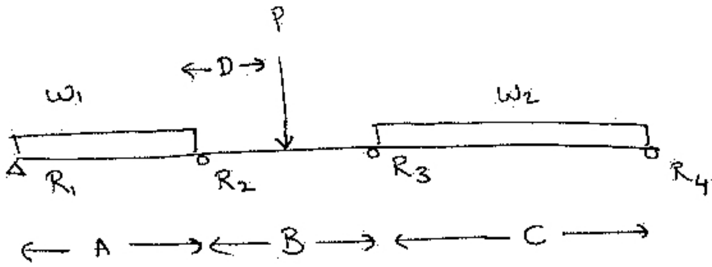
Three Moment Theorem

Feb 25 20

Given :

Span A - 29'
Span B - 28'
Span C - 26'
ω_1 - 3KLF
ω_2 - 4KLF
P - 40K
D - 14'



Q1 Moment at end supports is always 0

$\therefore$ $R_1 = R_4 = 0$ — $\boxed{\text{Ans 1 \& Ans 4}}$

Q2. Is it statically indeterminate?

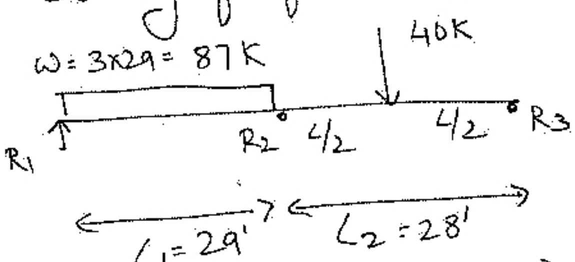
- It is asymmetrically loaded ✓
- It has more than 2 spans ✓

Yes.

To find the internal moments of a statically indeterminate beam, we use, 'Three moment equation'.

Given: $M_1L_1 + 2M_2(L_1+L_2) + M_3L_2 = 6\left[EI\theta_1 + EI\theta_2\right]$

Solving for first 2 spans:



$M_1 = 0$ ← Ans 1
$M_2 = ?$
$M_3 = ?$

$L_1 = 29'$
$L_2 = 28'$
$L_1 + L_2 = 57'$

Left of R_2 → $EI\theta_1 = \frac{wL^2}{24} = \frac{87 \times (29)^2}{24} = 3048.625$ — $\boxed{\text{Ans 2}}$

Right of R_2 → $EI\theta_2 = \frac{PL^2}{16} = \frac{40 \times (28)^2}{16} = 1960$ — $\boxed{\text{Ans 3}}$

Condition taken from the given chart,. In my case, load is at equal distances from R2 and R3

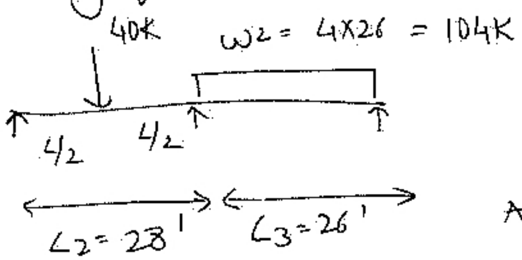
Using the given equation $\rightarrow$

$$0(29) + 2M_2(57) + M_3(28) = 6(3048.62 + 1960)$$

$$114 M_2 = 30051.72 - 28 M_3$$

$$M_2 = 263.611 - 0.2456 M_3 \quad \text{—— (1)}$$

Solving for next 2 spans



$M_2 = ?$ $\qquad L_2 = 28'$

$M_3 = ?$ $\qquad L_3 = 26'$

Ans4 $\rightarrow$ $M_4 = 0$ $\qquad L_2 + L_3 = 54'$

Left of R_3 $\longrightarrow$ $EI\theta_1 = \dfrac{PL^2}{16} = \dfrac{40(28)^2}{16} = 1960$ $\longrightarrow$ Ans5

Right of R_3 $\longrightarrow$ $EI\theta_2 = \dfrac{wL^2}{24} = \dfrac{104(26)^2}{24} = 2929.33$ — Ans6

using the given equation $\longrightarrow$

$$M_2(28) + 2M_3(54) + 0(26) = 6[1960 + 2929.33]$$

$$28 M_2 = 29335.58 - 108 M_3$$

$$M_2 = 1047.71 - 3.857 M_3 \quad \text{—— (2)}$$

From equations (1) & (2) $\rightarrow$

$$M_2 = 263.611 - 0.2456 M_3 = 1047.71 - 3.857 M_3$$

$$M_3 = \frac{783.799}{3.6114} = 217.03 \qquad \boxed{\text{Ans-8}}$$

Applying M_3 to eq ①:

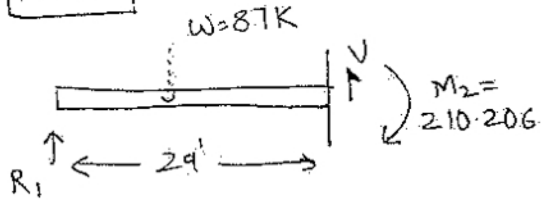
$$M_2 = 263.611 - (0.2456 \times 217.11)$$

$$M_2 = 263.611 - 53.322$$

$$M_2 = 210.2 \text{ KFT.} \quad \boxed{\text{Ans 7.}}$$

To find support reactions, draw Free Body Diagrams

FBD1



$\Sigma M @ R_2 = 0$

$R_1(29) - 87(14.5) + 210.206 = 0$

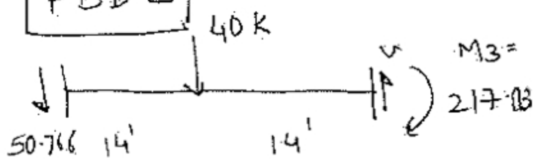
$R_1 = \dfrac{1261.5 - 210.206}{29} = 36.234$ — $\boxed{\text{Ans 9}}$

$\Sigma F_V = 0$

$R_1 - w + V = 0$

$V = w - R_1 = 87 - 36.234 = 50.766$

FBD2



$\Sigma M @ R_3 = 0$

$-210.206 + 217.11 - 50.761(28) - 40(14) + R_2(28) = 0$

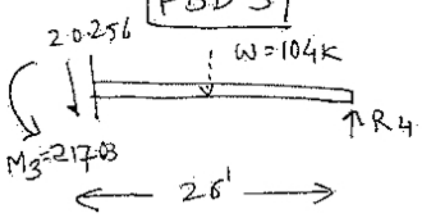
$R_2 = \dfrac{217.11 - 210.206 - 1421.288 - 560}{-28} = 70.51$. — $\boxed{\text{Ans 10}}$

$\Sigma F_V = 0$

$-V_2 + R_2 - 40 + V_3 = 0$.

$V_3 = 50.766 + 40 - 70.51 = 20.256$

FBD3



$\Sigma M @ R_3 = 0$

$104(13) - R_4(26) - 217.11 = 0$

$R_4 = 43.649$ — $\boxed{\text{Ans 12}}$

$\Sigma F_V = 0$

$-V_3 + R_3 - w + R_4 = 0$

$R_3 = 20.256 + 104 - 43.649$

$= 80.612 \text{ K}$ — $\boxed{\text{Ans 11}}$

MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT

NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"