# Structure II Recitation 3/15 

Concrete Beam Analysis

## Before we start ...

## Today's Tasks:

Homework Example (Concrete Beam Analysis) (15 Questions)
Lab Session (Flexural Strain)

## Reminder:

Tower Testing: 3/20, Wednesday!!!!!(Remember to sign up for your testing order)

## 8. Concrete Beam Analysis

Analyze the given composite floor system. Using a transformed section, determine peak stress values in both concrete and steel.

## DATASET: 1 -2-

simple span 17 FT
section width, b 14 IN
section height, h 20 IN
max. aggrigate size $\quad 0.75 \mathrm{IN}$
bar size number $\quad 7$
the number of bars 5
stirrup bar size number 3
concrete cover $\quad 1.5 \mathrm{IN}$
concrete ultimate strength, fc
steel yield strength, fy
60000 PSI


## Rectangular Beam Analysis

## Data:

- Section dimensions - b, h, (span)
- Steel area - As
- Material properties - f'c, fy

Required:

- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment - Mu
- Load capacity

$$
\begin{aligned}
& \mathrm{As}_{\text {min }} \text { : } \\
& \text { greater of (a) and (b) }
\end{aligned}
$$

$$
c=\frac{a}{\beta_{1}} \quad \varepsilon_{t}=\frac{d-c}{c} 0.003 \geq 0.005
$$

1. Calculate d
(a) $\frac{3 \sqrt{f_{c}^{\prime}}}{f_{y}} b_{w} d$
2. Check As min
(b) $\frac{200}{f_{y}} b_{w} d$

$$
a=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} b} \quad M_{n}=A_{s} f_{y}\left(d-\frac{a}{2}\right)
$$

3. Calculate a
4. Determine c

$$
\varphi M_{n} \geq M_{u}
$$

5. Check that $\varepsilon_{\mathrm{t}} \geq 0.005$ (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, $\phi \mathrm{Mn} \geq \mathrm{Mu}$ (if $\varepsilon_{\mathrm{t}} \geq 0.005$ then $\phi=0.9$ )

$$
\begin{aligned}
& M_{u}=\frac{\left(1.2 w_{D L}+1.6 w_{L L}\right) l^{2}}{8} \\
& 1.6 w_{L L}=\frac{M_{u} 8}{l^{2}}-1.2 w_{D L}
\end{aligned}
$$

8. Determine max. loading (or span)

Q1: Flexural Steel Bar Diameter (db)
Look at Table A.2, for my situation:
Bar Size Number $=\# 7, \underline{\mathbf{d b}}=\mathbf{0 . 8 7 5}$ in

## Q2: Stirrup Bar Diameter

Look at Table A.2, for my situation:
Stirrup Bar Size Number = \#3, $\underline{\text { Answer }=0.375 \text { in }}$
Q3: Distance from the Lower Beam Edge to Center of Flexural Steel (dc)
dc

$=$ Concrete Cover + Stirrup Bar Diameter $+\mathrm{db} / 2$
$=1.5+0.375+0.875 / 2$
$=\underline{2.3125}$ in
Q4: Distance from the Top Beam Edge to Center of Flexural Steel (d)
$\mathrm{d}=$ Section Height $-\mathrm{dc}=20-2.3125=\underline{\mathbf{1 7} .6875}$ in


| simple span | 17 FT |
| :--- | :---: |
| section width, b | 14 IN |
| section height, h | 20 IN |
| max. aggrigate size | 0.75 IN |
| bar size number | 7 |
| the number of bars | 5 |
| stirrup bar size number | 3 |
| concrete cover | 1.5 IN |
| concrete ultimate strength, fc | 8000 PSI |
| steel yeld strength, fy | 60000 PSI |

Course Slides 3/13 p5. (PDF)

| Table A.2 D |  | Course Slides 3/13 p5. (PDF) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Designations, Areas, Perimeters, and Weights of Standard Bars |  |  |  |  |
|  |  | stomary Un |  |  | SI Units |  |
| Bar No. | Diameter (in.) | Crosssectional Area (in. ${ }^{2}$ ) | Unit Weight ( $\mathrm{lb} / \mathrm{ft}$ ) | Diameter $(\mathrm{mm})$ | $\begin{gathered} \text { Cross- } \\ \text { sectional } \\ \text { Area }\left(\mathrm{mm}^{2}\right) \end{gathered}$ | Unit Weight (kg/m) |
| 3 | 0.375 | 0.11 | 0.376 | 9.52 | 71 | 0.560 |
| 4 | 0.500 | 0.20 | 0.668 | 12.70 | 129 | 0.994 |
| 5 | 0.625 | 0.31 | 1.043 | 15.88 | 200 | 1.552 |
| 6 | 0.750 | 0.44 | 1.502 | 19.05 | 284 | 2.235 |
| 7 | 0.875 | 0.60 | 2.044 | 22.22 | 387 | 3.042 |
| 8 | 1.000 | 0.79 | 2.670 | 25.40 | 510 | 3.973 |
| 9 | 1.128 | 1.00 | 3.400 | 28.65 | 645 | 5.060 |
| 10 | 1.270 | 1.27 | 4.303 | 32.26 | 819 | 6.404 |
| 11 | 1.410 | 1.56 | 5.313 | 35.81 | 1006 | 7.907 |
| 14 | 1.693 | 2.25 | 7.650 | 43.00 | 1452 | 11.384 |
| 18 | 2.257 | 4.00 | 13.600 | 57.33 | 2581 | 20.238 |

## Q5: Minimum Required Area of Steel (As min)

Calculate the two formulas and choose the bigger one:
$\left(3 \times\left(f c^{\prime}\right)^{0.5} / \mathrm{fy}\right) \times \mathrm{bxd}=3 \times 8000^{0.5} / 60000 \times 14 \times 17.6875=1.107412$
$200 /$ fy x b x d $=200 / 60000 \times 14 \times 17.6875=0.825$
Since $1.107412>0.825, \underline{\text { As } \min }=\mathbf{1 . 1 0 7 4 1 2}$ in $^{\underline{2}}$
Q6: Actual Area of Flexural Stress (As)

As = cross sectional area $x$ the number of bars
$\mathrm{As}=(0.6) \times 5=\underline{3 \text { in }^{2}}$

Get area from Table A-2
Check if As > As min, For my situation, It's a Pass!
(fc', fy, b, number of bars given from question, d from Q 4 )

## $\mathrm{As}_{\text {min }}$ :

greater of (a) and (b)
(a) $\frac{3 \sqrt{f_{c}^{\prime}}}{f_{y}} b_{w} d$
(b) $\frac{200}{f_{y}} b_{w} d$

## Q7: Depth of Concrete Stress Block (a)

$\quad$| $\mathrm{a}=(\mathrm{A} s \times \mathrm{fy}) /\left(0.85 \times \mathrm{fc}^{\prime} \times \mathrm{b}\right)$ |
| :--- |
| $=(3 \times 60000) /(0.85 \times 8000 \times 14)$ |
| $=\underline{\mathbf{1 . 8 9} \mathrm{in}}$ |

simple span
$\frac{\text { section width, } \mathrm{b}}{\text { section height } \mathrm{h}}$
section height, h


## Q10: Strain in Flexural Steel ( $\varepsilon \mathrm{t}$ )

$\underline{\varepsilon \mathrm{t}=(\mathrm{d}-\mathrm{c}) / \mathrm{cx}(0.003)}=(17.6875-2.9) / 2.9 \times 0.003=\underline{\mathbf{0 . 0 1 5 2}}$


## Q11: Strength Reduction Factor ( $\Phi$ )

Check if $\varepsilon$ t is bigger than 0.005 ,
Since my $\varepsilon$ t is bigger, its tension - controlled, $\boldsymbol{\Phi}=\mathbf{0 . 9}$

Q12: Tensile Force in the Flexural Steel (T)
$\underline{T}=$ As $\times \mathrm{fy}=3 \times 60000 / 1000=\underline{180 \mathrm{k}}$


14 IN 20 IN 0.75 IN

$$
\varepsilon_{t}=\frac{d-c}{c}(0.003)
$$



Q13: The Nominal Bending Moment (Mn)
$\operatorname{Mn}=\operatorname{Tx}(\mathrm{d}-(\mathrm{a} / 2))=180 \times(17.6875-1.89 / 2)=\underline{3013.6} \mathrm{k}-\mathrm{in}$


Q14: The Factored Bending Resistance ( $\mathbf{\Phi M n}$ )
$\Phi \mathrm{Mn}=0.9 \times 3013.6=\underline{\mathbf{2 7 1 2}} \mathbf{2} \mathbf{k}$ k-in


Q15: The Factored Design Moment (Mu)
$\mathrm{Mu}=\Phi \mathrm{Mn}=2712.2 / 12=\underline{\mathbf{2 2 6} \mathbf{k}-\mathrm{ft}}$

Covert Unit (in to ft )

$$
M_{n}=T\left(d-\frac{a}{2}\right)=A_{s} f_{y}\left(d-\frac{a}{2}\right)
$$

20 IN 0.75 IN
max. aggrigate size
,



#  





Flexural Strain

## Description

This project produces a graphic representation of the strain diagram for a tension controlled concrete beam.

## Goals

To plot the compression and tension strain levels in a concrete beam
To graphically determine the neutral axis.
To draw the ACI "Whitney" stress block showing C and T forces.
To compare plotted and calculated results.

## Procedure

1. For the tension controlled beam analysis discussed in lecture, construct the strain diagram with $\boldsymbol{\epsilon c u}=$ 0.003 and $\boldsymbol{\epsilon}_{\mathrm{t}}$ as calculated.
2. Use fc $=6000$ psi and fy $=60000 \mathrm{psi}$
3. Graphically determine the c distance from the top to the N.A on your diagram.
4. Make a second diagram to show the relationship of $C$ \& $T$ forces to the strains.
5. Draw the ACI - Whitney stress block at " $a$ " distance from the top.
6. Show the moment arm and calculate jusing $j d=z$. Due Calculate Moment
Sunday, March 28


## Lab Session:

Calculate $\beta 1$, a, c, $\varepsilon$ t, Bending Moment

## Moment:

T x Moment Arm = (As x Fy) x (d - a/2)
As = cross sectional area (\#6) $x$ number of bars

| $\boldsymbol{f}_{\mathbf{C}}^{\prime}$ | $\boldsymbol{\beta}_{1}$ |
| :---: | :---: |
| 0 | 0.85 |
| 1000 | 0.85 |
| 2000 | 0.85 |
| 3000 | 0.85 |
| 4000 | 0.85 |
| 5000 | 0.8 |
| 6000 | 0.75 |
| 7000 | 0.7 |
| 8000 | 0.65 |
| 9000 | 0.65 |
| 10000 | 0.65 |

$$
a=\frac{A_{s} f_{y}}{0.85 f_{c}^{\prime} b}
$$

$$
a=B_{1} c
$$

$$
\varepsilon_{t}=\frac{d-c}{c}(0.003)
$$

Table A. 2 Designations, Areas, Perimeter
Table

|  | Customary Units <br> Bar <br> Bo. |  |  |
| :---: | :---: | :---: | :---: |
| Diameter <br> (in.) | Cross- <br> (ectional <br> Area (in. $\left.{ }^{2}\right)$ | Unit <br> Weight <br> $(\mathrm{lb} / \mathrm{ft})$ |  |
| 3 0.375 0.11 0.376 <br> 4 0.500 0.20 0.668 <br> 5 0.625 0.31 1.043 <br> 6 0.750 0.44 1.502 <br> 7 0.875 0.60 2.044 <br> 8 1.000 0.79 2.670 <br> 9 1.128 1.00 3.400 <br> 10 1.270 1.27 4.303 <br> 11 1.410 1.56 5.313 <br> 14 1.693 2.25 7.650 <br> 18 2.257 4.00 13.600 |  |  |  |

