Recitation 10

Concrete Beam Design

Homework problem

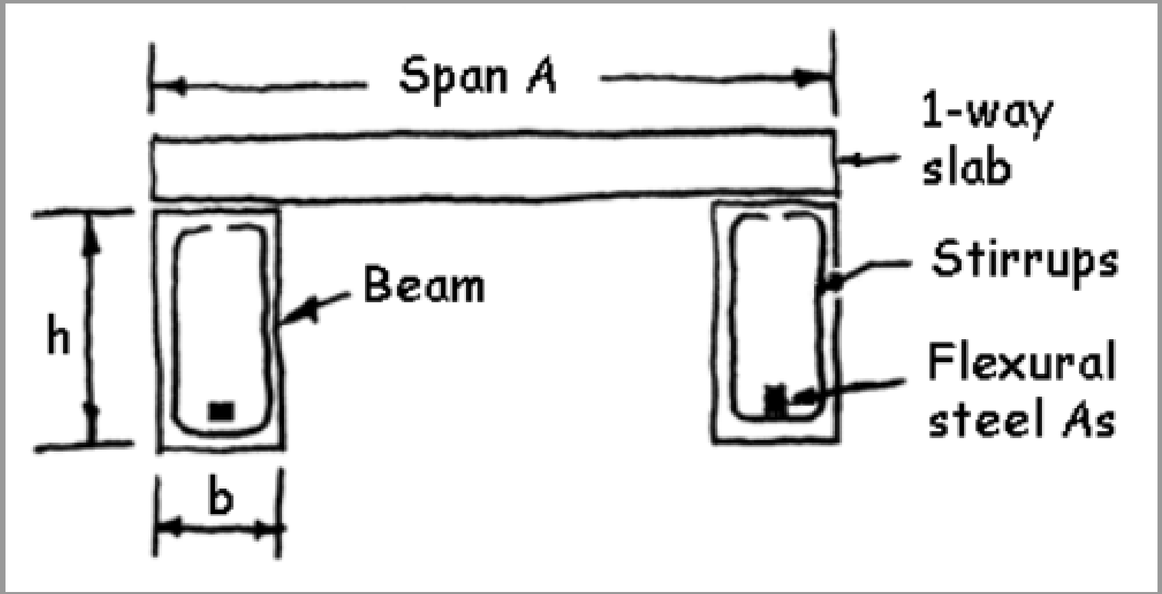
Concrete Beam Design

9. Concrete Beam Design

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

DATASET: 1 -2- -3-

Span of slab	14 FT
Span of beam	23 FT
Thickness of slab	9 IN
section width, b	10 IN
section height, h	18 IN
max. aggrigate size	0.75 IN
bar size number	8
stirrup bar size number	4
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	55 PSF



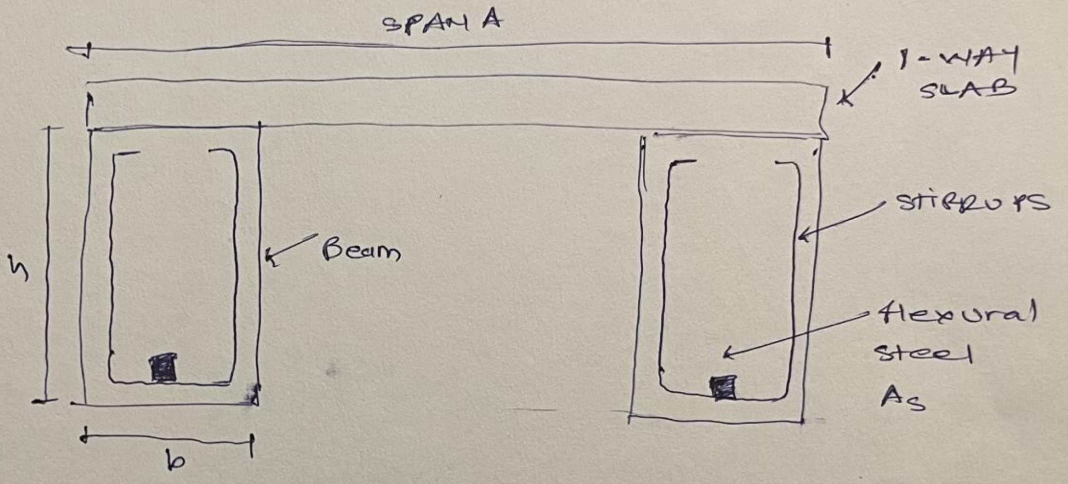
Q9) Concrete Beam Design.

Analyze given section to determine its safe moment capacity. (ϕM_u).

Check that section is tensioned controlled

⟶ (epsilon_t > 0.005)

and that the amount of steel, A_s is more than the minimum (A_{smin}).



Span of slab = 14ft

Span of beam = 28ft

Thickness of slab = 9IN

section width, b = 10IN.

section height, h = 18IN

max aggregate size = 0.75IN

bar size number = 8

stirrup bar size number = 4

concrete cover = 1.5 IN

concrete ultimate strength f'c = 5500 PSI

Steel yield strength = 60000 PSI

floor live load = 55 PSF

Q1) unfactored Dead Load on beam from slab :-

$$D_L = D_{concrete} \times \frac{L_{slab}}{2} \times t_{slab} \times \frac{1\,ft}{12\,in}$$

$$= 150\,\frac{lb}{ft^3} \times \frac{14\,ft}{2} \times 9\,in \times \frac{1\,ft}{12\,in}$$

$$= 787.5\ lb/ft.$$

Q2) unfactored Dead load on beam from the beam :-

$$W_{DL,\ Beam} = D_{concrete} \times b \times h$$

$$= 150\,\frac{lb}{ft^3} \times 10\,in \times \frac{1\,ft}{12\,in} \times 18\,in \times \frac{1\,ft}{12\,in}$$

$$= 187.5\ lb/ft.$$

Q3) unfactored live load on beam, LL :-

$$W_{LL} = \frac{LL \times l_{slab}}{2}$$

$$= 55\,lb/ft^3 \times \frac{14}{2}\,ft$$

$$= 385\ lb/ft$$

Q4) Total factored beam load, wu :-

$$W_u = 1.2\,W_{DL} + 1.6\,W_{LL}$$

$$= 1.2 \times (W_{DL\,SLAB} + W_{DL\,BEAM}) + 1.6\,W_{LL}$$

$$= 1.2 \times (787.5 + 187.5) + 1.6 \times (385)$$

$$= 1.2 \times 975 + 1.6\,(385)$$

$$= 1786 \text{ lb/ft}$$

Q5) Factored Design Moment from the loads, Mu :-

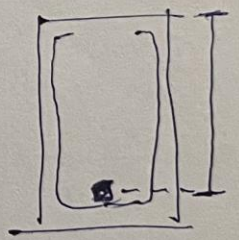
$$M_u = \frac{W_u L^2}{8} = \frac{W_u L_{beam}^2}{8}$$

$$= \frac{1786 \times 23^2}{8} \left[\frac{lb}{ft} \times ft^2 \times \frac{1\text{ KIP}}{1000\text{ lb}}\right]$$

unit conversion.

$$M_u = 118.09925 \text{ K-FT}$$

Q6) Distance from top beam edge to controid flexural steel, d :-



from $d = h - (cover + d_{stirrup} + \frac{1}{2} d_{bar})$

$$= 18 - (1.5 + 0.5 + \frac{1}{2} \times 1)$$

$$= 15.5 \text{ in}$$

Q7) The final calculated Area of steel req, As req :-

(from Q6).

Estimate moment arm: $z \simeq 0.95d$

$\therefore z = 0.95(15.5)$

$z = 14.725$ in.

Estimate As, from $A_s = \dfrac{M_u}{\phi f_y z}$
(Trial 1)

$= \dfrac{118.09925 \text{ K.ft}}{0.9 \times (60 \text{ KSI})(14.725 \text{ in})} \times \dfrac{12 \text{ in}}{1 \text{ FT}}$

$A_s = 1.7823 \text{ in}^2$

from $a = \dfrac{A_s f_y}{0.85 f'_c b} = \dfrac{1.7823 \text{ in}^2 \times 60 \text{ KSI}}{0.85 \times (5.5 \text{ KSI}) \times (10 \text{ in})}$

$a = 2.28744$ in.

Use "a" to find "As" :-

$A_s = \dfrac{M_u}{\phi f_y (d - \frac{a}{2})} \times \dfrac{12 \text{ in}}{1 \text{ FT}}$

$= \dfrac{118.09925 \text{ K.FT}}{0.9(60)(15.5 - \frac{2.28744}{2})} \times \dfrac{12 \text{ in}}{\text{ft}}$

$A_s = 1.8281 \text{ in}^2$

Q8) Number of rebars used :-

Bar size #8 ; Area = 0.79 in^2

$$\text{Number of rebars} = \frac{As_{req}}{Area} = \frac{1.8281}{0.79} = 2.314$$

≈ round upto higher whole number

∴ Number of rebar = 3

Q9) Actual area of rebars :-

$3 \times 0.79 = 2.37$ in^2

Q10) Min req area of steel, Asmin :-

(from Q6)

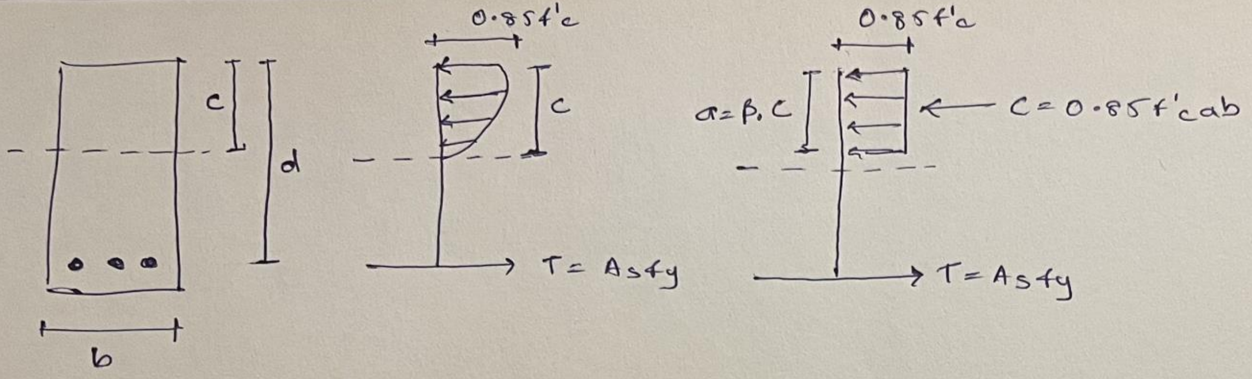
a) $$\frac{3\sqrt{f'c}\ bd}{fy} = \frac{3\sqrt{5500}}{60,000} \times 10 \times 15.5 = 0.5748 \text{ in}^2$$

b) $$\frac{200\ bd}{fy} = \frac{200(10)(15.5)}{60,000} = 0.5167 \text{ in}^2$$

} use greater of two values.

∴ Asmin = 0.5748 in^2

Q11) Depth of concrete stress block, a :-



$$C = T$$

$$0.85 f'_c ab = A_s f_y$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.37\,(60,000)}{0.85\,(5500)\,(10)} = \boxed{3.0417 \text{ in}}$$

Q12) The factor β_1 :-

$$\beta_1 = 0.85 - 0.05\left(\frac{f'c - 4000}{1000}\right)$$

$$\beta_1 = 0.85 - 0.05\left(\frac{5500 - 4000}{1000}\right)$$

$$\boxed{\beta_1 = 0.775}$$

Q13) Distance to Neutral Axis from top of beam, "c" :-

$a = \beta_1 c$

$c = \frac{a}{\beta_1} = \frac{3.0417}{0.775} = \boxed{3.9247 \text{ in}}$

Q14) strain in flexural steel, ϵ_t :-

$\epsilon_t = \frac{d-c}{c}(0.003)$

$= \left(\frac{15.5 - 3.9247}{3.9247}\right)(0.003)$

$\therefore \boxed{\epsilon_t = 0.008848} > 0.004$ $\therefore$ ok

$= 0.008848 > 0.005$, $\therefore$ tension controlled.

Q15) strength reduction factor, phi :-

$\phi = 0.9$

Q16) Tensile force in the flexural steel, T :-

$T = A_s f_y$

$= 2.37 \times 60$

$= 142.2$ KIP

Q17) Nominal Bending Moment (Mn) :-

$$M_n = T\left(d - \frac{a}{2}\right)$$

$$= 142.2\left(15.5 - \frac{3.0417}{2}\right)$$

$$\therefore \boxed{M_n = 1987.835 \text{ k.in.}}$$

Q18) Factored bending Resistance, phi Mn :-

$$\phi M_n = 0.9\,(1987.835)\text{ k.in} \times \frac{1\text{ft}}{12\text{in.}}$$

$$\boxed{\phi M_n = 149.0876 \text{ k.ft.}}$$

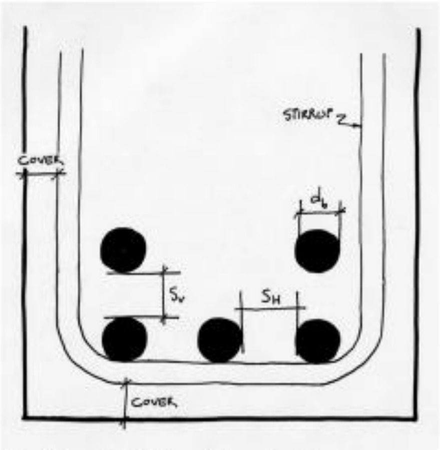
Goals

To determine bar diameters and horizontal spacing
To find the placement and dimensions of a shear stirrup.
To establish proper cover for reinforcement.
To draw all beam elements in the proper scale and location.

Procedure

1. For the example beam worked in class, determine the required spacing, s_v and s_h, for the bar size used.
2. For the given stirrup size determine the bend radius for a 90° bend.
3. Make a sketch showing the proper locations of bars and the stirrup including cover.
4. Draw and dimension the depth of the stress block, “a” and the distance to the N.A. from the top of the beam, “c”.
5. Dimension and label “d” and “d_c”.

LAB

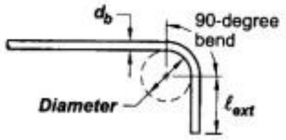
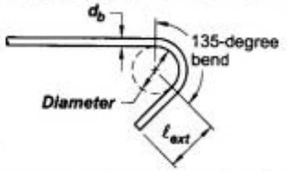
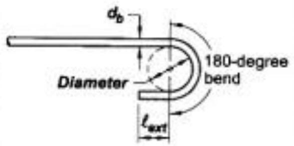


Horizontal Spacing in Beams

ACI 25.2.1

- 1 inch
- db
- 4/3 max aggregate

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

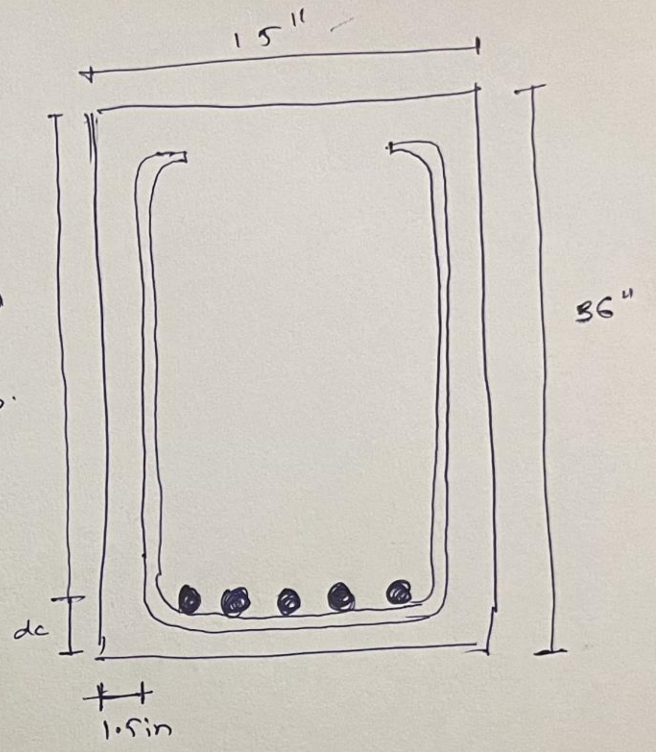
[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

CAB :- Reinforcement placement

Given :- Bar No. 4 for → stirrups

Bar No. 8 for → flexural steel.

15 × 36 → Dimension of beam
1.5 → cover on all sides.
} values from classnotes.



Total number of bars = 5

Stirrups → since bar No. = 4

∴ Diameter = 0.5"

flexural steel → since Bar No. = 8

∴ Diameter = 1.00"

d_c = cover + $d_{stirrup}$ + $d/2$ bar = 1.5 + 0.5 + 0.5
= 2.5"

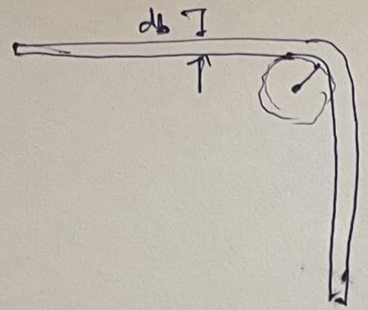
Dimension d → $d_c = h - d$

$d = h - d_c$

$d = 36 - 2.5$

$d = 33.5"$

Bend radius for a 90° bend :-



Minimum inside bend diameter, in :- $4 \times d_b$

$4 \times 0.5 = 2$ in.

$$a = \frac{A_s f_y}{0.85\, f'_c\, b}$$

$A_s = 5 \times 0.79 = 4.05\ in^2$

$b = 15''$, $f_y = 60,000$, $f'_c = 5500$

$$a = \frac{4.05 \times 60000}{0.85 \times 5500 \times 15} = \frac{243000}{70125} = 3.46 \text{ in}$$

Solve for 'c'

~~Assume~~ $\rightarrow$ $\beta_1 = 0.85 - 0.05 \dfrac{(f'_c - 4000)}{1000}$ (5500 $\swarrow$)

$\beta_1 = 0.775$

$$c = \frac{a}{\beta_1}$$

$$= \frac{3.46}{0.775}$$

$$c = 4.46 \text{ in}$$

Thankyou !!!