

Arch324

STRUCTURES II

Winter 2024
Recitation

FACULTY: Prof. Peter von Bülow
GSI: Mohsen Vatandoost

Arch324: STRUCTURES II

Welcome to Recitation session 03/08

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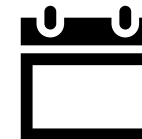
hours:

Fri: 11:30 – 14:30

Mon, Wed: 11:00 - 12:00

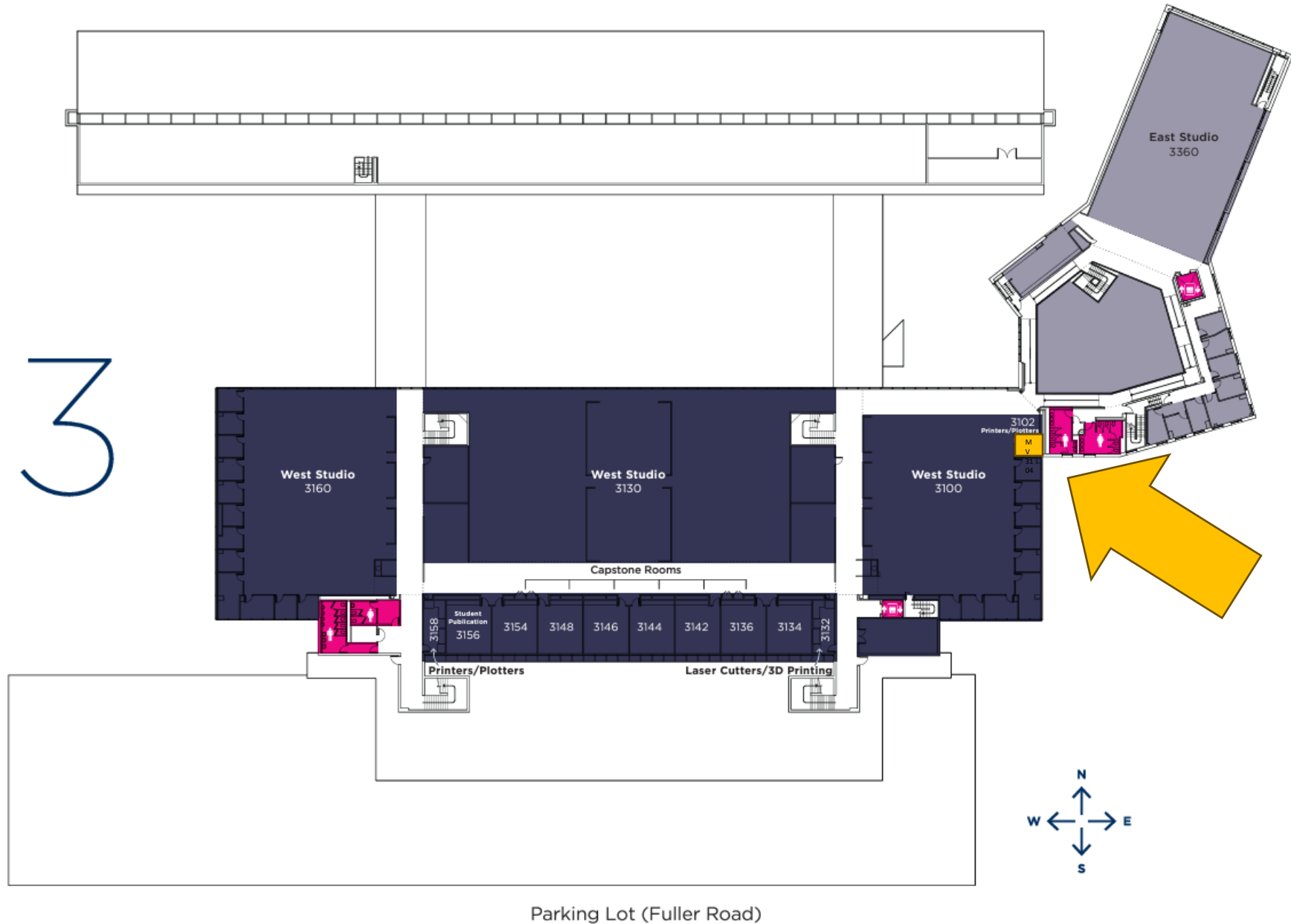
walk-ins welcome!

Please feel free to ask questions.



[Click here to make an appointment](#)

Where can you find me?



Arch324: STRUCTURES II

Welcome to Recitation session 03/08

Outline:

- Quick **Recap** of the week
- Provide the solution for the assignment (**Homework 7**)
- Answering student's questions
- Lab: **Continuous Beams**
- **Tower Project:** Preliminary report (revision) + Test date: **March 20**

Please feel free to ask questions.

Recap of the week

Continuous beams

Methods for solving internal forces in Continuous beams:

- Deflection Method
- Slope Method
- Three-Moment Theorem



two spans - simply supported



two spans - continuous

Statically indeterminate:

- Cannot be solved by the three equations of statics alone
- Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

Recap of the week

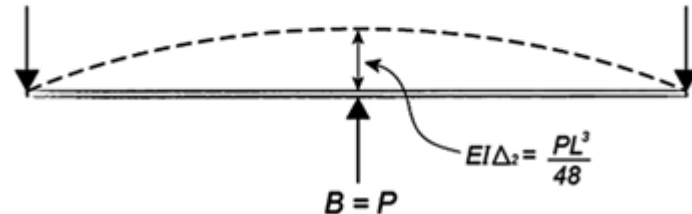
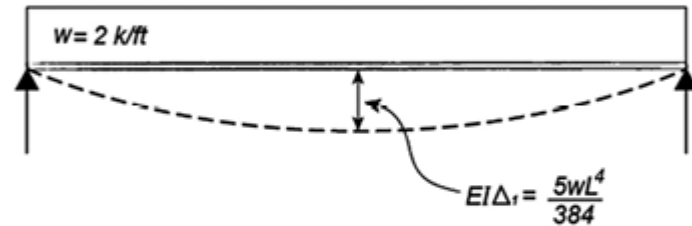
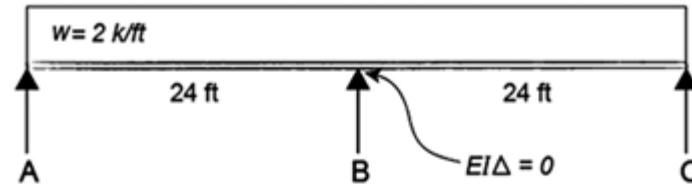
Continuous beams

Deflection Method

- Two continuous, symmetric spans
- Symmetric Load

Procedure:

1. Remove the center support, and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.



$$EI\Delta_1 + EI\Delta_2 = 0$$

Recap of the week

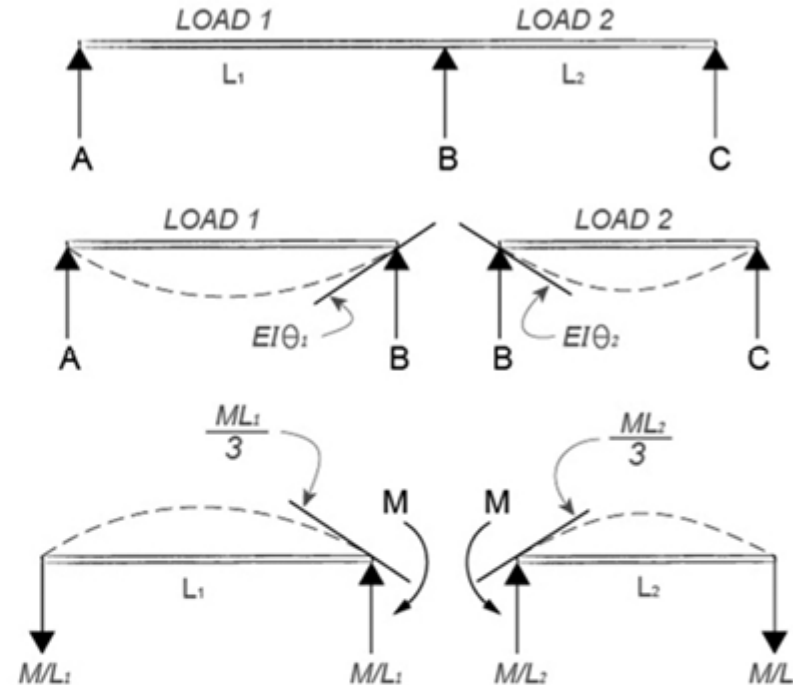
Continuous beams

Slope Method

- Two continuous spans
- Non-symmetric loads and spans

Procedure:

1. Break the beam into two halves at the interior support, and calculate the interior slopes of the two simple spans.
2. Use the Slope Equation to solve for the negative interior moment.
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.
5. Draw the shear and moment diagrams as usual.



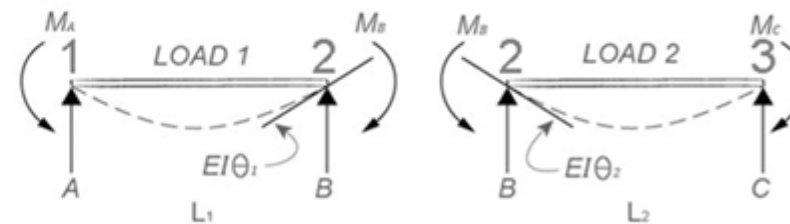
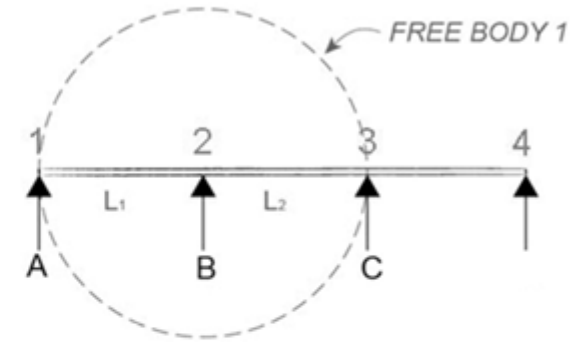
$$M = \frac{3}{L_1 + L_2} [EI\Theta_1 + EI\Theta_2]$$

Recap of the week

Continuous beams

Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric



$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Recap of the week

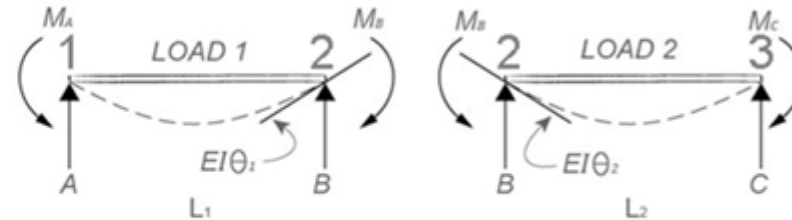
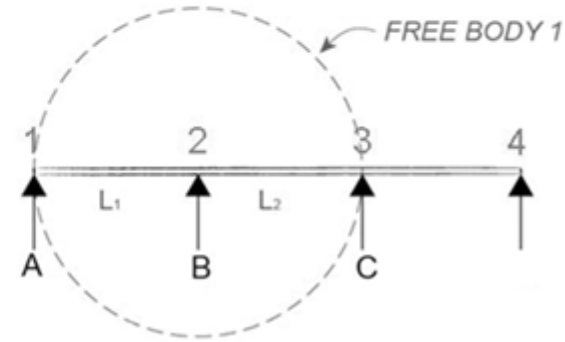
Continuous beams

Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric

Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L_1 and L_2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.



$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

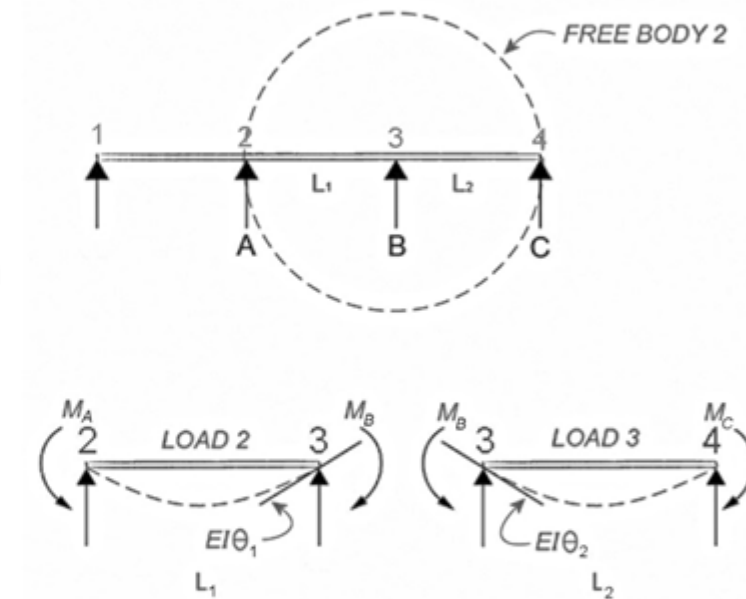
Recap of the week

Continuous beams

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.



$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

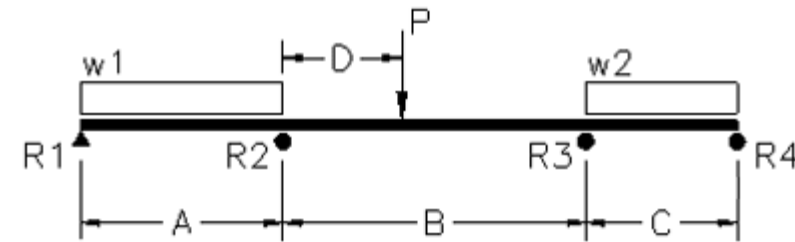
Provide the solution for the assignment – HW7

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

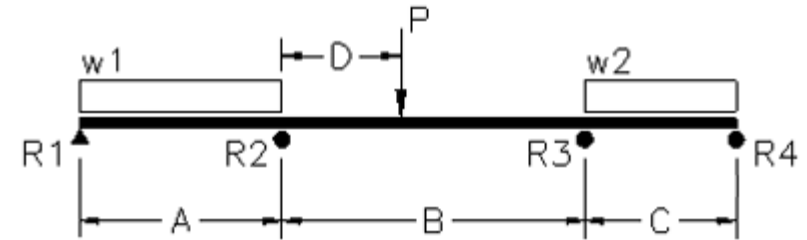
DATASET: 1 -2- -3-

| | |
|---|-------|
| Span A | 28 FT |
| Span B | 21 FT |
| Span C | 24 FT |
| Uniform load on span A, w_1 | 4 KLF |
| Uniform load on span C, w_2 | 5 KLF |
| Point load on span b, P | 60 K |
| Distance to point load P from R_2 , D | 14 FT |



Provide the solution for the assignment – HW7

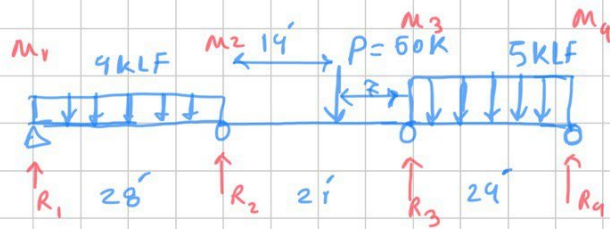
| # | Question | Your Response |
|----|--|---------------------------|
| 1 | Moment at support R1, M1 (- if tension on top) | <input type="text"/> K-FT |
| 2 | EI Theta on left side of R2 | <input type="text"/> |
| 3 | EI Theta on right side of R2 | <input type="text"/> |
| 4 | Moment at support R4, M4 (- if tension on top) | <input type="text"/> K-FT |
| 5 | EI Theta on left side of R3 | <input type="text"/> |
| 6 | EI Theta on right side of R3 | <input type="text"/> |
| 7 | Moment at support R2, M2 (- if tension on top) | <input type="text"/> K-FT |
| 8 | Moment at support R3, M3 (- if tension on top) | <input type="text"/> K-FT |
| 9 | Support reaction, R1 (- if downward) | <input type="text"/> K |
| 10 | Support reaction, R2 (- if downward) | <input type="text"/> K |
| 11 | Support reaction, R3 (- if downward) | <input type="text"/> K |
| 12 | Support reaction, R4 (- if downward) | <input type="text"/> K |



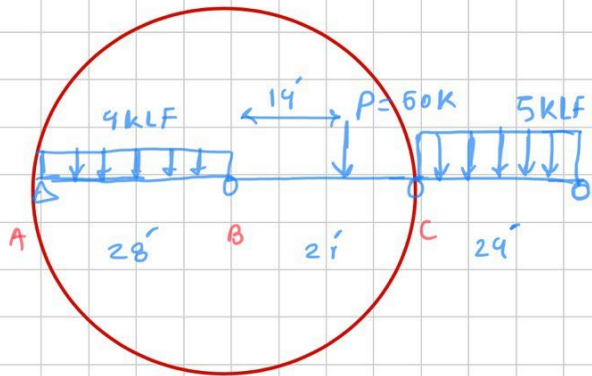
Provide the solution for the assignment – HW7

① Draw FBD

② Label

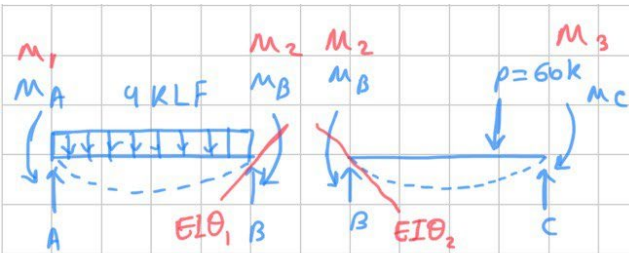


This is a simple supported beam, Moment at the supports = 0
($M_1 = M_4 = 0$)



$$\frac{4PL^2}{81} \quad \frac{5PL^2}{81}$$

Provide the solution for the assignment – HW7



$$m_A L_1 + 2m_B (L_1 + L_2) + m_C L_2 = 6 [EI\theta_1 + EI\theta_2]$$

$$m_A = \cdot \quad L_1 = 28 \quad EI\theta_1 = \frac{wl^2}{24} = \frac{(428)(28)^2}{24} = 3658.66$$

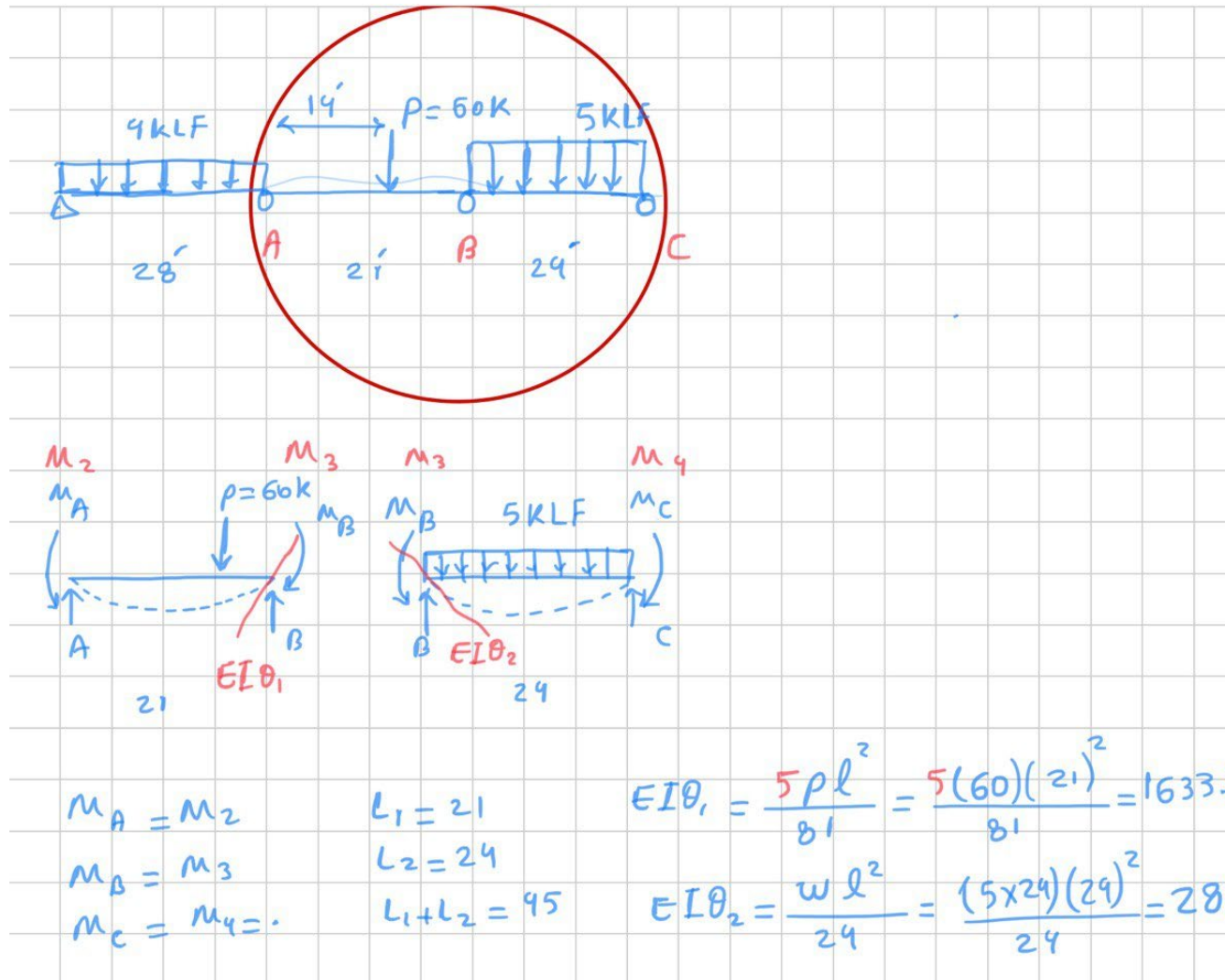
$$m_B = m_2 \quad L_2 = 21 \quad EI\theta_2 = \frac{4Pl^2}{81} = \frac{4(60)(21)^2}{81} = 1306.67$$

$$\bullet + 2M_2(20+21) + M_3(21) = 6[3658.66 + 1306.67]$$

$$2M_2(99) + M_3(21) = 29791.98 \quad \text{①}$$

$$m_2 = \frac{29791.98 - 21m_3}{98}$$

Provide the solution for the assignment – HW7



Provide the solution for the assignment – HW7

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = 6 [EI\theta_1 + EI\theta_2]$$

$$M_2 (21) + 2M_3 (45) + \cdot = 6 [1633.34 + 2880]$$

$$\underline{M_2 (21) + 2M_3 (45) = 27080.04} \quad (2)$$

Replace M_2 From (1)


$$\rightarrow \left(\frac{29791.98 - 21M_3}{98} \right) (21) + 2M_3 (45) = 27080.04$$

$$\rightarrow \begin{cases} M_3 = 242.82 \\ M_2 = 251.9669 \end{cases}$$

Remember M_2, M_3
→ were assumed to cause tension
on top, so don't
forget to use (-) sign!

Provide the solution for the assignment – HW7

Solve FBD For Reactions:



112 K

4 KLF

$M_2 = 251.9669$

28

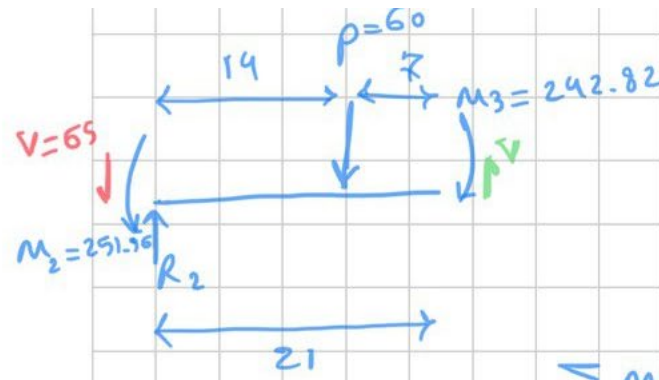
R_1

V

Clockwise +

$$\sum M @ R_2 = 0 \rightarrow R_1(28) - 112\left(\frac{28}{2}\right) + 251.9669 = 0$$
$$\rightarrow \boxed{R_1 = 47 \text{ K}}$$
$$\sum F_y = 0 \rightarrow R_1 - 112 + V = 0$$
$$\rightarrow 47 - 112 + V = 0 \rightarrow V = 65 \text{ K}$$

Provide the solution for the assignment – HW7



Free body diagram showing a beam segment with the following parameters:

- Length: 21
- Reaction force at left end: R_2
- Applied force at left end: $V = 65$
- Applied force at distance 7: $P = 60$
- Applied force at distance 14: $P = 60$
- Moment at right end: $M_3 = 242.82$
- Moment at left end: $M_2 = 251.96$

Sum of moments about R_3 :

$$\sum M @ R_3 = 0$$

$$R_2(21) - 65(21) - 60(7) + 242.82 - 251.9669 = 0$$

Solving for R_2 :

$$\rightarrow R_2 = 85.4355$$

Sum of vertical forces:

$$\sum F_y = 0 \rightarrow 85.4355 - 65 - 60 + V = 0$$

Solving for V :

$$\rightarrow V = 39.5645$$

Provide the solution for the assignment – HW7

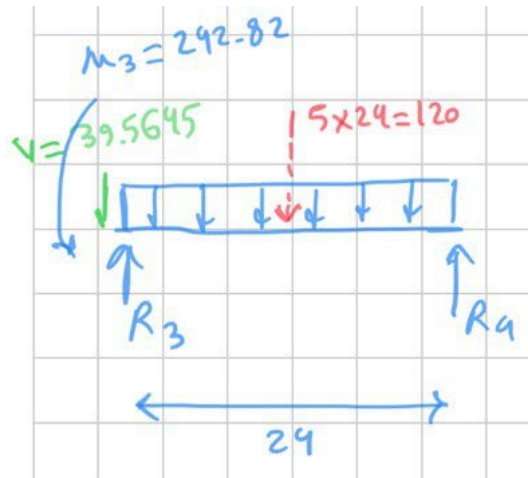
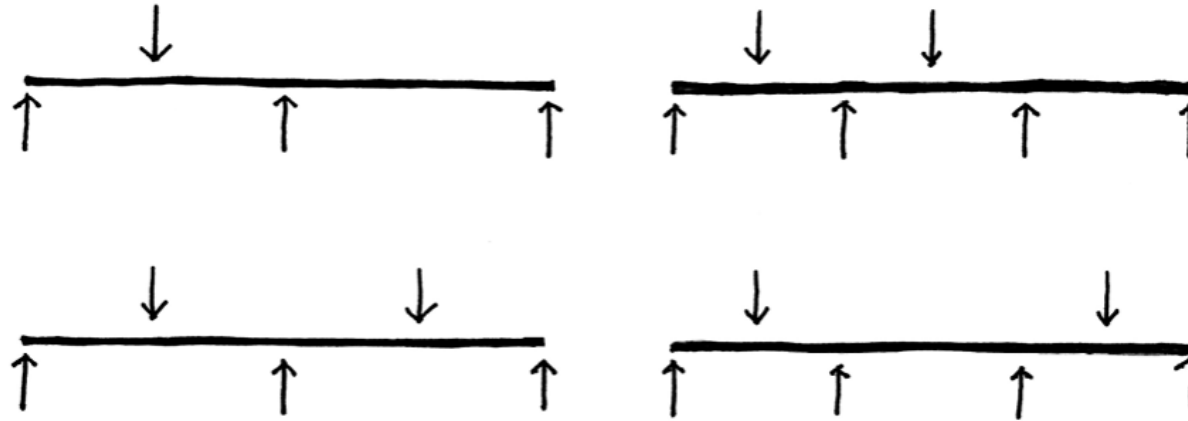


Diagram of a beam of length 24. The beam is supported by a pin at the left end and a roller at the right end. A uniformly distributed load of 5 is applied downwards over the entire length of 24, with a resultant of 120 acting at the center (24/2). At the left end, there is a counter-clockwise moment $M_3 = 242.82$ and a downward shear force $V = 39.5645$. At the left support, there is an upward reaction R_3 . At the right support, there is an upward reaction R_4 .

$$\sum M_{@R_4} = 0$$
$$-242.82 + R_3(24) - 120\left(\frac{24}{2}\right) - 39.5645(24) = 0$$
$$\rightarrow R_3 = 109.682$$
$$\sum F_y = 0 \rightarrow$$
$$109.682 - 39.5645 - 120 + R_4 = 0$$
$$\rightarrow R_4 = 49.8825$$

Lab : Continuous Beams



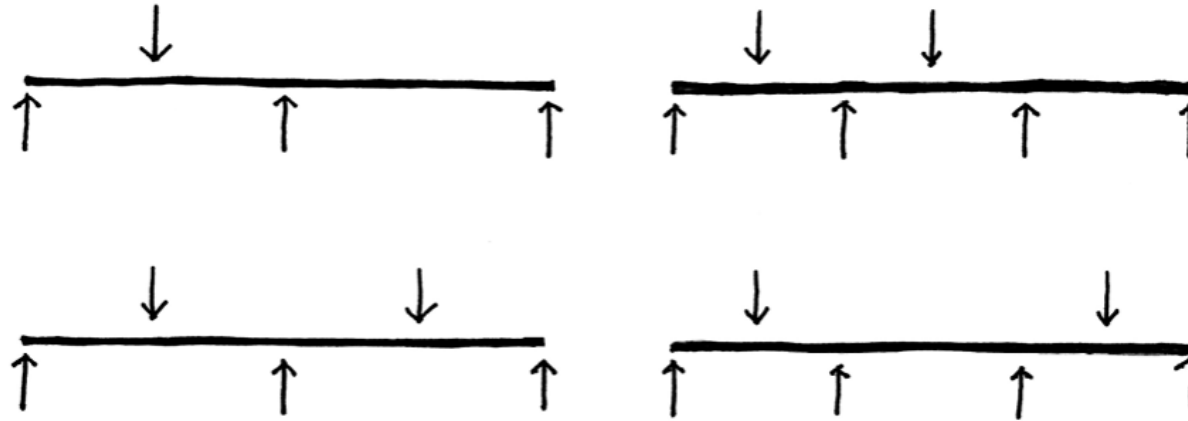
Description

This project uses observation to understand the behavior of beams continuous over multiple supports.

Goals

- To observe the behavior of continuous beams under different loadings
- To estimate locations of contraflexure and effective lengths
- To determine areas of positive and negative moment based on curvature

Lab : Continuous Beams



Procedure

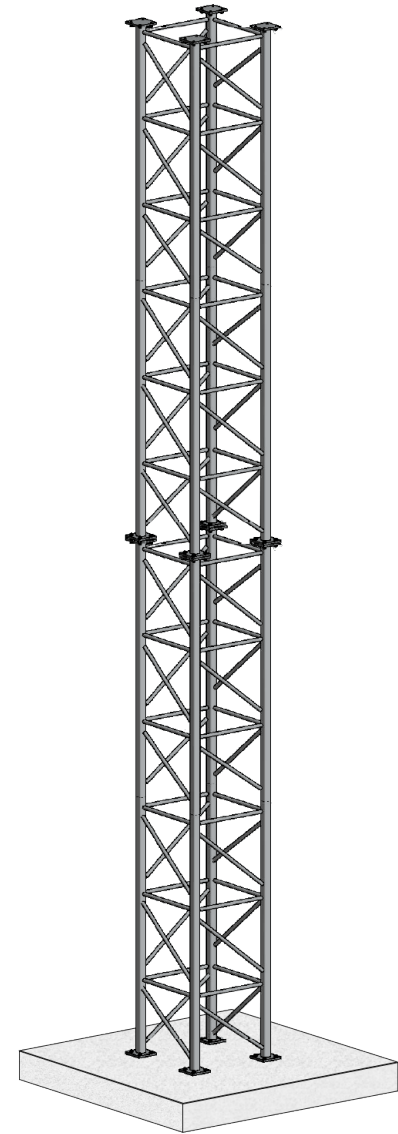
1. Using the 24 inch stick, position the supports and loads (with your finger) as shown in the diagrams below. Hold the beam down on the reactions if it lifts up.
2. For each case observe and draw the elastic curve.
3. Label + and - curvature (moment) and points of contraflexure.
4. Estimate the effective lengths, l_e , across the beam. (between points of $M=0$)

[Link](#) to Prof. von Buelow's description of this lab.

Tower Project:

Due date for the submission of **revision**
of the Preliminary report is **March 11**

Tower Test : **March 20**



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Thank you.

Any question?

Please feel free to ask questions.