Recitation 7

Three Moment Theorem

Homework problem

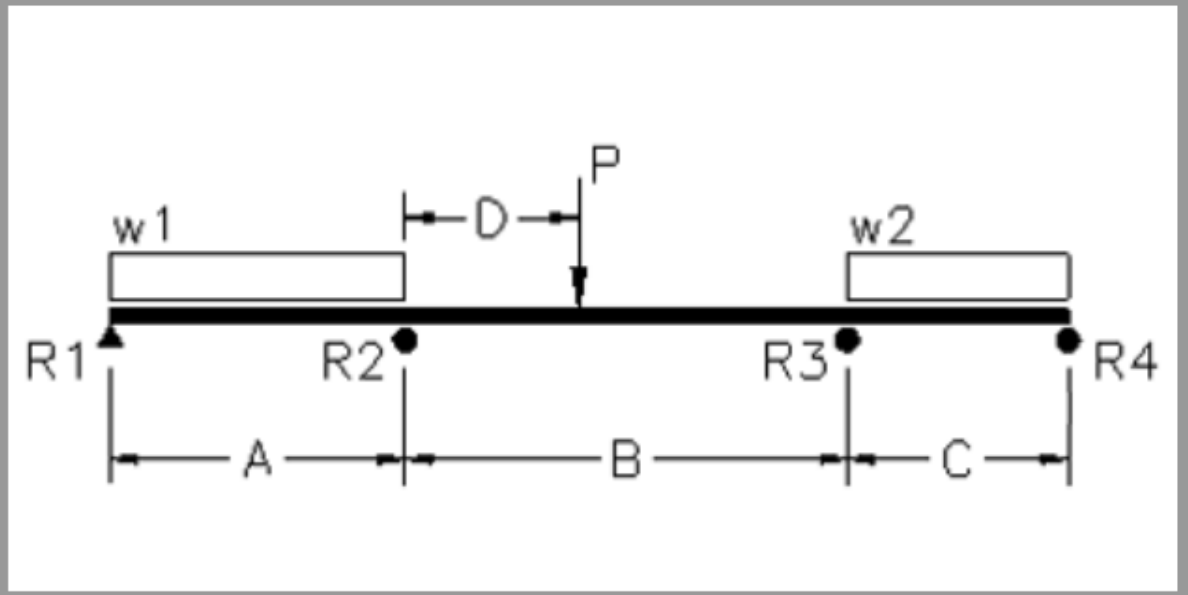
Three moment theorem

7. Three Moment Theorem

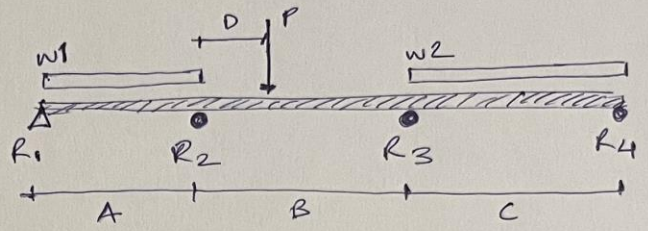
Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

DATASET: 1 -2- -3-

Span A	17 FT
Span B	30 FT
Span C	10 FT
Uniform load on span A, w1	5 KLF
Uniform load on span C, w2	4 KLF
Point load on span b, P	44 K
Distance to point load P from R2, D	10 FT



7) Three Moment theorem :-



To determine all reactions and support moments for the given continuous beam.

SPAN A = 17 ft
Span B = 30 ft
Span C = 10 ft

uniform load on SPAN A, w1 = 5 KLF
uniform load on span B, w2 = 4 KLF
point load on span B, P = 44 K
Distance to point load 'P' from R_2 = 10 ft.

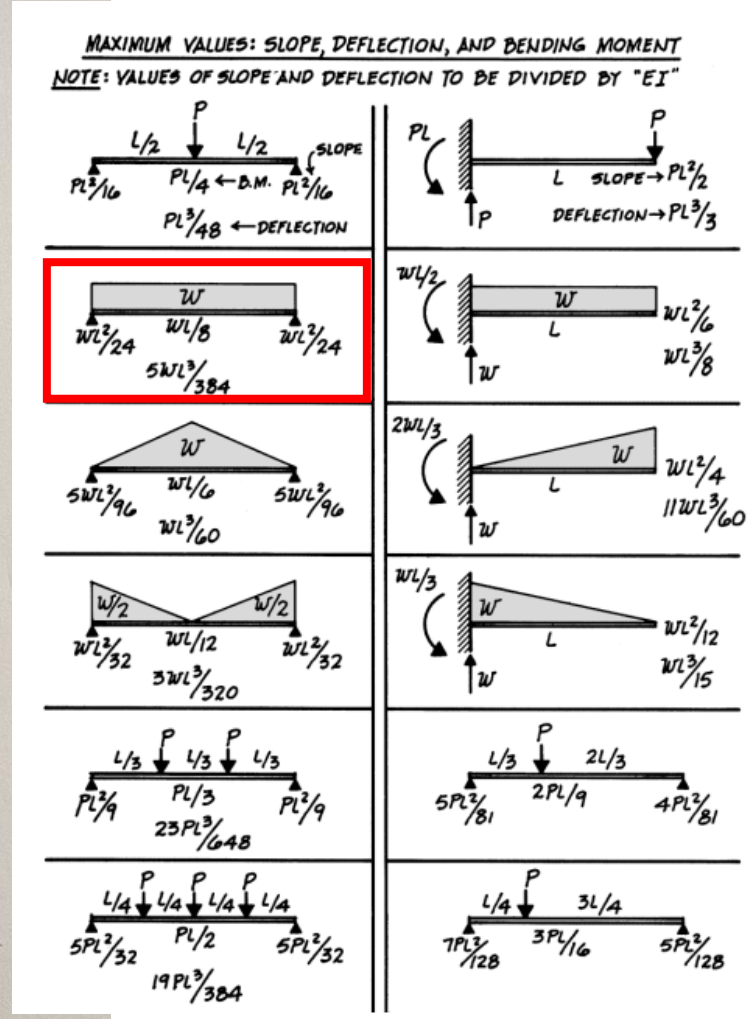
Q1) Moment at support R_1, M_1 :-

M_1 = 0 K-FT (pinned end reactions always have zero moment).

Q2) EI Theta on left side of R_2 :-

$$EI\theta = \frac{WL^2}{24}$$

$$= \frac{(5 \times 17) \times (17)^2}{24} = 1023.5416$$



Q3) EI Theta on right side of R_2 :-

$$EI\theta_4 = \frac{5PL^2}{81}$$

$$= \frac{5 \times 44 \times 30^2}{81}$$

$$= 2444.44$$

Q4) Moment at support R_4, M_4 :-

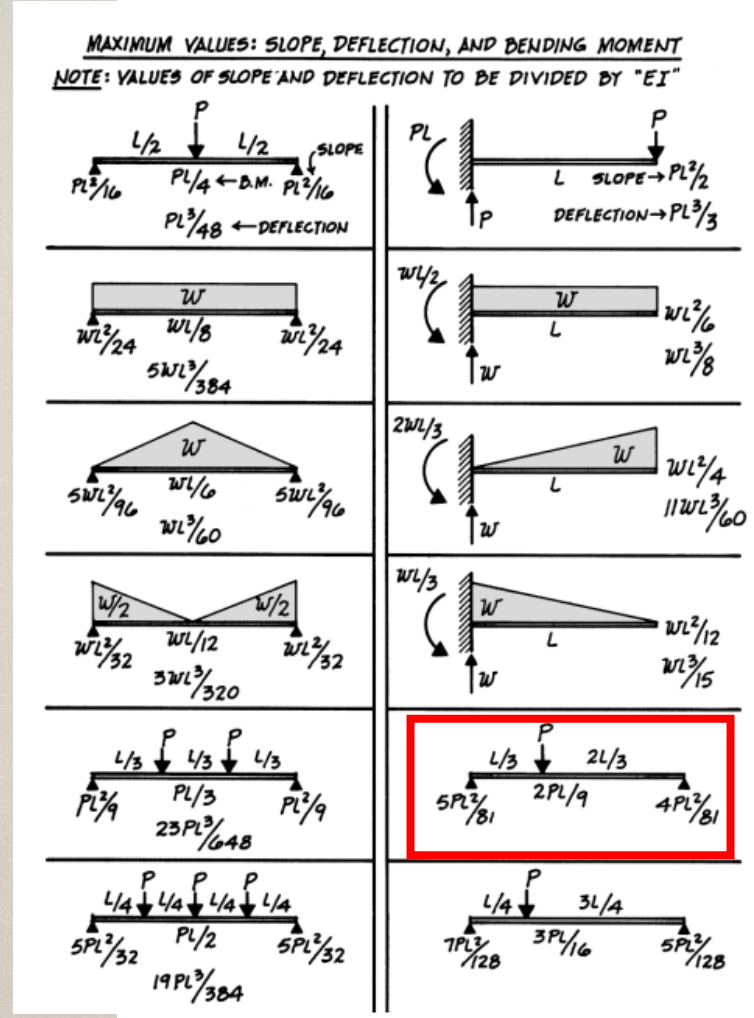
$M_4 = 0$ k-ft (pinned end reactions always have a zero moment).

Q5) EI Theta on left side of R_3 :-

$$EI\theta = \frac{4PL^2}{81} = \frac{4 \times 44 \times 30^2}{81}$$

$$= \frac{4 \times 44 \times 900}{81}$$

$$= 1955.55$$



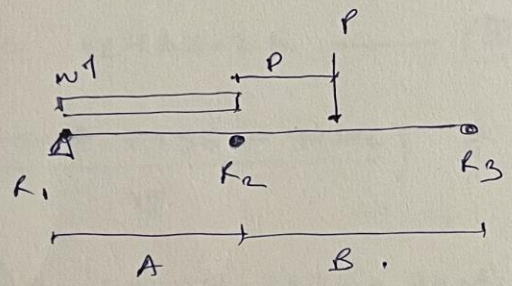
Q6) $EI\theta$ on right side of K_3 :-

$$EI\theta = \frac{wl^2}{24}$$

$$= \frac{4 \times (10) \times (10)^2}{24}$$

$$= 166.66$$

Q7) Moment at support K_2, M_2 :-



$$M_1(L_1) + 2M_2(L_1+L_2) + M_3(L_2) = 6(EI\theta_1 + EI\theta_2)$$

$$0(17) + 2M_2(17+30) + M_3(30) = 6(1023.5416 + 2444.44).$$

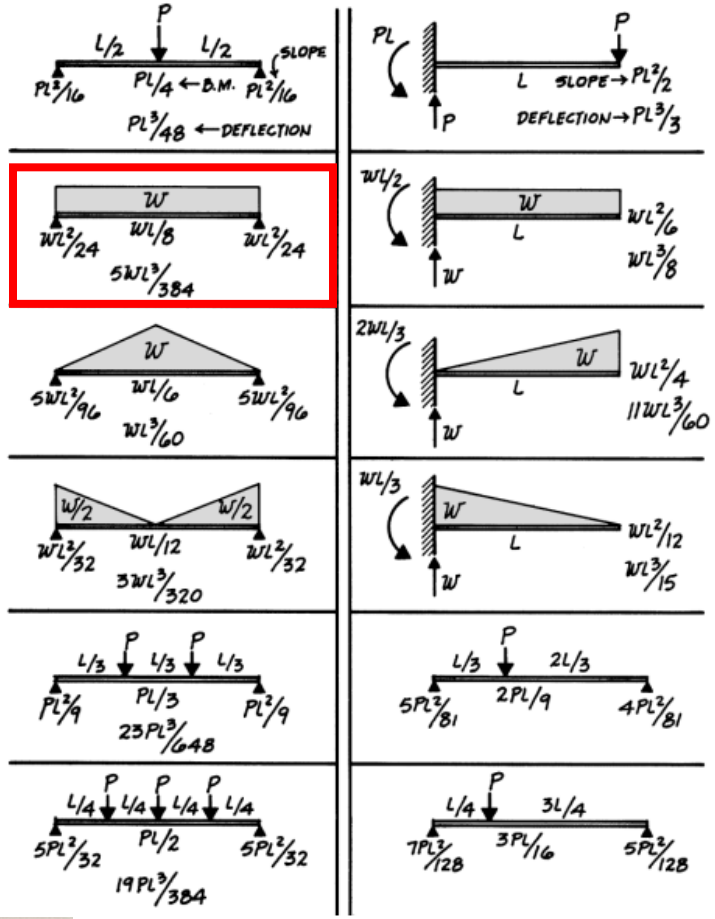
$$0 + 94M_2 + 30M_3 = 20807.8896$$

$$94M_2 + 30M_3 = 20807.8896 \quad \text{——} (1)$$

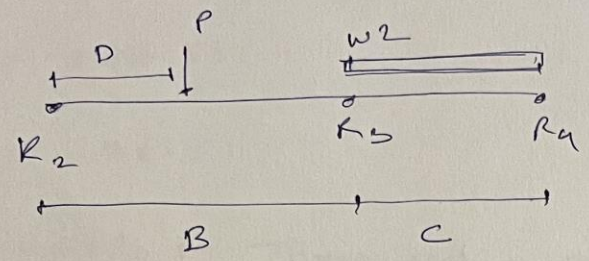
$$M_3 = \frac{20807.8896 - 94M_2}{30}$$

MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT

NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"



(08) Moment at support R_3, M_3 :-



$$M_2(L_1) + 2M_3(L_1+L_2) + M_4(L_2) = 6(EI\theta_1 + EI\theta_2).$$

$$M_2(30) + 2M_3(40) + 0(10) = 6(1955.55 + 166.66)$$

$$30M_2 + 80M_3 = 12733.26 \quad \text{——} \quad (2)$$

from (1) and (2)

$$30M_2 + 8\phi\left(\frac{20807.8896 - 94M_2}{3\phi}\right) = 12733.26.$$

$$90M_2 + 8(20807.8896 - 94M_2) = 38,199.78$$

$$90M_2 + 166,463.1168 - 752M_2 = 38,199.78$$

$$-662M_2 = -128\,263.3368$$

$$M_2 = 193.75$$

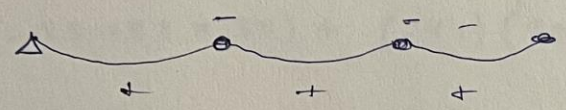
$$M_2 = -193.75$$

$$M_3 = \frac{20807.8896 - 94(193.75)}{30}$$

$$M_3 = +86.51$$

But $M_3 = -86.51$

(*) Remember of M_2 and M_3 :- These values are in tension. Refer to the graphic below. Because of that we have to write final value as negative.



g) Support reaction, R_1 :-

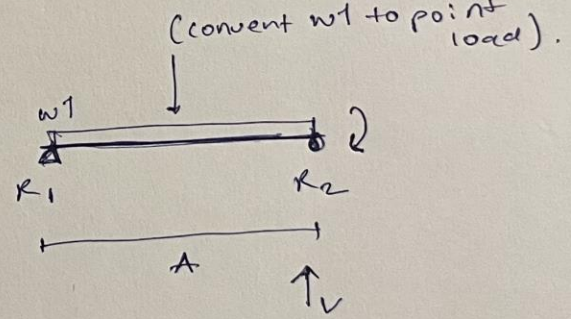
$$M = R_1(\text{Span A}) + (-k1)\left(\frac{\text{Span A}}{2}\right) + M_{R_2}$$

$$0 = R_1(17) + (-5)\times\left(\frac{17}{2}\right) + (-193.75)$$

$$0 = 17R_1 - 722.5 + 193.75$$

$$17R_1 = 528.75$$

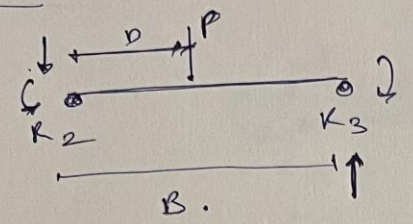
$$\boxed{R_1 = 31.102\ K}$$



$$F_V = 0 = 31.102\ K - 5\times 17 + V$$

$$V = 53.898.$$

(10) Support Reaction, R_2 :-



$V = 53.898$

Point load P

$M = 0 = R_2(\text{span B}) + (-53.898)(\text{span B}) + (-44)(\text{span B} - 10) + (-193.75) + 86.51$

$= R_2(30) + (-53.898 \times 30) + (-44)(30-10) + (-107.24).$

$30 R_2 = 1616.94 + 880 + 107.24$

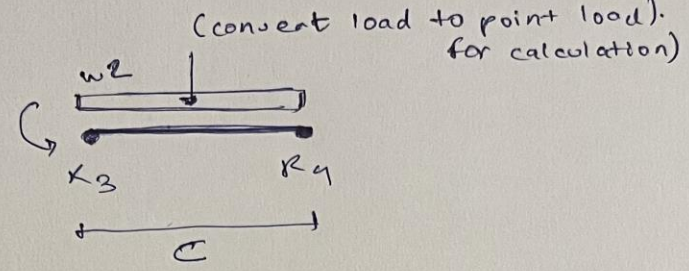
$30 R_2 = 2604.18$

$R_2 = 86.806 \text{ K}$

$F_V = 0 = -53.898 + 86.806 + V - 44$

$V = 11.092$

Q11) Support Reaction, R_3 :-



$V = 11.092$

(M3)

$M = 0 = -86.51 + (-11.092)(10) + R_3(\text{Span c}) + (-W_2)\left(\frac{\text{Span c}}{2}\right) + 0$

$= -86.51 + (-110.92) + R_3(10) + (-4 \times 10)\left(\frac{10}{2}\right) + 0$

$= -86.51 + (-110.92) + 10R_3 - 200$

$10R_3 = 200 + 86.51 + 110.92$

$R_3 = 39.743 \text{ K}$

Q12) Support Reaction, R_4 :-

$F = 0 = R_1 + R_2 + R_3 + R_4 - W1 - P - W2$

$0 = 31.102 + 86.06 + 39.743 + R_4 - 85 - 44 - 40$

$R_4 = +12.095 \text{ K}$

Thankyou !!!