Concrete Beam Design 3/29

HW – Concrete Beam Design

Lab – Reinforcement Placement

Structure II
Section 004

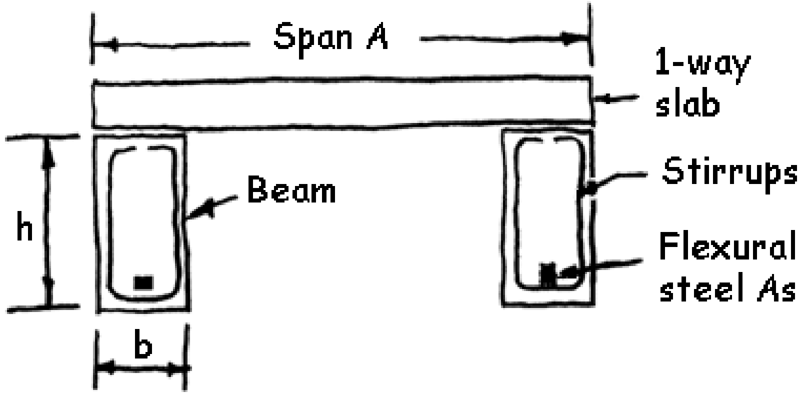
Yifan Ma
yifanma@umich.edu

HW - Concrete Beam Design

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

DATASET: 1 -2- -3-

Span of slab	19 FT
Span of beam	30 FT
Thickness of slab	12 IN
section width, b	18 IN
section height, h	39 IN
max. aggrigate size	0.75 IN
bar size number	9
stirrup bar size number	3
concrete cover	1.5 IN
concrete ultimate strength, f'c	5500 PSI
steel yield strength, fy	60000 PSI
Floor Live Load	45 PSF

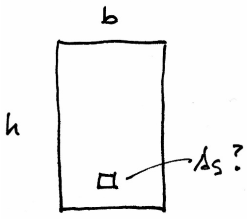


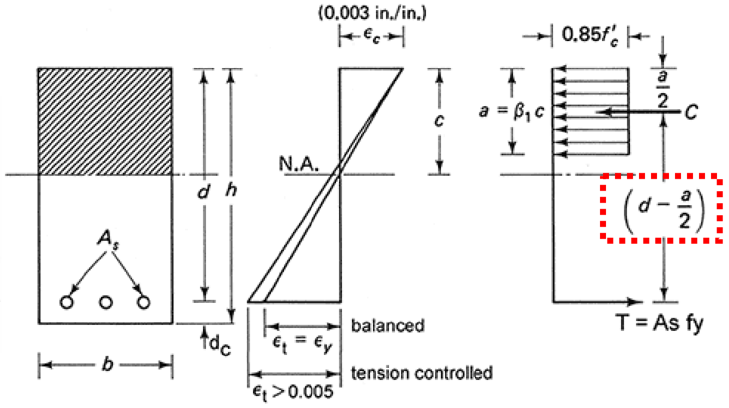
Data:

Load and Span

Material properties – f'c, fy

All section dimensions: h and b





Required:

Steel area - As

1. Calculate the factored load and find factored required moment, Mu
2. Find d = h – cover – stirrup – $d_b/2$ (one layer)
3. Estimate moment arm z = jd. For beams j ≈ 0.9 for slabs j ≈ 0.95
4. Estimate As based on estimate of jd.
5. Use As to find a
6. Use a to find As (repeat…until 2% accuracy)
7. Choose bars for As and check As max & min
8. Check that $\varepsilon_t \geq 0.005$
9. Check Mu ≤ ϕ Mn (final condition)

$$M_u = \frac{(\gamma\, w_{DL} + \gamma\, w_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi\, f_y \boxed{\left(d - \frac{a}{2}\right)}}$$

moment arm

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

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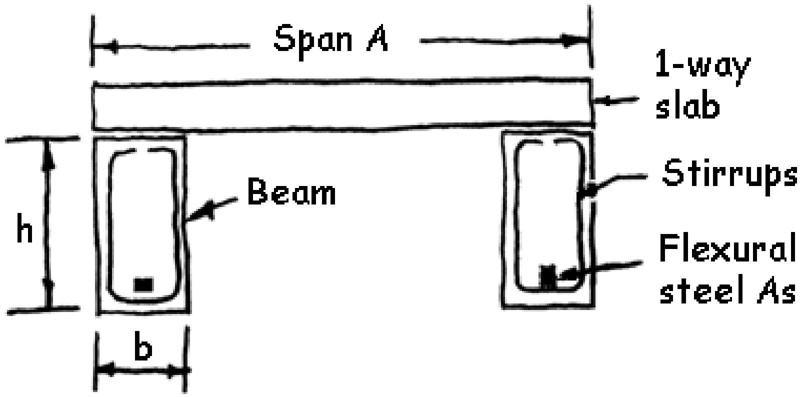

1. Unfactored dead load on beam from slab

½* Span A * Slab thickness * 150 pcf
= ½*19*12/12*150
=1425 plf

2. Unfactored dead load on beam from the beam(PLF)(beam selfweight)

b * h * 150 = 18/12 * 39/12 *150= 731.25 plf

3. Unfactored live load on beam, LL

LL = LLfloor * (Span A) /2 = 45*19/2 = 427.5 plf

4. Total factored beam load, wu

wu = 1.2*DL + 1.6*LL = 1.2 *(1425+731.25)+1.6*427.5 = 3271.5 plf

5. Factored design moment from the loads, Mu

Mu = 1/8 * wu*(Spanbeam) ^ 2 = 1/8* 3271.5*30^2 = 368.04 ft-k

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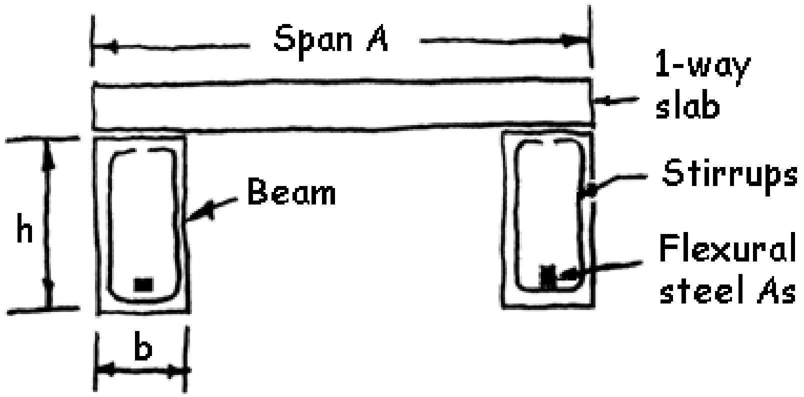
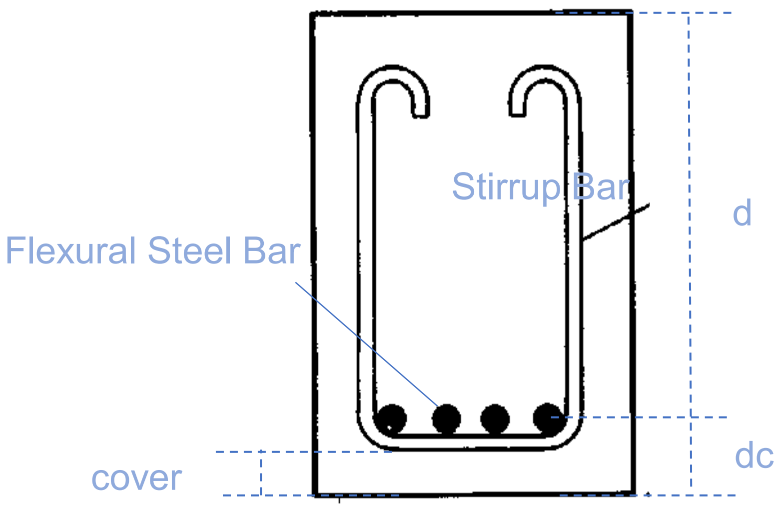


6. Distance from top beam edge to centroid of flexural steel,d



Bar size designa-tion	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257

dc = cover + stirrup bar diameter + 1/2 flexural steel bar diameter

= 1.5+0.375+0.5*1.128

= 2.439 in

d= h – dc = 39 - 2.439 = 36.561 in

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

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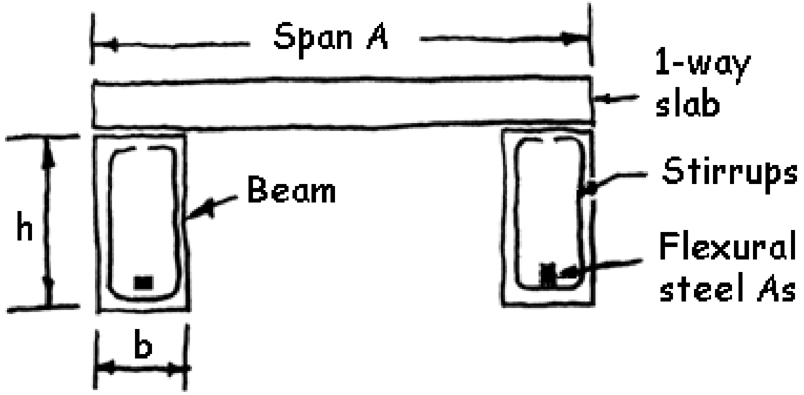
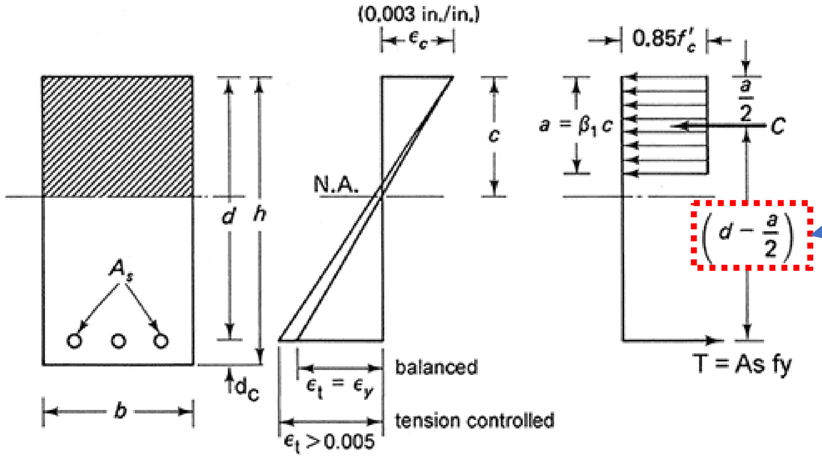


7. The final calculated area of steel required , As,req



ϕ Mn = ϕ T * moment arm = Mu

T = As * fy

(1) Estimate moment arm Z = jd

For beams j ≈ 0.9

Zset = 0.9*36.561 = 32.905”

(2) Estimate As based on estimate Z

ϕ = 0.9

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

≈ Mu / (0.9 * fy * Zset)

= 368.04 * 12 *1000 / (0.9*60000*32.905) = 2.486 in²

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

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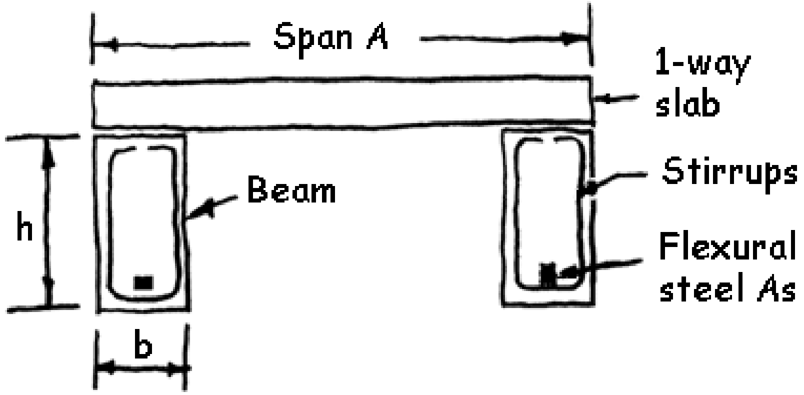
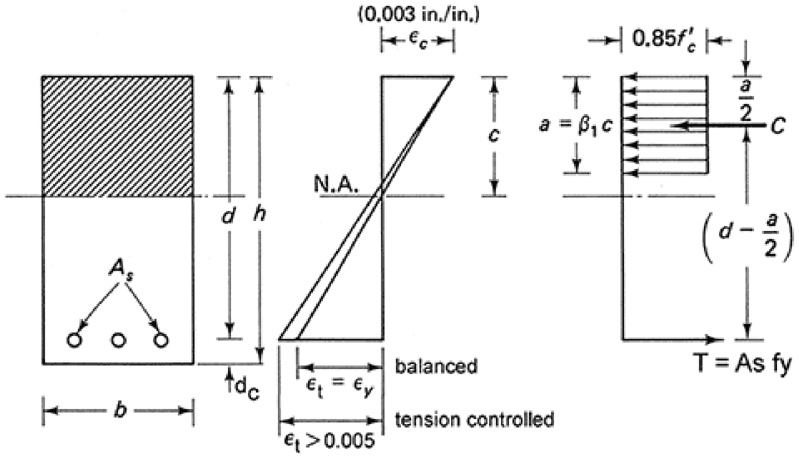


7. The final calculated area of steel required , As,req

(3) Use As1 to find a1

$$a = \frac{A_s f_y}{0.85 f'_c b}$$ = 2.486*60000/ (0.85*5500*18)= 1.773 in

(4) Use a1 to find As2



Z = d – a/2
= 36.561- 1.773 /2
= 35.674 in

$$A_s = \frac{M_u}{\phi\, f_y \left(d - \dfrac{a}{2}\right)}$$

As2 = 368.04 * 12 *1000 / (0.9*60000* 35.674) = 2.293 in²

(5) Compare As1 and As2

|As2-As1| / As2 = | 2.293 - 2.486 | / 2.293 > 2%

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

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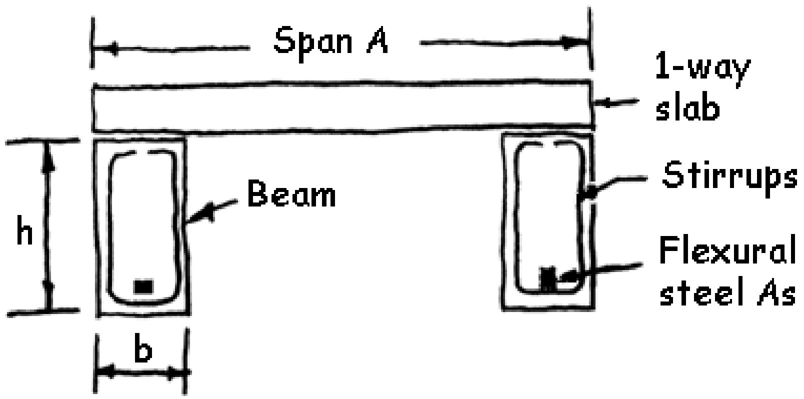

7. The final calculated area of steel required , As,req

(6) Repeat, use As2 to find a2

$$a = \frac{A_s f_y}{0.85 f'_c b}$$ = 2.293 *60000/ (0.85*5500*18)= 1.635 in

(7) Use a2 to find As3

Z = d – a/2
= 36.561- 1.635 /2
= 35.744 in

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$ = 368.04 * 12 *1000 / (0.9*60000* 35.744) = 2.288 in²

(8) Compare As3 and As2

|As3-As2| / As3 = | 2.288 - 2.293 | / 2.288 < 2% Pass! As req = 2.288 in²

Using the Ultimate Strength Method, analyze the given section to determine its safe moment capacity, Mu, based on the given parameters. Check that the section is tension controlled (epsilon_t > 0.005), and that the amount of steel, As is more than the minimum, As_min.

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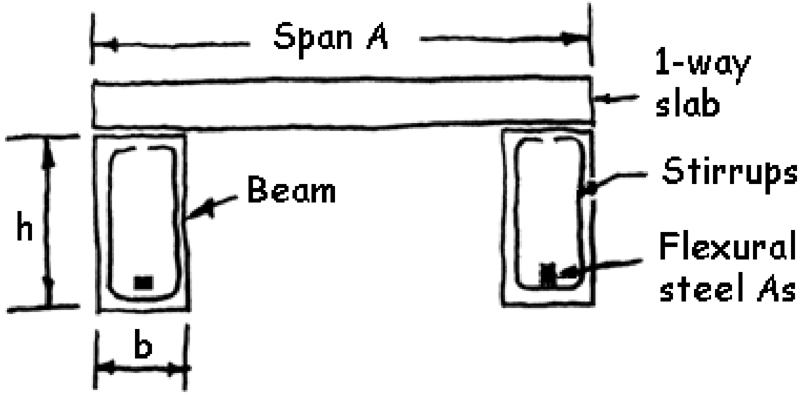

8. Numberof rebars used

Choose bars for As and check As min

Table A.4 Areas of Groups of StandardBars (in.^{2})

Bar No.	Number of Bars 2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

As req = 2.288 in² N=3

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9. Actual, final area of flexural steel used, As, used

As used = Ab * N= 1.000*3 = 3 in²

10. Minimum required area of steel, As,min

$$\text{(a)} \quad \frac{3\sqrt{f'_c}}{f_y} b_w d \quad = \; 3 * \frac{\sqrt{5500}}{60000} * 18 * 36.561 = 2.440 \text{ in} \quad \surd$$

$$\text{(b)} \quad \frac{200}{f_y} b_w d \quad = \; \frac{200}{60000} * 18 * 36.561 = 2.194 \text{ in}$$

As,min: **greater** of a and b

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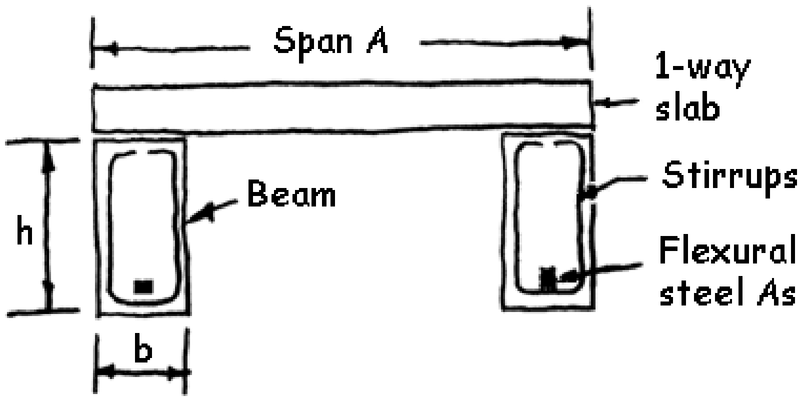


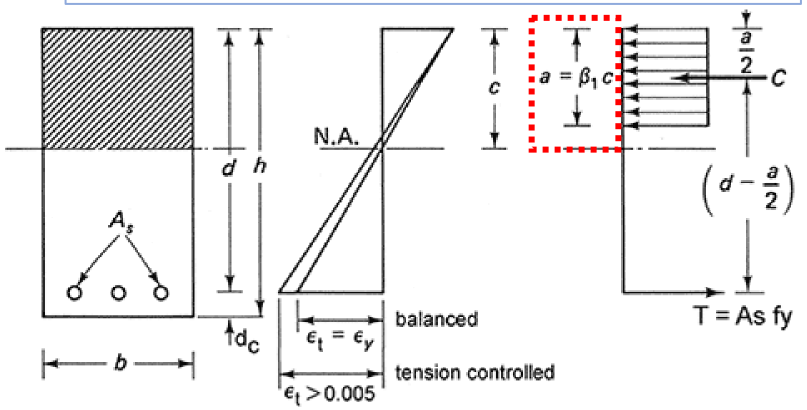
11. Depth of concrete stress block, a

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

As used

= 3 *60000/ (0.85*5500*18)
= 2.139 in

Check that $\epsilon_t \geq 0.005$ (tension controlled)

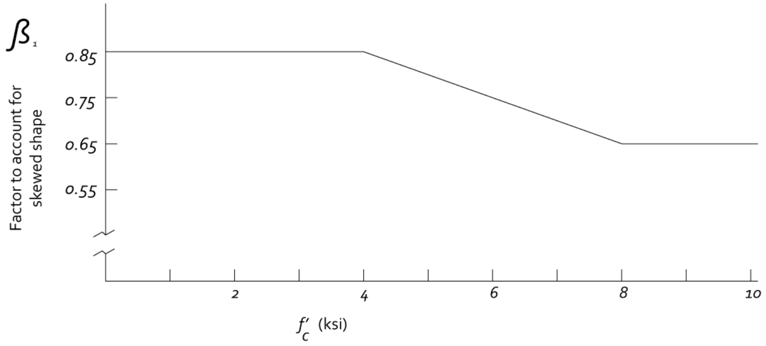


12. The factor beta_1

β_1 is a factor to account for the non-linear shape of the compression stress block.

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

5500



$$0.85 \geq \boxed{0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)} \geq 0.65$$

β1 = 0.775

13. Distance to Neutral Axis from top of beam, c

$$c = \frac{a}{\beta_1}$$ = 2.139 / 0.775 = 2.76 in

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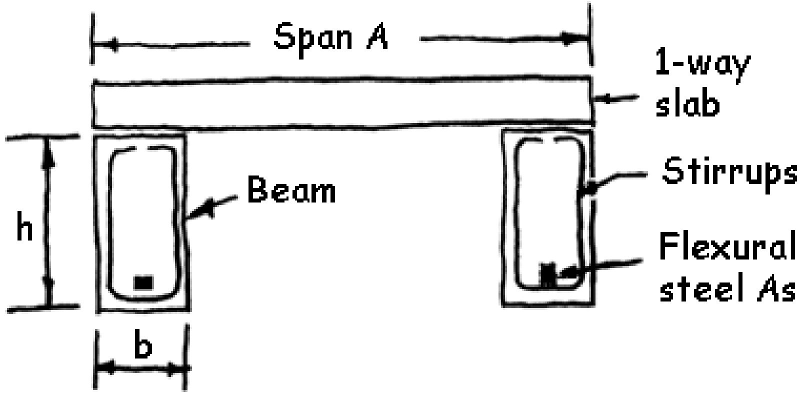
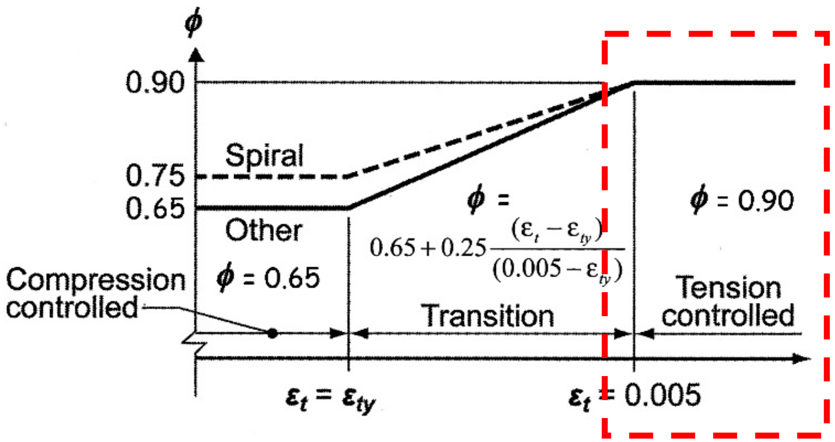


14. Strain in flexural steel, epsilon_t

Check that $\epsilon_t \geq 0.005$ (tension controlled)



$$\varepsilon_t = \frac{d-c}{c}0.003 \geq 0.005$$

= (36.561-2.76) / 2.76 * 0.003

= 0.037

15. Strength reduction factor, phi

$$\varepsilon_t = \frac{d-c}{c}0.003 \geq 0.005$$

ϕ = 0.9

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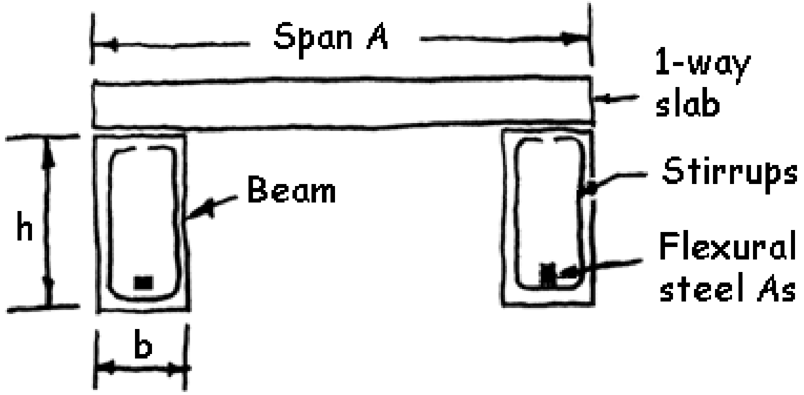


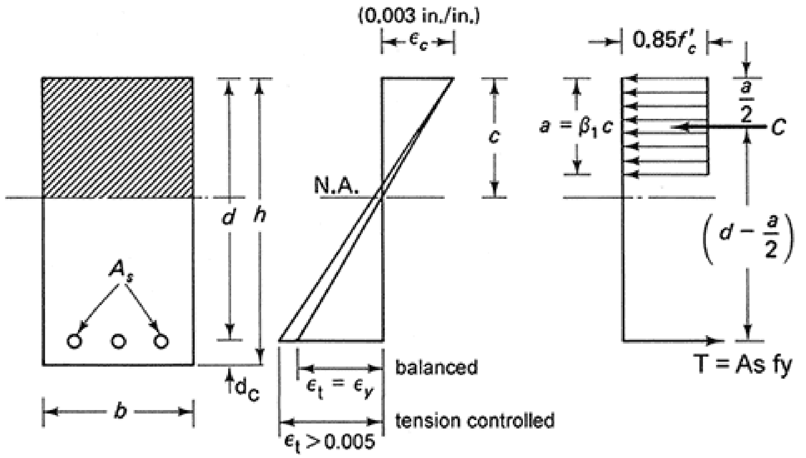
Check Mu≤ φMn

16. Tensile force in the flexural steel, T

T = fy* As = 60000*3/1000 = 180 k

17. Nominal bending moment, Mn

Mn = fy* As *(d –a/2)
= 180* (36.561-2.139/2)
= 6388.47 k-in



18. Factored bending resistance, phi Mn

ϕ *Mn = 0.9 * 6388.47/12 = 479.14 k-ft > Mu = 368.044 k-ft

Final Report

Tower Project Score Sheet

PRELIMINARY REPORT (re-submit with final report)	**40**	
TESTING	**60**	
Tower weight ≤ 4oz (15 pts); height = 48" (5 pts); holds ≥ 50 lbs (5 pts) Correct Materials (5 pts) (scaled if doesn't meet requirements)	30	
Efficiency (4/weight OZ)+(load LBS/50)+(load LBS/weight OZ)x1.5 (scaled based on class rank)	30	
FINAL REPORT REQUIREMENTS	**150**	
Preliminary Design Development	**20**	
How cross-sectional design of preliminary tower was chosen	4	
How elevation of preliminary tower was developed (e.g. bracing, taper, etc.)	4	
Why/how cross-section was or was not adjusted from preliminary report	4	
Why/how elevation of tower was or was not adjusted from preliminary report	4	
Discussion of how basic principles of columns supported these decisions	4	
Revised/Tested Tower Design Analysis [SHOW WORK AND UNITS!]	**50**	
Calculated/modeled axial forces and derivation of required member cross-sectional areas from axial forces (consider both crushing and buckling)	10	
Estimated weight calculation using actual member sizes used – include weight from members, glue, and gussets, etc.	7	
Member properties table: A, r, L, slenderness ratio (L/r), utilization ratio (actual load / allowable load)	7	
Indicate critical member (largest utilization ratio)	8	
Tower stability (as a whole) - buckling calculation	8	
Prediction of capacity of tower and mode of failure	10	
Illustration of Final/Tested Design	**20**	
Cross-section and elevations(s) of tower	5	
Perspective(s) or isometric of tower (no screenshots!)	5	
Overall dimensions labeled (height, width, etc.) with units	5	
Member sizes labeled (cross-sectional area, length of vertical members and cross-bracing) with units	5	
Testing Results	**30**	
Final weight and height of tower	6	
Tested capacity of tower	6	
Observations of testing (loading, any buckling observed, etc.)	6	
Description of mode of failure	6	
Images of failure	6	
Post-Testing Analysis	**30**	
Comparison of testing results with predicted capacity and modes of failure	10	
Discussion of discrepancies between results	10	
Suggested improvements for future designs with reasoning discussed	10	
FINAL GRADE	**250**	

(Note: re-submit your Preliminary Design Proposal with your Final Report.)

Due Apr 12th

LAB - Reinforcement Placement

Description

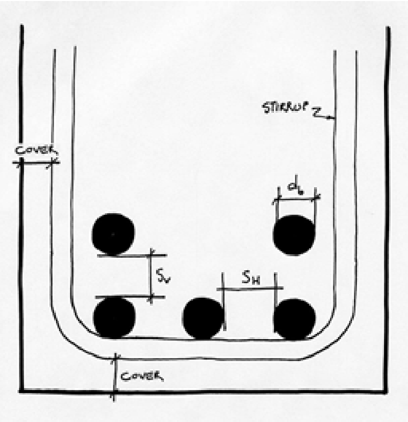
This project produces a graphic representation of the reinforcing layout of a concrete beam.

Goals

To determine bar diameters and horizontal spacing.
To find the placement and dimensions of a shear stirrup.
To establish proper cover for reinforcement.
To draw all beam elements in the proper scale and location.

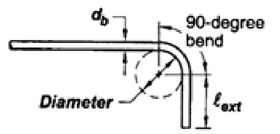
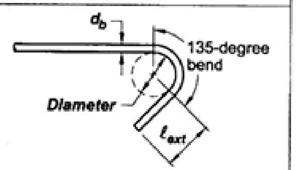
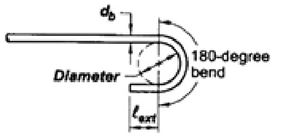
Procedure

1. For the example beam worked in class, determine the required spacing, s_v and s_h, for the bar size used.
2. For the given stirrup size determine the bend radius for a 90° bend.
3. Make a sketch showing the proper locations of bars and the stirrup including cover.
4. Draw and dimension the depth of the stress block, “a” and the distance to the N.A. from the top of the beam, “c”.
5. Dimension and label “d” and “d_c”.

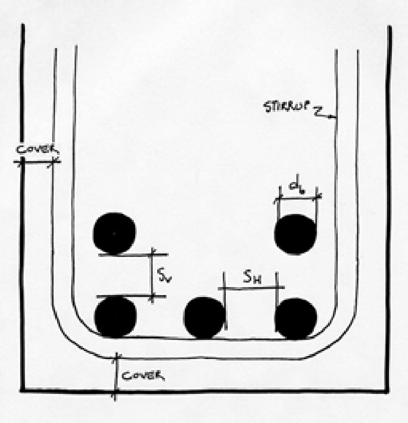


Horizontal Spacing in Beams
ACI 25.2.1
- 1 inch
- db
- 4/3 max aggregate

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

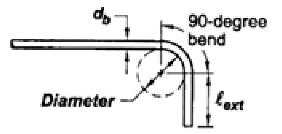
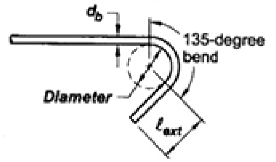
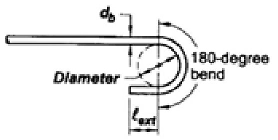
Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

[1] A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.



Horizontal Spacing in Beams
ACI 25.2.1
- 1 inch
- db
- 4/3 max aggregate

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

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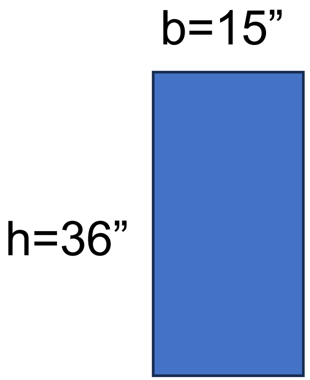


Flexural bar #9
db=1.128"
5 Flexural bars

Stirrup bar #4
db=0.5"

Aggregate = ¾"

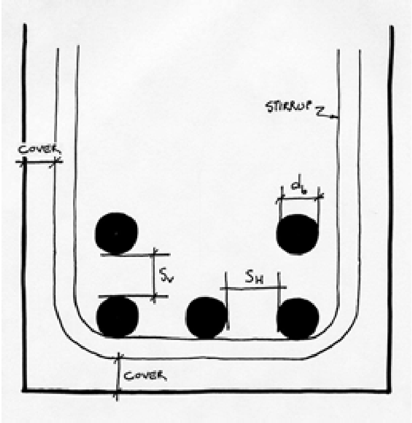
Cover 1.5"

- For the example beam, determine the required spacing, Sv and Sh, for the bar size used.

- **horizontal spacing in beams**
  ACI 25.2.1
  - 1 inch
  - d_b (Flexural bar)
  - 4/3 max aggregate

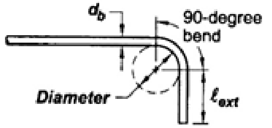
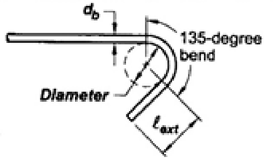
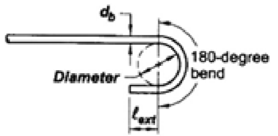
Sh greater than

- **vertical spacing in beams**
  ACI 25.2.2
  - 1 inch



Horizontal Spacing in Beams
ACI 25.2.1
- 1 inch
- db
- 4/3 max aggregate

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

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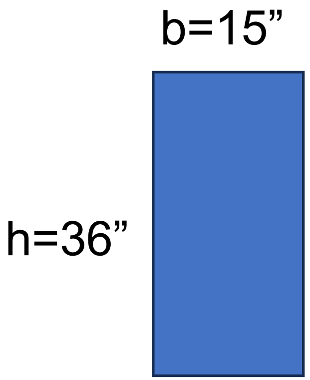


Flexural bar #9
db=1.128”
5 Flexural bars

Stirrup bar #4
db=0.5”

Aggregate = ¾”

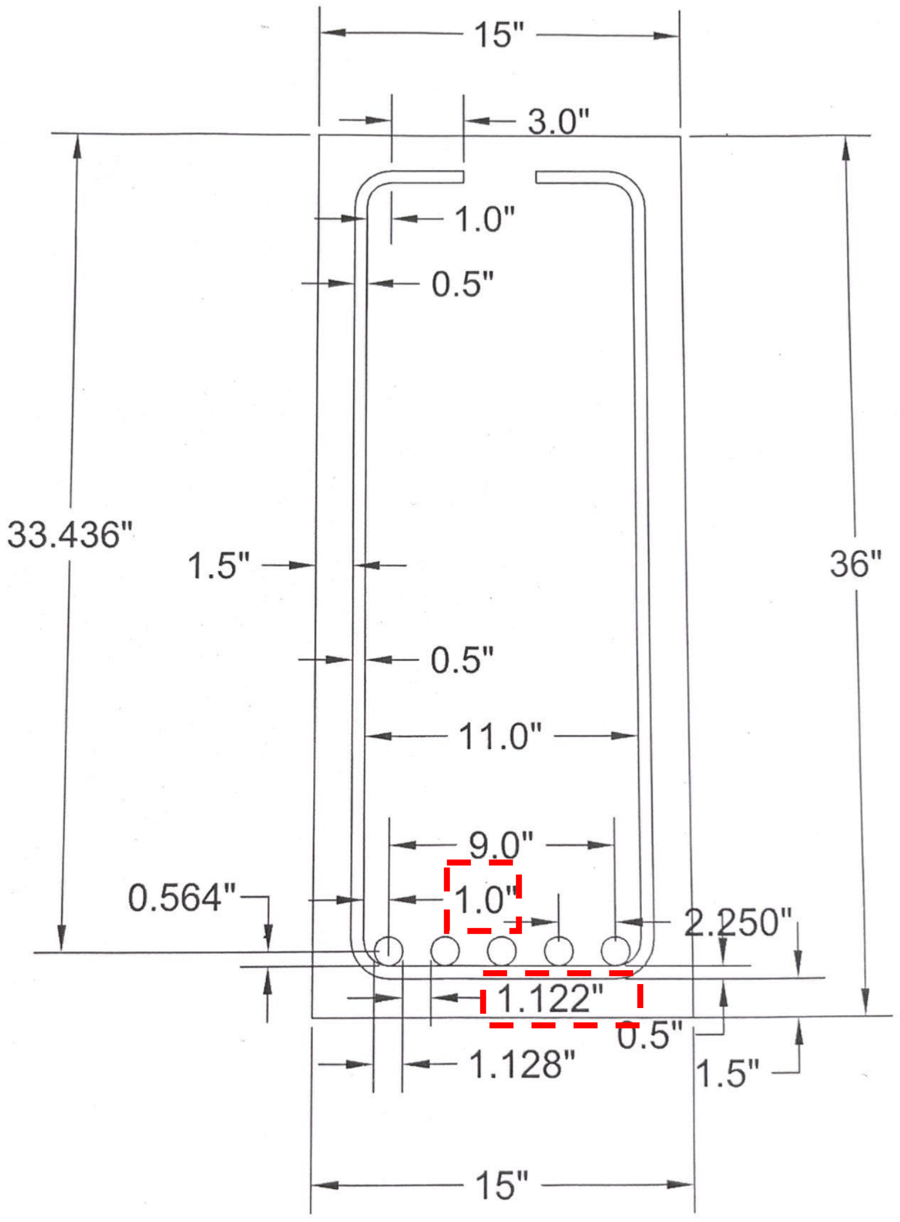
Cover 1.5”

- For the given stirrup size determine the bend radius for a 90° bend.

Minimum inside bend diameter = 4db

Minimum inside bend radius?

(Flexural bars can’t be set inside the radius)

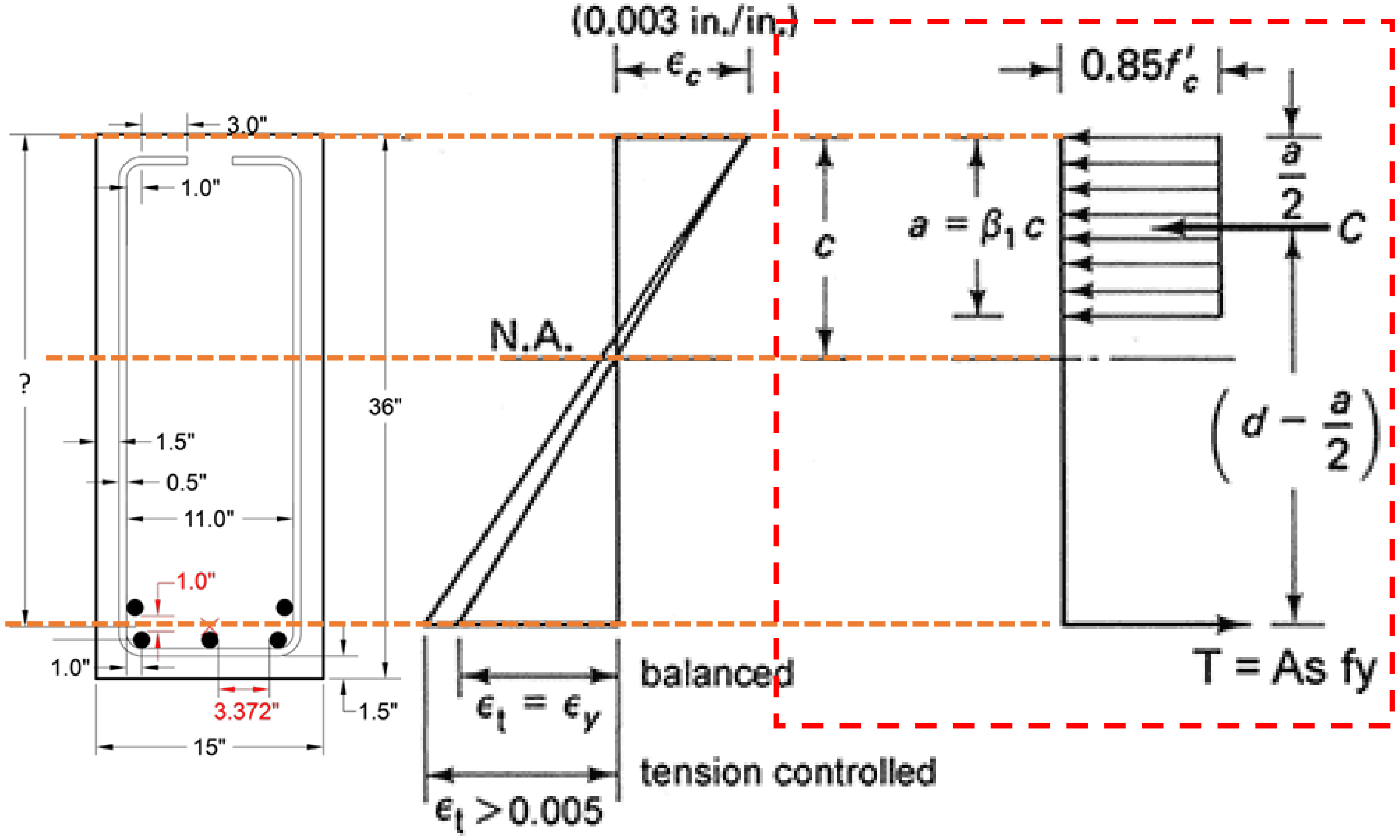


- Make a sketch showing the proper locations of bars and the stirrup including cover.

Determine put 5 bars in one layer or two layers.

Meet the spacing requirement.

One layer does not pass!



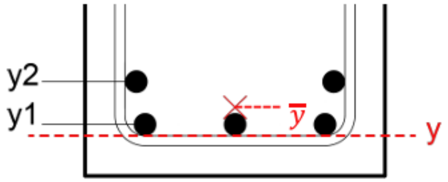
- Make a sketch showing the proper locations of bars and the stirrup including cover.

Put 5 bars in two layers!

- Draw and dimension the depth of the stress block, "a" and the distance to the N.A. from the top of the beam, "c"

a =7.843" c =9.227"

- Dimension and label "d" and "dc".



y1= db/2

y2= db + Sv + db/2

the centroid $\bar{y} = \frac{\sum A \times d_x}{\sum A}$ = (3y1+2y2)/5

dc= cover + stirrup bar diameter + $\bar{y}$

d= h-dc

Any Questions?

yifanma@umich.edu

Thank You!