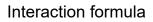
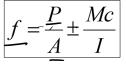
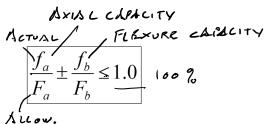


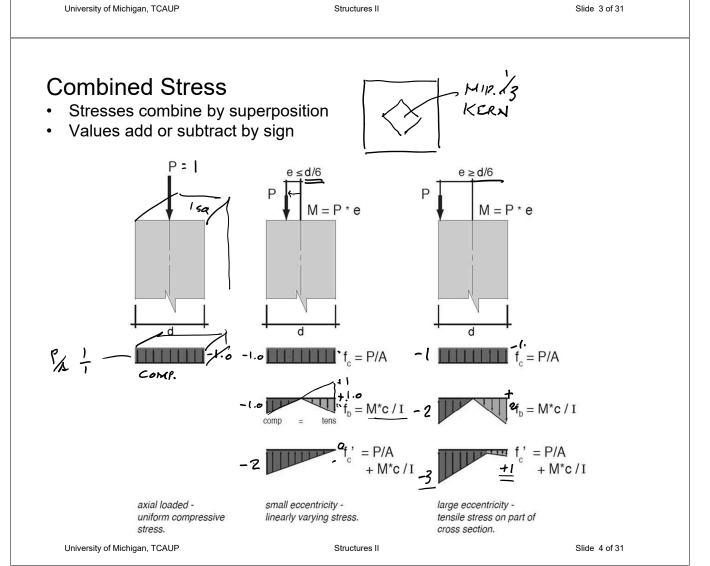
# **Eccentric Loads**

- · Load is offset from centroid
- Bending Moment = P e
- Total load = P + M









y

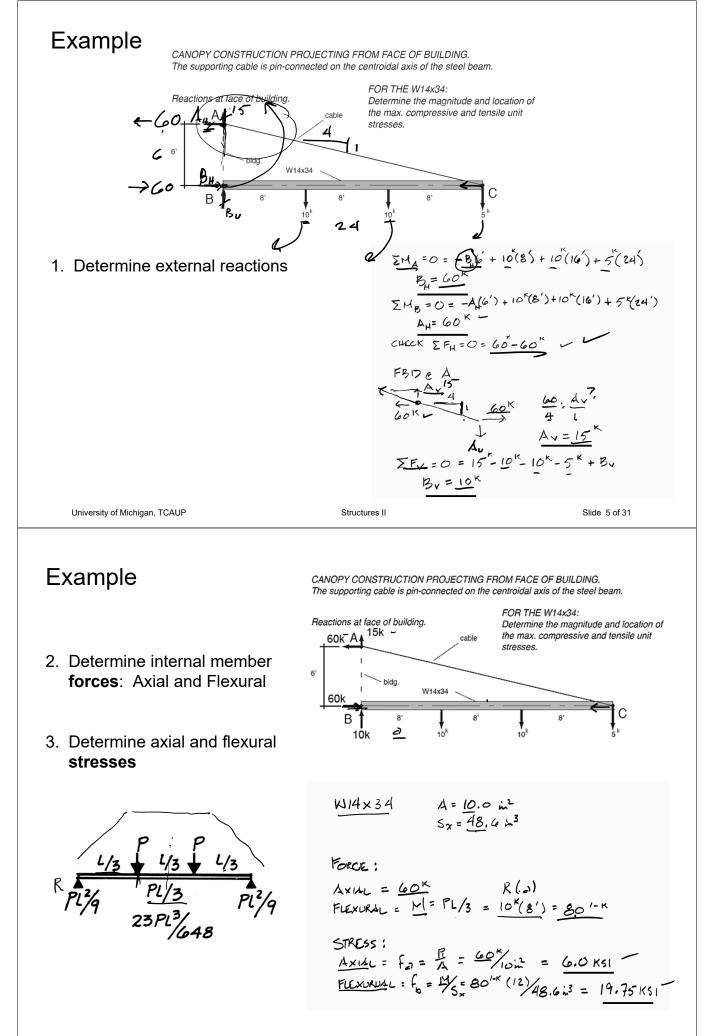
Х

+

ተ

e

M=Pe



# Example

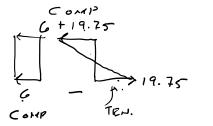
2. Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

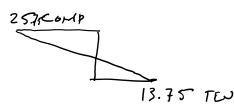
COMBINED STRESS

TOP SIDE ;

$$f_{2} + f_{b} = 6.0 + 19.75 = 25.75 \text{ KSI}(comp)$$

BOTTOM SIDE: fa - fb = 6.0 - 19.75 = -13.75 KSI (TENS)





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Structures II

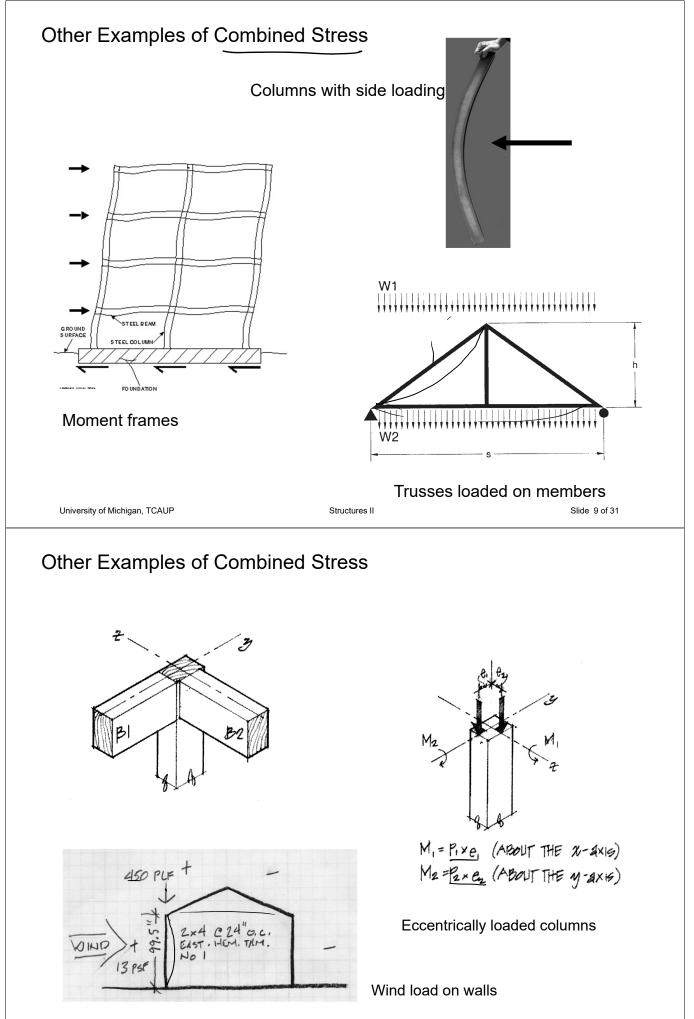
Slide 7 of 31

# Second Order Stress "P Delta Effect"

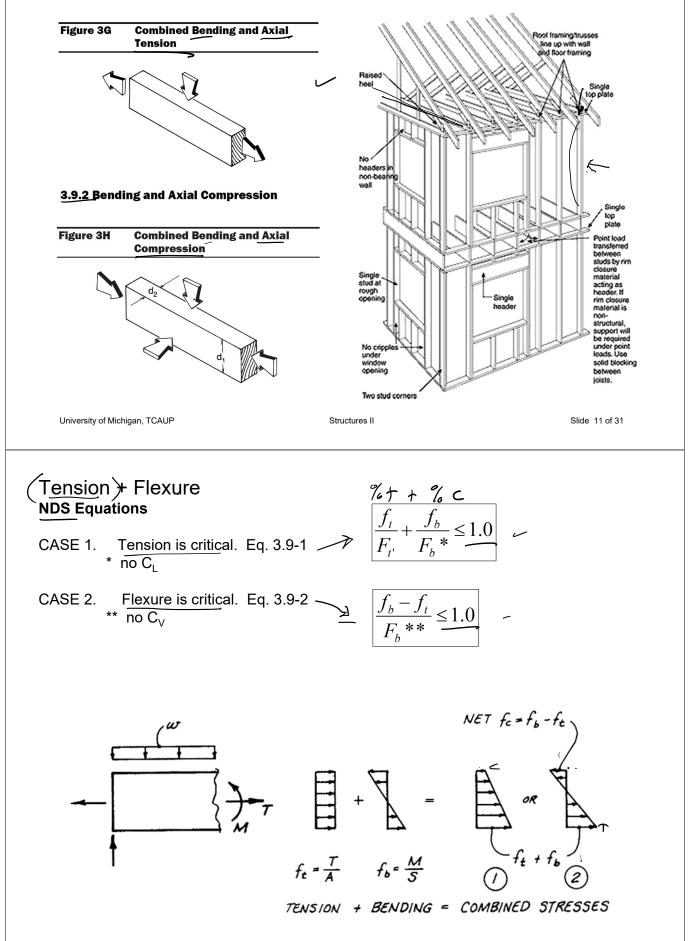
With larger deflections this can become significant.

- 1. Eccentric load causes bending moment
- 2. Bending moment causes deflection,  $\Delta$
- 3. P x  $\Delta$  causes additional moment





# **Combined Stress in NDS**



### **3.9.1 Bending and Axial Tension**

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_{t}}{F_{t}} + \frac{f_{b}}{F_{b}} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$

and

$$\frac{f_{b} - f_{t}}{F_{b}^{\star}} \leq 1.0 \qquad \text{FLEXURE CRIT.} \qquad (3.9-2)$$

where:

- $F_{b}^{\cdot}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{L}$
- $F_{b}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{v}$ \_

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Structures II

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## **Example Problem**

Given: Queen Post truss Hem-Fir No.1 & Better  $F_b = 1100 \text{ psi}$  $F_t = 725 \text{ psi}$  $F_c = 1350 \text{ psi}$  $E_{min} = 550000 \text{ psi}$ 

> span = 30 ft. spaced 48" o.c. D + S Load = 44 psf (projected) D (attic + ceiling) = 8 psf

bottom chord: 2x8 top chord: 2x10

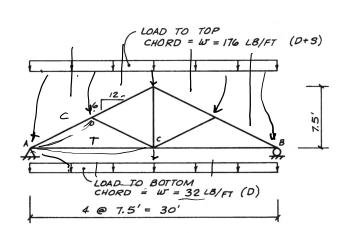
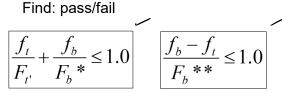
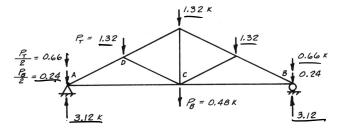


Figure 3G Combined Bending and Axial Tension



1. Determine truss joint loading



## Example (cont.)

- 2. Determine the external end reactions of the whole truss. The geometry and loads are symmetric, so each reaction is  $\frac{1}{2}$  of the total load.
- 3. Use an FBD of the reaction joint to find the chord forces. Sum the forces horizontal and vertical to find the components.

Top chord = 4.96 k compression Bottom chord = 4.44 k tension

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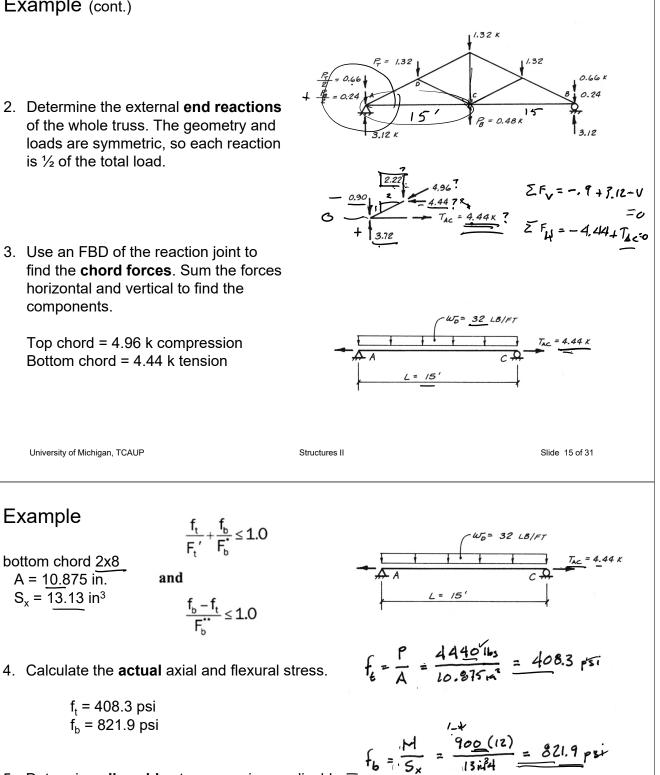
Example

bottom chord 2x8 A = 10.875 in.

 $S_x = 13.13 \text{ in}^3$ 

f<sub>t</sub> = 408.3 psi

f<sub>b</sub> = 821.9 psi



5. Determine **allowable** stresses using applicable factors:

and

(tension: D+S)  

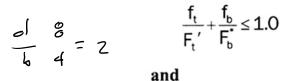
$$\rightarrow F_{t}' = F_{t} (C_{D} C_{F})$$
  
 $F_{t}' = 725 (1.15 1.2) = 1000 \text{ psi} > 408.3$   
(flexure: D+S)  
 $F_{b}' = F_{b} (C_{D} C_{L} C_{F})$   
 $F_{b}' = 1100 (1.15 1.0 1.2) = 1518 \text{ psi} > 821.9 \text{ psi}$ 

$$M = \frac{\omega l^{2}}{8} = \frac{32(15)^{2}}{8} = \frac{900}{8} + \frac{32}{8}$$

$$S_{x} = 13.14 \text{ m}^{3}$$

# Example

bottom chord 2x8



# $\frac{f_{b} - f_{t}}{F_{b}^{**}} \leq 1.0$

5. Determine **allowable** stresses using applicable factors:

(tension: D+S)  $F_t = F_t (C_D C_F)$   $F_t = 725 (1.15 1.2) = 1000 \text{ psi} > 408.3$ 54

(flexure: D+S)  

$$F_b' = F_b (C_D C_L C_F)$$
  
 $F_b' = 1100 (1.15 1.0 1.2) = 1518 \text{ psi} > 821.9 \text{ psi}$   
 $F_b' = 250 \text{ psi}$ 

Structures II

#### 4.4.1 Stability of Bending Members

4.4.1.1 Sawn lumber bending members shall be designed in accordance with the lateral stability calculations in 3.3.3 or shall meet the lateral support requirements in 4.4.1.2 and 4.4.1.3.

4.4.1.2 As an alternative to 4.4.1.1, rectangular sawn lumber beams, rafters, joists, or other bending members, shall be designed in accordance with the following provisions to provide restraint against rotation or lateral displacement. If the depth to breadth, d/b, based on nominal dimensions is:

(a)  $d/b \le 2$ ; no lateral support shall be required. (b)  $2 \le d/b \le 4$ ; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

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# Example

bottom chord 2x8

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### **3.9.1 Bending and Axial Tension**

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{-f_{t}}{F_{t}} + \frac{f_{b}}{F_{b}} \le 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$
and
$$\frac{f_{b} - f_{t}}{F_{t}} \le 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2)$$

and

$$F_{b}^{t} \leq 1.0$$
 FLEXURE CRIT.

where:

- $F_{b}^{*}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{L}$
- $F_{b}$  = reference bending design value multiplied by all applicable adjustment factors except  $C_{v}$

 $f_{b} = 821.9 \text{ psi} \qquad f_{t} = 408.3 \text{ psi}$   $F'_{b} = 1518 \text{ psi} \qquad F'_{t} = 1000 \text{ psi}$  (3.9-1)  $\frac{408.3}{1000} + \frac{821.9}{1518}$  0.4083 + 0.5414 = 0.95 75% 0.95 < 1.0 pass (3.9-2)  $\frac{621.9 - 408.3}{1518} = 0.2724$  0.27 < 1.0 pass

# **Bending + Axial Compression**

#### 3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

$$\begin{bmatrix} \frac{f_{o}}{F_{o}'} \end{bmatrix}^{2} + \frac{f_{b1}}{F_{b1}' [1 - (f_{o}/F_{cE1})]} + \frac{f_{b2}}{F_{b2}' [1 - (f_{o}/F_{cE2}) - (f_{b1}/F_{bE})^{2}]} \le 1.0$$
(3.9-3)

where:

$$f_{c} < F_{cE1} = \frac{0.822 E_{min}'}{(\ell_{e1} / d_{1})^{2}}$$
 for either uniaxial  
edgewise bending or  
biaxial bending

and

$$f_{c} < F_{cE2} = \frac{0.822 E_{min}'}{(\ell_{e2} / d_{2})^{2}} \quad \begin{array}{c} \text{for uniaxial } \underline{flatwise} \\ \text{bending or biaxial} \\ \text{bending} \end{array}$$

and

Structures II

(3.9-4)

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}'}{(R_{B})^2}$$
 for biaxial bending

- f<sub>b1</sub> = actual edgewise bending stress (bending load applied to narrow face of member), psi
- f<sub>b2</sub> = actual flatwise bending stress (bending load applied to wide face of member) , psi
- $d_1$  = wide face dimension (see Figure 3H), in.
- d<sub>2</sub> = narrow face dimension (see Figure 3H), in.

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Example

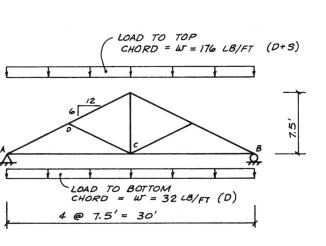
and

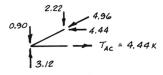
top chord 2x10

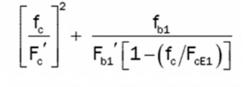
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 $\frac{f_c}{F_{cE2}} + \left(\frac{f_{b1}}{F_{bE}}\right)^2 < 1.0$ 

- 4. Calculate the **actual** axial and flexural stress. 2x10:  $A = 13.875 \text{ in}^2$   $f_c = 357.5 \text{ psi}$ 
  - $\begin{array}{ll} {\sf A} = 13.875 \mbox{ in}^2 & {\sf f}_{\rm c} = 357.5 \mbox{ psi} \\ {\sf Sx} = 21.39 \mbox{ in}^3 & {\sf f}_{\rm b1} = 694.2 \mbox{ psi} \end{array}$







$$\omega = 17G PLF$$

$$4960^{*} - \sqrt{1111111} \leftarrow 4960^{*}$$

$$7.5'$$

$$(Referred)$$

$$f_{c} = \frac{P}{A} = \frac{4960^{*}}{1.5 \times 9.25} = 357.5 \text{ psi}$$

$$f_{b} = \frac{M}{5_{x}} = \frac{1237.5 (12)}{21.39} = 694.2 \text{ psi}$$

$$M = \frac{\omega P^{2}}{5} = \frac{176 \text{ Pur}(7.5')^{2}}{8} = 1237.5'^{-4}$$

$$S_{x} = 21.39 \text{ m}^{3}$$

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Example top chord 2x10

$$\left[\frac{f_{c}}{F_{c}^{'}}\right]^{2} + \frac{f_{b1}}{F_{b1}^{'} \left[1 - \left(f_{c}^{'}/F_{cE1}\right)\right]}$$

5. Determine **allowable** stresses using applicable factors:

(compression: D+S)  $F_c' = F_c (C_D C_F C_P)$  $F_c' = 1350 (1.15 \ 1.0 \ 0.897) = 1392.6 \text{ psi} > 357.5$ 

(flexure: D+S)  $F_b' = F_b (C_D C_L C_F)$  $F_b' = 1100 (1.15 \ 1.0 \ 1.1) = 1391.5 \text{ psi} > 694.2$ 

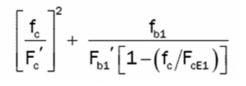
$$C_{p} = \frac{1 + (F_{cE}/F_{c}^{*})}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_{c}^{*})}{2c}\right]^{2} - \frac{F_{cE}/F_{c}^{*}}{c}} \quad (3.7-1)$$
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$$C_{P}$$

$$I_{C} = \frac{0.822}{(f_{e}/d)^{2}} = \frac{0.822}{(f_{e}/d)^{2}} = \frac{0.822}{(f_{e}/d)^{2}} = \frac{0.822}{(f_{e}/d)^{2}} = \frac{0.822}{(f_{e}/d)^{2}} = \frac{3920}{(f_{e}/d)^{2}} = \frac{3920}{(f_{e}/d)^{2}} = \frac{3920}{(f_{e}/d)^{2}} = \frac{2.46}{(f_{e}/d)^{2}} = \frac{2.46}{(f_{e}/d)^{2}}$$

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Example top chord 2x10



	Size F	actors, C <sub>F</sub>				
		F <sub>b</sub>	Ft	F <sub>c</sub>		
	Width (depth	Thickness (b)				
Grades		2" & 3"	4"			
	2", 3", & 4"	1.5	1.5	1.5	1.15	
Select	5"	1.4	1.4	1.4	1.1	
Structural,	6"	1.3	1.3	1.3	1.1	
No.1 & Btr,	8"	1.2	1.3	1.2	1.05	
No.1, No.2,	10"	1.1	1.2	1.1	1.0	
No.3	12"	1.0	1.1	1.0	1.0	
	14" & wider	0.9	1.0	0.9	0.9	
	2", 3", & 4"	1.1	1.1	1.1	1.05	
Stud	5" & 6"	1.0	1.0	1.0	1.0	
	8" & wider	Use No.3				
Construction.	2", 3", & 4"	1.0	1.0	1.0	1.0	
Standard						
Utility	4"	1.0	1.0	1.0	1.0	
	2" & 3"	0.4		0.4	0.6	

5. Determine **allowable** stresses using applicable factors:

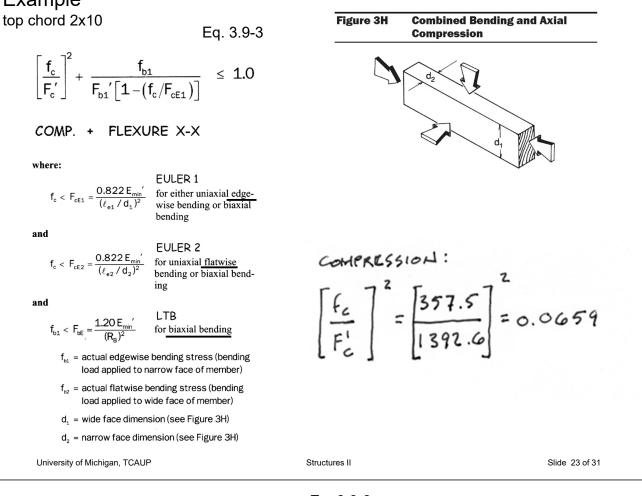
(compression: D+S)  $F_c' = F_c (C_D C_F C_P)$  $F_c' = 1350 (1.15 1.0 0.897) = 1392.6 psi > 357.5$ 

(flexure: D+S)  $F_b' = F_b (C_D C_L C_F)$  $F_b' = 1100 (1.15 1.0 1.1) = 1391.5 \text{ psi} > 694.2$ 

#### 3.3.3 Beam Stability Factor, CL

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation,  $C_L = 1.0$ .

## Example



Eq. 3.9-3

$$\left\lfloor \frac{f_{c}}{F_{c}'} \right\rfloor + \frac{f_{b1}}{F_{b1}' \left[ 1 - \left( f_{c}/F_{cE1} \right) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

EULER 1

-2

$$f_c < F_{cE1} = \frac{0.822 E_{min}'}{(\ell_{e1} / d_1)^2}$$

for either uniaxial edgewise bending or biaxial bending

- f<sub>b1</sub> = actual edgewise bending stress (bending load applied to narrow face of member)
- d<sub>1</sub> = wide face dimension (see Figure 3H)
- d<sub>2</sub> = narrow face dimension (see Figure 3H)

$$f_{c} = \frac{P}{A} = \frac{4960^{*}}{1.5 \times 9.25} = 357.5 \text{ psi}$$

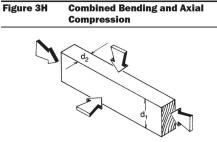
$$f_{c} = \frac{M}{S_{x}} = \frac{1237.5}{1.5 \times 9.25} = 357.5 \text{ psi}$$

$$f_{b} = \frac{M}{S_{x}} = \frac{1237.5}{21.39} = 694.2 \text{ psi}$$

$$f_{c} = \frac{W}{S} = \frac{176 \text{ Pur}(7.5')^{2}}{21.39} = 1237.5'^{-1} \text{ le}_{1} = 9.25''$$

$$f_{c} = \frac{W}{S} = \frac{176 \text{ Pur}(7.5')^{2}}{8} = 1237.5'^{-1} \text{ le}_{2} = \frac{8.385(12)}{9.25''} = 10.882$$

$$f_{c} = 1 = \frac{0.822(55000)}{10.88^{2}}$$



FLEXURE :

$$\frac{f_{61}}{F_{61}} = \frac{694.2}{1392} = 0.4987$$

AMPLIFICATION FACTOR :

$$\frac{1}{1 - (357.5/3820)} = \frac{1}{0.906}$$
  
0.4987 (1.103) = 0.550

COMBINATION :

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= 3820 PSI

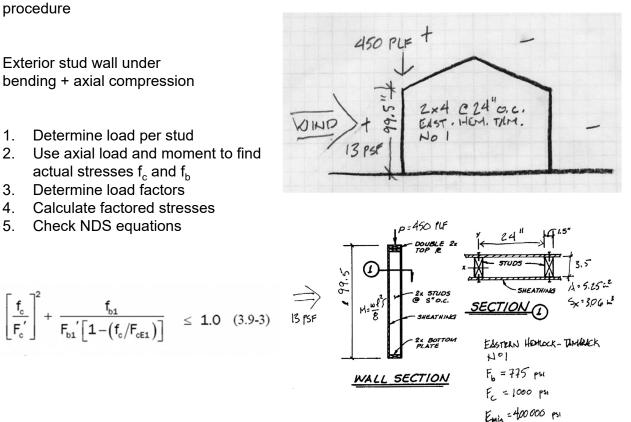
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## Combined Stress in NDS procedure

Exterior stud wall under bending + axial compression

- Determine load per stud 1.
- Use axial load and moment to find 2. actual stresses f<sub>c</sub> and f<sub>b</sub>
- 3. **Determine load factors**
- Calculate factored stresses 4.
- 5. **Check NDS equations**



**Combined Stress in NDS** 

University of Michigan, TCAUP

example

Exterior stud wall under

bending + axial compression

$$\left[\frac{f_{c}}{F_{c}'}\right]^{2} + \frac{f_{b1}}{F_{b1}' \left[1 - (f_{c}/F_{cE1})\right]} \leq 1.0 \quad (3.9-3)$$

- Determine load per stud 1.
- Use axial load and moment to find 2. actual stresses fc and fb

$$\frac{1}{1000} = \frac{1}{1000} = \frac{1$$

P = LOAD/STUD  $P = 450 \text{ PLF } \frac{OC}{12} = 450 \frac{C4}{12} = 900 \text{ LBS}$  $\omega = 13PSF \ oc = 13 \ \frac{24}{12} = 26 \ PLF/STUD$  $M_{x} = \frac{\omega l^{2}}{8} = \frac{26 (99.5/12)^{2}}{8} = [223.4 - *]$  $f_c = \frac{P}{A} = \frac{900}{5125} = 171.43 \text{ Psi}$  $f_b = M_s = \frac{223.4(12)}{3.06} = 875.5 \text{ psi}$ 

Structures II

Structures II

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Combined Stress in NDS	Size Factors, C <sub>F</sub>						
example			F <sub>b</sub> Thickness (breadth)		Ft	F <sub>c</sub>	
example							
Exterior stud wall under	Grades	Width (depth)	2" & 3"	4"			
bending + axial compression		2", 3", & 4"	1.5	1.5	1.5	1.15	
	Select	5"	1.4	1.4	1.4	1.1	
10=450 ME x and 1045"	Structural,	6"	1.3	1.3	1.3	1.1	
TOP R 24	No.1 & Btr,	8"	1.2	1.3	1.2	1.0	
x - 57UD5 - 3.5	No.1, No.2,	10"	1.1	1.2	1.1	1.0	
0- 4=5.25 <sup>2</sup>	No.3	12"	1.0	1.1	1.0	1.0	
		14" & wider	0.9	1.0	0.9	0.9	
13 PSF ME T SHEATHING DECTION (1)		2", 3", & 4"	1.1	1.1	1.1	1.0	
PLATE EDSTEAN HOMOCK-TAMERICK	Stud	5" & 6"	1.0	1.0	1.0	1.0	
$\frac{1}{100}$		8" & wider	Use No.3				
$\frac{WALL SECTION}{F_{c} = 1000 \text{ psi}}$	Construction	2", 3", & 4"	1.0	1.0	1.0	1.0	
-	Standard						
Emia = 400 000 psi	Utility	4"	1.0	1.0	1.0	1.0	
		2" & 3"	0.4		0.4	0.6	
3. Determine load factors (bending)	FL = 77			Emíh =	400 0	00 19	
	$C_F = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	1.5 (FOR 1.0 (BRACE 1.15 (\$ 24'	Fb) (1.1. > BY SHEAT	T(FOR HING)	F <sub>c</sub> )		
	Structures II Slide 2						

# Combined Stress in NDS example

Exterior stud wall under bending + axial compression

$$\left[\frac{f_{c}}{F_{c}^{'}}\right]^{2} + \frac{f_{b1}}{F_{b1}^{'} \left[1 - \left(f_{c}^{'}/F_{cE1}\right)\right]}$$

$$F_b = 775$$
 psi  
 $C_D = 1.6$   $C_F = 1.5$   
 $C_M = 1.0$   $C_{F_0} = 1.0$   
 $C_4 = 1.6$   $C_1 = 1.0$   
 $C_L = 1.0$   $C_F = 1.15$ 

4. Calculate factored stresses (bending stress)

$$F_{b}^{1} = 775(1.4)(1.5)(1.15)$$
  
= 2139 psi

Structures II

## **Combined Stress in NDS** example

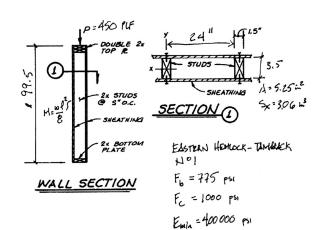
 $C_{p} = \frac{1 + (F_{cE}/F_{c}^{*})}{2c} - \sqrt{\left[\frac{1 + (F_{cE}/F_{c}^{*})}{2c}\right]^{2}} - \frac{F_{cE}/F_{c}^{*}}{c}$ Exterior stud wall under bending + axial compression  $F_{cE} = \frac{0.822 E_{min}}{(\ell_{o}/d)^{2}}$ P= 900# P+W c = 0.8 for sawn lumber 99.5" K=1.0 Cp F#= 1000(1.6×1.15)=1840  $F_{CE} = \frac{0.822(40000)}{(99.5/3.5)^2} = 406.0$ 6= 0.0 **Determine load factors** (compression) Cp = 0.21 Structures II University of Michigan, TCAUP Slide 29 of 31

## **Combined Stress in NDS** example

Exterior stud wall under bending + axial compression

3.

$$\!\!\left[\frac{f_{_{c}}}{F_{_{c}}^{'}}\right]^{2}+\frac{f_{_{b1}}}{F_{_{b1}}^{'}\left[1\!-\!\left(f_{_{c}}^{'}\!/\!F_{_{cE1}}^{'}\right)\right]}$$



Calculate stresses 4. (compression stress)

Actual Stress  
$$f_{c} = \frac{P}{A} = \frac{900}{5.25} = 171.4 \text{ psi}$$

## Factored Allowable Stress

Combined Stress in NDS example

$$\left[\frac{f_{c}}{F_{c}'}\right]^{2} + \frac{f_{b1}}{F_{b1}' \left[1 - (f_{c}/F_{cE1})\right]} \leq 1.0$$

Exterior stud wall under bending + axial compression

$$f_{c} = \frac{P}{A} = \frac{900}{5.25} = 171.43 \text{ Psi}$$

$$f_{b} = \frac{11}{3} = \frac{223.4(12)}{3.06} = 875.5 \text{ Psi}$$

5. Combined Stress Calculation (eq. 3.9-3)

$$F_{cE} = \frac{0.822 E_{min}'}{(\ell_e / d)^2}$$

$$F_{CE} = \frac{0.822(40000)}{(99.5/3.5)^2} = 406.0$$

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$$\left[\frac{f_{c}}{F_{c}'}\right]^{2} + \frac{f_{61}}{F_{61}'} \frac{1}{1 - \left(\frac{f_{c}}{F_{c}E_{1}}\right)} \leq 1.0$$

$$\left[\frac{171.4}{386.4}\right]^{2} + \frac{876}{2139} \frac{1}{1 - \left(\frac{171.4}{406.8}\right)}$$

$$0.1967 + (0.4095)(1.728) = 0.9045 \leq 1.0 \text{ Vok}$$

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Structures II