - GFRC

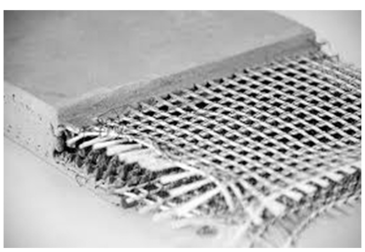


## Carbon Fiber

## Bamboo



## Textile Reinforced Concrete (TRC)



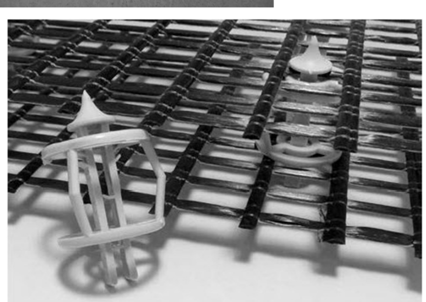


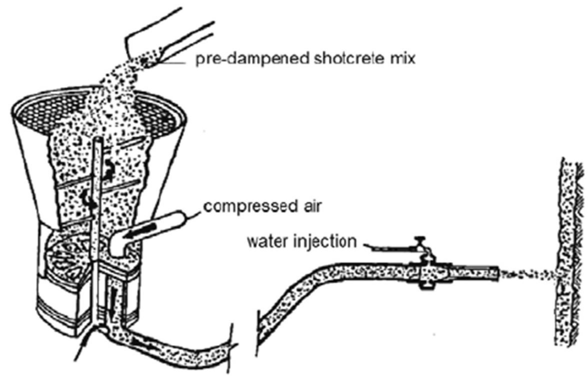
***Figure 12:*** *distTEX: special spacers for textile grids [photo: Frank Schladitz, TU Dresden]*

***Figure 13:*** *Manufacturing of the TRC hypar-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]*


***Figure 10:*** *Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]*

# Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels






# 3D-Print Evolution


https://www.youtube.com/watch?v=awpmJriWcEw