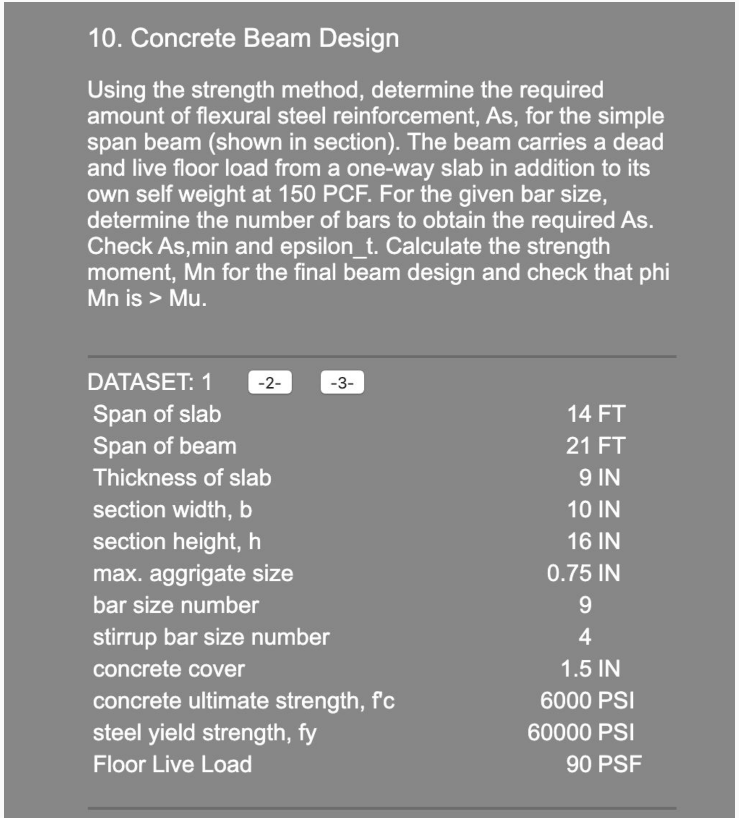
Lovejeet Gehlot

Concrete Beam Design

Lab Recitation #10
Group #3

March 25 2020
University of Michigan, TCAUP

Concrete Beam Design

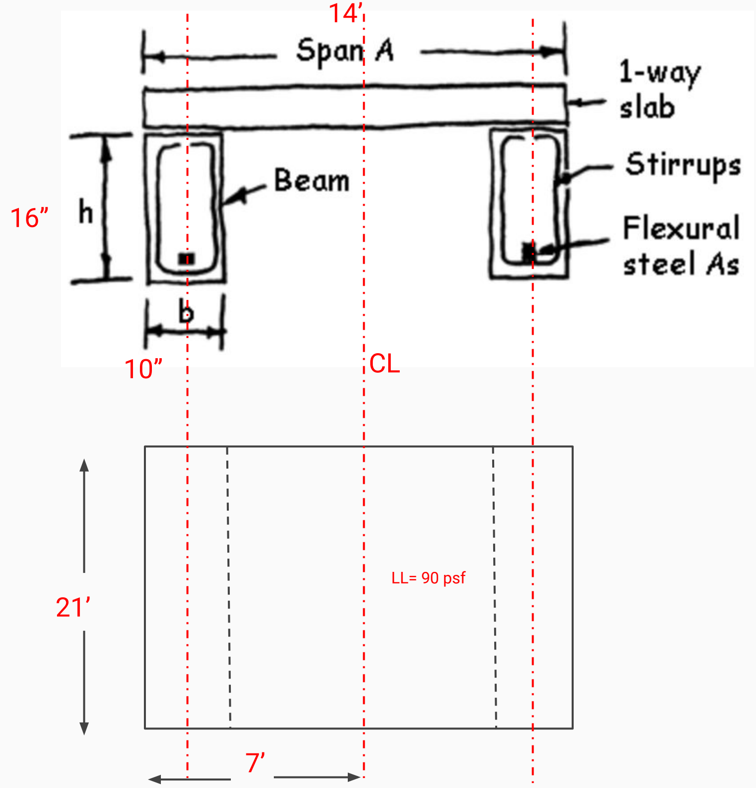


10. Concrete Beam Design

Using the strength method, determine the required amount of flexural steel reinforcement, As, for the simple span beam (shown in section). The beam carries a dead and live floor load from a one-way slab in addition to its own self weight at 150 PCF. For the given bar size, determine the number of bars to obtain the required As. Check As,min and epsilon_t. Calculate the strength moment, Mn for the final beam design and check that phi Mn is > Mu.

DATASET: 1 -2- -3-

Span of slab	14 FT
Span of beam	21 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	16 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6000 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	90 PSF



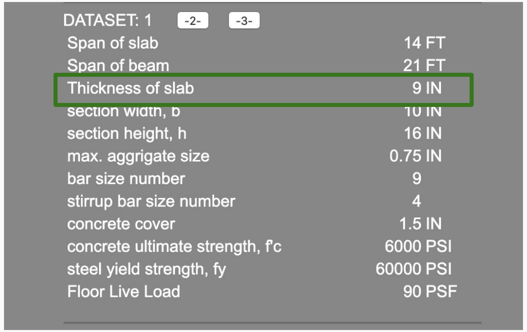
Q#1 Unfactored dead load on beam from slab

Given that, ρ of concrete =150 lb/cu ft

Unfactored load of Slab = $\textit{Density of concrete} \times \textit{slab thickness in feet} \times \textit{tributary length}$

$$\text{Unfactored load of Slab} = \underset{\text{(given)}}{150} \times \underset{\text{(given)}}{9''} \underset{\text{(convert to feet)}}{\div 12} \times \overset{\text{(span of slab / 2)}}{7'}$$

Unfactored load of Slab = **787.5 PLF**



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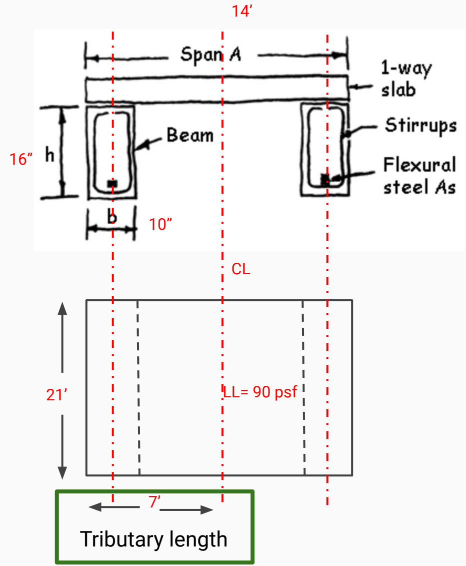


Q#2 Unfactored dead load on beam from the beam itself

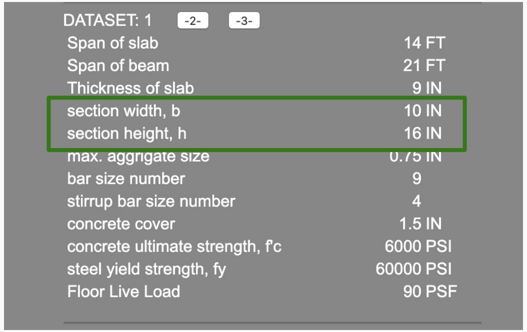
Given that, ρ of concrete =150 lb/cu ft

Unfactored load of beam = *Density of concrete* × *beam width in feet* × *beam depth in feet*

Unfactored load of beam = $150 \times (10'' \div 12) \times (16'' \div 12)$

(given) (given) (convert to feet) (given) (convert to feet)

Unfactored load of beam = **166.66 PLF**



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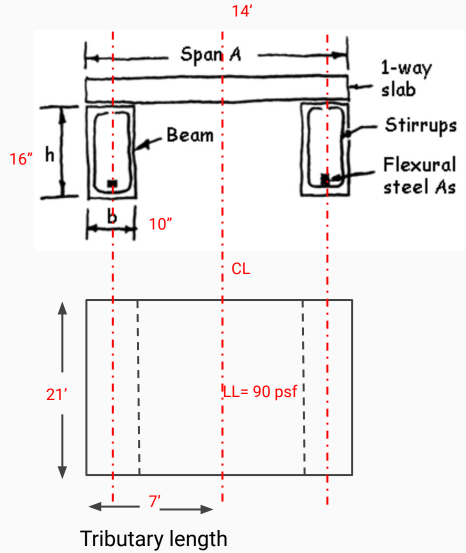
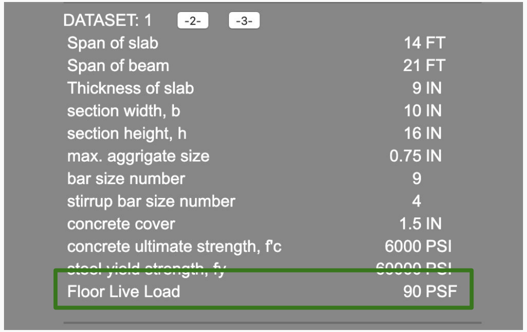


Q#3 Unfactored live load on beam, LL

Unfactored live load on Beam = Floor live load x Tributary length

Unfactored live load on Beam = 90 PSF x 7’

Unfactored live load on Beam = **630 PLF**



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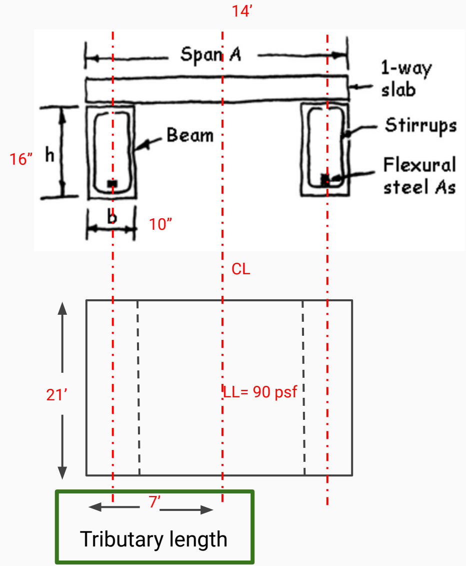


Q#4 Total factored beam load, wu

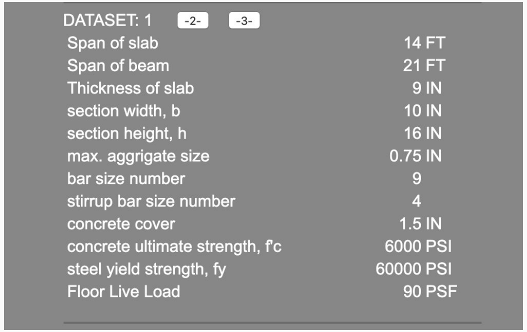
Total factored beam load w_u = 1.2 (D_L) + 1.6 (L_L)

Total factored beam load w_u = 1.2 (787.5 + 166.66) + 1.6 (630)

(Ans1) (Ans2) (Ans3)

Total factored beam load w_u = 1144.992 + 1008

Total factored beam load w_u = **2152.992 PLF**



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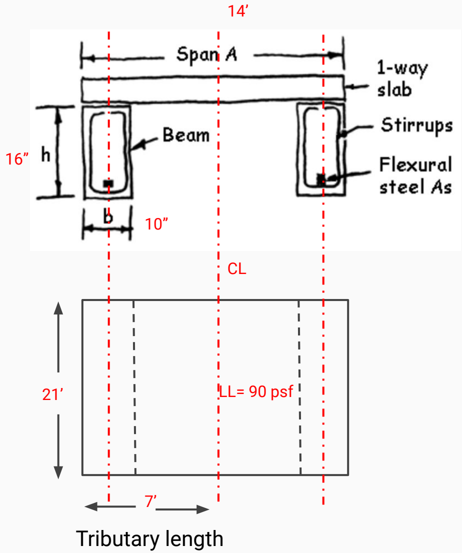


Q#5 Factored design moment from the loads, M_u

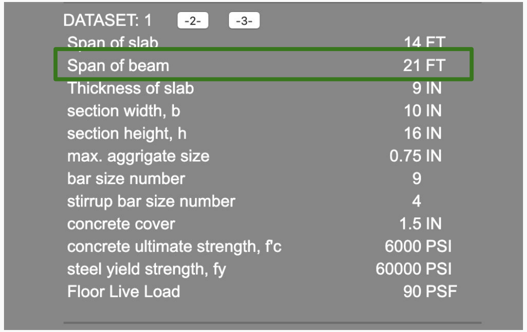
Factored design moment from the load M_u $=WL^2/8$

$= 2153 \times 21^2 \div 8$

(Ans4) (span of beam)

$=$ 118684.125 lb-ft / 1000 (convert to k-ft)

$=$ **118.684125 k-ft**



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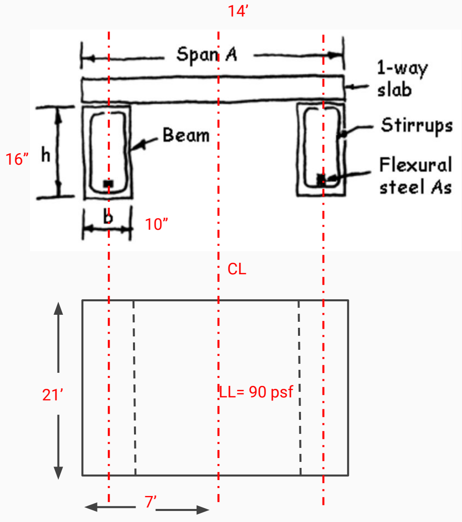


Tributary length

Q#6 Distance from the top beam edge to centroid of flexural steel, d

$$d_c = 1.5 + d_{\text{stirrup}} + (0.5 \times d_{\text{steel bar}})$$

(concrete cover to protect steel from weather)

(given)

(given)

$$d_c = 1.5'' + 0.5'' + (0.5 \times 1.125)''$$

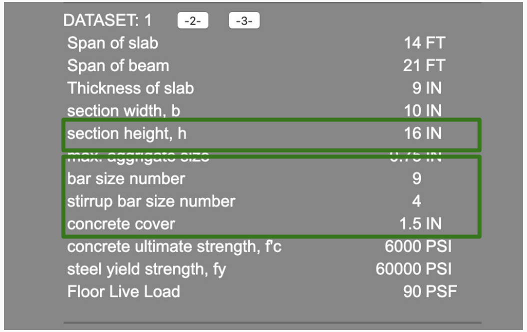
(Bar diameter = bar size number x ⅛”)

$$d_c = 2.5625''$$

$$d = h - d_c$$

$$d = 16 - 2.5625$$

$$d = \mathbf{13.4375''}$$



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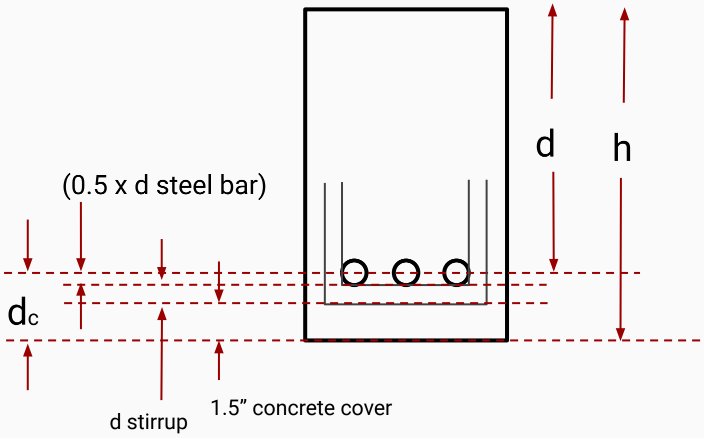


Q#7 The final calculated area of steel required , As, min

EQUATION 1

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

= moment arm = jd = z

Estimating z to be 0.9 times of d

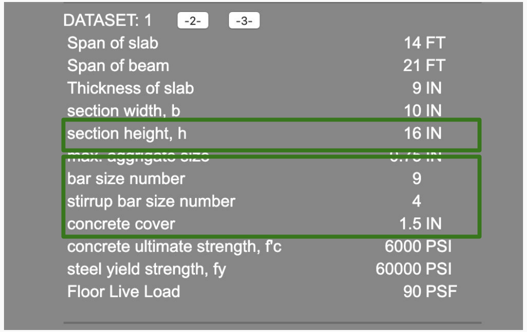
z = 0.9 x 13.4375

(Ans6)

z = 12.09

EQUATION 2

$$a = \frac{A_s f_y}{0.85 f_c' b}$$



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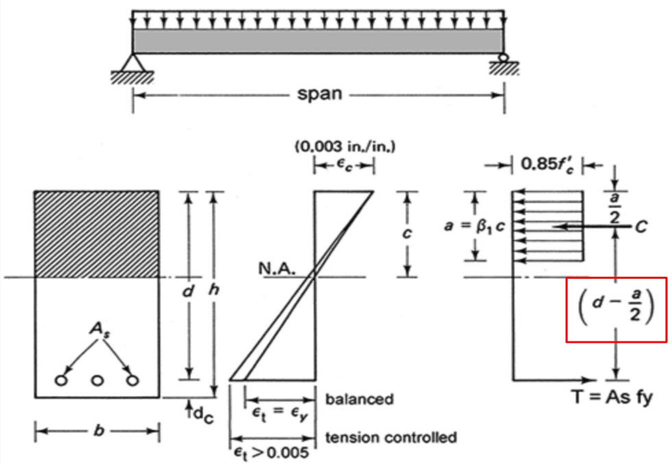


Q#7 The final calculated area of steel required , A_s, min

Step 1

Use Ƶ = 12.09 in Equation 1

$$A_s = \frac{M_u}{\phi \, f_y \left(12.09 \right)}$$

A_s = (118.68 × 12) ÷ (0.9 × 60 × 12.09) = **2.1814**

(Ans5 in inches) (table 21.2.1) (f_y in KIPS) (Ƶ)

Use this in Equation 2

$$a = \frac{2.18 \, f_y}{0.85 f_c' b}$$

a = (2.18 × 60) ÷ (0.85 × 6 × 10) = 2.5663

Put this 'a' back in equation 1 to confirm if the new A_s is in the range of 2% to previous A_s

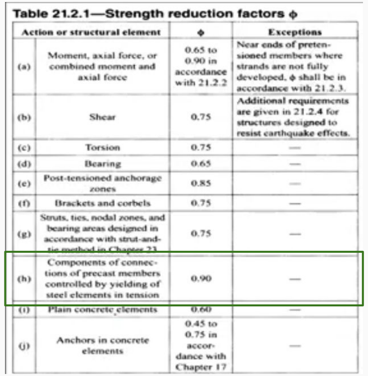
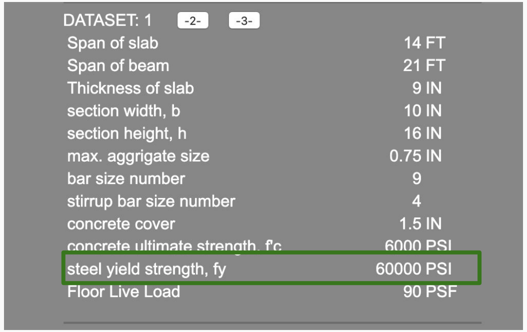


Table 21.2.1—Strength reduction factors ϕ

Action or structural element		ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—



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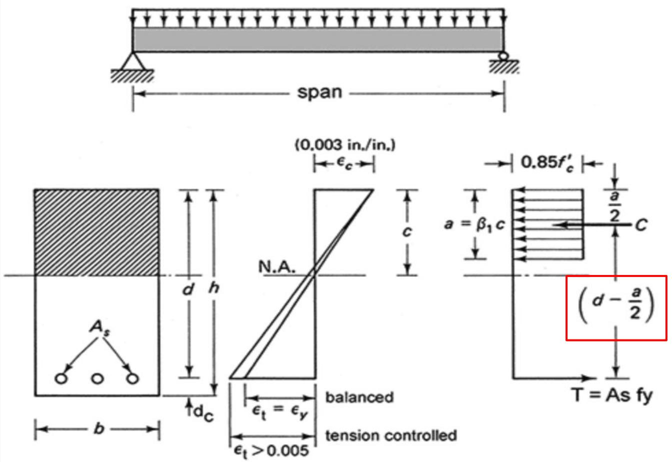


Q#7 The final calculated area of steel required , A_s, min

Step 2

Use a = 2.5663 in Equation 1

$$A_s = \frac{M_u}{\phi f_y\left(d - \frac{a}{2}\right)}$$

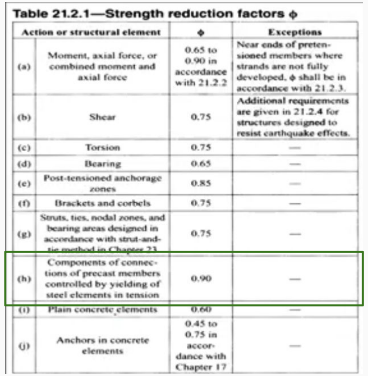
$$A_s = (118.68 \times 12) \div (0.9 \times 60 \times (13.4375 - (0.5 \times 2.5563)))$$

(Ans5 in inches) (table 21.2.1) (fy in KIPS) (Ans 6) (a)

= 2.1698

Is this within 2% of A_s = 2.1814?
Yes.
Hence, this is the answer

= **2.1698**



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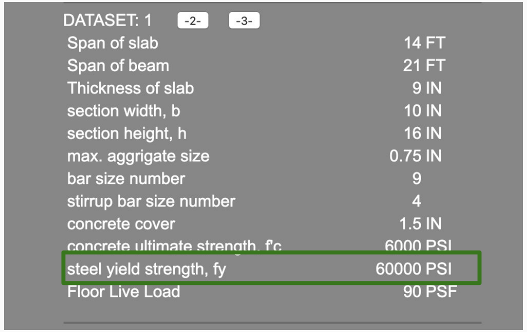

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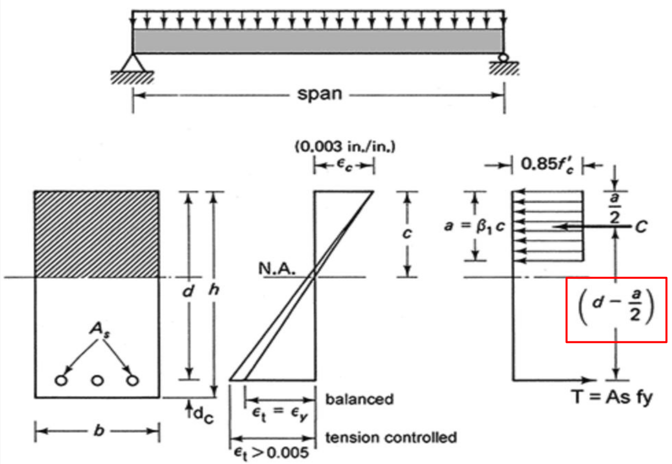


Q#8 Number of rebars used

$A_{\text{steel bar}}$ #9 = 1.00

(ASTM chart)

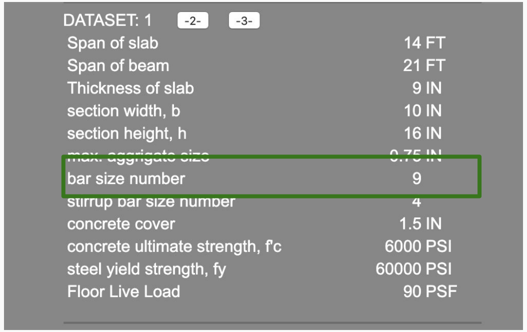
Required number of bars = $A_{s\,\text{req}} / A_{\text{steel bar}}$

Required number of bars = 2.1698 / 1

(Ans 7)

= 2.1698

= **3** (round off to higher integer)



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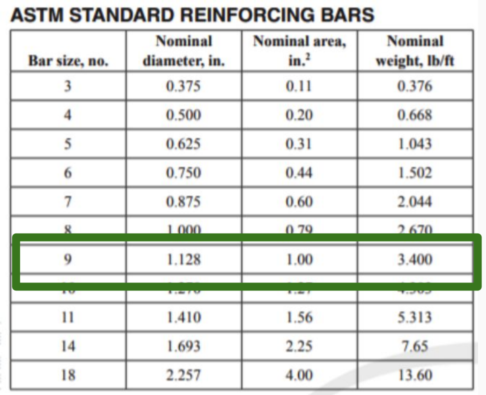


ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.65
18	2.257	4.00	13.60

Q#9 Final area of flexural steel used, A_s, used

$A_{\text{steel bar}}$ #9 = 1.00

↑

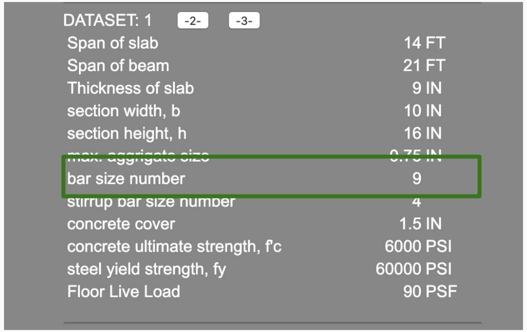
(ASTM chart)

A_s used = Bars used x $A_{\text{steel bar}}$

A_s used = 3 x 1

(Ans 8)

A_s used = **3 sq.in**



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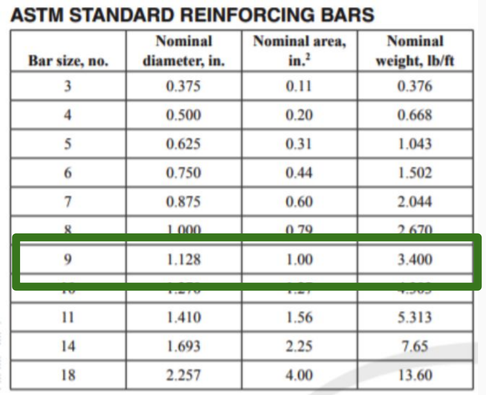

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9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.65
18	2.257	4.00	13.60

Q#10 Minimum required area of steel, A_s, min

$A_{s,}\min =$ $3\sqrt{f'c}\,(bd) \div fy$ OR $200 \times (bd) \div fy$

(WHICHEVER IS HIGHER)

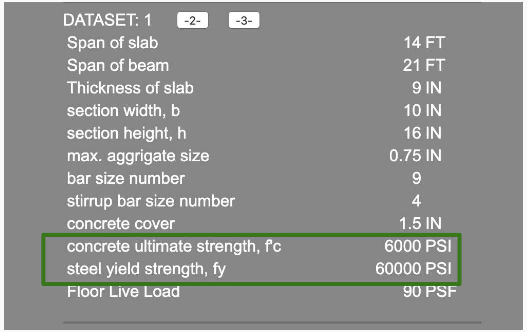
$3\sqrt{6000}$

$= 232.37$ > 200

Hence, we use $3\sqrt{f'c}\,(bd) \div 200$

$= 232.37\ (10)\ (13.4375) / 60000$

= **0.5204**



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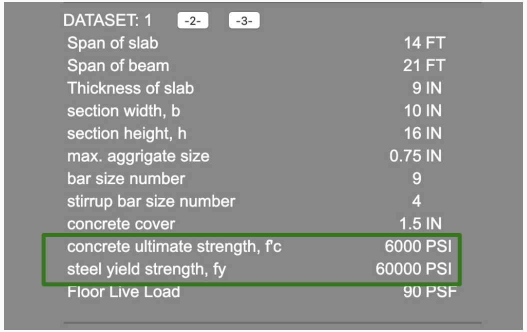
Q#11 Depth of concrete stress block, a

Calculate ‘a’ for A_s used (Ans 9)

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

= 3(60) / 0.85 (6) (10)

= **3.5294 in**



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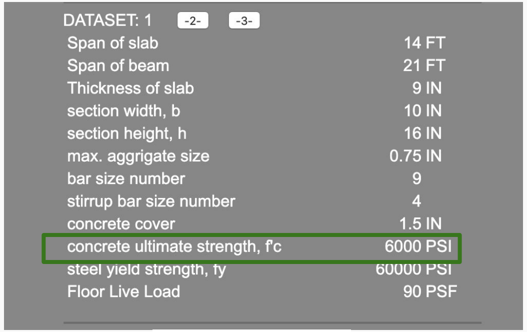
Q#12 Factor beta_1

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right)$$

β_1 = **0.75**



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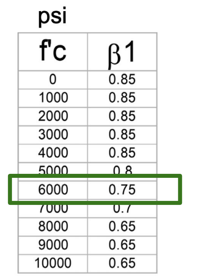
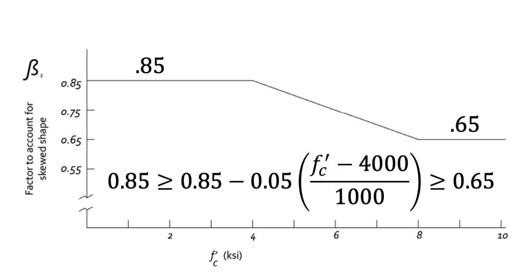


psi

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



$$0.85 \geq 0.85 - 0.05\left(\frac{f_c' - 4000}{1000}\right) \geq 0.65$$

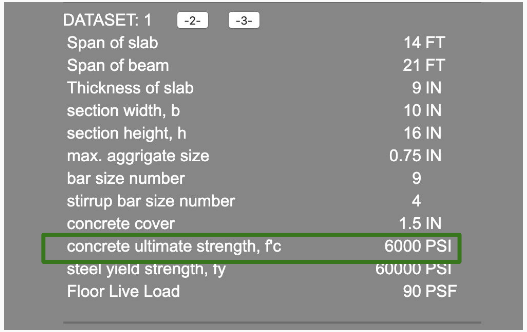
Q#13 Distance to neutral axis from top of the beam, c

$c = a / \beta_1$

$c = 3.5294 / 0.75$

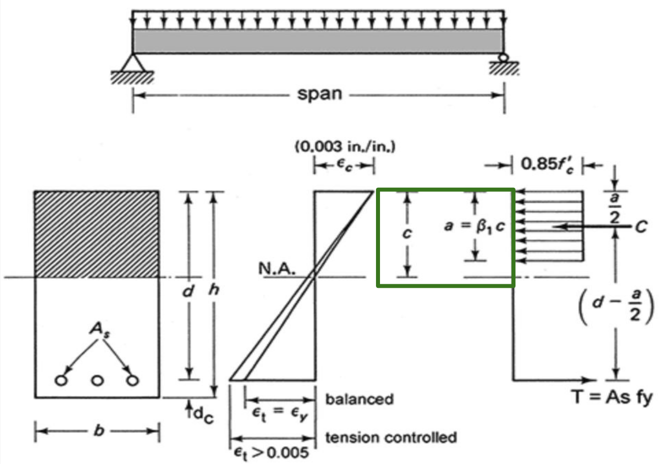
(Ans 11) (Ans 12)

= **4.705**



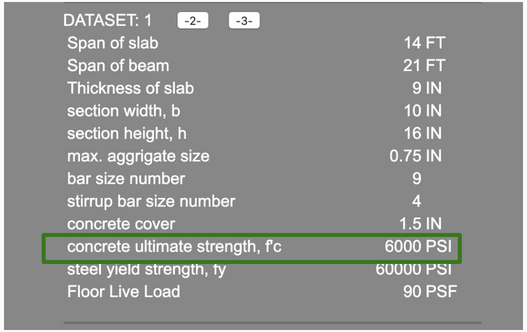
DATASET: 1 -2- -3-

Span of slab	14 FT
Span of beam	21 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	16 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6000 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	90 PSF



Q#14 Strain in flexural steel, epsilon_t

$$\epsilon_t = \frac{d - c}{c}(0.003)$$



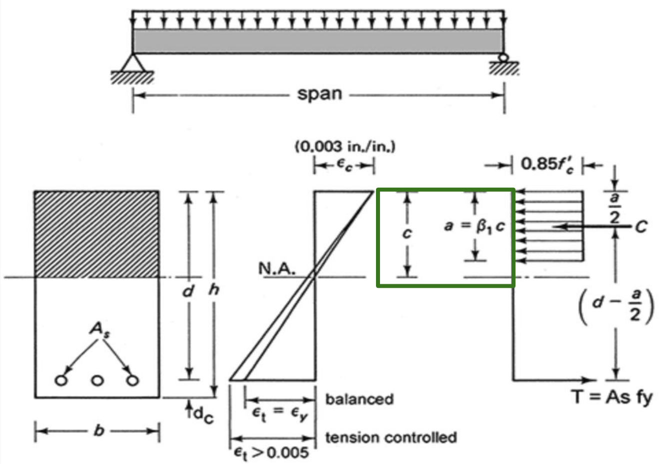
DATASET: 1 -2- -3-

Span of slab	14 FT
Span of beam	21 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	16 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6000 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	90 PSF

= ((13.4375 - 4.705) x (0.003)) / 4.705

(Ans 6) (Ans 13) (Ans 13)

= **0.0055** > .0005 (Hence under Tension control)



Q#15 Strength reduction factor, phi

Since the member is under Tension control, we use phi = **0.9** as our strength reduction factor

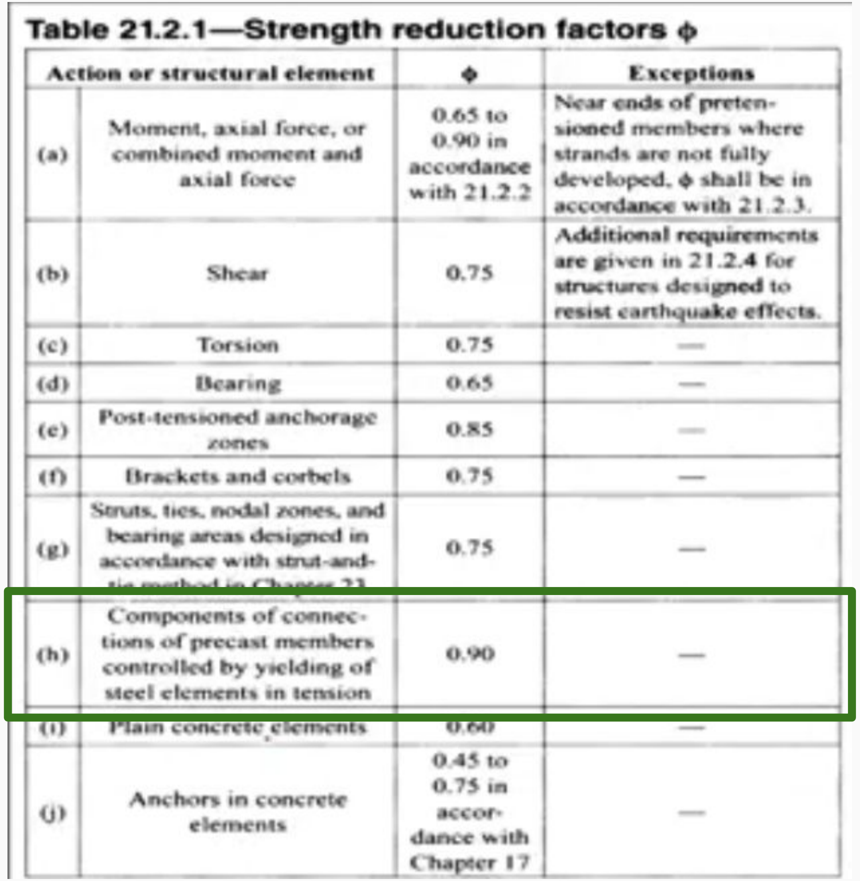


Table 21.2.1—Strength reduction factors ϕ

Action or structural element		ϕ	Exceptions
(a)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post-tensioned anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

Q#16 Tensile force in the flexural steel, T

$$T = A_s f_y$$

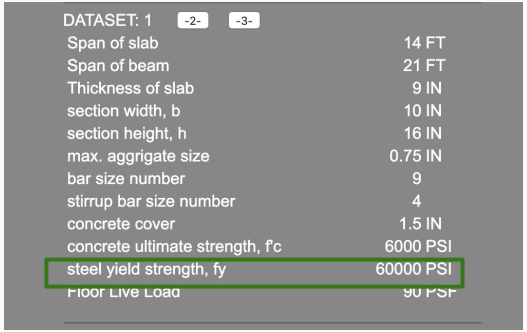
(Ans 9)

(given)

$$= (3 \times 60000) / 1000$$

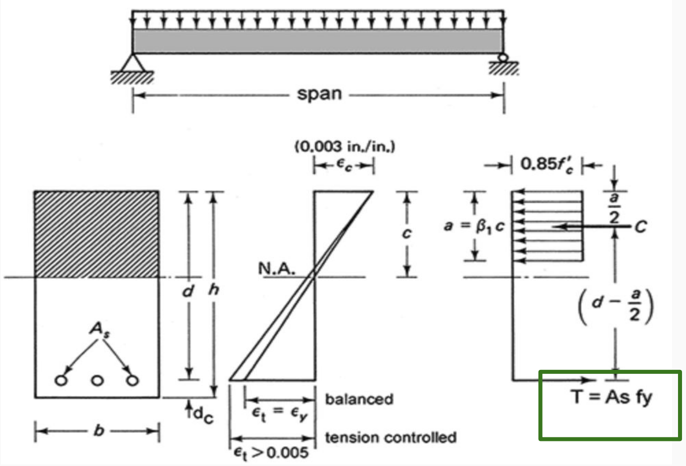
(convert to ksi)

$$= 180$$



DATASET: 1 -2- -3-

Span of slab	14 FT
Span of beam	21 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	16 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	6000 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	90 PSF



Q#17 The nominal bending moment, Mn

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

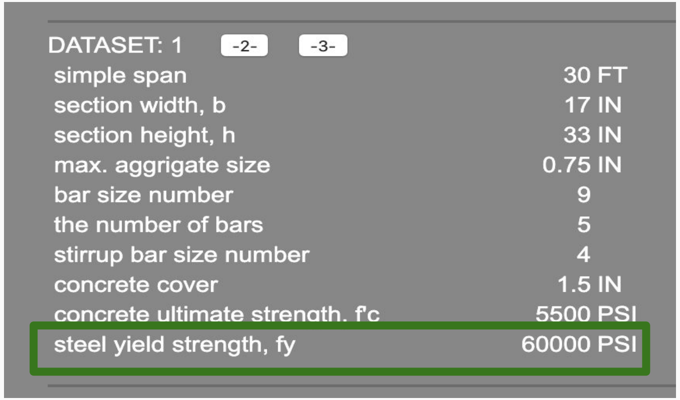
(Ans 11)

(Ans 12)

(Ans 6)

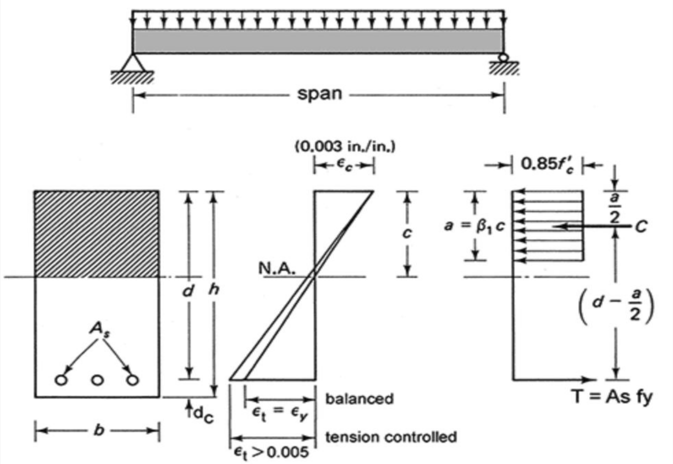
= 180 (13.4375 - (3.5294/2))

= **2100.83**



DATASET: 1 -2- -3-

simple span	30 FT
section width, b	17 IN
section height, h	33 IN
max. aggrigate size	0.75 IN
bar size number	9
the number of bars	5
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI



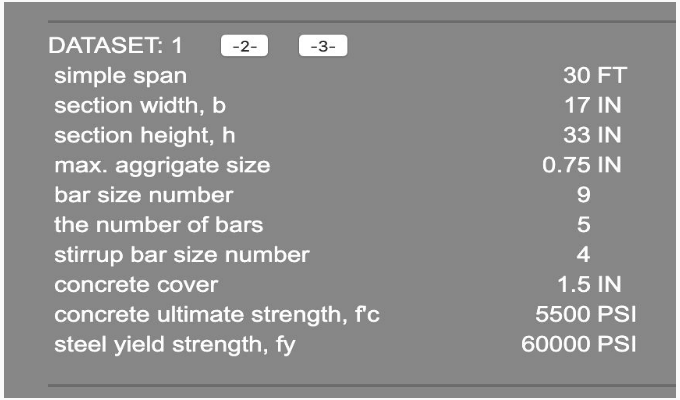
Q#18 The factored bending resistance, phi Mn

= (phi x Mn) / 12

(Ans 15) (Ans 17) (convert to K-FT)

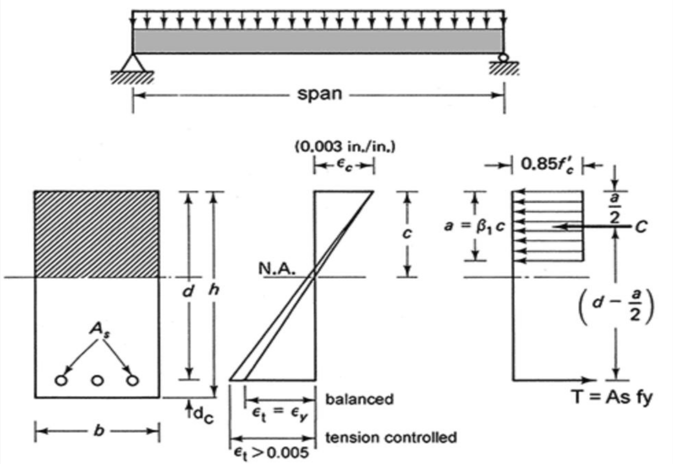
= (0.9 x 2100.83) / 12

= 157.56



DATASET: 1 -2- -3-

simple span	30 FT
section width, b	17 IN
section height, h	33 IN
max. aggrigate size	0.75 IN
bar size number	9
the number of bars	5
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI



Any Questions?

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