

# Recitation 8

Concrete Beam Analysis

# Homework problem

Concrete Beam Analysis

## 8. Concrete Beam Analysis

Analyze the given composite floor system. Using a transformed section, determine peak stress values in both concrete and steel.

DATASET: 1

-2-

-3-

simple span

section width,  $b$

section height,  $h$

max. aggregate size

bar size number

the number of bars

stirrup bar size number

concrete cover

concrete ultimate strength,  $f'_c$

steel yield strength,  $f_y$

26 FT

16 IN

23 IN

0.75 IN

8

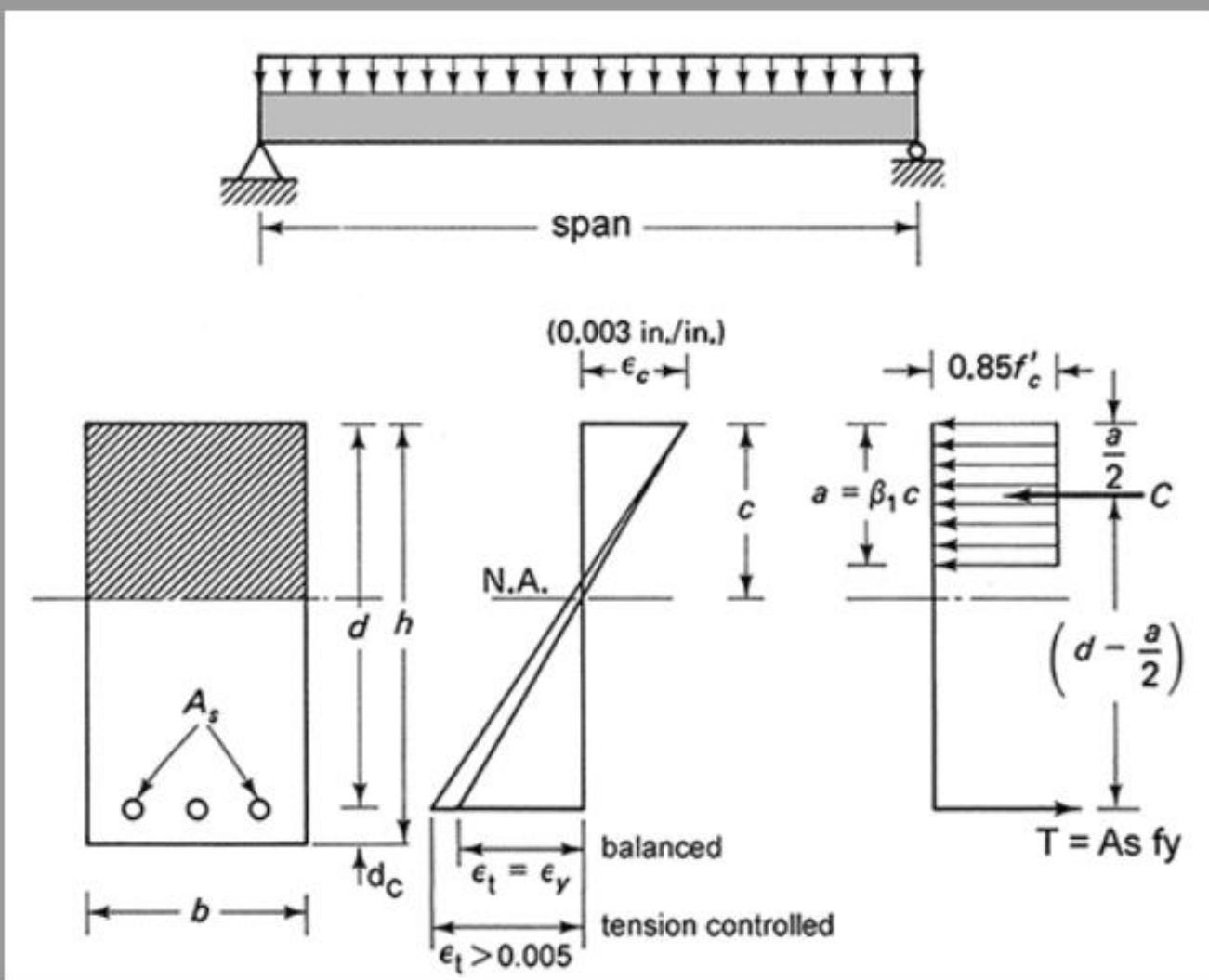
6

4

1.5 IN

6500 PSI

60000 PSI



## Q8) Concrete Beam Analysis

Analyze the given composite floor system.

Determine peak stress values in concrete and steel.

Simple span = 26 ft

Section width = 16 in (b)

Section height = 28 in (h)

max. aggregate size = 0.75 in

bar size number = 8

the no. of bars = 6

stirrup bar size number = 4

Concrete cover = 1.5 in

concrete ultimate strength ( $f'_c$ ) = 6500 psi

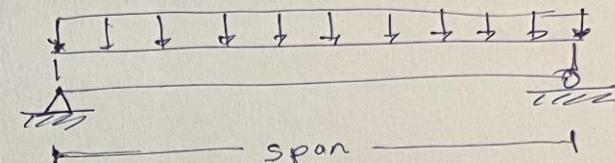
steel yield strength, ( $f_y$ ) = 60 000 psi

Q1) flexural steel bar diameter,  $d_b$  :-

Steel bars  $\rightarrow$   $\boxed{6}$  x  $\boxed{8}$   
no. of bars      bar size

from Table A.2, ~~db~~  

$$\boxed{d_b(\#8) = 1.0 \text{ in.}}$$



**Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars**

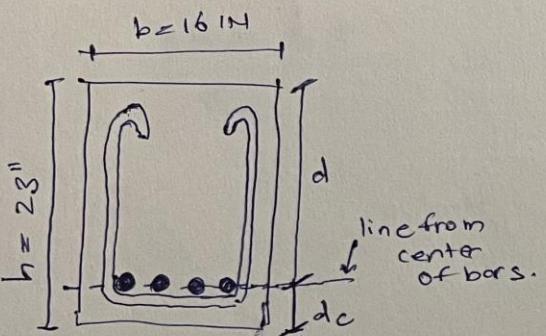
Bar No.	Diameter (in.)	Customary Units		SI Units	
		Cross-sectional Area (in. <sup>2</sup> )	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm <sup>2</sup> )
3	0.375	0.11	0.376	9.52	71
4	0.500	0.20	0.668	12.70	129
5	0.625	0.31	1.043	15.88	200
6	0.750	0.44	1.502	19.05	284
7	0.875	0.60	2.044	22.22	387
8	1.000	0.79	2.670	25.40	510
9	1.128	1.00	3.400	28.65	645
10	1.270	1.27	4.303	32.26	819
11	1.410	1.56	5.313	35.81	1006
14	1.693	2.25	7.650	43.00	1452
18	2.257	4.00	13.600	57.33	2581
					20.238

Q2) stirrup bar diameter :-

stirrup bar size number = 4.

from table A.2, [stirrup bar diameter = 0.5 in]

Q3) distance from lower beam edge to center of flexural steel,  $d_c$  :-



$$d_c = \text{cover} + \text{stirrup} + \frac{1}{2} (d_b)$$

$$= 1.5 + 0.5 + \frac{1}{2} (1)$$

$$\boxed{d_c = 2.5 \text{ in}}$$

Q4) distance from top beam edge to center of flexural steel,  $d$  :-

$$d = h - d_c$$

$$= 28 - 2.5$$

$$\boxed{d = 20.5 \text{ in}}$$

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Diameter (in.)	Customary Units		SI Units		
		Cross-sectional Area (in. <sup>2</sup> )	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm <sup>2</sup> )	
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

Q5) minimum required area of steel,

( $A_s$ )<sub>min</sub> :-

\* Take greater of the two following values:-

(a) 
$$\frac{3\sqrt{f'c}}{f_y} bd = \frac{3\sqrt{6500}}{60,000} \times 16 \times 20.5$$
  $\downarrow$  from previous question  
(rest all values given in question)

$$= 1.322 \text{ in}^2$$

(b) 
$$\frac{200 bd}{f_y} = \frac{200 (16) (20.5)}{60,000}$$

$$= 1.093 \text{ in}^2$$

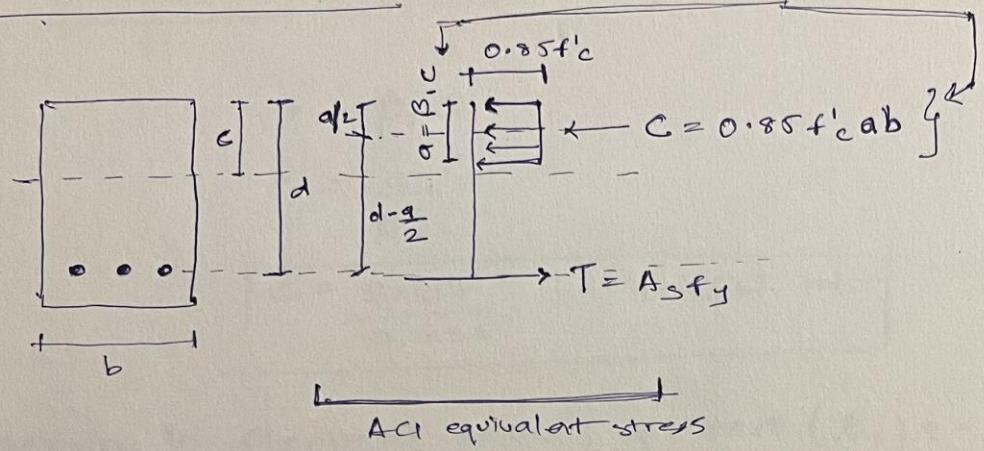
Q6) Actual Area of flexural steel,  $A_s$  :-

$$A_s = (\text{Area of steel bar}) \times (\text{Number of bars}).$$

from table A.2

$$\left| \begin{aligned} A_s &= 0.79 \times 6 \\ A_s &= 4.74 \text{ in}^2 \end{aligned} \right.$$

Q4) Depth of concrete stress block (a) :-



$$c = T$$

$$0.85 f'_c ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.74 \times 60,000}{0.85 \times (6500) (16)}$$
$$= 3.217 \text{ in}$$

Q8) Factor Beta 1, ( $\beta_1$ ) :-

$$\beta_1 = 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right)$$

$$= 0.85 - 0.05 \left( \frac{6500 - 4000}{1000} \right)$$

$$\therefore \beta_1 = 0.725$$

Q9) Distance of Neutral Axis from Top of beam, (C) :-

$$a = \beta_1 c$$

$$c = \frac{a}{\beta_1}$$

$$\boxed{C = \frac{3.217}{0.725} = 4.4317 \text{ in}}$$

Q10) strain in flexural steel, epsilon-t (E\_t) :-

$$\epsilon_t = \frac{d - c}{c} (0.003) \geq 0.005$$

$$\boxed{\epsilon_t = 0.01086 > 0.005}$$

∴ tension controlled.

Q11) strength reduction factor (phi) :-

since  $E_t > 0.005$ , beam is tension controlled

$$\boxed{\phi = 0.9}$$

Q<sub>12</sub>) Tensile force in the flexural steel (T) :-

from

$$T = As f_y$$

$$= \frac{4.74 \times 60,000 \text{ psi}}{1} \times \frac{1\text{k}}{1000 \text{ kip}}$$

$$\boxed{T = 284.4 \text{ kip}},$$

Q<sub>13</sub>) The nominal bending Moment, (M<sub>n</sub>) :-

$$M_n = T \left( d - \frac{a}{2} \right)$$

$$= 284.4 \left( 20.5 - \frac{3.217}{2} \right)$$

$$\boxed{M_n = 5372.24 \text{ k-in}}$$

Q<sub>14</sub>) The factored bending resistance, (phi M<sub>n</sub>) :-

$$\phi M_n = 0.9 \times 5372.24$$

$$\boxed{\phi M_n = 4835.47 \text{ k-in}}$$

Q15) The factored Design Moment,  $M_o$  :-

$$M_o = \phi M_n \times \frac{1 \text{ ft}}{12 \text{ in}}$$

$$= \frac{4835.47}{12}$$

$$\boxed{M_o = 402.96 \text{ k-ft}}$$

## Flexural Strain

**Description**

This project produces a graphic representation of the strain diagram for a tension controlled concrete beam.

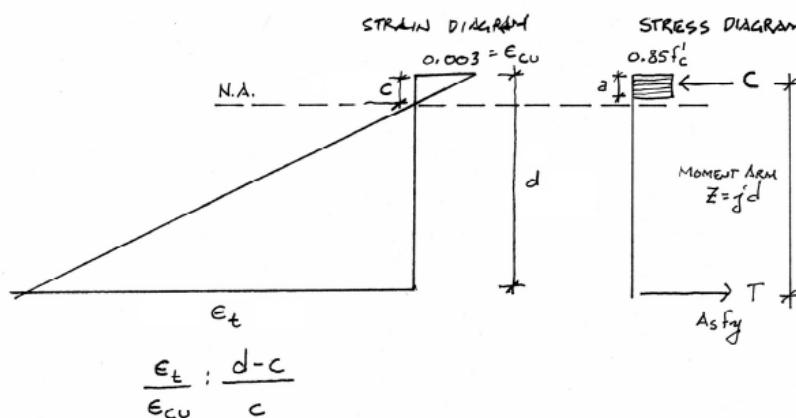
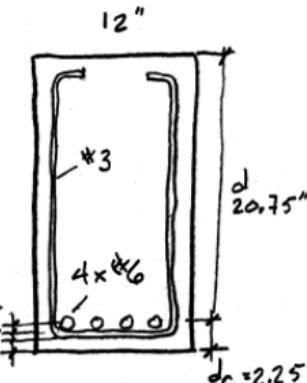
**Goals**

- To plot the compression and tension strain levels in a concrete beam
- To graphically determine the neutral axis.
- To draw the ACI "Whitney" stress block showing C and T forces.
- To compare plotted and calculated results.

**Procedure**

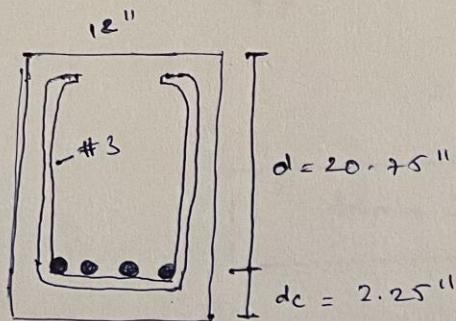
1. For the tension controlled beam analysis discussed in lecture, construct the strain diagram with  $\epsilon_{cu} = 0.003$  and  $\epsilon_t$  as calculated.
2. Use  $f_c = 6000$  psi and  $f_y = 60000$  psi
3. Graphically determine the c distance from the top to the N.A. on your diagram.
4. Make a second diagram to show the relationship of C & T forces to the strains.
5. Draw the ACI - Whitney stress block at "a" distance from the top.
6. Show the moment arm and calculate  $j$  using  $jd = z$ .

**Due**  
Sunday, March 28



# LAB – FLEXURAL STRAIN

LAB (flexural strain) :-



No. of bars = 4  
since  $d_b = 0.75$   
 $\therefore$  Bar No = 6.

1) cover = 1.5

2) stirrup bar diameter = 0.375

3)  $d/2$  of bar = 0.375

$$\therefore d = 0.375 \times 2 \\ = 0.75''$$

4)  $e_{co} = 0.003$ .

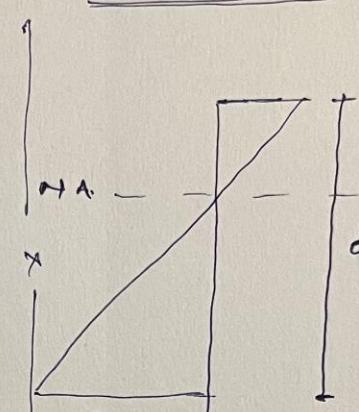
5)  $f'_c = 6000$  PSI

6)  $f_y = 60000$  PSI

To find :-

- 1)  $\epsilon_t$
- 2)  $c$ .
- 3)  $d$  (given)
- 4)  $a$ .
- 5) moment
- 6) T

Strain diagram



stress diagram

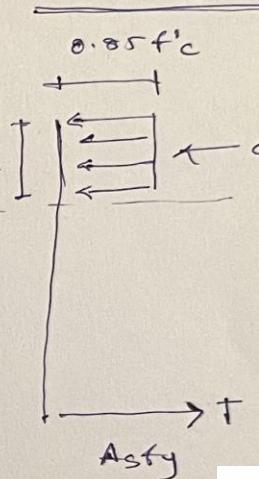


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Answer)

1) Min. req. area of steel,  $(A_s)_{min}$  :- (take greater of two).

$$\textcircled{a} \quad \frac{\sqrt{f'_c}}{f_y} bd = \frac{3\sqrt{6000}}{60000} \times 12 \times 20.75 = \frac{3 \times 77.459 \times 12 \times 20.75}{60,000} \\ = 0.964 \text{ IN}^2$$

$$\textcircled{b} \quad \frac{200 \text{ bd}}{f_y} = \frac{200(12)(20.75)}{60,000}$$

$$= 0.83 \text{ in}^2$$

$$\boxed{\therefore A_{\min} = 0.964 \text{ in}^2}$$

c) Actual area of flexural steel, ( $A_s$ ) :-

$$A_s = (\text{Area of steel bar}) \times (\text{Number of bars})$$

$$= 0.44 \times 4$$

$$\boxed{A_s = 1.76 \text{ in}^2}$$

$$C = T$$

$$0.85 f'_c ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$a = \frac{1.76 (60,000)}{0.85 (6000) (12)} = \frac{17.6}{0.85 \times 12} = \frac{17.6}{10.2} = 1.72 \text{ in}$$

$$\boxed{a = 1.72 \text{ in}}$$

$$\beta_1 = 0.85 - 0.05 \left( \frac{f'_c - 4000}{1000} \right)$$

$$= 0.85 - 0.05 \left( \frac{6000 - 4000}{1000} \right)$$

$$= 0.85 - 0.05 \left( \frac{2000}{1000} \right)$$

$$= 0.85 - 0.1$$

$$\boxed{\beta_1 = 0.75}$$

$$a = \beta_1 c$$

$$\boxed{c = \frac{a}{\beta_1} = \frac{1072}{0.75} = 2.293 \text{ in.}}$$

$$e_t = \frac{d - c}{c} (0.003)$$

$$= \frac{20.75 - 2.293}{2.293} (0.003)$$

$$\boxed{e_t = 0.024 > 0.005} \quad \therefore \text{tension is controlled.}$$

Tensile force in flanged steel ( $T$ ) :-

$$T = A_s f_y$$

$$= 1.76 \times 60000 \times \frac{1}{1000}$$

$$= 1.76 \times 60$$

$$\boxed{T = 105.6 \text{ kip}}$$

$$\overbrace{M_n}^{\text{(nominal bending moment)}} = T \left( d - \frac{a}{2} \right) \quad \textcircled{1}$$

$$= 105.6 \left( 20.75 - \frac{1.72}{2} \right)$$

$$= 105.6 (20.75 - 0.86)$$

$$= 105.6 \times 19.86$$

$$\boxed{M_n = 2100.38}$$

$$\textcircled{2} \quad (\text{factored bending resistance}) / (\phi M_n = 0.9 \times 2100.38 = \underline{\underline{1890.34 \text{ k-in}}})$$

since  $\epsilon_f > 0.005 \therefore \phi = 0.9$

The factored design moment ( $M_u$ ): -

$$M_u = \phi M_n$$

$$= \phi M_n.$$

$$= \cancel{0.75} \times 1890.34 \times \frac{1}{12}$$

$$\boxed{M_u = 157.52 \text{ k-ft}}$$

Thankyou !!!