## Structures II

## Steel Beam Analysis

- Steel Codes: ASD vs. LRFD
- Analysis Methods



## Steel Beams by LRFD

Yield Stress Values

- A36 Carbon Steel $F_{y}=36 \mathrm{ksi}$
- A992 High Strength $F_{y}=50 \mathrm{ksi}$


## Elastic Analysis for Bending

- Plastic Behavior (zone 1)
$M_{n}=M_{p}=F_{y} Z$
- Braced against LTB $\left(\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}}\right)$
- Inelastic Buckling "Decreased" (zone 2)
$M_{n}=C_{b}\left(M_{p}-\left(M_{p}-M_{r}\right)\left[\left(L_{b}-L_{p}\right) /\left(L_{r}-L_{p}\right)\right]<M_{p}\right.$
- $L_{p}<L_{b}<L_{r}$
- Elastic Buckling "Decreased Further" (zone 3)
$M_{c r}=C_{b}{ }^{*} \pi / L_{b} \sqrt{ }\left(E^{*} I_{y}{ }^{*} G^{*} J+\left(\pi^{*} E / L_{b}\right)^{2}{ }^{*} I_{y} C_{w}\right)$
- $L_{b}>L_{r}$



## Steel Beams by LRFD

## Analysis for Bending

- Plastic Behavior (zone 1)
$M_{n}=M_{p}=F_{y} Z$
- Braced against LTB $\left(\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}}\right)$
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- $L_{p}<L_{b}<L_{r}$
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- Lb > Lr




## Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

$$
\begin{aligned}
& F_{v}=V / A_{w} \\
& A_{w}=d^{*} t_{w}
\end{aligned}
$$

To adjust the stress a reduction factor of 0.6 is applied to $F_{y}$

$$
\begin{aligned}
& F_{v}=0.6 F_{y} \\
& \text { so, } V_{n}=0.6 F_{y} A_{w} \quad \text { (Zone 1) }
\end{aligned}
$$



The equations for the 3 stress zones:
( $\phi$ in all cases = 1.0)

## Zone 1:

WEB YIELDING (Most beam sections fall into this category)
if $\frac{h}{t_{\mathrm{x}}} \leq 2.45 \sqrt{E / F_{y}}=59$ (for 50 ksi steel)
then: $\quad V_{n}=0.6 F_{y} A_{m}$

## Zone 2:

INELASTIC WEB BUCKLING
if $2.45 \sqrt{E / F_{y}}<\frac{h}{t_{w}} \leq 3.07 \sqrt{E / F_{y}}=74$ (for 50 ksi steel)
then: $\quad V_{n}=0.6 F_{y} A_{w}(2.45 \sqrt{E / F}) / \frac{h}{t_{w}}$

## Zone 3:

ELASTIC WEB BUCKLING
if $3.07 \sqrt{E / F_{y}}<\frac{\mathrm{h}}{\mathrm{t}_{\mathrm{w}}} \leq 260$
then: $\quad V_{n}=A_{w}\left[\frac{4.25 E}{\left(\frac{h}{t_{w}}\right)^{2}}\right]$

## Procedure - Analysis of Steel Beams - for Zone $1 L_{b}<L_{p}$ Pass/Fail

Given: yield stress, steel section, loading, bracing ( $\mathrm{L}_{\mathrm{b}}$ )
Find: pass/fail of section

1. Calculate the factored design load $w_{u}$ $w_{u}=1.2 w_{\mathrm{DL}}+1.6 \mathrm{w}_{\mathrm{LL}}$
2. Determine the design moment $\mathrm{M}_{\mathrm{u}}$. $\mathrm{M}_{\mathrm{u}}$ will be the maximum beam moment using the factored loads
3. SIMPLE BEAM-UNIFORMLY DISTRIBUTED LOAD


4. Insure that $\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}}$ (zone 1)

$$
\mathrm{L}_{\mathrm{p}}=1.76 \mathrm{r}_{\mathrm{y}} \sqrt{E / F y}
$$

4. Determine the nominal moment, Mn $M_{n}=F_{y} Z_{x}$ (look up $Z_{x}$ for section)
5. Factor the nominal moment $\varnothing \mathrm{M}_{\mathrm{n}}=0.90 \mathrm{M}_{\mathrm{n}}$
6. Check that $M_{u}<\varnothing M_{n}$
7. Check shear
8. Check deflection

Example: Pass/Fail Analysis of Steel Beams - for Zone $1 \quad L_{b}<L_{p}$

Given: yield stress, steel section, loading, braced 24 " oc.

Find: pass/fail of section

1. Calculate the factored design load $\mathrm{w}_{\mathrm{u}}$

$$
w_{\mathrm{u}}=1.2 \mathrm{w}_{\mathrm{DL}}+1.6 \mathrm{w}_{\mathrm{LL}}
$$

2. Determine the design moment $M_{u}$. $M_{u}$ will be the maximum beam moment using the factored loads.

3. SIMPLE BEAM-UNIFORMLY DISTRIBUTED LOAD



FROM TABLE $1-1$ dISC $Z_{x}=95.4 \mathrm{~m}^{3}$

$$
\begin{aligned}
& \omega_{u}=1.2(1+.044)+1.6(3)=6.05 \mathrm{kLF} \\
& M_{u}=\frac{\omega_{u} l^{2}}{8}=\frac{6.05 \mathrm{kLF} \times 21^{\prime 2}}{8}=333.5 \mathrm{~K}^{-1}
\end{aligned}
$$

Example: Pass/Fail Analysis of Steel Beams - for Zone $1 \quad L_{b}<L_{p}$
3. Insure that $\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}}$ (zone 1)

$$
\begin{aligned}
& \mathrm{L}_{\mathrm{p}}=1.76 \mathrm{r}_{\mathrm{y}} \sqrt{E / F y} \\
& \mathrm{~L}_{\mathrm{p}}=1.76(1.26) \sqrt{29000 / 50} \\
& \mathrm{~L}_{\mathrm{p}}=53.4 \mathrm{in} .>24 \mathrm{in} . \mathrm{ok}
\end{aligned}
$$

4. Determine the nominal moment, $\mathrm{M}_{\mathrm{n}}$
$M_{n}=M_{p}=F_{y} Z_{x} \quad$ (for zone 1)
(look up $Z_{x}$ for section)
5. Factor the nominal moment $\varnothing \mathrm{M}_{\mathrm{n}}=0.90 \mathrm{M}_{\mathrm{n}}$

$$
\begin{aligned}
& M_{n}=F_{y} Z=50 \mathrm{ksi} 95.4 \mathrm{~m}^{3}=4770 \mathrm{k}-11 \\
& M_{n}=4770 \mathrm{k-11} / 12=397.5 \mathrm{k}-1 \\
& \phi H_{n}=0.9(397.5)=357.7 \mathrm{k-1}
\end{aligned}
$$

6. Check that $M_{u}<\varnothing M_{n}$


FROM TABLE $1-1$ MISC $Z_{x}=95.4 \mathrm{~m}^{3}$

## Steel Beams by LRFD

## Analysis for Bending

- Plastic Behavior (zone 1)
$M_{n}=M_{p}=F_{y} Z$
- $L_{p}=4.45 \mathrm{ft}=53.4 \mathrm{in} .>24 \mathrm{in}$. oc.
- $\emptyset_{\mathrm{b}} \mathrm{M}_{\mathrm{px}}=358 \mathrm{k}-\mathrm{ft}>\mathrm{M}_{\mathrm{u}}=333.5 \mathrm{k}-\mathrm{ft}$





## Example: Pass/Fail Analysis of Steel Beams - for Zone $1 \quad L_{b}<L_{p}$

7. Check shear for W21x44

Zone 1:
WEB YIELDING (Most beam sections fall into this category)
if $\frac{h}{t_{x}} \leq 2.45 \sqrt{E / F_{y}}=59$ (for 50 ksi steel)
then: $\quad V_{n}=0.6 F_{y} A_{\nu}$


Table 1-1 (continued)
W-Shapes
Dimensions


CHECK SHEAR:
FROM DISC TABLE $1-1$
$h / t_{w}=53.6<59$ (zone 1)


Example: Pass/Fail Analysis of Steel Beams - for Zone $1 \quad L_{b}<L_{p}$
7. Check shear (zone 1)

FROM AISC TABLE 1-1

$$
h / t_{w}=53.6<59 \text { (zone 1) }
$$

## Zone 1:

## WEB YIELDING (Most beam sections fall into this category)

if $\frac{h}{t_{\mathrm{z}}} \leq 2.45 \sqrt{E / F_{y}}=59$ (for 50 ksi steel)
then: $\quad V_{n}=0.6 F_{y} A_{x}$


FROM TABLE $1-1$ MISC $Z_{x}=95.4 \mathrm{~m}^{3}$
$\omega_{u}=1.2(1+.044)+1.6(3)=6.05 \mathrm{kLF}$

CHECK SHEAR:

$$
V_{u}=\frac{\omega_{0} l}{2}=\frac{6.05(21)}{2}=63.5^{\mathrm{K}}
$$

FROM AISC TABLE $1-1$

$$
h / t_{w}=53.6<59 \text { (zone 1) }
$$

$$
V_{n}=0.6 F_{y} \Lambda_{w}=0.6(50)(20.7 \times 0.35)
$$

$$
V_{n}=217.35 \mathrm{~K}
$$

$$
\phi V_{n}=1.0(217.35)=217.35^{k}
$$

$$
V_{u}=63.5^{k}<217.3^{k}=\phi V_{n}
$$

Therefore, pass.

## Example: Pass/Fail Analysis of Steel Beams - for Zone $1 \quad L_{b}<L_{p}$

8. Check deflection

$$
\frac{P}{360}=\frac{21(12)}{360}=0.7^{\prime \prime}
$$

$$
\Delta_{\text {ACTUAL }}=0.535^{\prime \prime}<0.7^{\prime \prime}=\Delta_{\text {ALLowaBLE }}
$$



FROM TABLE $1-1$ MISC $Z_{x}=95.4 \mathrm{~m}^{3}$

$$
\omega_{u}=1.2(1+.044)+1.6(3)=6.05 \mathrm{KLF}
$$

TABLE 1604.3 DEFLECTION LIMITs ${ }^{\text {a, }} \mathrm{b}, \mathrm{c}, \mathrm{h}, \mathrm{i}$


## Procedure - Analysis of Steel Beam - Capacity

Given: yield stress, steel section, bracing
Find: moment or load capacity

1. Determine the unbraced length of the compression flange ( $L_{b}$ ).
2. Find the $L_{p}$ and $L_{r}$ values from the AISC $Z_{x}$ Table 3-2
3. SIMPLE BEAM-UNIFORMLY DISTRIBUTED LOAD


## Example - Analysis of Steel Beam - Capacity

Find applied live load capacity, $\mathrm{w}_{\mathrm{LL}}$ in KLF

$$
\begin{aligned}
& \mathrm{w}_{\mathrm{u}}=1.2 \mathrm{w}_{\mathrm{DL}}+1.6 \mathrm{w}_{\mathrm{LL}} \\
& \mathrm{w}_{\mathrm{DL}}=\text { beam }+ \text { floor }=44 \mathrm{plf}+1500 \mathrm{plf}
\end{aligned}
$$

Fy $=50$ ksi, Fully Braced

1. Find the Plastic Modulus $\left(Z_{x}\right)$ for the given section from the AISC table 1-1
2. Check that $\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{p}}$ (fully braced -ok )
3. Determine $M_{n}=M_{p}=F_{y} Z_{x}$
4. Set $M_{u}=\phi_{b} M_{n}$ $\phi_{b}=0.90$

## Steel Beams by LRFD

## Analysis for Bending

- Plastic Behavior (zone 1)
$M_{n}=M_{p}=F_{y} Z$
- $\mathrm{L}_{\mathrm{p}}=4.45 \mathrm{ft}=53.4 \mathrm{in}$.
- $\emptyset_{\mathrm{b}} \mathrm{M}_{\mathrm{px}}=358 \mathrm{k}-\mathrm{ft}>\mathrm{M}_{\mathrm{u}}=333.5 \mathrm{k}-\mathrm{ft}$




## Example - Analysis of Steel Beam - Capacity

6. Using the maximum moment equation, solve for the factored distributed loading, $\mathrm{w}_{\mathrm{u}}$

$$
M_{0}=\frac{\omega_{0} l^{2}}{8} \Rightarrow \omega_{0}=\frac{8 M_{0}}{l^{2}}
$$

$$
\omega_{0}: \frac{8 \times 357.75 \mathrm{kF7}}{20 \mathrm{ct}^{2}}
$$

$$
\omega_{0}: 7.155 \mathrm{k} / \mathrm{FT}
$$

7. The applied (unfactored) load

$$
\begin{aligned}
& w=w_{u} /(\gamma \text { factors }) \\
& w_{u}=1.2 w D L+1.6 w L L
\end{aligned}
$$

$$
\begin{aligned}
& \omega_{U}=7.155 \mathrm{kLL}=1.2(0.044+1.5)+1.6\left(\omega_{L L}\right) \\
& \omega_{0}=1.853+1.6 \omega_{L L}=7.155 \mathrm{kLF} \\
& \omega_{L L}=3.31 \mathrm{kLF}
\end{aligned}
$$

## Steel Beams by LRFD

## Moment Capacity vs. $L_{b}$ Graphs

## Analysis for Bending

- Plastic Behavior (zone 1)
$M_{n}=M_{p}$
Braced against LTB $\left(L_{b}<L_{p}\right)$
- Inelastic Buckling "Decreased" (zone 2) $M_{n}<M_{p}$ $\mathrm{L}_{\mathrm{p}}<\mathrm{L}_{\mathrm{b}}<\mathrm{L}_{\mathrm{r}}$
- Elastic Buckling "Decreased Further" (zone 3) $M_{n}=M_{\text {cr }}$ $L_{b}>L_{r}$


## Pass/Fail Analysis of Steel Beams

 for Zone $1 \quad L_{b}<L_{p}$
## Example:

Given: yield stress, steel section, loading, braced @ 24" o.c.

Find: pass/fail of section


## Steel Beams by LRFD

Moment Capacity Graphs


> For A WZ1x44 FROM TAOLE $Z_{x}=95.4 \mathrm{n}^{3}$
$M_{N} \cdot F_{Y} Z_{x}=50_{m} \times 95.4: 4,770_{\mathrm{mw}}$
$\mu_{0}=\phi_{b} \cdot \mu_{N}=0.9 \times 4,770 \mathrm{k}-\mathrm{N}$
$M_{0}: 4,293 \mathrm{k}-\mathrm{w}=357.75 \mathrm{kr}$

