

Continuous Beams 3/8

HW – Three Moment Theorem

Lab – Continuous Beams

Structure II
Section 004

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Tower Project

You can Revise the Preliminary Report

Send the updated Report by Email

By Mar11

Tower Testing

Mar 20 Wednesday

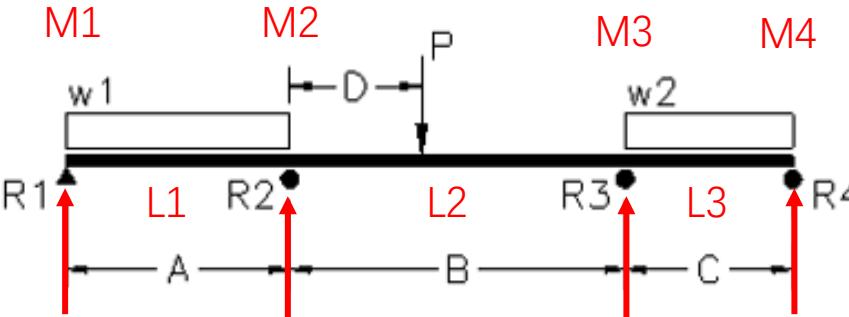
Buy long sticks (48") ahead of Time

MAR 4	Continuous Beams	I. Engel Ch. 17, Schodek 8
MAR 6	Gerber Beams	Schodek 8.4.4
MAR 8	Recitation [5-Continuous Beams]	7. Three Moment Theorem
MAR 11	Intro to Concrete – PCA video.	
MAR 13	Concrete Beams	Schodek 6.4.4 – 6.4.6
MAR 15	Recitation [6-Stress vs Strain]	
MAR 18	Concrete Beams	I. Engel Ch.15
MAR 20	Tower Testing **** Tower Testing **** Tower Testing **** Tower Testing ****	
MAR 22	Recitation	8. Concrete Beam Analysis
MAR 25	Concrete Beams	
MAR 27	Concrete Columns	Schodek 7.4.5
MAR 29	Recitation [7-Concrete Reinforcing]	9. Concrete Beam Design
APR 1	Composite Sections	TMS 402
APR 3	Masonry Walls	TMS 402
APR 5	Recitation [8-Composite Sections]	10. Composite Sections
APR 8	Masonry Walls	TMS 402
APR 10	Shells and Vaults	Schodek 12
APR 12	Recitation [9-Lateral Stability] Final Tower Report Due	

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

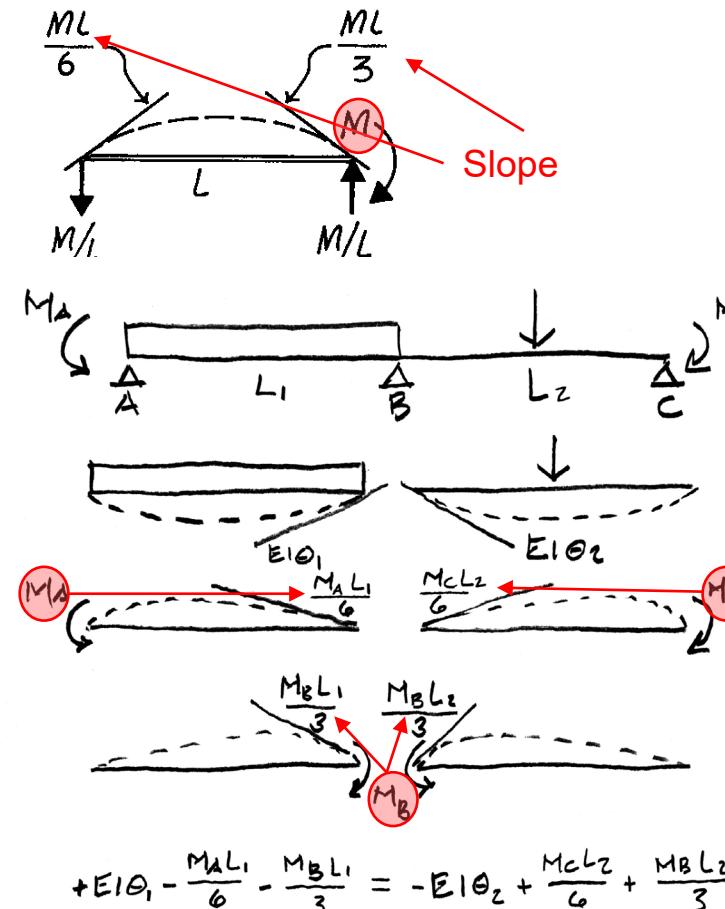
DATASET: 1	-2-	-3-
Span A	21 FT	
Span B	16 FT	
Span C	20 FT	
Uniform load on span A, w1	4 KLF	
Uniform load on span C, w2	4 KLF	
Point load on span b, P	62 K	
Distance to point load P from R2, D	4 FT	



HW - Three Moment Theorem

3-Moment Theorem:

Any number of continuous spans
Non-Symmetric Load and Spans



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

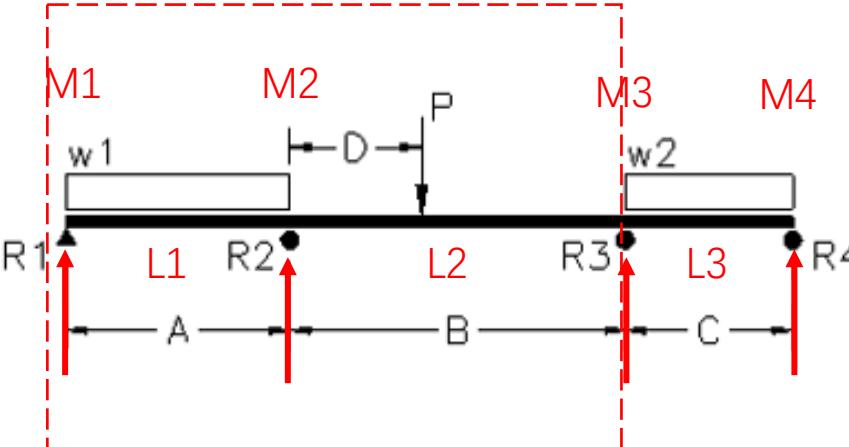
MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT

NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"

7. Three Moment Theorem

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1. Moment at support R1, M1(-if tension on top)

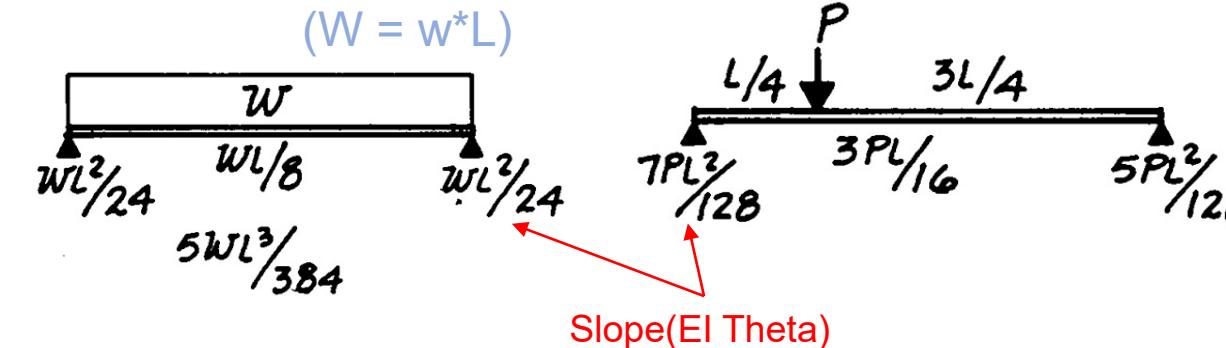
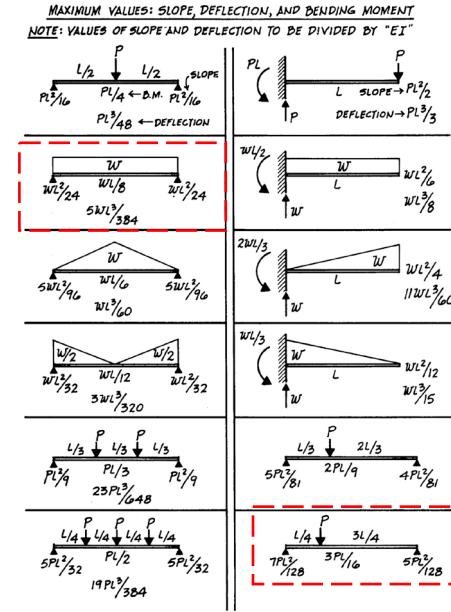
4. Moment at support R4, M4 (-if tension on top)

$$M1 = M4 = 0 \text{ (Exterior end moments = 0)}$$

2. EI Theta on left side of R2

3. EI Theta on right side of R2

$$D/\text{SpanB} = 1/4 \rightarrow \text{(From the chart)}$$



$$W1 = w1 * \text{spanA} = 4 * 21 = 84 \text{ k}$$

$$\text{EI Theta on left} = W1 * \text{SpanA}^2 / 24 = 84 * 21^2 / 24 = 1543.5 \text{ k-ft}^2$$

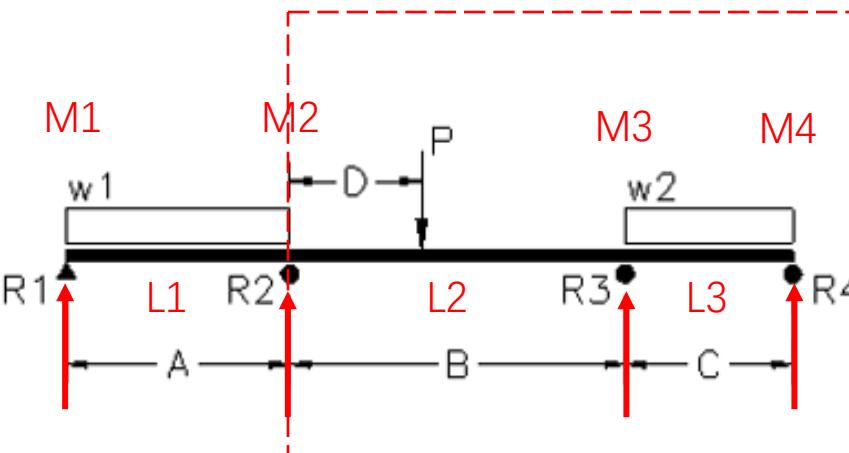
$$\text{EI Theta on right} = 7/128 * P * \text{spanB}^2 = 7/128 * 62 * 16^2 = 868 \text{ k-ft}^2$$

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

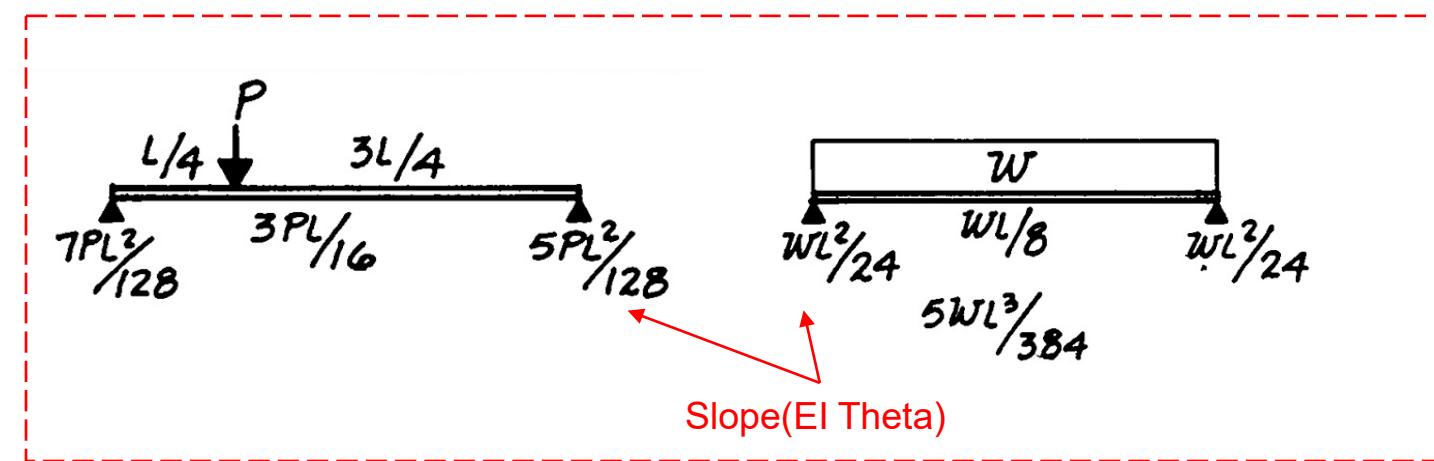
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5. EI Theta on left side of R3

6. EI Theta on left side of R3



$$W_2 = w_2 * \text{spanC} = 4 * 20 = 80 \text{ kip}$$

$$\text{EI Theta on left} = 5/128 * P * \text{spanB}^2 = 5/128 * 62 * 16^2 = 620 \text{ k-ft}^2$$

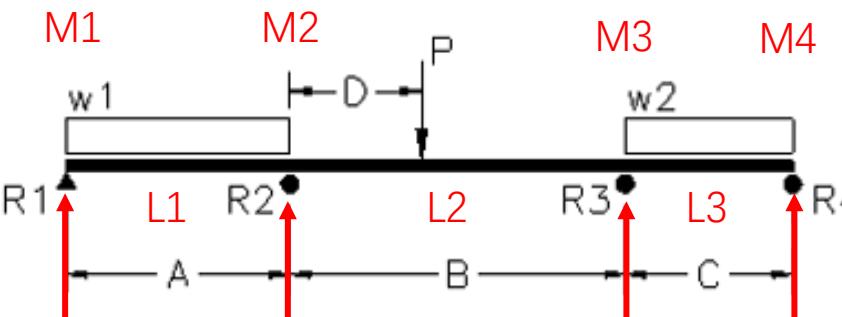
$$\text{EI Theta on right} = W_2 * \text{SpanC}^2 / 24 = 80 * 20^2 / 24 = 1333.33 \text{ k-ft}^2$$

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

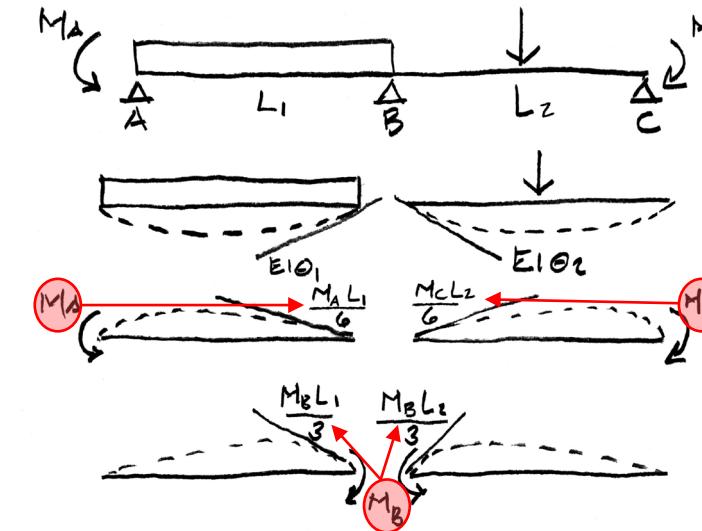
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Distance to point load P from R ₂ , D	4 FT



7. Moment at support R₂, M₂

8. Moment at support R₃, M₃



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

$$+EI\theta_1 - \frac{M_A L_1}{6} - \frac{M_B L_1}{3} = -EI\theta_2 + \frac{M_C L_2}{6} + \frac{M_B L_2}{3}$$

$$\boxed{M_1 L_1 + 2M_2(L_1 + L_2) + M_3 L_2 = 6[EI\theta_1 + EI\theta_2]}$$

$$\boxed{M_2 L_2 + 2M_3(L_2 + L_3) + M_4 L_3 = 6[EI\theta_1 + EI\theta_2]}$$

$$\boxed{0 + 2M_2 * (21 + 16) + M_3 * 16 = 6 * [1543.5 + 868]}$$

$$\boxed{M_2 * 16 + 2M_3 * (16 + 20) + 0 = 6 * [620 + 1333.33]}$$

$$M_2 = -168.42 \text{ k-ft} \quad M_3 = -125.35 \text{ k-ft}$$

7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

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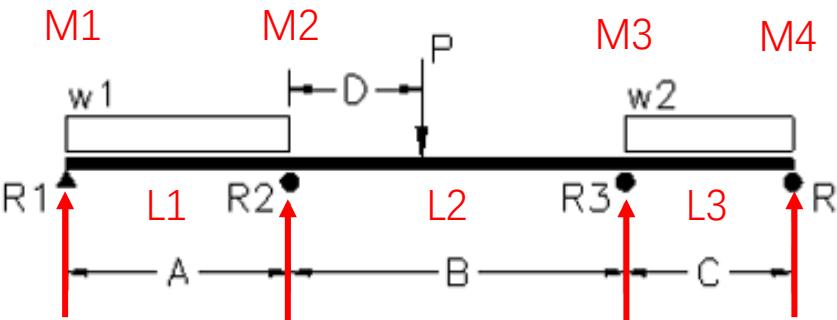
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9. Support reaction, R1(-if downward)



$$\sum M @ R_2 = R_1 * L_A - w_1 * L_A * (L_A / 2) + M_2 = 0$$

$$R_1 * 21 - 4 * 21 * (21 / 2) + 168.42 = 0$$
$$R_1 = 33.98 \text{ k}$$



$$\sum F_y = R_1 + V_2 - w_1 * L_A = 0$$

$$V_2 = 50.02 \text{ k}$$

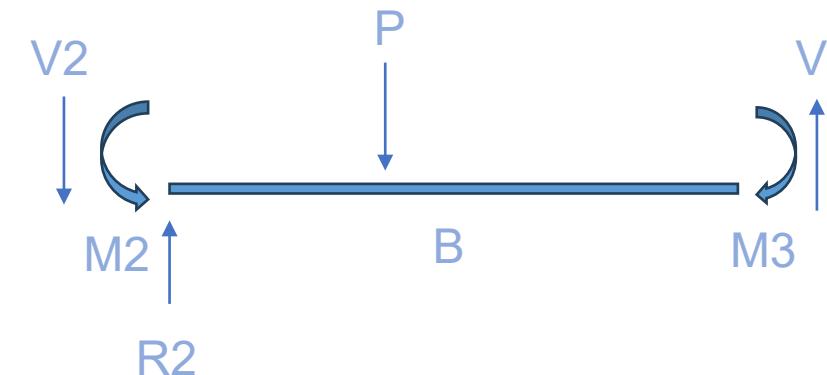
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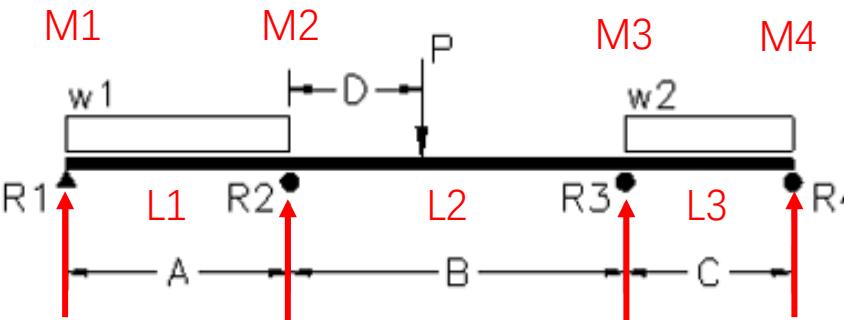
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Point load on span b, P	62 K
Distance to point load P from R ₂ , D	4 FT

10. Support reaction, R₂(-if downward)



$$\sum M @ R_3 = -V_2 * LB + R_2 * LB - P * (LB - LD) - M_2 + M_3 = 0$$

$$- 50.02 * 16 + R_2 * 16 - 62(16 - 4) - 168.42 + 125.35 = 0 \\ R_2 = 99.21 \text{ k}$$



$$\sum F_y = R_2 - V_2 - P + V_3 = 0$$

$$V_3 = 12.81 \text{ k}$$

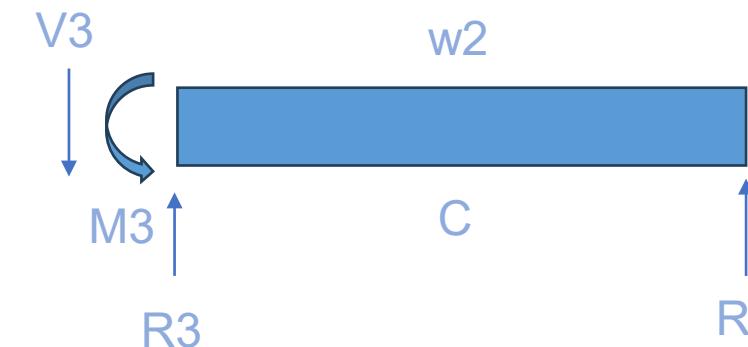
7. Three Moment Theorem

Use the Three Moment Theorem to determine all reactions and support moments for the given continuous beam.

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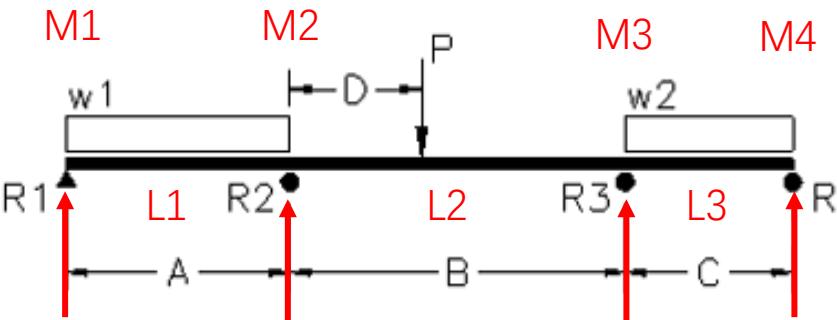
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Distance to point load P from R2, D	4 FT

11. Support reaction, R3(-if downward)



$$\sum M @ R_4 = -V_3 \cdot LC - M_3 + R_3 \cdot LC - w_2 \cdot LC \cdot (LC/2) = 0$$

$$- 12.81 \cdot 20 - 125.35 + R_3 \cdot 20 - 4 \cdot 20 \cdot 20 / 2 = 0 \\ R_3 = 59.08 \text{ k}$$



12. Support reaction, R4(-if downward)

$$\sum F_y = R_3 - V_3 - w_2 \cdot LC + R_4 = 0 \\ R_4 = 33.73 \text{ k}$$

LAB - Continuous Beams

Description

This project uses observation to understand the behavior of beams continuous over multiple supports.

Goals

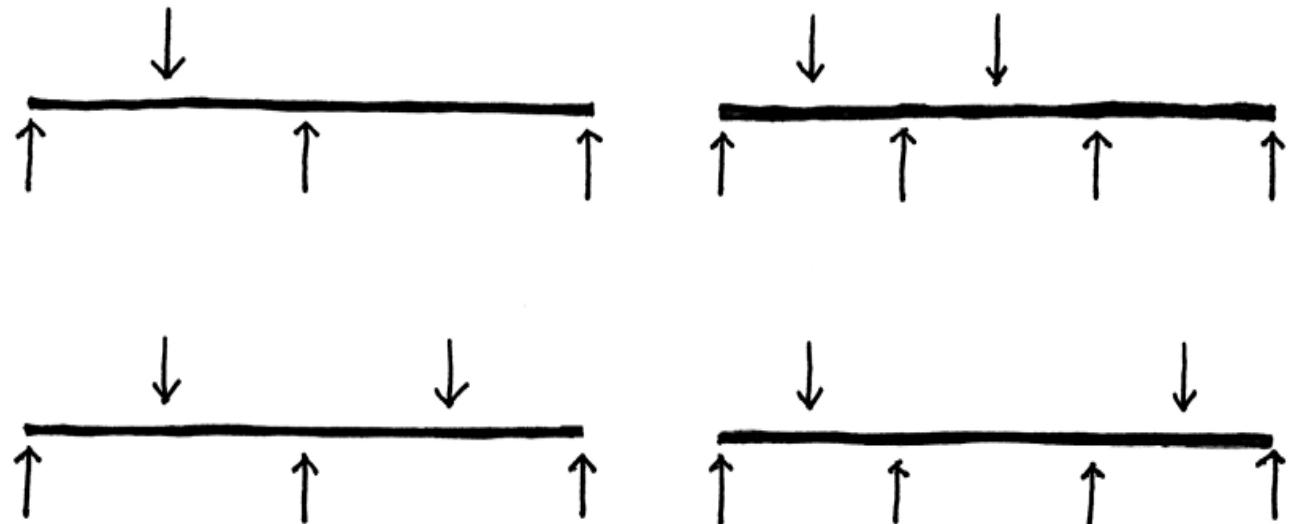
To observe the behavior of continuous beams under different loadings

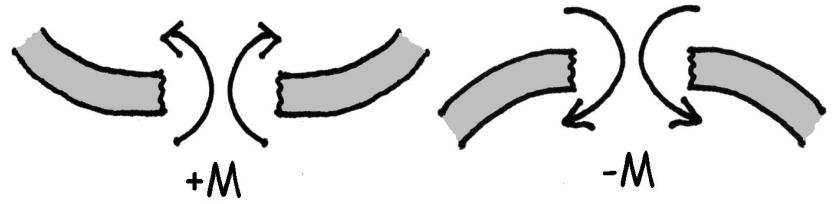
To estimate locations of contraflexure and effective lengths

To determine areas of positive and negative moment based on curvature

Procedure

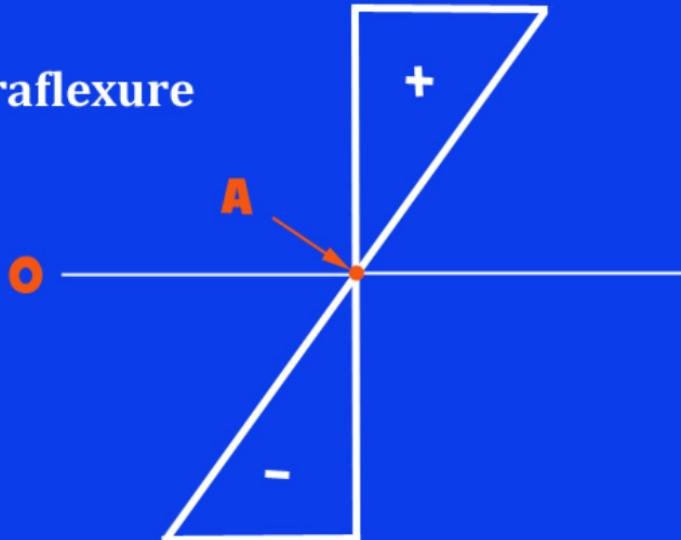
1. Using the 24 inch stick, position the supports and loads (with your finger) as shown in the diagrams below. Hold the beam down on the reactions if it lifts up.
2. For each case observe and draw the elastic curve.
3. Label + and – curvature (moment) and points of contraflexure.
4. Estimate the effective lengths, l_e , across the beam. (between points of $M=0$)





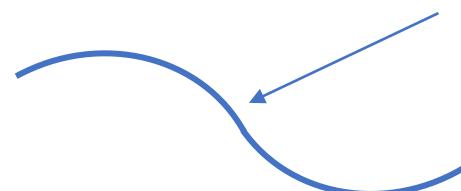
Point of contraflexure

'A' is the point of contraflexure



The point of contraflexure (PoC) occurs where bending is zero and at the point of change between positive and negative (or between compression and tension).

https://www.designingbuildings.co.uk/wiki/Point_of_contraflexure



Any Questions?

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Thank You!

