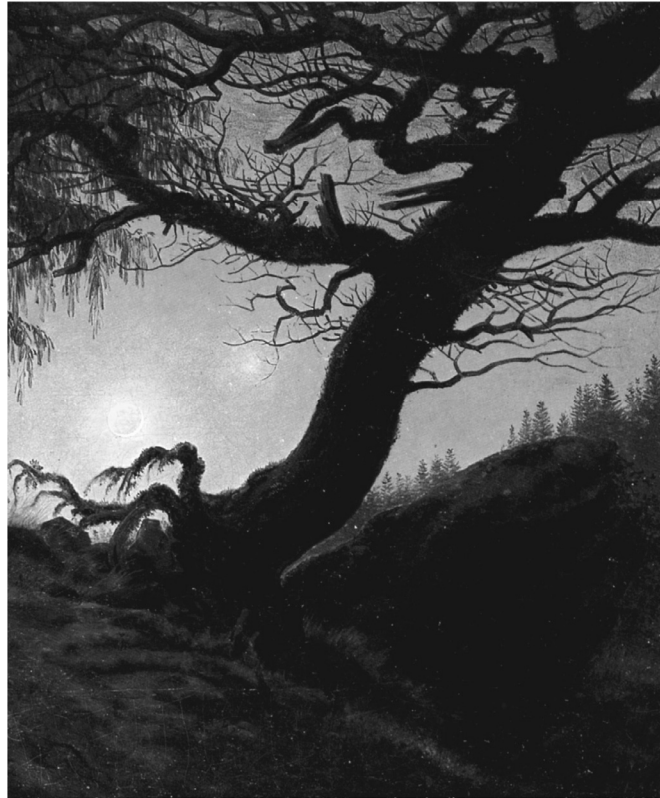


## Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas



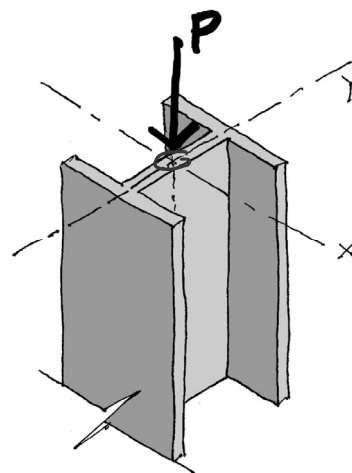
from "Man und Frau den Mond betrachtend"  
1830-35 by Caspar David Friedrich  
Alte Nationalgalerie, Berlin

## Axial Stress

- Loads pass through the centroid of the section, i.e. axially loaded
- Member is straight
- Load less than buckling load

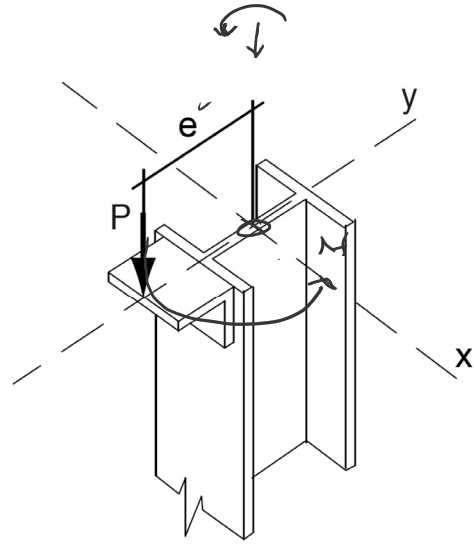
Then:

$$f_a = \frac{P}{A}$$



# Eccentric Loads

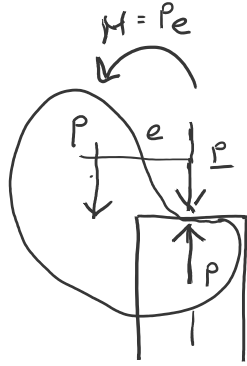
- Load is offset from centroid
- Bending Moment =  $P e$
- Total load =  $P + M$



Interaction formula

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$

AXIAL    BEND

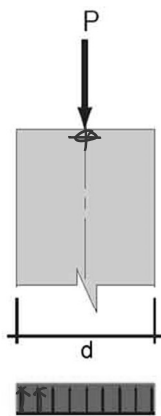
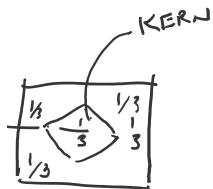


% AXIAL    % BEND

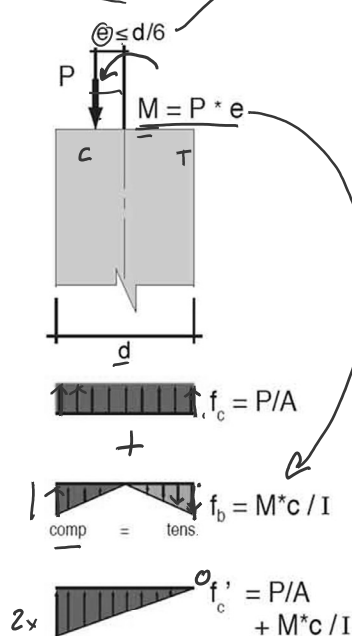
$$\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0 \quad 100\%$$

# Combined Stress

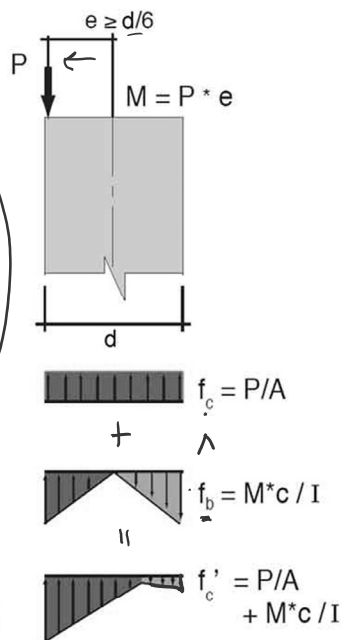
- Stresses combine by superposition
- Values add or subtract by sign



axial loaded - uniform compressive stress.



small eccentricity - linearly varying stress.



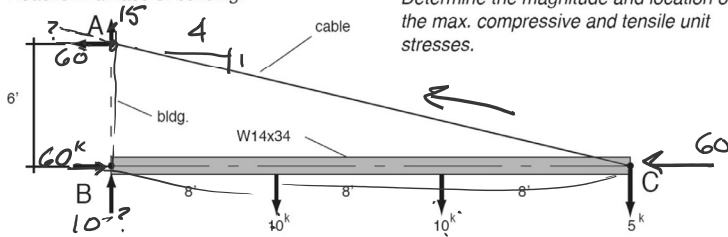
large eccentricity - tensile stress on part of cross section.

# Example

CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.  
The supporting cable is pin-connected on the centroidal axis of the steel beam.

$$\frac{60}{15} = \frac{4}{1}$$

Reactions at face of building.



FOR THE W14x34:  
Determine the magnitude and location of the max. compressive and tensile unit stresses.

## 1. Determine external reactions

$$\sum M_A = 0 = -B_H(6') + 10^k(8') + 10^k(16') + 5^k(24')$$

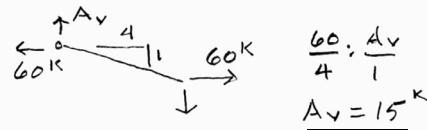
$$B_H = 60^k$$

$$\sum M_B = 0 = -A_H(6') + 10^k(8') + 10^k(16') + 5^k(24')$$

$$A_H = 60^k$$

$$\text{CHECK } \sum F_H = 0 = 60^k - 60^k \quad \checkmark$$

FBD @ A



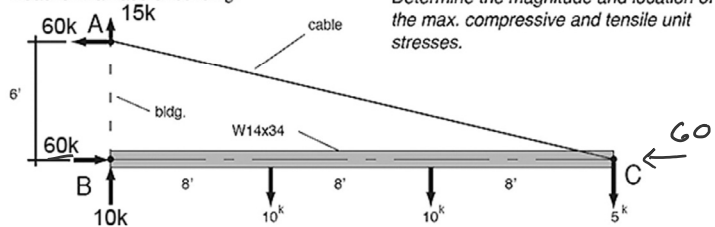
$$\sum F_V = 0 = 15^k - 10^k - 10^k - 5^k + B_V$$

$$B_V = 10^k$$

# Example

CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.  
The supporting cable is pin-connected on the centroidal axis of the steel beam.

Reactions at face of building.



FOR THE W14x34:  
Determine the magnitude and location of the max. compressive and tensile unit stresses.

## 2. Determine internal member forces: Axial and Flexural

## 3. Determine axial and flexural stresses

W14x34

$$A = 10.0 \text{ in}^2$$

$$S_x = 48.4 \text{ in}^3$$

FORCE:

$$\text{AXIAL} = 60^k \quad \checkmark$$

$$\text{FLEXURAL} = M = \frac{PL}{3} = 10^k(8') = 80 \text{ k-ft}$$

STRESS:

$$\text{AXIAL} = f_a = \frac{P}{A} = \frac{60^k}{10 \text{ in}^2} = 6.0 \text{ KSI}$$

$$\text{FLEXURAL} = f_b = \frac{M}{S} = \frac{80 \text{ k-ft} (12 \text{ in/ft})}{48.4 \text{ in}^3} = 19.75 \text{ KSI}$$

## Example

2. Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

### COMBINED STRESS

TOP SIDE:

$$f_c \pm f_b = 6.0 + 19.75 = \underline{25.75 \text{ KSI (COMP)}}$$

BOTTOM SIDE:

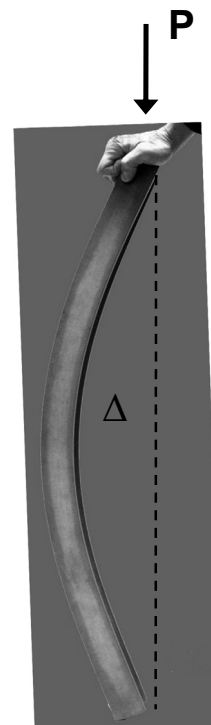
$$f_c - f_b = 6.0 - 19.75 = \underline{-13.75 \text{ KSI (TENS)}}$$

## Second Order Stress "P Delta Effect"

With larger deflections this can become significant.

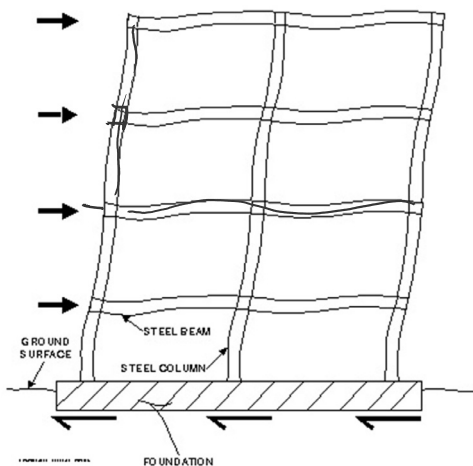
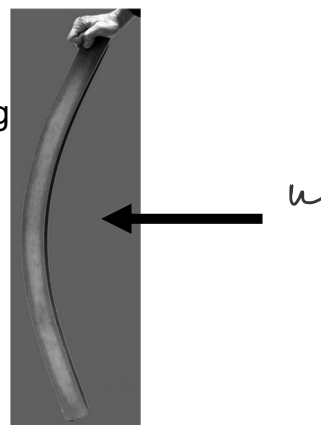
1. Eccentric load causes bending moment
2. Bending moment causes deflection,  $\Delta$
3.  $P \times \Delta$  causes additional moment

$M$

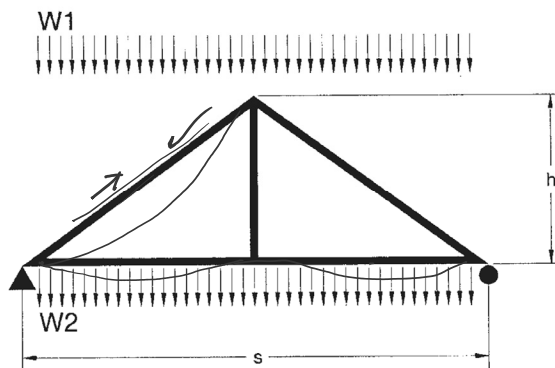


# Other Examples of Combined Stress

Columns with side loading

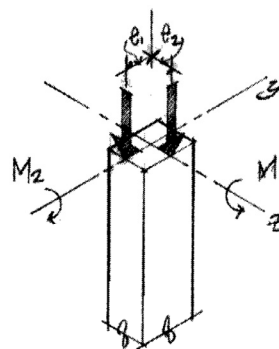
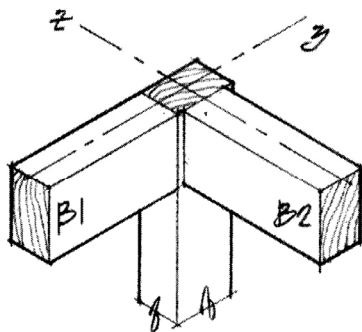


Moment frames



Trusses loaded on members

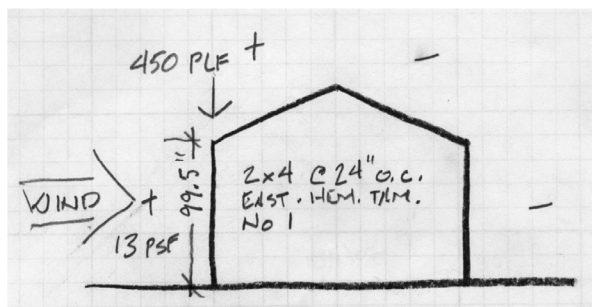
# Other Examples of Combined Stress



$$M_1 = P_1 \times e_1 \text{ (ABOUT THE } z\text{-axis)}$$

$$M_2 = P_2 \times e_2 \text{ (ABOUT THE } y\text{-axis)}$$

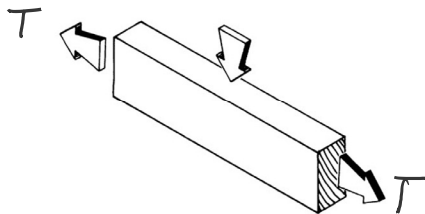
Eccentrically loaded columns



Wind load on walls

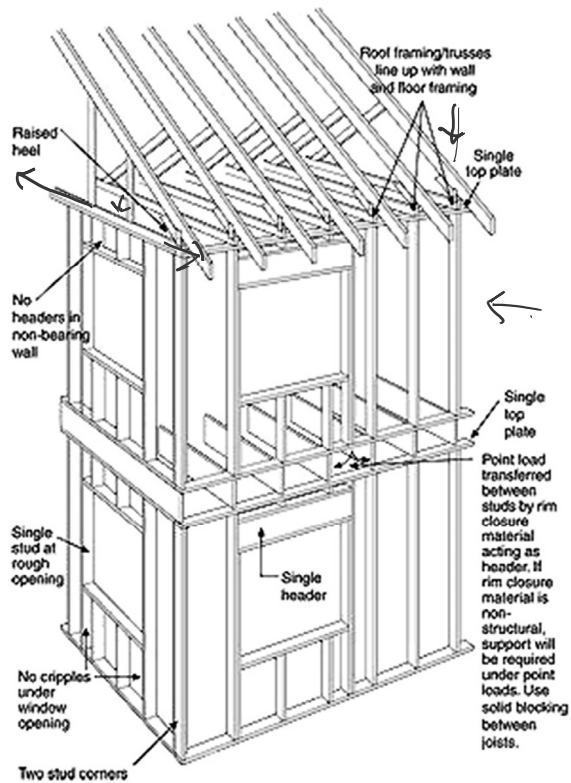
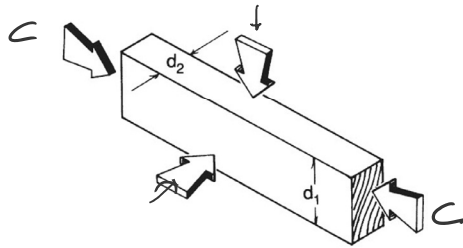
# Combined Stress in NDS

**Figure 3G Combined Bending and Axial Tension**



## 3.9.2 Bending and Axial Compression

**Figure 3H Combined Bending and Axial Compression**



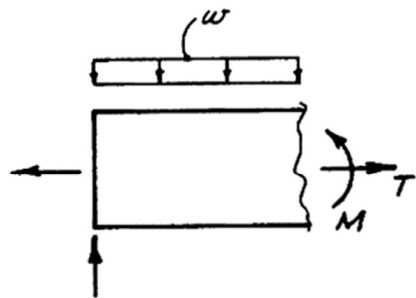
## Tension + Flexure NDS Equations

CASE 1. Tension is critical. Eq. 3.9-1  
\* no  $C_L$

CASE 2. Flexure is critical. Eq. 3.9-2  
\*\* no  $C_V$

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0$$

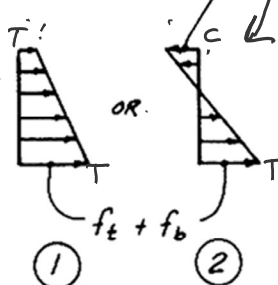
$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0$$



$$f_t = \frac{T}{A}$$

$$f_b = \frac{M}{S}$$

NET  $f_c = f_b - f_t$



TENSION + BENDING = COMBINED STRESSES

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$

and

$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2)$$

where:

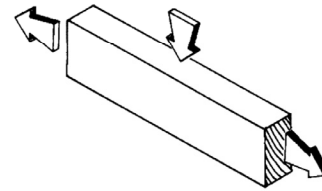
$F_b^*$  = reference bending design value multiplied by all applicable adjustment factors except

$$\frac{C_L}{1}$$

$F_b^{**}$  = reference bending design value multiplied by all applicable adjustment factors except

$$\frac{C_V}{1}$$

**Figure 3G Combined Bending and Axial Tension**



### Example Problem

Given: Queen Post truss

Hem-Fir No.1 & Better

$F_b = 1100$  psi

$F_t = 725$  psi

$F_c = 1350$  psi

$E_{min} = 550000$  psi

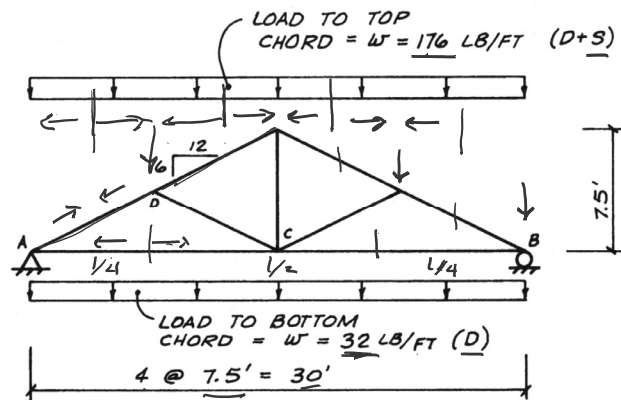
span = 30 ft. spaced 48" o.c.

D + S Load = 44 psf (projected)

D (attic + ceiling) = 8 psf

bottom chord: 2x8

top chord: 2x10

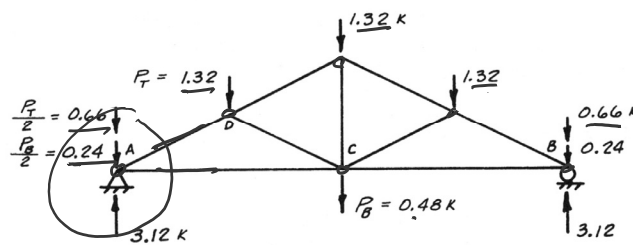


Find: pass/fail

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b^*} \leq 1.0$$

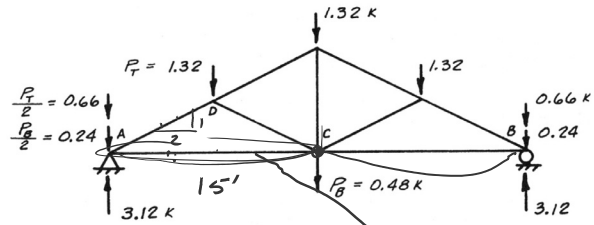
$$\frac{f_b - f_t}{F_b^{**}} \leq 1.0$$

- Determine truss joint loading



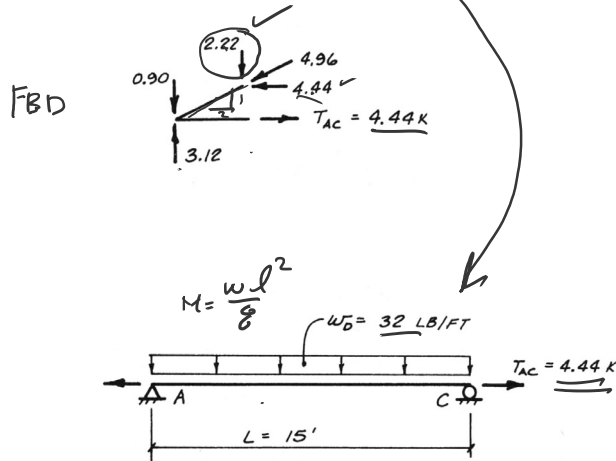
## Example (cont.)

- Determine the external **end reactions** of the whole truss. The geometry and loads are symmetric, so each reaction is  $\frac{1}{2}$  of the total load.



- Use an FBD of the reaction joint to find the **chord forces**. Sum the forces horizontal and vertical to find the components.

Top chord = 4.96 k compression  
Bottom chord = 4.44 k tension



Example  
bottom chord 2x8

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0$$

and

$$\frac{f_b - f_t}{F_b''} \leq 1.0$$

- Calculate the **actual** axial and flexural stress.

$$f_t = 408.3 \text{ psi}$$

$$f_b = 821.9 \text{ psi}$$

- Determine **allowable** stresses using applicable factors:

(tension: D+S)

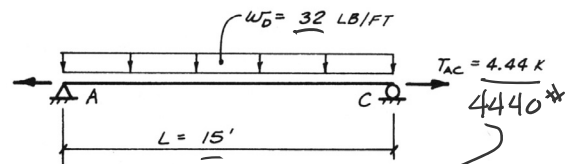
$$F_t' = F_t (C_D C_F)$$

$$F_t' = 725 (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3$$

(flexure: D+S)

$$F_b' = F_b (C_D C_L C_F)$$

$$F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi}$$



$$f_t = \frac{P}{A} = \frac{4440 \text{ lbs}}{10.875 \text{ in}^2} = \underline{408.3 \text{ psi}}$$

$$f_b = \frac{M}{S_x} = \frac{900 (12)}{13.14 \text{ in}^3} = \underline{821.9 \text{ psi}}$$

$$M = \frac{w l^2}{8} = \frac{32 (15)^2}{8} = \underline{900 \text{ ft} \cdot \text{lb}}$$

$$S_x = \underline{13.14 \text{ in}^3}$$

$C_L$  is 1.0 BY 4.4.1  
 $d/b = 4$ , ENDS ARE HELD



## Example

bottom chord 2x8

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0$$

and

$$\frac{f_b - f_t}{F_b''} \leq 1.0$$

5. Determine **allowable** stresses using applicable factors:

(tension: D+S) ✓  $S = 1.15$

$$F_t' = F_t (C_D C_E)$$

$$F_t' = 725 (1.15 \cdot 1.2) = 1000 \text{ psi} > 408.3$$

(flexure: D+S) ✓

$$F_b' = F_b (C_D C_L C_F)$$

TABLE ✓

$$F_b' = 1100 (1.15 \cdot 1.0 \cdot 1.2) = 1518 \text{ psi}$$

Size Factors,  $C_F$

Grades	Width (depth)	Thickness (breadth)		$F_t$	$F_c$
		2" & 3"	4"		
		Select	2", 3", & 4"		
Structural, No.1 & Btr, No.1, No.2, No.3	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
Stud	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
Construction Standard	8" & wider	Use No.3			
	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

$C_L$  is 1.0 BY 4.4.1 ✓  
 $d/b = 4$ , ENDS ARE HELD

## Example

bottom chord 2x8

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$\frac{f_t}{F_t'} + \frac{f_b}{F_b'} \leq 1.0 \quad \text{TENSION CRIT.} \quad (3.9-1)$$

and

$$\frac{f_b - f_t}{F_b''} \leq 1.0 \quad \text{FLEXURE CRIT.} \quad (3.9-2)$$

where:

$F_b'$  = reference bending design value multiplied by all applicable adjustment factors except  $C_L$

$F_b''$  = reference bending design value multiplied by all applicable adjustment factors except  $C_v$

(3.9-1)

$$f_t \frac{408.3}{1000} + \frac{821.9}{1518} f_b$$

$$0.4083 + 0.5414 = 0.95$$

$$0.95 < 1.0 \quad \checkmark \text{ PASS } \checkmark$$

(3.9-2)

$$f_b \frac{821.9 - 408.3}{1518} = 0.2724$$

$$0.27 < 1.0 \quad \checkmark \text{ PASS}$$

# Bending + Axial Compression

## 3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3H) shall be so proportioned that:

$$\left[ \frac{f_c}{F_c'} \right]^2 + \frac{f_{b1}}{F_{b1}' \left[ 1 - (f_c/F_{cE1}) \right]} + \frac{f_{b2}}{F_{b2}' \left[ 1 - (f_c/F_{cE2}) - (f_{b1}/F_{bE}) \right]} \leq 1.0 \quad (3.9-3)$$

*AMPLIFICATION FACTOR*  
*PΔ EFFECT*

where:

$$f_c < F_{cE1} = \frac{0.822 E_{min}'}{(l_{e1}/d_1)^2} \quad \text{for either uniaxial edgewise bending or biaxial bending}$$

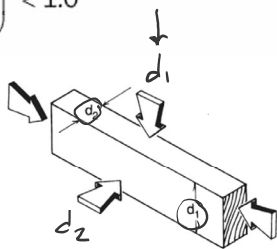
$$f_c < F_{cE2} = \frac{0.822 E_{min}'}{(l_{e2}/d_2)^2} \quad \text{for uniaxial flatwise bending or biaxial bending}$$

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}'}{(R_B)^2} \quad \text{for biaxial bending}$$

and

$$\frac{f_c}{F_{cE2}} + \left( \frac{f_{b1}}{F_{bE}} \right)^2 < 1.0 \quad (3.9-4)$$



$f_{b1}$  = actual edgewise bending stress (bending load applied to narrow face of member), psi

$f_{b2}$  = actual flatwise bending stress (bending load applied to wide face of member), psi

$d_1$  = wide face dimension (see Figure 3H), in.

$d_2$  = narrow face dimension (see Figure 3H), in.

## Example

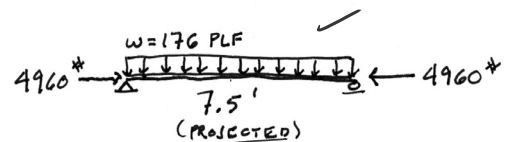
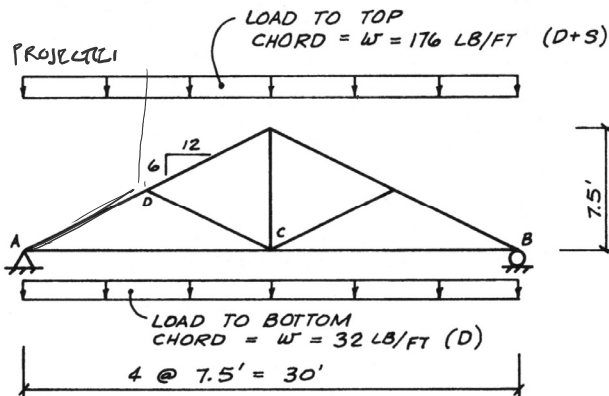
top chord 2x10

4. Calculate the **actual** axial and flexural stress.

$$f_c = 357.5 \text{ psi}$$

$$f_b = 694.2 \text{ psi}$$

$$\left[ \frac{f_c}{F_c'} \right]^2 + \frac{f_{b1}}{F_{b1}' \left[ 1 - (f_c/F_{cE1}) \right]}$$

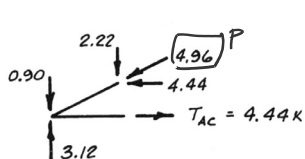


$$f_c = \frac{P}{A} = \frac{4960^*}{1.5 \times 9.25} = 357.5 \text{ psi}$$

$$f_b = \frac{M}{S_x} = \frac{1237.5 (12)}{21.39} = 694.2 \text{ psi}$$

$$M = \frac{w l^2}{8} = \frac{176 \text{ PLF} (7.5')^2}{8} = 1237.5 \text{ ft-lb}$$

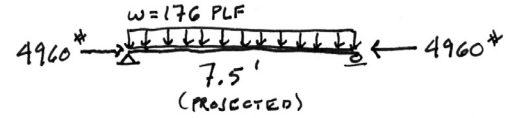
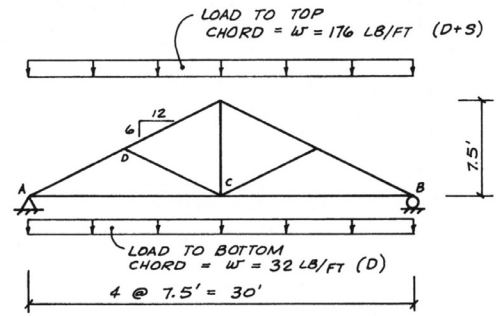
$$S_x = 21.39 \text{ in}^3$$



# Example

top chord 2x10

$$\left[ \frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[ 1 - (f_c / F_{cE1}) \right]}$$



5. Determine **allowable** stresses using applicable factors:

(compression: D+S)

$$F'_c = F_c (C_D C_F C_P)$$

$$F'_c = 1350 (1.15 \cdot 1.0 \cdot 0.897) = 1392.7 \text{ psi} > 357.5$$

(flexure: D+S)

$$F'_{b1} = F_b (C_D C_L C_F)$$

$$F'_{b1} = 1100 (1.15 \cdot 1.0 \cdot 1.1) = 1392 \text{ psi} > 694.2$$

$C_P$

$$l_e = 8.385' \quad d = 9.25''$$

$$l_e/d = \frac{8.385(12)}{9.25} = 10.88$$

EULER

$$F_{cE} = \frac{0.822 E_{min}}{(l_e/d)^2} = \frac{0.822(550000)}{10.88^2} = 3820 \text{ psi}$$

$$F_c^* = 1350 (1.15 \cdot 1.0) = 1552.5 \text{ psi}$$

$$F_{cE}/F_c^* = \frac{3820}{1552} = 2.46 \quad \alpha = 0.8$$

$$C_P = 0.897$$

# Example

top chord 2x10

Eq. 3.9-3

$$\left[ \frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[ 1 - (f_c / F_{cE1}) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

where:

EULER 1

$$f_c < F_{cE1} = \frac{0.822 E_{min}}{(l_{e1} / d_1)^2} \text{ for either uniaxial edge-wise bending or biaxial bending}$$

and

EULER 2

$$f_c < F_{cE2} = \frac{0.822 E_{min}}{(l_{e2} / d_2)^2} \text{ for uniaxial flatwise bending or biaxial bending}$$

and

LTB

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}}{(R_B)^2} \text{ for biaxial bending}$$

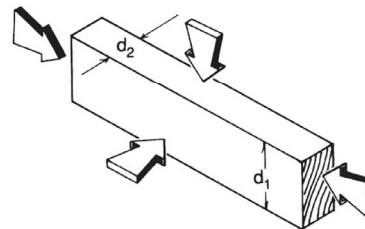
$f_{b1}$  = actual edgewise bending stress (bending load applied to narrow face of member)

$f_{b2}$  = actual flatwise bending stress (bending load applied to wide face of member)

$d_1$  = wide face dimension (see Figure 3H)

$d_2$  = narrow face dimension (see Figure 3H)

Figure 3H Combined Bending and Axial Compression



COMPRESSION:

$$\left[ \frac{f_c}{F'_c} \right]^2 = \left[ \frac{357.5}{1392.7} \right]^2 = 0.0659$$

# Example

top chord 2x10

Figure 3H Combined Bending and Axial Compression

Eq. 3.9-3

$$\left[ \frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[ 1 - (f_c / F_{cE1}) \right]} \leq 1.0$$

COMP. + FLEXURE X-X

where:

$$f_c < F_{cE1} = \frac{0.822 E_{min}}{(\ell_{e1} / d_1)^2}$$

EULER 1 for either uniaxial edge-wise bending or biaxial bending

and

$$f_c < F_{cE2} = \frac{0.822 E_{min}}{(\ell_{e2} / d_2)^2}$$

~~EULER 2 for uniaxial flatwise bending or biaxial bending~~

and

$$f_{b1} < F_{bE} = \frac{1.20 E_{min}}{(R_B)^2}$$

~~LTB for biaxial bending~~

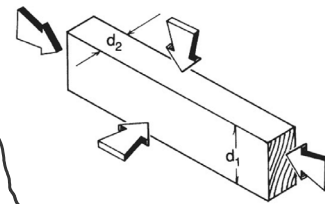
$f_{b1}$  = actual edgewise bending stress (bending load applied to narrow face of member)

$f_{b2}$  = actual flatwise bending stress (bending load applied to wide face of member)

$d_1$  = wide face dimension (see Figure 3H)

$d_2$  = narrow face dimension (see Figure 3H)

$f_c = 357.5 \text{ psi}$   
 $E_{min} = 550,000 \text{ psi}$   
 $\ell_{e1} = 8.385'$   
 $d_1 = 9.25''$   
 $\ell_{e1} / d_1 = \frac{8.385(12)}{9.25} = 10.88$   
 $F_{cE1} = \frac{0.822(550,000)}{10.88^2} = 3820 \text{ psi}$



FLEXURE:

$$\frac{f_{b1}}{F'_{b1}} = \frac{694.2}{1392} = 0.4987$$

AMPLIFICATION FACTOR:

$$\frac{1}{1 - (357.5 / 3820)} = 1.103$$

$$0.4987 (1.103) = 0.550$$

COMBINATION:

$$0.0659 + 0.550 = 0.616$$

$$0.616 < 1.0 \checkmark \text{ PASS}$$

## Combined Stress in NDS procedure

Exterior stud wall under bending + axial compression

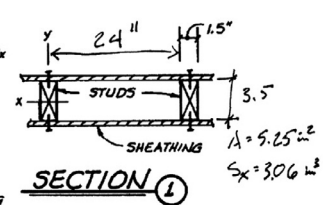
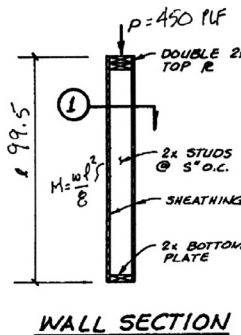
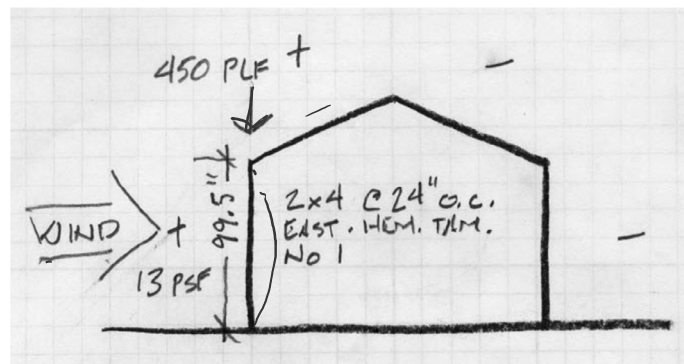
1. Determine load per stud
2. Use axial load and moment to find actual stresses  $f_c$  and  $f_b$
3. Determine load factors
4. Calculate factored stresses
5. Check NDS equations

NDS

COMPRESSION + FLEXURE

$$\left[ \frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[ 1 - (f_c / F_{cE1}) \right]} \leq 1.0 \quad (3.9-3)$$

13 PSF

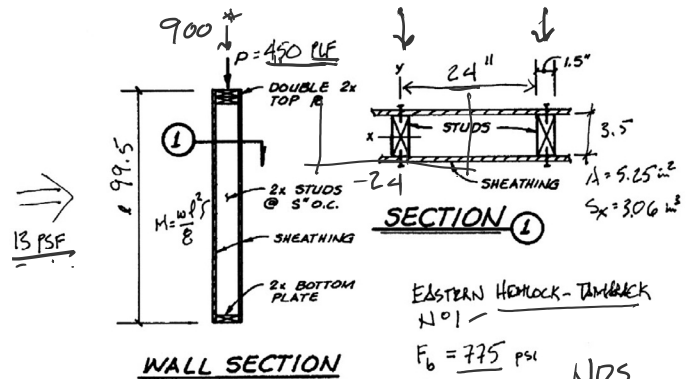


EASTERN HEMLOCK-TAMARACK  
 N1  
 $F_b = 775 \text{ psi}$   
 $F_c = 1000 \text{ psi}$   
 $E_{min} = 400,000 \text{ psi}$

# Combined Stress in NDS

example

Exterior stud wall under bending + axial compression



1. Determine load per stud
2. Use axial load and moment to find actual stresses  $f_c$  and  $f_b$

$$P = \text{LOAD/STUD}$$

$$P = 450 \text{ PLF} \frac{OC}{12} = 450 \frac{24}{12} = \boxed{900 \text{ LBS}}$$

$$w = 13 \text{ PSF} \frac{OC}{12} = 13 \frac{24}{12} = \boxed{26 \text{ PLF/STUD}}$$

$$M_x = \frac{w l^2}{8} = \frac{26 (99.5/12)^2}{8} = \boxed{223.4 \text{ ft-lb}}$$

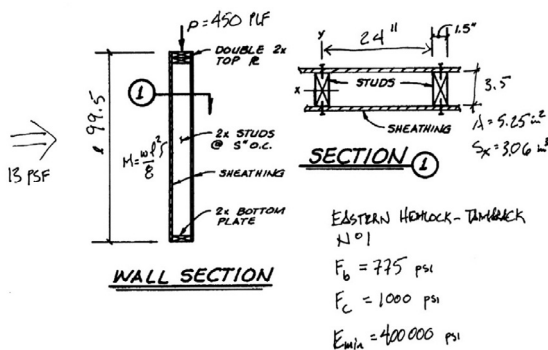
$$f_c = \frac{P}{A} = \frac{900 \text{ lb}}{5.25 \text{ in}^2} = \boxed{171.43 \text{ psi}}$$

$$f_b = \frac{M}{S_x} = \frac{223.4 (12)}{3.06 \text{ in}^3} = \boxed{875.5 \text{ psi}}$$

# Combined Stress in NDS

example

Exterior stud wall under bending + axial compression



Size Factors,  $C_F$

Grades	Width (depth)	Thickness (breadth)		$F_t$	$F_c$
		2" & 3"	4"		
Select	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
Stud	14" & wider	0.9	1.0	0.9	0.9
	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
Construction Standard	8" & wider	Use No.3			
	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

NDS TABLE  $\rightarrow F_b = 775 \text{ psi}$   $F_c = 1000 \text{ psi}$   $E_{min} = 400,000 \text{ psi}$

3. Determine load factors (bending)

FACTORS :

$$C_D = \boxed{1.6} \text{ (WIND)}$$

$$C_F = \boxed{1.5} \text{ (FOR } F_b) \quad \boxed{1.15} \text{ (FOR } F_c)$$

$$C_L = \boxed{1.0} \text{ (BRACED BY SHEATHING)}$$

$$C_r = \boxed{1.15} \text{ (} \leq 24" \text{ o.c.)}$$

## Combined Stress in NDS

example

Exterior stud wall under  
bending + axial compression

$$\left[ \frac{f_c}{F'_c} \right]^2 + \frac{f_{b1}}{F'_{b1} \left[ 1 - (f_c / F_{cE1}) \right]}$$

$$F_b = 775 \text{ psi}$$

$$C_D = 1.6 \text{ wind}$$

$$C_F = 1.5 \text{ 2x4}$$

$$C_M = 1.0$$

$$C_{Fu} = 1.0 \times$$

$$C_t = 1.0$$

$$C_i = 1.0 \times$$

$$C_L = 1.0 \checkmark$$

$$C_r = 1.15 \checkmark$$

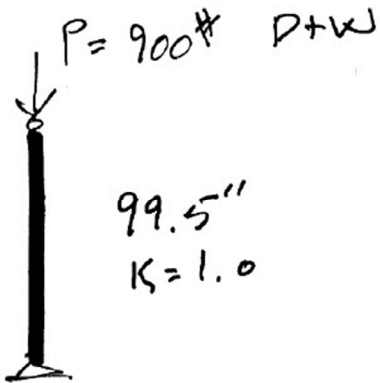
4. Calculate factored stresses  
(bending stress)

$$F'_b = 775 (1.6) (1.5) (1.15) \\ = \underline{2139 \text{ psi}}$$

## Combined Stress in NDS

example

Exterior stud wall under  
bending + axial compression



$$C_p = \frac{1 + (F_{cE} / F_c^*)}{2c} - \sqrt{\left[ \frac{1 + (F_{cE} / F_c^*)}{2c} \right]^2 - \frac{F_{cE} / F_c^*}{c}}$$

$$C_p$$

$$F_c^* = 1000 \overset{C_D}{(1.6)} \overset{C_F}{(1.15)} = \underline{1840}$$

$$\text{Euler } F_{cE} = \frac{0.822 (400000)}{(99.5/3.5)^2} = \underline{406.8}$$

$$c = 0.8$$

$$C_p = 0.21 \checkmark$$

3. Determine load factors  
(compression)

