## Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas Alte Nationalgalerie, Berlin



## Axial Stress

- Loads pass through the centroid of the section, i.e. axially loaded
- Member is straight
- Load less than buckling load

Then:

$$
f_{a}=\frac{P}{A}
$$



## Eccentric Loads

- Load is offset from centroid
- Bending Moment $=\mathrm{Pe}$
- Total load = P + M

Interaction formula

$f=\frac{P}{A} \pm \frac{M c}{I}$
AXIS C CAPACITY
Actual $/$ Flexure caidcity
$f_{a}+\frac{f_{b}}{F_{b}} \leq 1.0$
$\frac{f_{a}}{F_{a}} \pm \frac{f_{b}}{F_{b}} \leq 1.0$
slow.

## Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign

 stresses.

1. Determine external reactions

## Example

2. Determine internal member forces: Axial and Flexural
3. Determine axial and flexural stresses


CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING. The supporting cable is pin-connected on the centroidal axis of the steel beam.

$W 14 \times 34$

$$
\begin{aligned}
& A=10.0 \mathrm{~m}^{2} \\
& S_{x}=48.6 \mathrm{~m}^{3}
\end{aligned}
$$

Force:
$A \times 1 A L=60^{K} \quad R(\sigma)$
FLEXURSL $=M=P L / 3=10^{k}\left(8^{\prime}\right)=80^{1-K}$
STRESS:
$A \times 1 \mathrm{sc}=F_{2}=\frac{P}{A}=60 \mathrm{k} / 10 \mathrm{~m}^{2}=6.0 \mathrm{ks1}$
FLCXURUAL $: f_{b}=H / S_{x}=80^{1-k}(12) / 48.6 \mathrm{~m}^{3}=19.75 \mathrm{kS1}$

## Example

2. Use interaction formula to determine combined stresses at key locations (egg. extreme fibers)
(ovA) ${ }^{2}$


Combined STRESS
TOP SIDE:

$$
f_{2}+f_{6}=6.0+19.75=25.75 \mathrm{ks1} \text { (comp) }
$$

BOTTOM SIDE:

$$
f_{a}-f_{b}=6.0-19.75=-13.75 \mathrm{kSI}(\text { (ENS) }
$$

## Second Order Stress <br> "P Delta Effect"

With larger deflections this can become significant.

1. Eccentric load causes bending moment
2. Bending moment causes deflection, $\Delta$
3. $\mathrm{P} \times \Delta$ causes additional moment



Trusses loaded on members

## Other Examples of Combined Stress



## Combined Stress in NDS



## (Tension) \& Flexure NDS Equations

CASE 1. Tension is critical. Eq. 3.9-1

$$
\rightarrow \frac{\% t+\% c}{} \begin{aligned}
& \frac{f_{t}}{F_{t^{\prime}}}+\frac{f_{b}}{F_{b} *} \leq 1.0
\end{aligned}
$$

CASE 2. Flexure is critical. Eq. 3.9-2



TENSION + BENDING $=$ COMBINED STRESSES

## Tension + Flexure

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so proportioned that:

$$
\begin{equation*}
\frac{\mathrm{f}_{\mathrm{t}}}{\mathrm{~F}^{\prime}+\frac{\mathrm{f}_{\mathrm{b}}}{\mathrm{E}^{*}} \leq 1.0 \quad \text { TENSION CRIT. } . \text {. }} \tag{3.9-1}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{f_{b}-f_{t}}{F_{b}^{* 2}} \leq 1.0 \tag{3.9-2}
\end{equation*}
$$

FLEXURE CRIT.
where:

$$
\begin{aligned}
\mathrm{F}_{\mathrm{b}}^{*}= & \text { reference bending design value multiplied } \\
& \text { by all applicable adjustment factors except }
\end{aligned}
$$

## Example Problem

Given: Queen Post truss
Hem-Fir No. 1 \& Better
$\mathrm{F}_{\mathrm{b}}=1100 \mathrm{psi}$
$\mathrm{F}_{\mathrm{t}}=725 \mathrm{psi}$
$\mathrm{F}_{\mathrm{c}}=1350 \mathrm{psi}$
$\mathrm{E}_{\text {min }}=550000 \mathrm{psi}$
span $=30 \mathrm{ft}$. spaced $48^{\prime \prime}$ o.c.
D + S Load $=44$ psf (projected)
D (attic + ceiling) $=8 \mathrm{psf}$

bottom chord: $2 \times 8$
top chord: $2 \times 10$
Find: pass/fail

$$
\frac{f_{t}}{F_{t^{\prime}}}+\frac{f_{b}}{F_{b} *} \leq 1.0
$$



1. Determine truss joint loading

Figure 3G Combined Bending and Axial Tension


## Example (cont.)

2. Determine the external end reactions of the whole truss. The geometry and loads are symmetric, so each reaction
 is $1 / 2$ of the total load.
3. Use an FBD of the reaction joint to find the chord forces. Sum the forces horizontal and vertical to find the components.

Top chord $=4.96 \mathrm{k}$ compression
Bottom chord $=4.44 \mathrm{k}$ tension


## Example

bottom chord $2 \times 8$

$$
\frac{f_{t}}{F_{t}^{\prime}}+\frac{f_{b}}{F_{b}^{*}} \leq 1.0
$$

$$
\begin{aligned}
& A=\frac{10.875 i n .}{13.13 i n^{3}} \quad \text { and } \\
& S_{x}=\underline{i_{b}-f_{t}} \\
& F_{b}^{\prime \prime}
\end{aligned} 1.0
$$


4. Calculate the actual axial and flexural stress. $\quad f_{t}=\frac{p}{A}=\frac{4440^{2} \mathrm{lhs}}{10.875 \mathrm{~m}^{2}}=408.3 \mathrm{p}$ si

$$
\begin{aligned}
& \mathrm{f}_{\mathrm{t}}=408.3 \mathrm{psi} \\
& \mathrm{f}_{\mathrm{b}}=821.9 \mathrm{psi}
\end{aligned}
$$

5. Determine allowable stresses using applicable factors:

$$
f_{b}=\frac{M}{S_{x}}=\frac{1-*}{900(12)} \frac{83 i^{3} 4}{=81.9 p s i}
$$

(tension: $\mathrm{D}+\mathrm{S}$ )
$\rightarrow \underline{F}_{t}^{\prime}=F_{t}\left(\underline{C}_{D} C_{F}\right)$
$S_{x}=13.14 \mathrm{~m}^{3}$
$F_{t}^{\prime}=725\left(\frac{1.15}{S_{L}} \underline{1.2}\right)=1000 \mathrm{psi}>408.3$
(flexure: $\mathrm{D}+\mathrm{S}$ )
$F_{b}{ }^{\prime}=F_{b}\left(C_{D} C_{L} C_{F}\right)$
$\mathrm{F}_{\mathrm{b}}{ }^{\prime}=1100(1.151 .01 .2)=1518 \mathrm{psi}>821.9 \mathrm{psi}$
$C_{L}$ is 1.0 By 4.4.1
$d / b<4$, ENDS ARE HELD

## Example

bottom chord $2 \times 8$

and
5. Determine allowable stresses using applicable factors:
(tension: $\mathrm{D}+\mathrm{S}$ )
$F_{t}^{\prime}=F_{t}\left(C_{D} C_{F}\right)$
$\underline{F}_{\dot{\prime}}^{\prime}=725(1.15 \underline{1.2})=\underline{1000 \mathrm{psi}>408.3}$
SC
(flexure: $\mathrm{D}+\mathrm{S}$ )
$F_{b}{ }^{\prime}=F_{b}\left(C_{D} C_{L} C_{F}\right)$
$F_{b}{ }^{\prime}=1100(\overline{1.15} 1.01 .2)=1518 \mathrm{psi}>821.9 \mathrm{psi}$


### 4.4.1 Stability of Bending Members

4.4.1.1 Sawn lumber bending members shall be designed in accordance with the lateral stability calculatons in 3.3.3 or shall meet the lateral support requirements in 4.4.1.2 and 4.4.1.3.
4.4.1.2 As an alternative to 4.4.1.1, rectangular sawn lumber beams, rafters, joists, or other bending members, shall be designed in accordance with the following provisions to provide restraint against rotation or lateral displacement. If the depth to breadth, $\mathrm{d} / \mathrm{b}$, based on nominal dimensions is:
(a) $\mathrm{d} / \mathrm{b} \leq 2$; no lateral support shall be required.
(b) $2<\mathrm{d} / \mathrm{b} \leq 4$; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

## Example

bottom chord 2x8

$$
\begin{array}{ll}
f_{b}=821.9 \mathrm{psi} & f_{t}=408.3 \mathrm{psi} \\
F_{b}^{\prime}=1518 \mathrm{psi} & F_{t}^{\prime}=1000 \mathrm{psi}
\end{array}
$$

### 3.9.1 Bending and Axial Tension

Members subjected to a combination of bending and axial tension (see Figure 3G) shall be so propertoned that:

and



FLEXURE CRIT.
TENSION CRIT.
(3.9-1)


$$
\begin{gathered}
0.4083+0.5414=0.9595 \% \\
0.95<1.0 \quad \text { pass }
\end{gathered}
$$

$$
\begin{aligned}
F_{b}^{*}= & \text { reference bending design value multiplied } \\
& \text { by all applicable adjustment factors except } \\
& C_{L}
\end{aligned}
$$

$F_{b}{ }^{\prime \prime}=$ reference bending design value multiplied by all applicable adjustment factors except $\underline{C_{v}}$
where:

$$
\begin{gathered}
\frac{821.9-408.3}{1518}=0.2724 \\
0.27<1.0 \checkmark \text { pASs }
\end{gathered}
$$

## Bending + Axial Compression

### 3.9.2 Bending and Axial Compression

Members subjected to a combination of bending about one or both principal axes and axial compression (see Figure 3 H ) shall be so proportioned that:
where:

## Buckuine

$$
\mathrm{f}_{\mathrm{c}}<\mathrm{F}_{\mathrm{oE} 1}=\frac{0.822 \mathrm{E}_{\min }^{\prime}}{\left(\ell_{\mathrm{e} 1} / \mathrm{d}_{1}\right)^{2}}
$$

for either uniaxial edgewise bending or biaxial bending for biaxial bending

$$
\begin{equation*}
\frac{f_{c}}{F_{\mathrm{cE} 2}}+\left(\frac{f_{\mathrm{b} 1}}{F_{\mathrm{bE}}}\right)^{2}<1.0 \tag{3.9-4}
\end{equation*}
$$


$f_{b 1}=$ actual edgewise bending stress (bending load applied to narrow face of member) , psi
$\mathrm{f}_{\mathrm{b} 2}=$ actual flatwise bending stress (bending load applied to wide face of member), psi
$d_{1}=$ wide face dimension (see Figure $3 H$ ), in.
$\mathrm{d}_{2}=$ narrow face dimension (see Figure 3 H ), in.

## Example

top chord $2 \times 10$
4. Calculate the actual axial and flexural stress.

| $2 \times 10:$ |  |
| :--- | :--- |
| $\left[\begin{array}{ll}A=9.25 \mathrm{in}^{2} & f_{c}=357.5 \mathrm{psi} \\ S x=21.39 \mathrm{in}^{3} & f_{b 1}=694.2 \mathrm{psi}\end{array}\right.$ |  |



## Example

top chord $2 \times 10$

$$
\left[\frac{f_{c}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}\left[1-\left(f_{c} / F_{c E 1}\right)\right]}
$$


5. Determine allowable stresses using applicable factors:
NDS
(compression: $\mathrm{D}+\mathrm{S}$ )
$F_{c}{ }^{\prime}=F_{c}\left(C_{D} C_{F} C_{p}\right)$
$F_{c}^{\prime}=\frac{1350}{\text { NDS }}\left(\frac{1.151 .00 .897}{S L}\right)=1392.6 \mathrm{psi}>357.5$
$\frac{C_{p}}{l_{e}}=8.385^{\prime} \quad d=9.25^{\prime \prime}$
(flexure: D+S)
$\mathrm{le} / d=\frac{8.385(12)}{9.25}=10.88$
$F_{b}^{\prime}=F_{b}\left(C_{D} C_{L} C_{F}\right)$
$F_{b}^{\prime}=\frac{1100}{\operatorname{NDS}}(1.151 .01 .1)=1391.5 \mathrm{psi}>694.2$
$\left.\hat{C}_{\mathrm{C}}\right)=\frac{1+\left(\mathrm{F}_{c \mathrm{E}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}-\sqrt{\left[\frac{1+\left(\mathrm{F}_{c \mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}}\right]^{2}-\frac{\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\mathrm{c}}}$
(3.7-1)


## Example

top chord $2 \times 10$

$$
\left[\frac{f_{c}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}\left[1-\left(f_{c} / F_{c E 1}\right)\right]}
$$

5. Determine allowable stresses using applicable factors:
(compression: D+S)

(flexure: $\mathrm{D}+\mathrm{S}$ )
$F_{b}^{\prime}=F_{b}\left(C_{D} C_{L} C_{F}\right)$
$\vec{F}_{b}{ }^{\prime}=1100\left(1.15 \frac{1.0}{S_{L}} \frac{1.1)}{}=\underline{f_{6}}\right.$

Size Factors, $\mathrm{C}_{\mathrm{F}}$

| Grades | Width (depth | $\mathrm{F}_{\mathrm{b}}$ |  | $\mathrm{F}_{\mathrm{t}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Thickness (breadth) |  |  |  |
|  |  | 2" \& 3" | 4" |  |  |
| Select <br> Structural, <br> No. 1 \& Btr, <br> No.1, No.2, <br> No. 3 | 2", $3^{\prime \prime}$, \& 4" | 1.5 | 1.5 | 1.5 | 1.15 |
|  | 5 | 1.4 | 1.4 | 1.4 | 1.1 |
|  | $6 "$ | 1.3 | 1.3 | 1.3 | 1.1 |
|  | 8" | 1.2 | 1.3 | 1.2 | 1.05 |
|  | $10^{\prime \prime}$ | 1.1 | 1.2 | 1.1 | (1.0) |
|  | 12 " | 1.0 | 1.1 | 1.0 | 1.0 |
|  | 14" \& wider | 0.9 | 1.0 | 0.9 | 0.9 |
| Stud | 2", $3^{\prime \prime}$, \& 4" | 1.1 | 1.1 | 1.1 | 1.05 |
|  | $5 " \& 6^{\prime \prime}$ | 1.0 | 1.0 | 1.0 | 1.0 |
|  | $8{ }^{\prime \prime}$ \& wider | Use No. 3 |  |  |  |
| Construction Standard | $2^{\prime \prime}, 3^{\prime \prime}, \& 4 "$ | 1.0 | 1.0 | 1.0 | 1.0 |
| Utility | 4" | 1.0 | 1.0 | 1.0 | 1.0 |
|  | 2" \& 3" | 0.4 | - | 0.4 | 0.6 |


,

## Example

top chord $2 \times 10$

Eq. 3.9-3
$\leq 1.0$

where:
. 0659

| Figure 3H | Combined Bending and Axial <br> Compression |
| :--- | :--- |


compression:


EULER 1
$\mathrm{f}_{\mathrm{c}}<\mathrm{F}_{\mathrm{cE} 1}=\frac{0.822 \mathrm{E}_{\min }^{\prime}}{\left(\ell_{\mathrm{e} 1} / \mathrm{d}_{1}\right)^{2}}$
for either uniaxial edge-
bending
and

and

$f_{b 1}=$ actual edgewise bending stress (bending load applied to narrow face of member)
$\mathrm{f}_{\mathrm{b} 2}=$ actual flatwise bending stress (bending load applied to wide face of member)
$d_{1}=$ wide face dimension (see Figure $3 H$ )
$d_{2}=$ narrow face dimension (see Figure $3 H$ )

Eq. 3.9-3
Example top chord $2 \times 10$

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Figure 3H Combined Bending and Axial Compression


COMP. + FLEXURE EULER 1
$\mathrm{f}_{\mathrm{c}}<\mathrm{F}_{\mathrm{cE} 1}=\frac{0.822 \mathrm{E}_{\text {min }}}{\left(\ell_{\text {ell }} / \mathrm{d}_{1}\right)^{2}}$ for either uniaxial edgewise bending or biaxial bending
$f_{01}=$ actual edgewise bending stress (bending load applied to narrow face of member)
$d_{1}=$ wide face dimension (see Figure $3 H$ )
$d_{2}=$ narrow face dimension (see Figure 3H)

## ACTUAL

$f_{c}=\frac{P}{A}=\frac{4960^{*}}{1.5 \times 9.25}=357.5 \mathrm{pst}$
$f_{b}=\frac{M}{S_{x}}=\frac{1237.5(12)}{21.39}=694.2 \mathrm{ps} 1$
$H=\frac{\omega l^{2}}{\dot{g}}=\frac{176 \mathrm{PLF}\left(7.5^{\prime}\right)^{2}}{8}=1237.5^{1-4}$
$S_{x}=21.39 \mathrm{~m}^{3}$


## Combined Stress in NDS

procedure

Exterior stud wall under bending + axial compression

1. Determine load per stud
2. Use axial load and moment to find actual stresses $f_{c}$ and $f_{b}$

3. Determine load factors
4. Calculate factored stresses
5. Check NDS equations
$\left[\frac{f_{c}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}\left[1-\left(f_{c} / F_{c E 1}\right)\right]} \leq 1.0$
$\Rightarrow$


## Combined Stress in NDS example

Exterior stud wall under bending + axial compression

$$
\begin{equation*}
\left[\frac{f_{c}}{F_{\mathrm{c}}^{\prime}}\right]^{2}+\frac{\mathrm{f}_{\mathrm{b} 1}}{\mathrm{~F}_{\mathrm{b} 1}^{\prime}\left[1-\left(\mathrm{f}_{\mathrm{c}} / \mathrm{F}_{\mathrm{cE} 1}\right)\right]} \leq 1.0 \tag{3.9-3}
\end{equation*}
$$

1. Determine load per stud
2. Use axial load and moment to find actual stresses fo and fob


$$
\begin{aligned}
& P=\operatorname{Cos} D / S T U D \\
& P=450 \mathrm{PLF} \frac{24}{12}=450 \frac{24}{12}=900 \text { LBS } \\
& \omega=13 P S F \frac{o c}{12}=13 \frac{24}{12}=26 \text { PLF/STUD } \\
& M_{x}=\frac{w l^{2}}{8}=\frac{26(99.5 / 12)^{2}}{0}=223.41-x \\
& f_{c}=\frac{p}{\lambda}=\frac{900}{2 \times 4}=\frac{171.43 \mathrm{ps1}}{1-\ldots} \\
& f_{b}=1 / \sin _{\text {sin }}=\frac{1-*(123)}{3.06 \mathrm{ma}^{3}}=875.5 \mathrm{psi}
\end{aligned}
$$

Combined Stress in NDS example

Exterior stud wall under bending + axial compression

3. Determine load factors (bending)

Size Factors, $\mathrm{C}_{\mathrm{F}}$


## Combined Stress in NDS

example

Exterior stud wall under bending + axial compression
$F_{b}=775 \mathrm{mi}$

4. Calculate factored stresses (bending stress)

$$
\begin{aligned}
F_{b}^{\prime} & =775(1.6)(1.5)(1.15) \\
& =2139 \text { psi }
\end{aligned}
$$

## Combined Stress in NDS

 exampleExterior stud wall under bending + axial compression

$$
\xi^{P=900^{*} D+W} \begin{aligned}
& 99.5^{\prime \prime} \\
& K=1.0
\end{aligned}
$$

3. Determine load factors (compression)

$$
\begin{aligned}
& \begin{aligned}
\frac{C_{p}}{i}=\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{2 \mathrm{c}} & \sqrt{\left[\frac{1+\left(\mathrm{F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}\right)}{\int_{2}^{2}}\right]^{2}} \rightarrow \frac{\mathrm{~F}_{\mathrm{cE}} / \mathrm{F}_{\mathrm{c}}^{*}}{\mathrm{c}}
\end{aligned} \\
& c=0.8 \text { for awn lumber } \\
& \text { Cp } \\
& F_{c}^{*}=1000(1.6 \times(1.15)=1840 \\
& F_{C E}=\frac{0.822(400000)}{(99.5 / 3.5)^{2}}=406.0^{\circ} \\
& \begin{array}{l}
c_{1}=0.8 \\
C_{p}=0.21
\end{array}
\end{aligned}
$$

## Combined Stress in NDS

 exampleExterior stud wall under bending + axial compression

$$
\left[\frac{f_{c}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}\left[1-\left(f_{c} / F_{c E 1}\right)\right]}
$$


4. Calculate stresses (compression stress)


## Actual Stress

$$
f_{C}=\frac{p}{A}=\frac{900^{*}}{5.25}=171.4 \mathrm{psi}
$$

## Comp

Factored Allowable Stress

$$
F_{C}^{\prime}=\frac{1000}{F_{C}}(1.6)(1.15)(0.21)=C_{C_{D}}^{386.4 \mathrm{psi}}
$$

Combined Stress in NDS example

$$
\left[\frac{f_{c}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}\left[1-\left(f_{c} / F_{c E 1}\right)\right]} \leq 1.0
$$

Exterior stud wall under
COMP. + FLEXURE XXX
bending + axial compression

## Actuse

$$
\begin{aligned}
& f_{c}=\frac{p}{\lambda}=\frac{900}{5.25}=171.43 \mathrm{ps1} \\
& f_{b}=1 / s_{x}=\frac{223.4(12)}{3.06}=875.5 \mathrm{ps1}
\end{aligned}
$$

5. Combined Stress Calculation (eq. 3.9-3)
$\mathrm{F}_{\mathrm{cE}}=\frac{0.822 \mathrm{E}_{\text {min }}^{\prime}}{\left(\ell_{\mathrm{e}} / \mathrm{d}\right)^{2}}$
$\left[\frac{f_{c}^{\prime}}{F_{c}^{\prime}}\right]^{2}+\frac{f_{b 1}}{F_{b 1}^{\prime}} \frac{1}{1-\left(f_{c} / F_{c} E_{1}\right)} \leqslant 1.0$
$F_{C E}=\frac{0.822(400000)}{(99.5 / 3.5)^{2}}=\frac{\left[\frac{171.4}{386.4}\right]^{2}+\frac{876}{2139} \frac{1}{1-(171.4 / 406.8)}}{0.1967+(0.4095)(1.728)}=$
$0.1967+0.7077=0.9045 \leqslant 1.0 \mathrm{Vok}$
