Structure II Recitation 3/8

Three Moment Theorem

Before we start ...

Today's Tasks:

Homework Example (Three Moment Theorem) (12 Questions)

Lab Session (Continuous Beam)

Reminder:

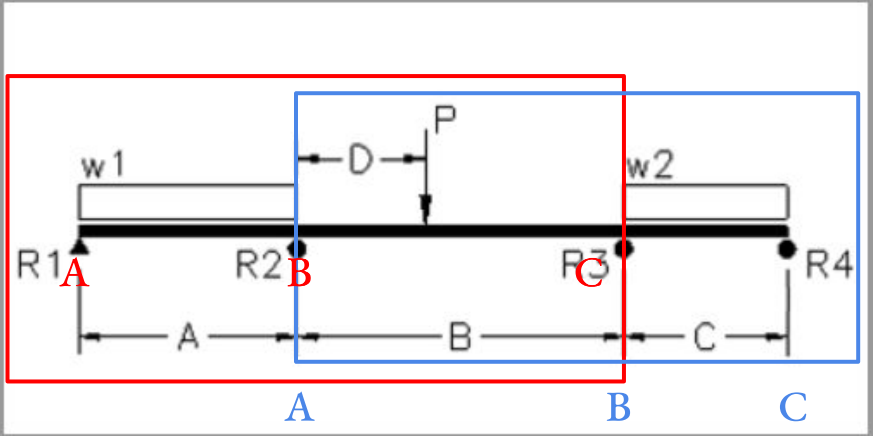
Preliminary Report Resubmission: 3/11, Monday (Email Submission)

Tower Testing: 3/20, Wednesday

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

DATASET: 1 -2- -3-

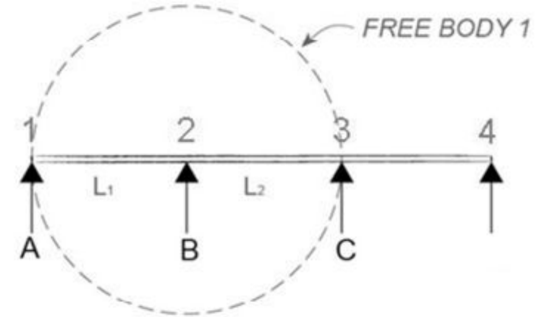
Span A	16 FT
Span B	24 FT
Span C	15 FT
Uniform load on span A, w1	7 KLF
Uniform load on span C, w2	3 KLF
Point load on span b, P	68 K
Distance to point load P from R2, D	18 FT



$$\boxed{M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]}$$

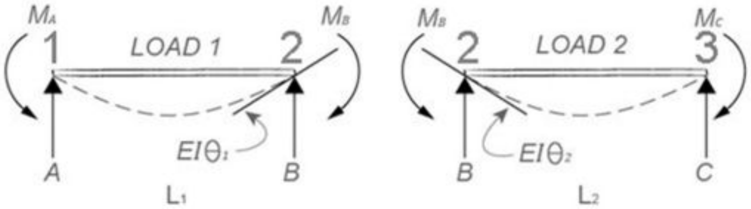
Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric



Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.

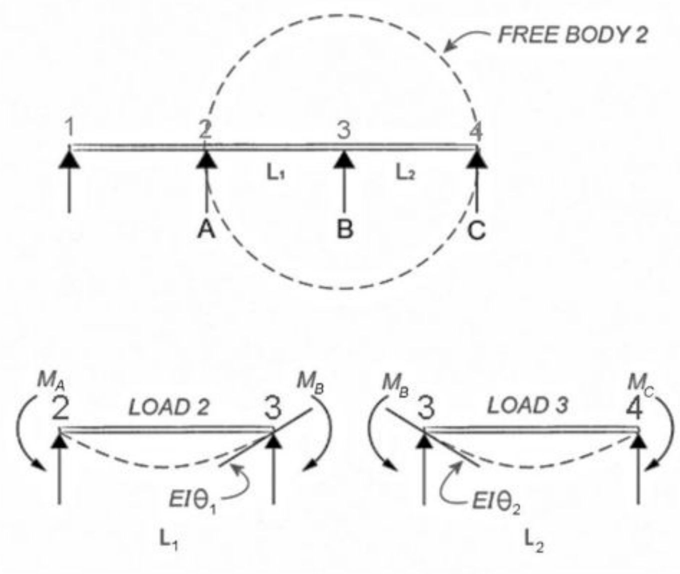


$$\boxed{M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]}$$

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.



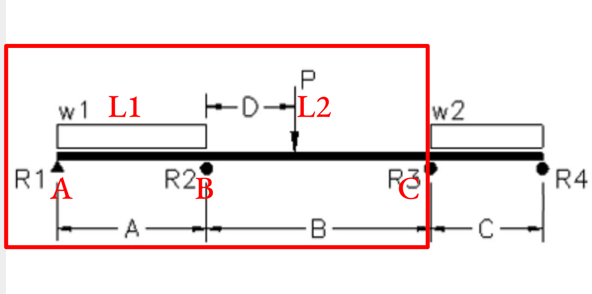
$$\boxed{M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]}$$

Q1: Moment at support R1 (M1)

Since its end support, **$M1 = 0$**

Continuous Beam

- Exterior end moments = 0
- Interior support moments are usually negative
- Mid-span moments are usually positive
- End + Mid = $0.125wL^2$



Q2: EIθ on Left Side of R2

Based on the given data:

$M_A = 0$ (From Q1)

$L1$ = Span A = 16 ft

$L2$ = Span B = 24 ft

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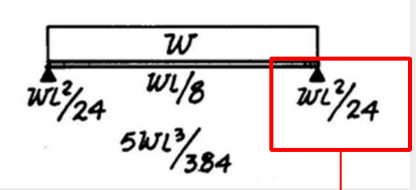
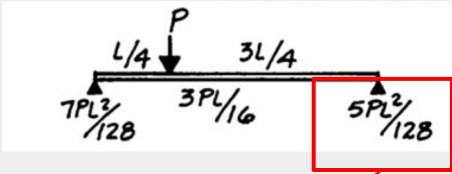
Use the slope diagram to find the EIθ (Course Slides P.8, 3/4)

$EI\theta(\text{Left}) = W \times L^2 / 24 = (7 \times 16) \times 16^2 / 24 =$ **1194.667**

Q3: EIθ on Right Side of R2

$EI\theta(\text{Right}) = 5 \times P \times L^2 / 128 = 5 \times 68 \times 24^2 / 128 =$ **1530**





$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Q4: Moment at support R4 (M4)

Since its end support, **<u>M4 =0</u>**

Q5: EIθ on Left Side of R3

Based on the given data:

$M_C = 0$ (From Q4)

L1 = Span B = 24 ft

L2 = Span C = 15 ft

Use the slope diagram to find the EIθ:

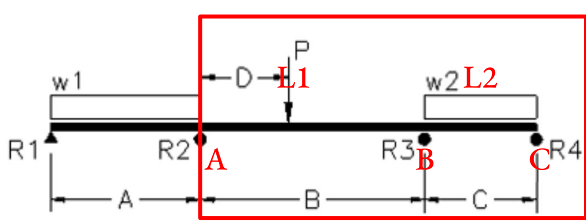
$EI\theta(\text{Left}) = 7 \times P \times L^2 / 128 = 7 \times 68 \times 24^2 / 128 = \underline{2142}$

Q6: EIθ on Right Side of R3

$EI\theta(\text{Right}) = W \times L^2 / 24 = (3 \times 15) \times 15^2 / 24 = \underline{421.875}$

Continuous Beam

- Exterior end moments = 0
- Interior support moments are usually negative
- Mid-span moments are usually positive
- End + Mid = $0.125wL^2$



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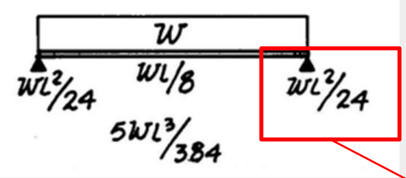

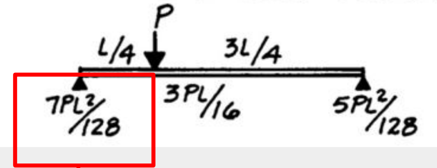

$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Q7 & Q8 (M_2 & M_3)

Plug in the values into the Three Moment Theorem:

<u>First Formula:</u>

$M_A = 0$, $L1 = 16$ ft , $L2 = 24$ ft, $EI\theta_1 = 1194.667$, $EI\theta_2 = 1530$, $\mathbf{M_B = M_2}$, $\mathbf{M_C = M_3}$

$0 \times 16 + 2 \times M_2 \times (16 + 24) + M_3 \times 24 = 6 \times (1194.667 + 1530)$

$\mathbf{80 \times M_2 + 24 \times M_3 = 16348.002}$

<u>Second Formula:</u>

$M_C = 0$, $L1 = 24$ ft , $L2 = 15$ ft, $EI\theta_1 = 2142$, $EI\theta_2 = 421.875$, $\mathbf{M_A = M_2}$, $\mathbf{M_B = M_3}$

$M_2 \times 24 + 2 \times M_3 \times (24 + 15) + 0 \times 15 = 6 \times (2142 + 421.875)$

$\mathbf{24 \times M_2 + 78 \times M_3 = 15383.25}$

Both tension on top:

<u>**M2 = -148.00635 k-ft , M3 = -159.948 k-ft**</u>

Sign convention

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Span B	24 FT
Span C	15 FT
Uniform load on span A, w1	7 KLF
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Point load on span b, P	68 K
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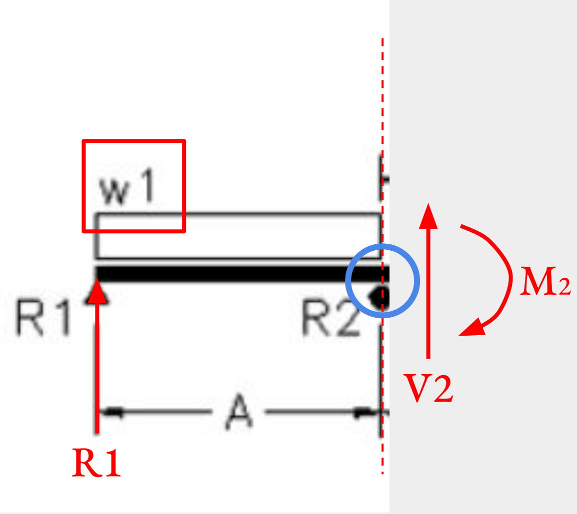
Q9 Support Reaction R1

Draw Free Body Diagram



ΣM at R2 = 0:

$\underline{R1 \times \text{Span A} + M_2 - (w1 \times \text{Span A}) \times (\text{Span A} / 2) = 0}$

$R1 \times 16 + 148.00635 - (7 \times 16) \times (16 / 2) = 0$

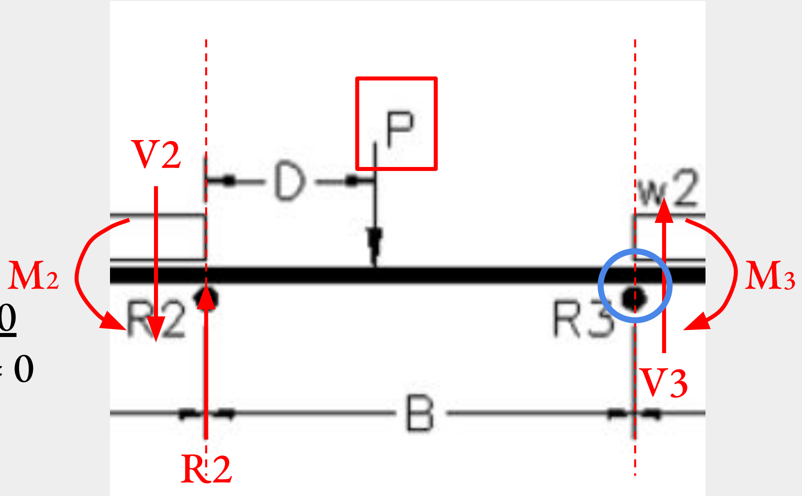
$R1 = (112 \times 8 - 148.00635) / 16 =$ **46.75 k**

$R1 - (w1 \times L) + V2 = 0$

$V2 = (7 \times 16) - 46.75 = 65.25$

Q10: Support Reaction R2

Draw Free Body Diagram



ΣM at R3 = 0:

$\underline{R2 \times \text{Span B} + M3 - P \times (\text{Span B} - D) - M2 - V2 \times (\text{Span B}) = 0}$

$R2 \times 24 + 159.948 - 68 \times (24 - 18) - 148.00635 - 65.25 \times 24 = 0$

$R2 = (408 + 148.00635 + 1566 - 159.948) / 24 =$ **81.75 k**

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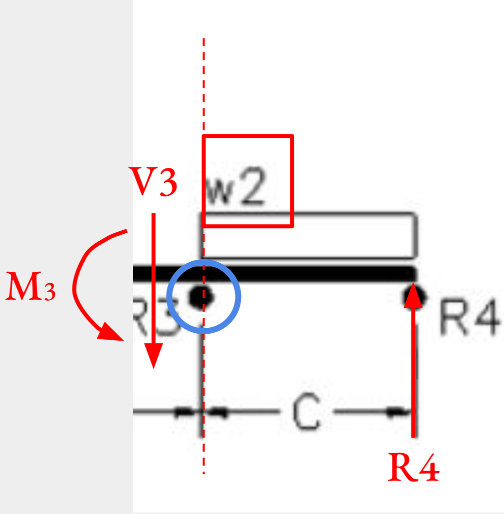
Span B	24 FT
Span C	15 FT
Uniform load on span A, w1	7 KLF
Uniform load on span C, w2	3 KLF
Point load on span b, P	68 K
Distance to point load P from R2, D	18 FT

Q12: Support Reaction R4

Draw Free Body Diagram



ΣM at R3 = 0:

<u>(w2 x Span C) x (Span C / 2) - M3 - R4 x Span C= 0</u>

(3 x 15) x (15 / 2) - 159.948 - R4 x 15 = 0

R4 = (337.5 -159.948) / 15 = <u>**11.837 k**</u>

Q11: Support Reaction R3

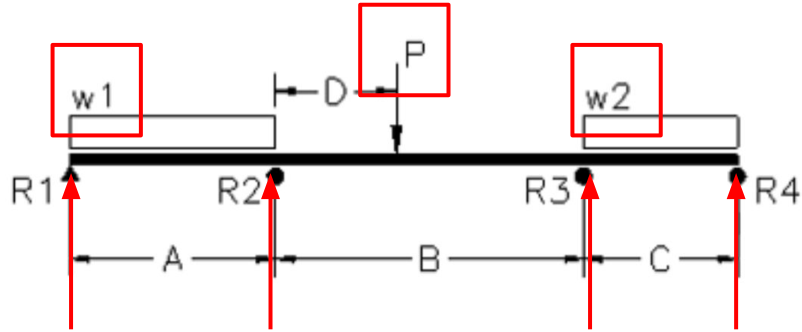
Look at the whole beam

$\Sigma Fy = 0$:

<u>(R1 + R2 + R3 + R4) = (w1 x Span A) + P + (w2 x Span C)</u>

(46.75 + 81.75 + R3 + 11.837) = 7 x 16 + 68 + 3 x 15

R3 = <u>**84.663 k**</u>

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Continuous Beams

Description

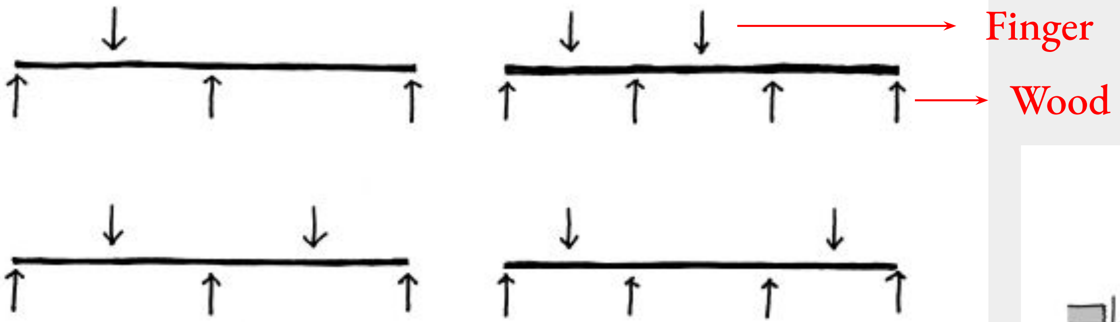
This project uses observation to understand the behavior of beams continuous over multiple supports.

Goals

To observe the behavior of continuous beams under different loadings
To estimate locations of contraflexure and effective lengths
To determine areas of positive and negative moment based on curvature

Procedure

1. Using the 24 inch stick, position the supports and loads (with your finger) as shown in the diagrams below. Hold the beam down on the reactions if it lifts up.
2. For each case observe and draw the elastic curve.
3. Label + and – curvature (moment) and points of contraflexure.
4. Estimate the effective lengths, ℓ_e, across the beam. (between points of M=0)



Lab Session:

Step 1: Draw the elastic curve
Step 2: Label curvature with + or –
Step 3: Label the points of contraflexure
Step 4: Estimate the effective lengths
(Total L is 24 inch)

Sign convention