TAUBMAN COLLEGE
ARCHITECTURE & URBAN PLANNING
UNIVERSITY OF MICHIGAN

Arch324
STRUCTURES II

Winter 2024
Recitation

FACULTY: Prof. Peter von Bülow
GSI: Mohsen Vatandoost

Arch324: STRUCTURES II

Welcome to Recitation session 01/26

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Office: Room 3104
hours:
Fri: 11:30 – 14:30
Mon, Wed: 11:00 - 12:00
walk-ins welcome!

Please feel free to ask questions.

Arch324: STRUCTURES II

Welcome to Recitation session 01/26

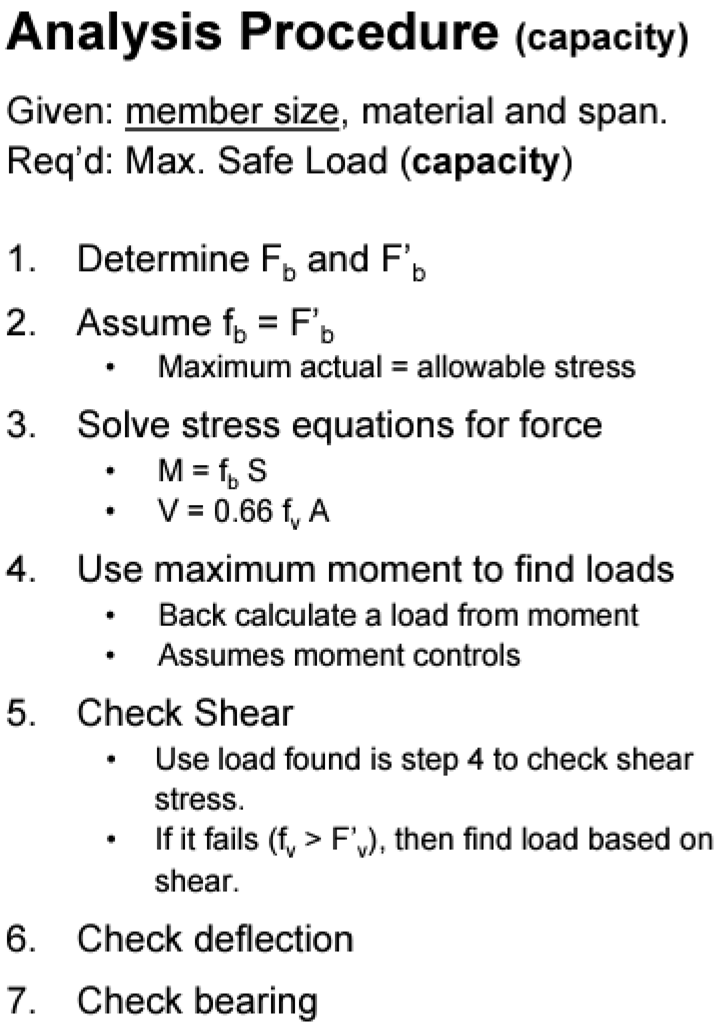
Outline:

- Quick **Recap** of the week
- Provide the solution for the assignment **(Homework 2)**
- Answering student’s questions
- Lab: **---**

Please feel free to ask questions.

Recap of the week

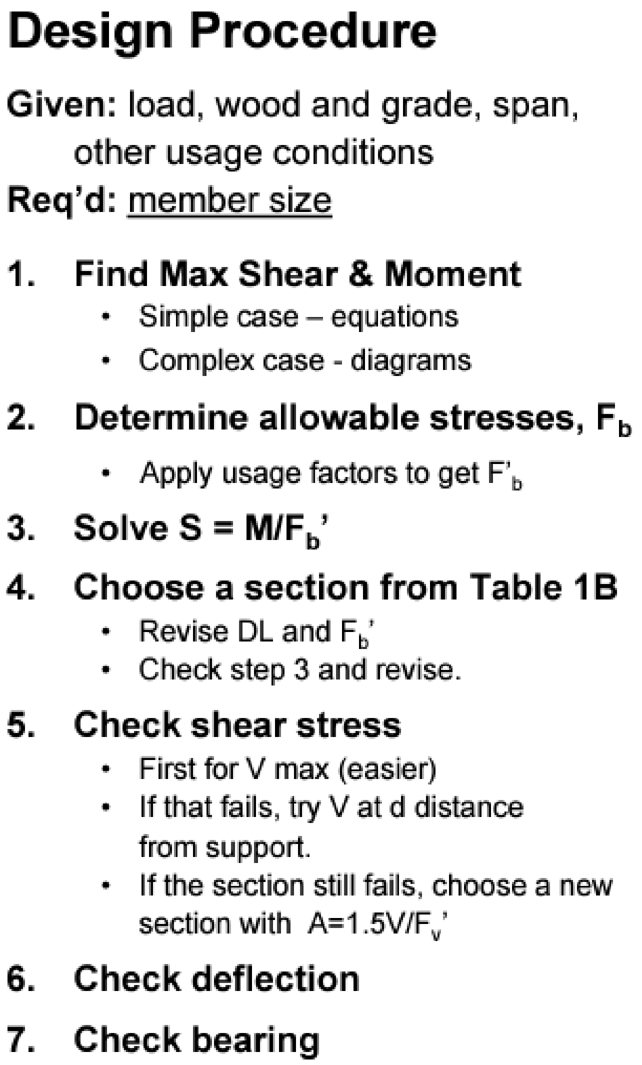
Wood Beam Analysis / Design



Analysis Procedure (capacity)

Given: member size, material and span.
Req'd: Max. Safe Load (**capacity**)

1. Determine F_b and F'_b
2. Assume $f_b = F'_b$
   - Maximum actual = allowable stress
3. Solve stress equations for force
   - $M = f_b S$
   - $V = 0.66 f_v A$
4. Use maximum moment to find loads
   - Back calculate a load from moment
   - Assumes moment controls
5. Check Shear
   - Use load found is step 4 to check shear stress.
   - If it fails ($f_v > F'_v$), then find load based on shear.
6. Check deflection
7. Check bearing



Design Procedure

Given: load, wood and grade, span, other usage conditions
Req'd: member size

1. **Find Max Shear & Moment**
   - Simple case – equations
   - Complex case - diagrams
2. **Determine allowable stresses, F_b**
   - Apply usage factors to get F'_b
3. **Solve $S = M/F_b'$**
4. **Choose a section from Table 1B**
   - Revise DL and F_b'
   - Check step 3 and revise.
5. **Check shear stress**
   - First for V max (easier)
   - If that fails, try V at d distance from support.
   - If the section still fails, choose a new section with $A=1.5V/F_v'$
6. **Check deflection**
7. **Check bearing**

Recap of the week

Wood Column Analysis

AXIAL

$$\boxed{f_c = \frac{P}{A}}$$

$$C_P = \frac{1+\left(F_{cE}/F_c^*\right)}{2c} - \sqrt{\left[\frac{1+\left(F_{cE}/F_c^*\right)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}}$$

Bearing (crushing limit)

Short Columns – fail by crushing

Analysis

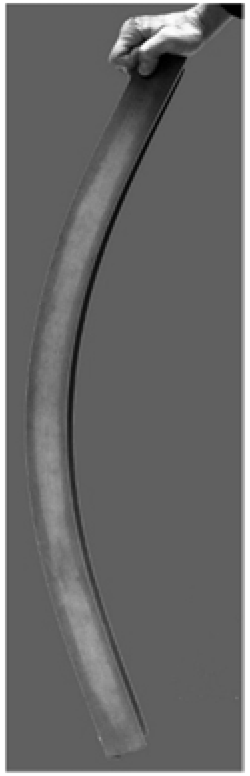
$$\boxed{f_c = \frac{P}{A} \leq F'_c}$$

Design

$$\boxed{A = \frac{P}{F'_c}}$$

- f_c = Actual compressive stress
- A = Cross-sectional area of column (in^2)
- P = Load on the column
- F'_c = Allowable compressive stress per codes

Long Columns – fail by buckling



$$\boxed{r = d/\sqrt{12}}$$

NDS Equation

$$\boxed{F_{cE} = \frac{0.822E'_{min}}{\left(\frac{l_e}{d}\right)^2}}$$

- E'min = reduced E modulus (psi)
- le = Ke l_u (inches)
- d (inches)
- $0.822 = \pi^2/12$

Recap of the week

Adjustment Factors

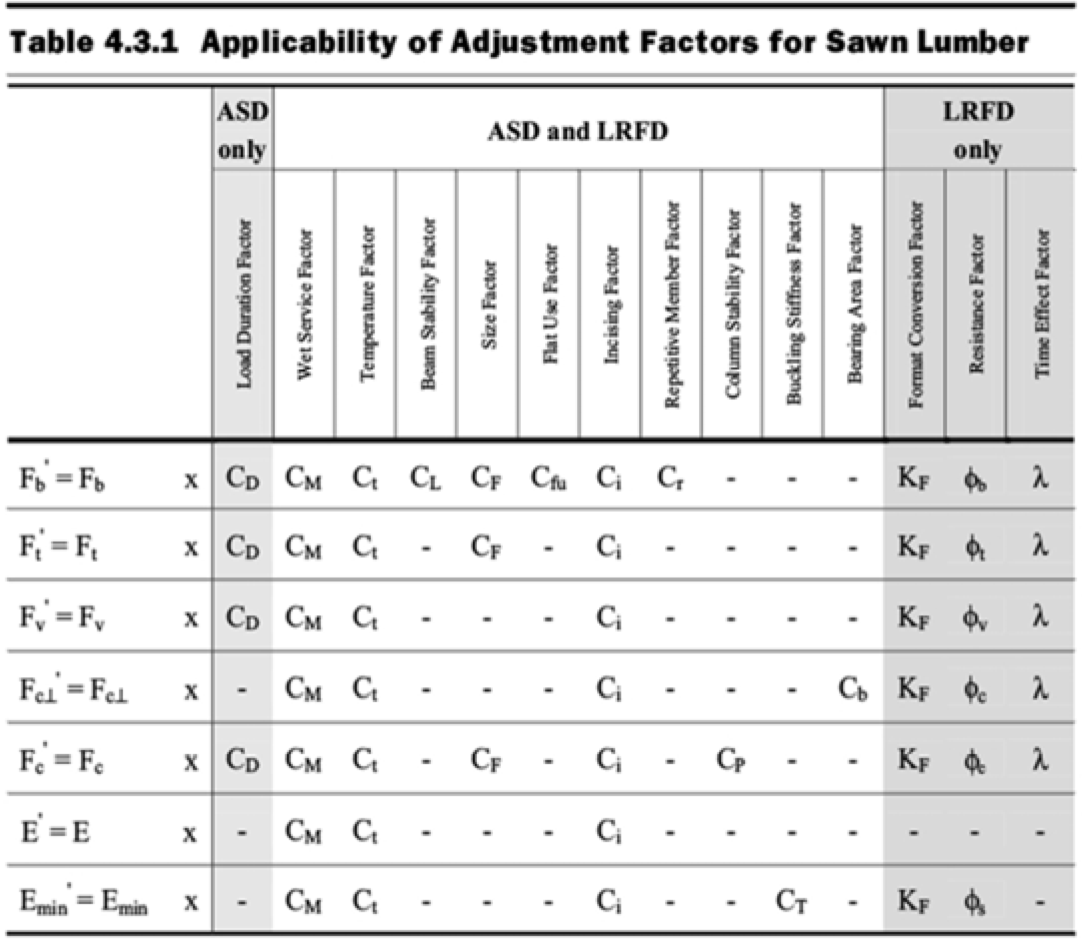
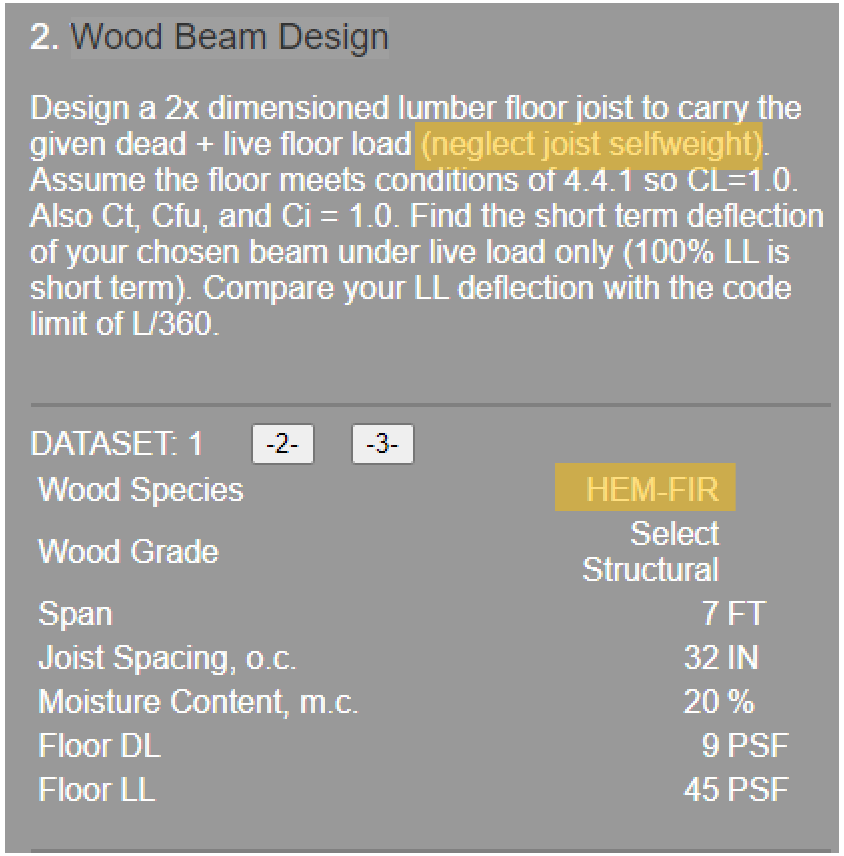


Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	-	-	-	K_F	ϕ_b	λ
$F_t' = F_t$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	-	-	K_F	ϕ_t	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	K_F	ϕ_v	λ
$F_{c\perp}' = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_b	K_F	ϕ_c	λ
$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	-	-	-
$E_{min}' = E_{min}$	x	-	C_M	C_t	-	-	-	C_i	-	-	C_T	-	K_F	ϕ_s	-

Provide the solution for the assignment – **HW2**



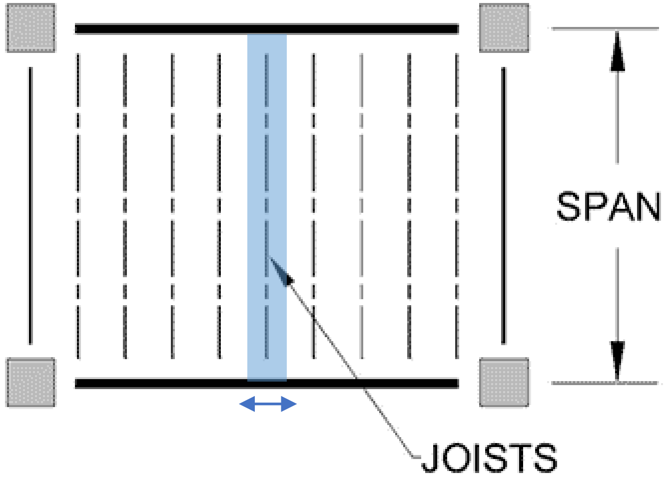
2. Wood Beam Design

Design a 2x dimensioned lumber floor joist to carry the given dead + live floor load (neglect joist selfweight). Assume the floor meets conditions of 4.4.1 so CL=1.0. Also Ct, Cfu, and Ci = 1.0. Find the short term deflection of your chosen beam under live load only (100% LL is short term). Compare your LL deflection with the code limit of L/360.

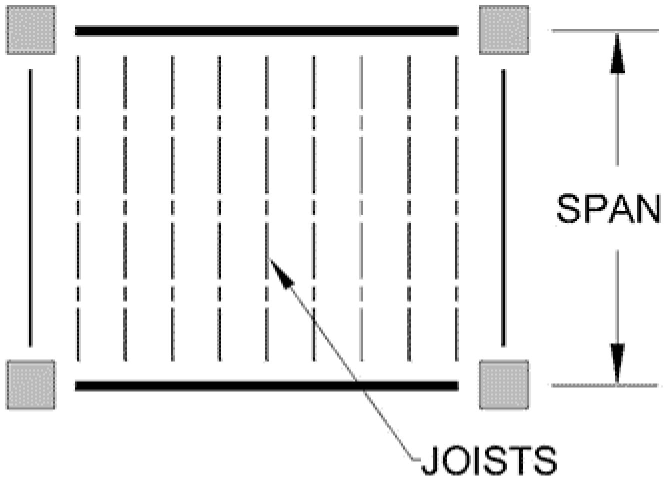
DATASET: 1 -2- -3-

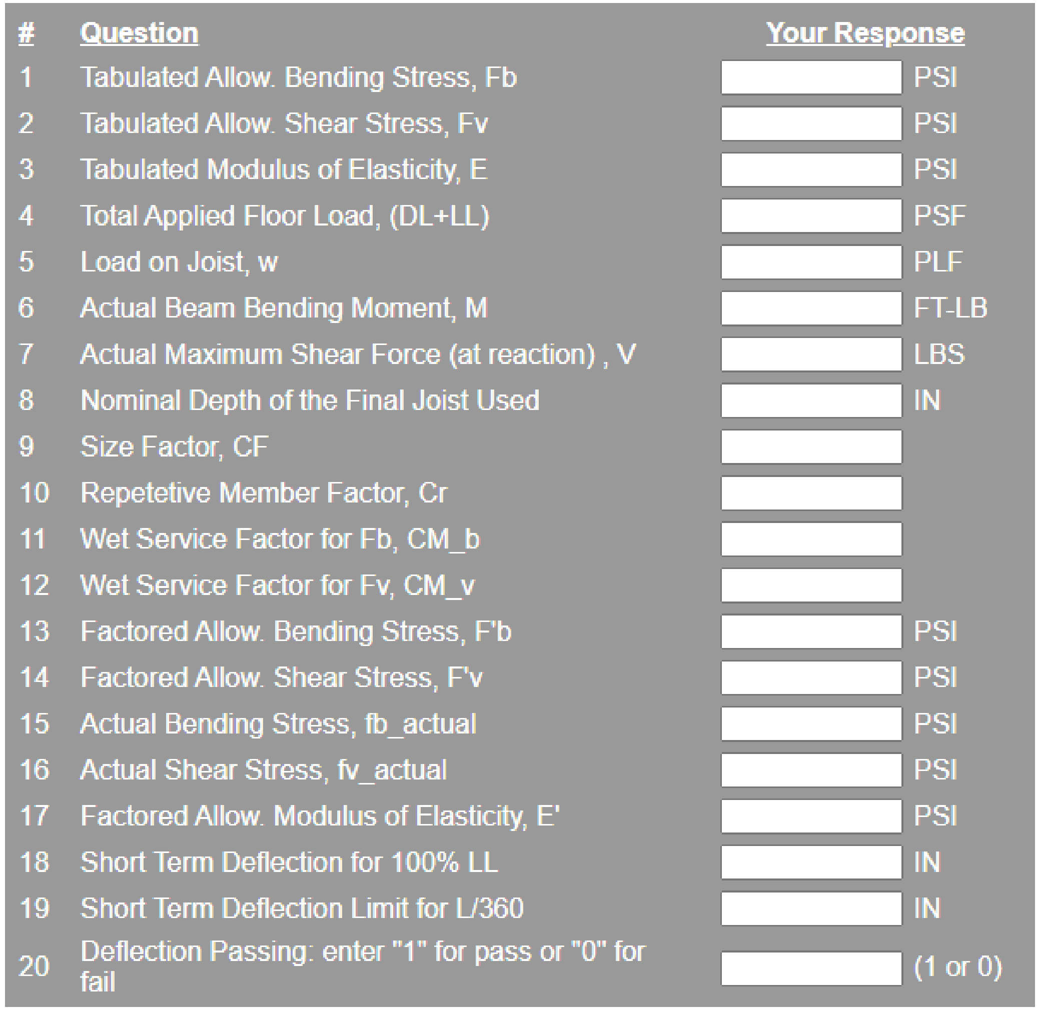
Wood Species	HEM-FIR
Wood Grade	Select Structural
Span	7 FT
Joist Spacing, o.c.	32 IN
Moisture Content, m.c.	20 %
Floor DL	9 PSF
Floor LL	45 PSF

- **Problem:**



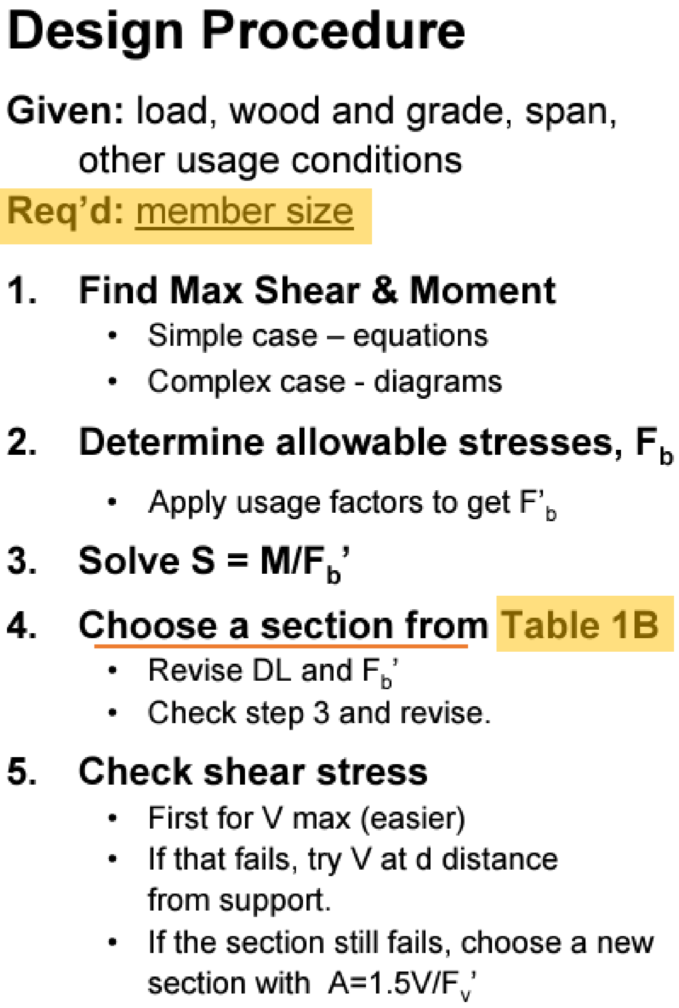
Provide the solution for the assignment – **HW2**





#	Question	Your Response	
1	Tabulated Allow. Bending Stress, Fb		PSI
2	Tabulated Allow. Shear Stress, Fv		PSI
3	Tabulated Modulus of Elasticity, E		PSI
4	Total Applied Floor Load, (DL+LL)		PSF
5	Load on Joist, w		PLF
6	Actual Beam Bending Moment, M		FT-LB
7	Actual Maximum Shear Force (at reaction) , V		LBS
8	Nominal Depth of the Final Joist Used		IN
9	Size Factor, CF		
10	Repetetive Member Factor, Cr		
11	Wet Service Factor for Fb, CM_b		
12	Wet Service Factor for Fv, CM_v		
13	Factored Allow. Bending Stress, F'b		PSI
14	Factored Allow. Shear Stress, F'v		PSI
15	Actual Bending Stress, fb_actual		PSI
16	Actual Shear Stress, fv_actual		PSI
17	Factored Allow. Modulus of Elasticity, E'		PSI
18	Short Term Deflection for 100% LL		IN
19	Short Term Deflection Limit for L/360		IN
20	Deflection Passing: enter "1" for pass or "0" for fail		(1 or 0)

Provide the solution for the assignment – **HW2**



Design Procedure

Given: load, wood and grade, span, other usage conditions

Req'd: member size

1. **Find Max Shear & Moment**
    - Simple case – equations
    - Complex case - diagrams
2. **Determine allowable stresses, F_b**
    - Apply usage factors to get F'_b
3. **Solve $S = M/F_b'$**
4. **Choose a section from Table 1B**
    - Revise DL and F_b'
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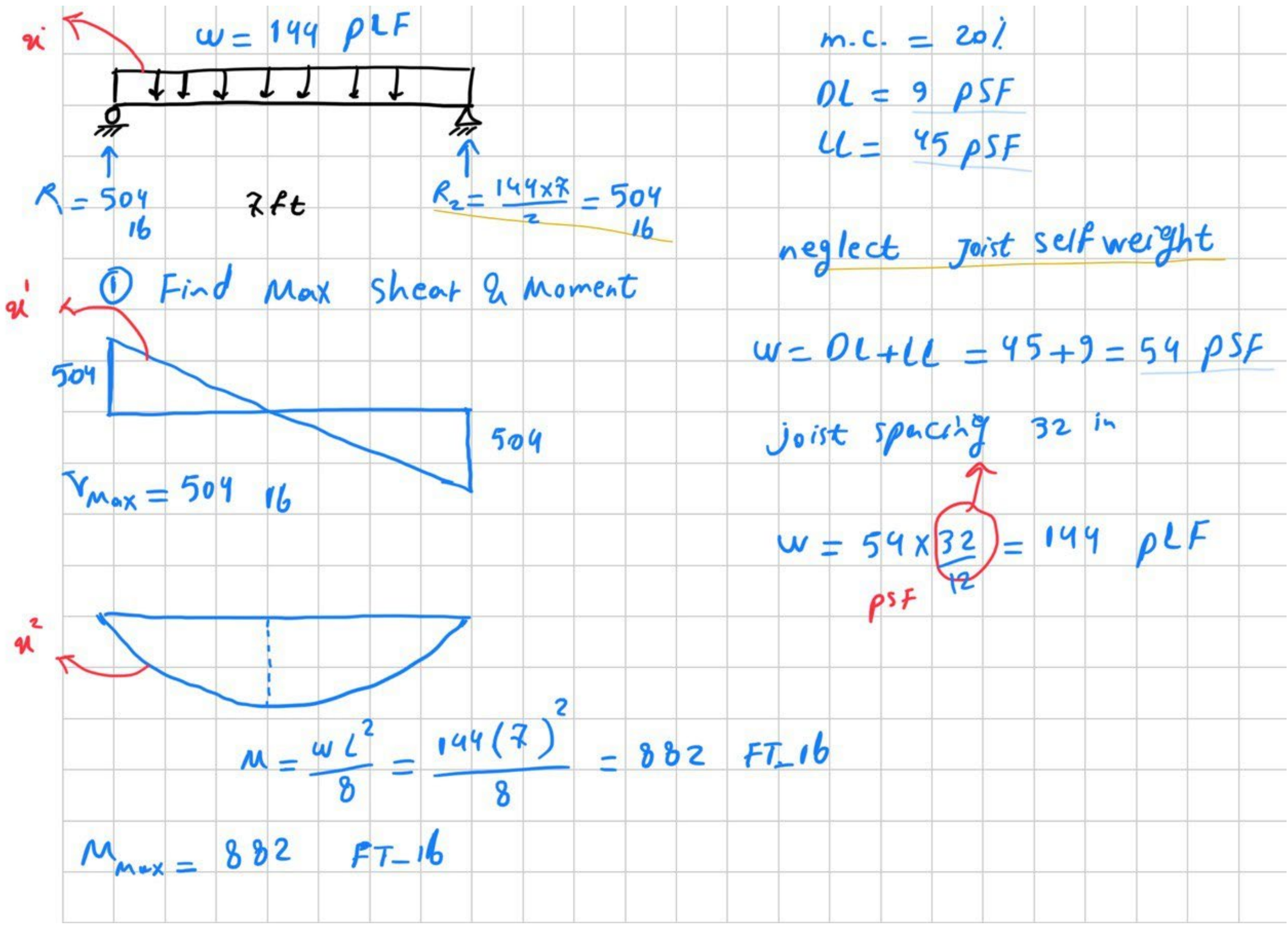
5. **Check shear stress**
    - First for V max (easier)
    - If that fails, try V at d distance from support.
    - If the section still fails, choose a new section with $A=1.5V/F_v'$

6. **Check deflection**
7. **Check bearing**

Provide the solution for the assignment – **HW2**



$w = 144$ pLF

$R_1 = 504$ lb

$R_2 = \frac{144 \times 7}{2} = 504$ lb

① Find Max Shear & Moment

$V_{Max} = 504$ lb

$M = \frac{wL^2}{8} = \frac{144(7)^2}{8} = 882$ FT_lb

$M_{Max} = 882$ FT_lb

m.c. = 20%

DL = 9 pSF

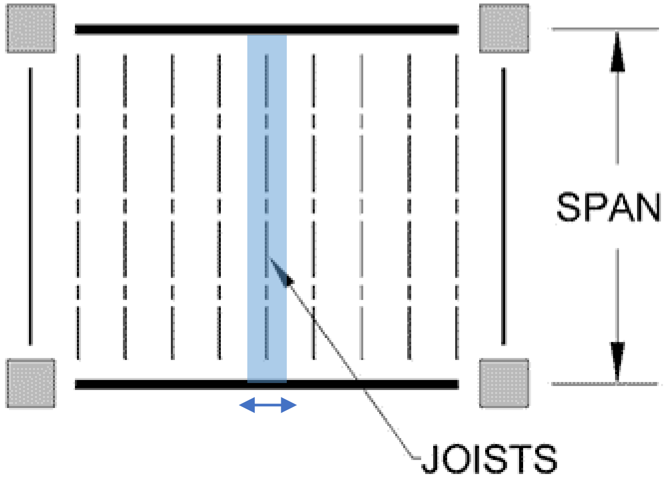
LL = 45 pSF

neglect Joist self weight

$w = DL + LL = 45 + 9 = 54$ pSF

joist spacing 32 in

$w = \underset{psf}{54} \times \frac{32}{12} = 144$ pLF



Provide the solution for the assignment – **HW2**

NDS Supplement

Wood Species → HEM-FIR
Wood Grade → SELECT Structural

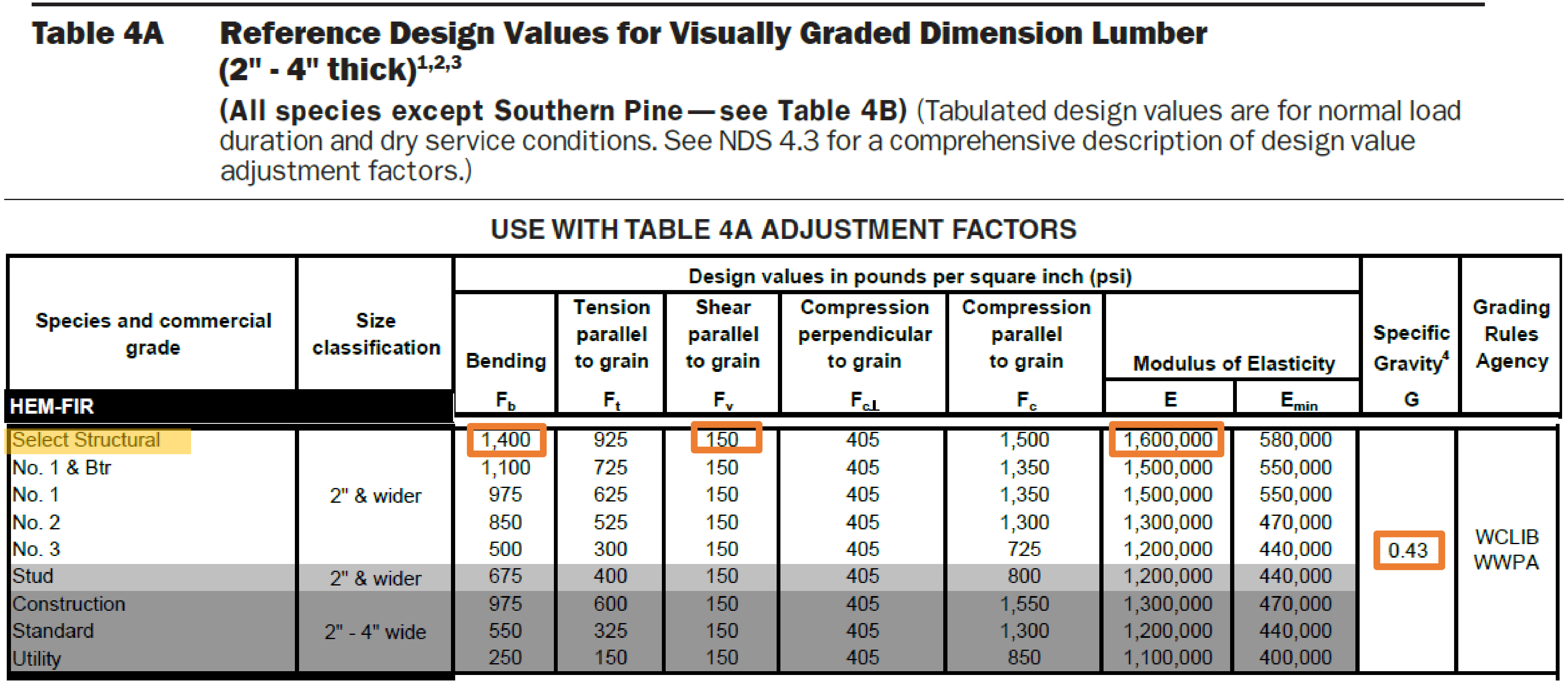


Table 4A **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

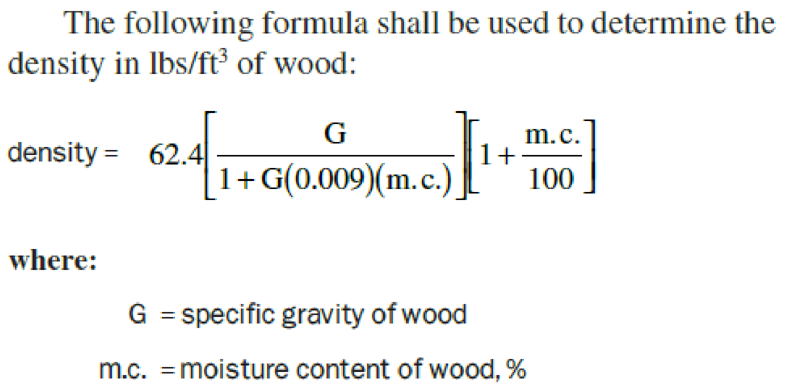
(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific Gravity[4]	Grading Rules Agency
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity			
		F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}	G	
HEM-FIR										
Select Structural	2" & wider	1,400	925	150	405	1,500	1,600,000	580,000	0.43	WCLIB WWPA
No. 1 & Btr		1,100	725	150	405	1,350	1,500,000	550,000		
No. 1		975	625	150	405	1,350	1,500,000	550,000		
No. 2		850	525	150	405	1,300	1,300,000	470,000		
No. 3		500	300	150	405	725	1,200,000	440,000		
Stud	2" & wider	675	400	150	405	800	1,200,000	440,000		
Construction	2" - 4" wide	975	600	150	405	1,550	1,300,000	470,000		
Standard		550	325	150	405	1,300	1,200,000	440,000		
Utility		250	150	150	405	850	1,100,000	400,000		

Provide the solution for the assignment – **HW2**

[NDS- Supplement- p 12]



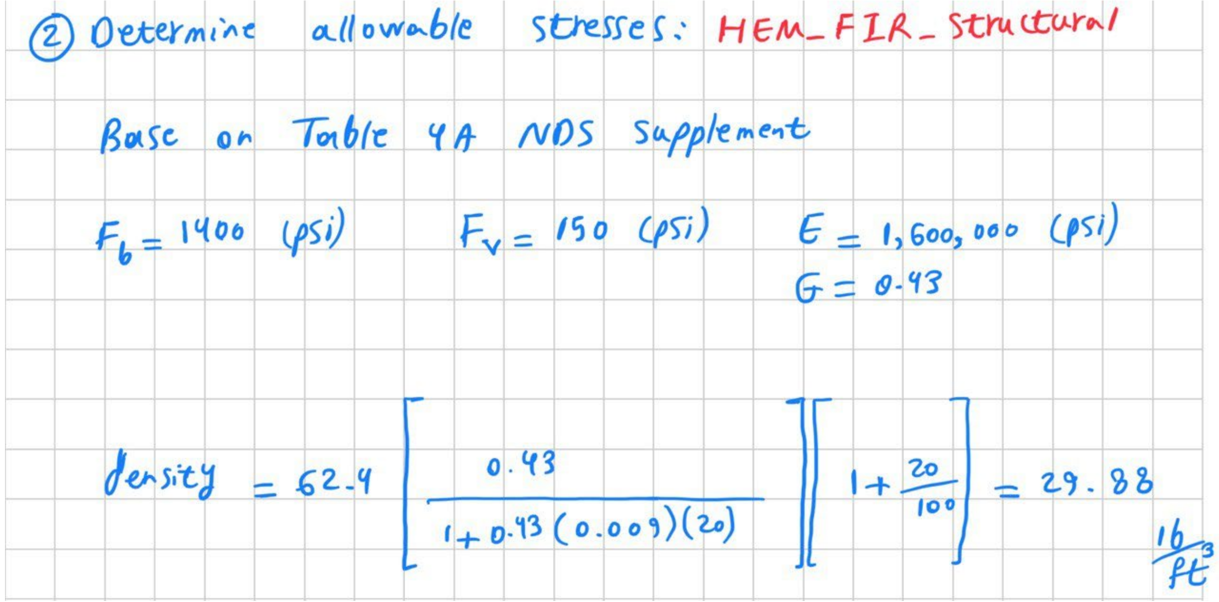
The following formula shall be used to determine the density in lbs/ft^3 of wood:

$$\text{density} = 62.4\left[\frac{G}{1+G(0.009)(\text{m.c.})}\right]\left[1+\frac{\text{m.c.}}{100}\right]$$

where:

G = specific gravity of wood

m.c. = moisture content of wood, %



② Determine allowable stresses: HEM_FIR_Structural

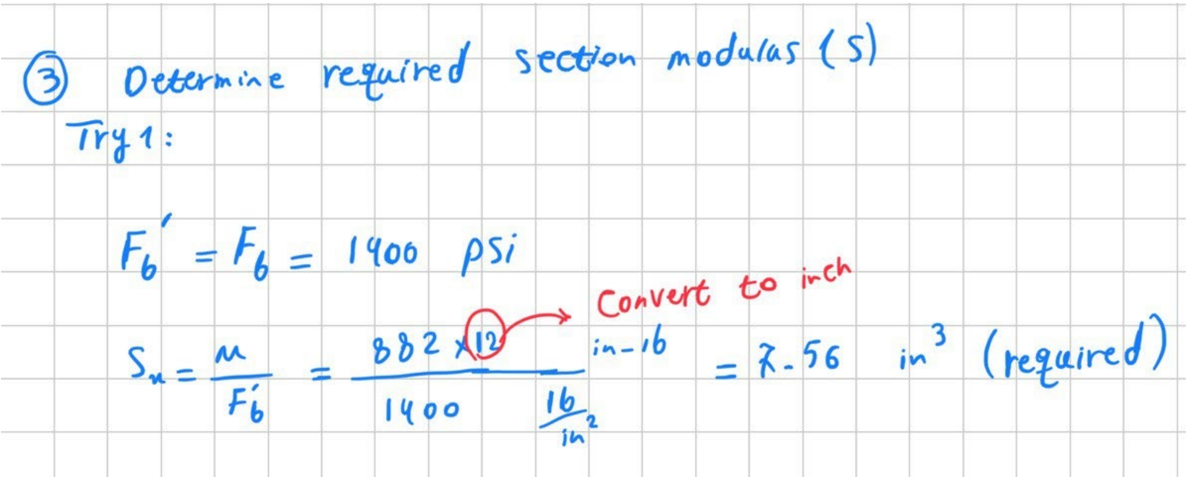
Base on Table 4A NDS supplement

$F_b = 1400$ (psi) $F_v = 150$ (psi) $E = 1{,}600{,}000$ (psi)

$G = 0.43$

$$\text{density} = 62.4\left[\frac{0.43}{1+0.43(0.009)(20)}\right]\left[1+\frac{20}{100}\right] = 29.88\ \frac{lb}{ft^3}$$

Provide the solution for the assignment – **HW2**



③ Determine required section modulas (S)

Try 1:

$$F_b' = F_b = 1400 \text{ psi}$$

$$S_x = \frac{M}{F_b'} = \frac{882 \times \boxed{12} \text{ in-lb}}{1400 \ \frac{lb}{in^2}} = 7.56 \text{ in}^3 \text{ (required)}$$

Convert to inch

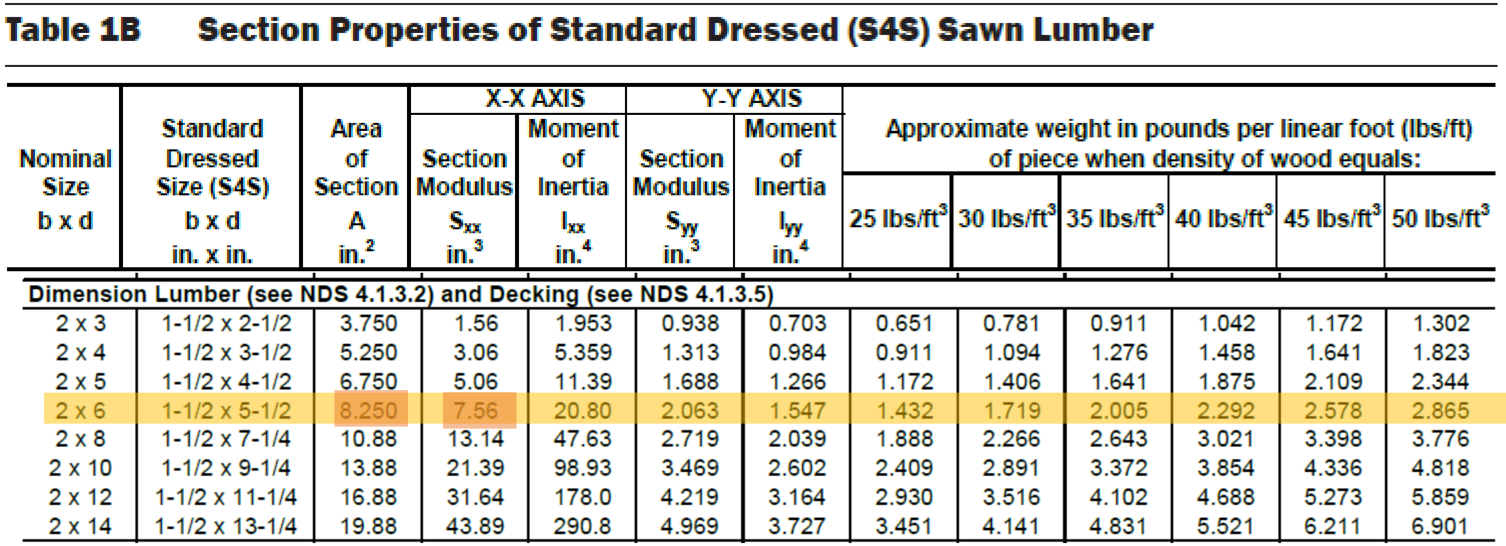
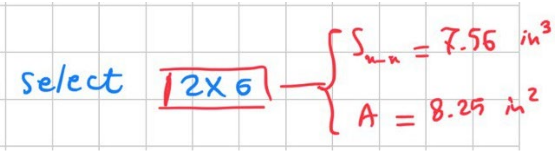


Table 1B **Section Properties of Standard Dressed (S4S) Sawn Lumber**

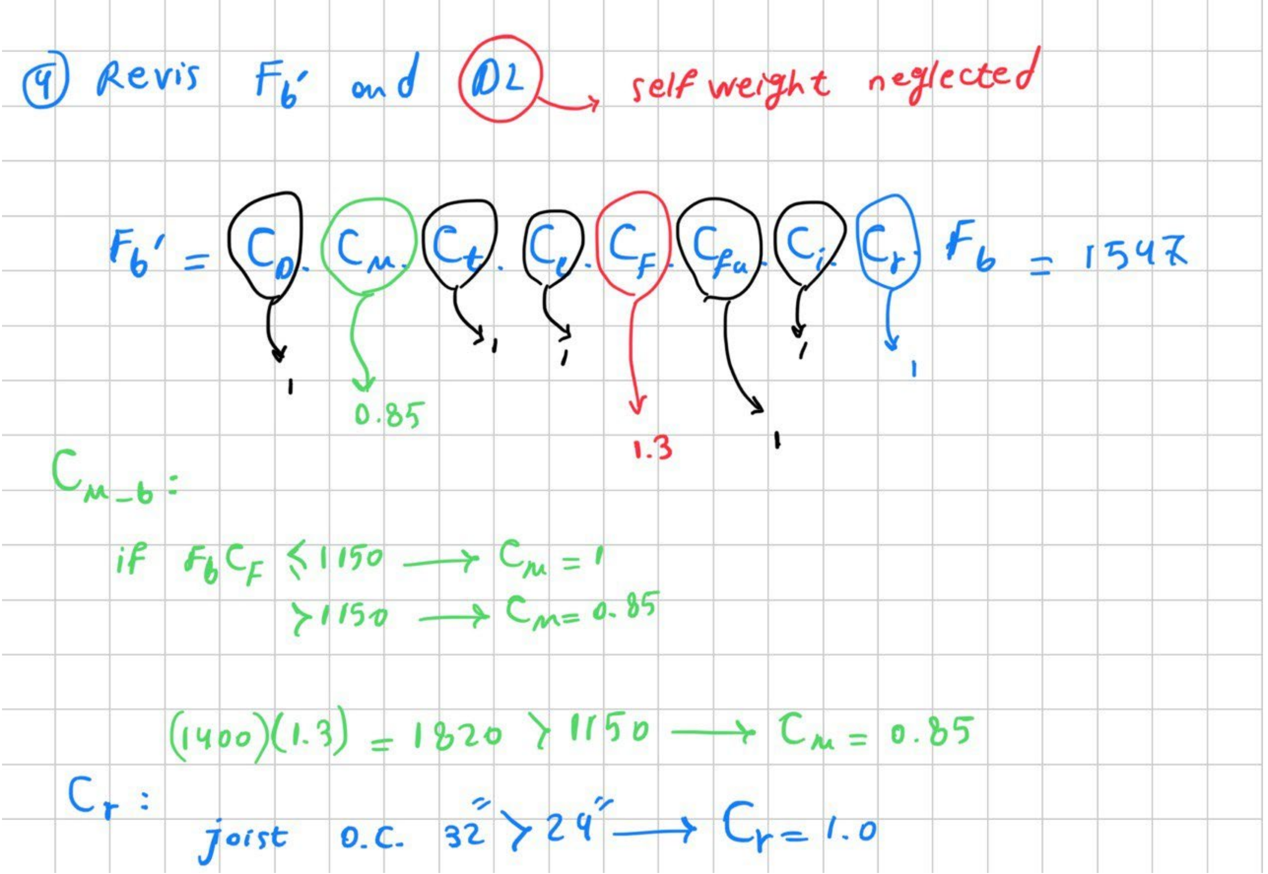
			X-X AXIS		Y-Y AXIS		Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals:					
Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	Section Modulus S_{xx} in.^{3}	Moment of Inertia I_{xx} in.^{4}	Section Modulus S_{yy} in.^{3}	Moment of Inertia I_{yy} in.^{4}	25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)												
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109	2.344
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901



select $\boxed{2 \times 6}$ $\begin{cases} S_{xx} = 7.56 \text{ in}^3 \\ A = 8.25 \text{ in}^2 \end{cases}$

[NDS- Supplement]

Provide the solution for the assignment – **HW2**



④ Revis F_b' and (D2) → self weight neglected

$$F_b' = \underbrace{C_D}_{1} \cdot \underbrace{C_M}_{0.85} \cdot \underbrace{C_t}_{1} \cdot \underbrace{C_L}_{1} \cdot \underbrace{C_F}_{1.3} \cdot \underbrace{C_{fu}}_{1} \cdot \underbrace{C_i}_{1} \cdot \underbrace{C_r}_{1} \cdot F_b = 1547$$

C_{M_b}:

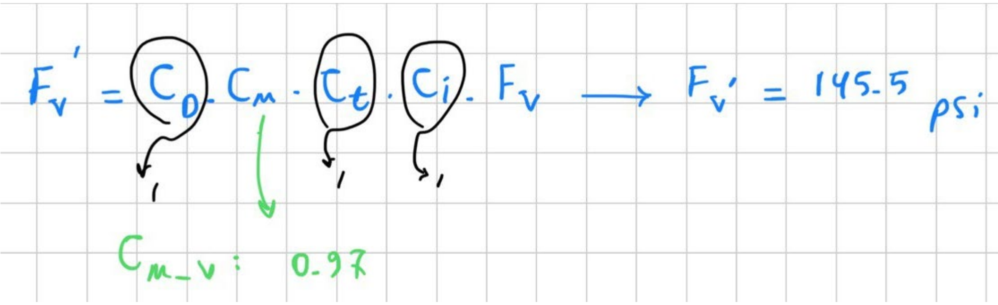
if $F_b C_F \leq 1150 \longrightarrow C_M = 1$

$> 1150 \longrightarrow C_M = 0.85$

$(1400)(1.3) = 1820 > 1150 \longrightarrow C_M = 0.85$

C_r:

joist o.c. $32'' > 24'' \longrightarrow C_r = 1.0$



$$F_v' = \underbrace{C_D}_{1} \cdot \underbrace{C_M}_{C_{M\_v}:\ 0.97} \cdot \underbrace{C_t}_{1} \cdot \underbrace{C_i}_{1} \cdot F_v \longrightarrow F_v' = 145.5 \text{ psi}$$

Provide the solution for the assignment – **HW2**

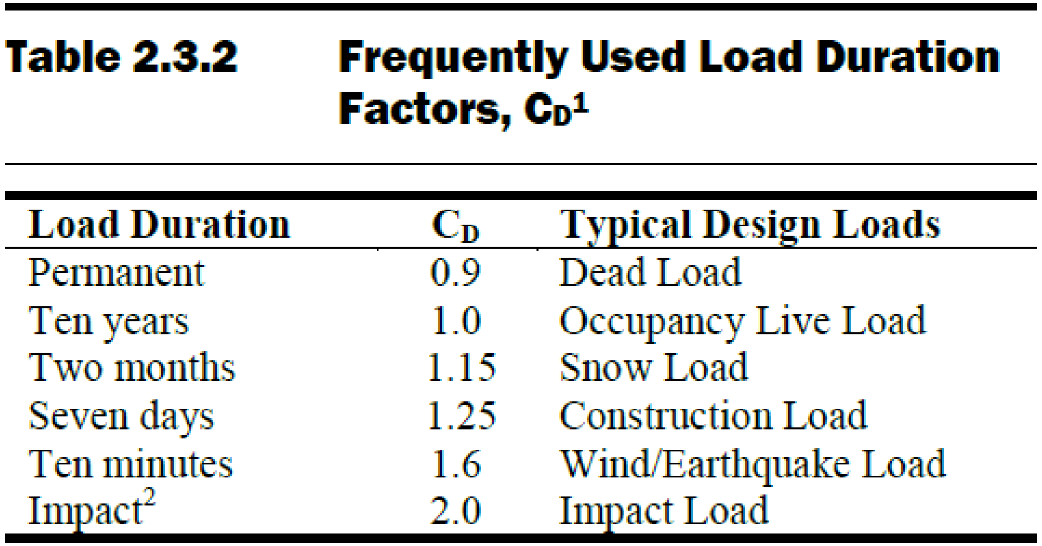


Table 2.3.2 Frequently Used Load Duration Factors, C_D[1]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact[2]	2.0	Impact Load

Load duration is based on the live load ($C_D = 1.0$)

Provide the solution for the assignment – **HW2**

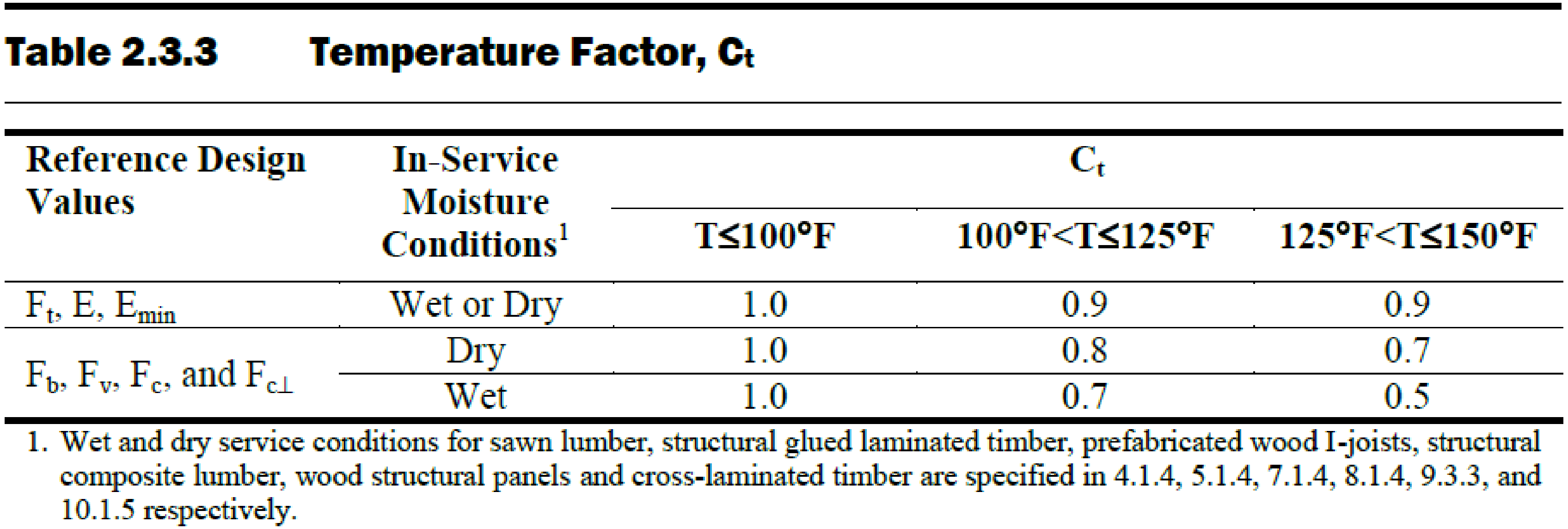


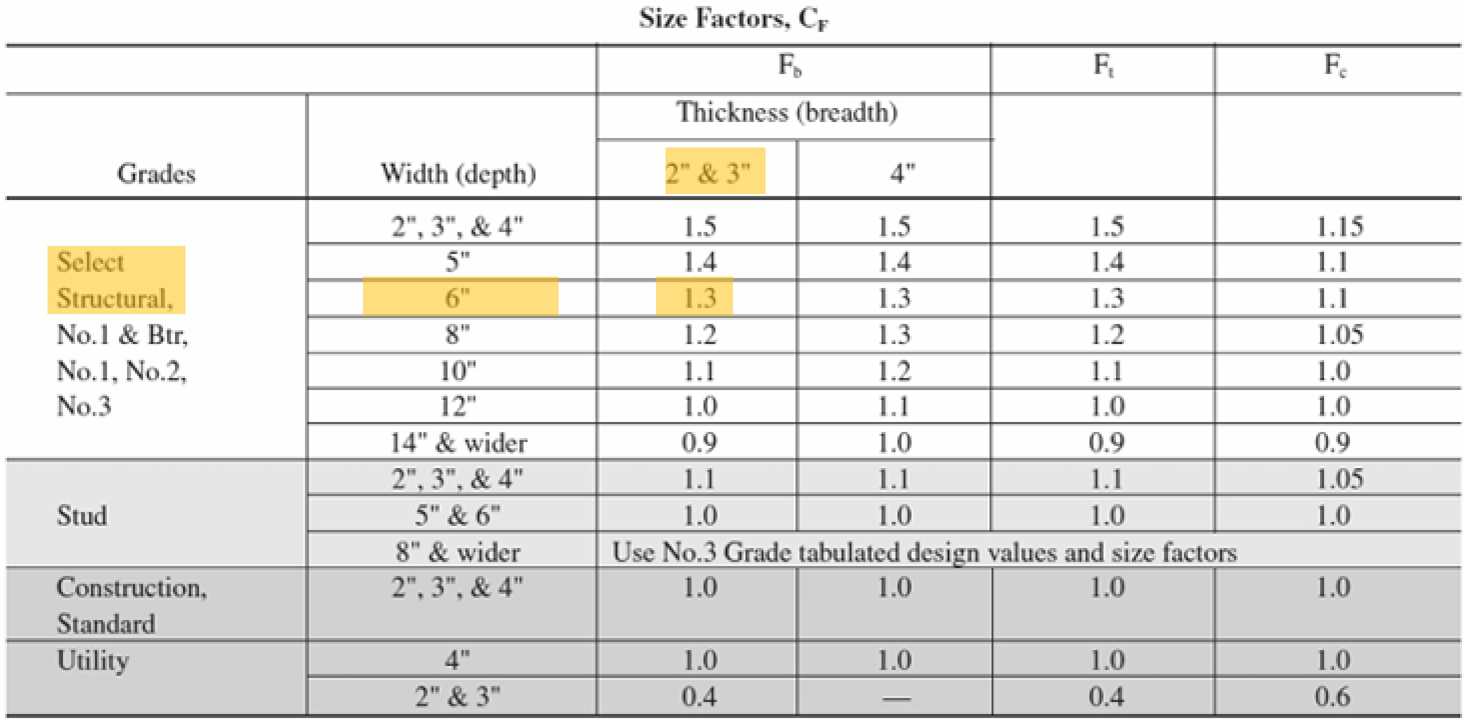
Table 2.3.3 Temperature Factor, C_t

Reference Design Values	In-Service Moisture Conditions[1]	C_t		
		$T \leq 100°F$	$100°F < T \leq 125°F$	$125°F < T \leq 150°F$
F_t, E, E_{min}	Wet or Dry	1.0	0.9	0.9
F_b, F_v, F_c, and $F_{c\perp}$	Dry	1.0	0.8	0.7
	Wet	1.0	0.7	0.5

1. Wet and dry service conditions for sawn lumber, structural glued laminated timber, prefabricated wood I-joists, structural composite lumber, wood structural panels and cross-laminated timber are specified in 4.1.4, 5.1.4, 7.1.4, 8.1.4, 9.3.3, and 10.1.5 respectively.

normal temperature, $C_t = 1.0$

Provide the solution for the assignment – **HW2**



Size Factors, C_F

		F_b		F_t	F_c
		Thickness (breadth)			
Grades	Width (depth)	2" & 3"	4"		
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Size Factors, C_F = 1.3

Provide the solution for the assignment – **HW2**

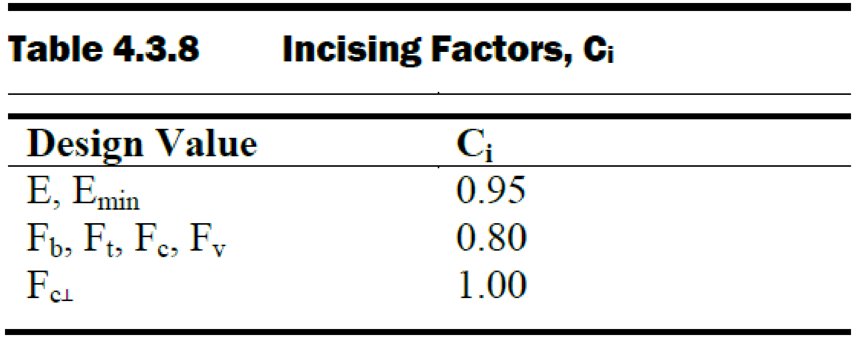
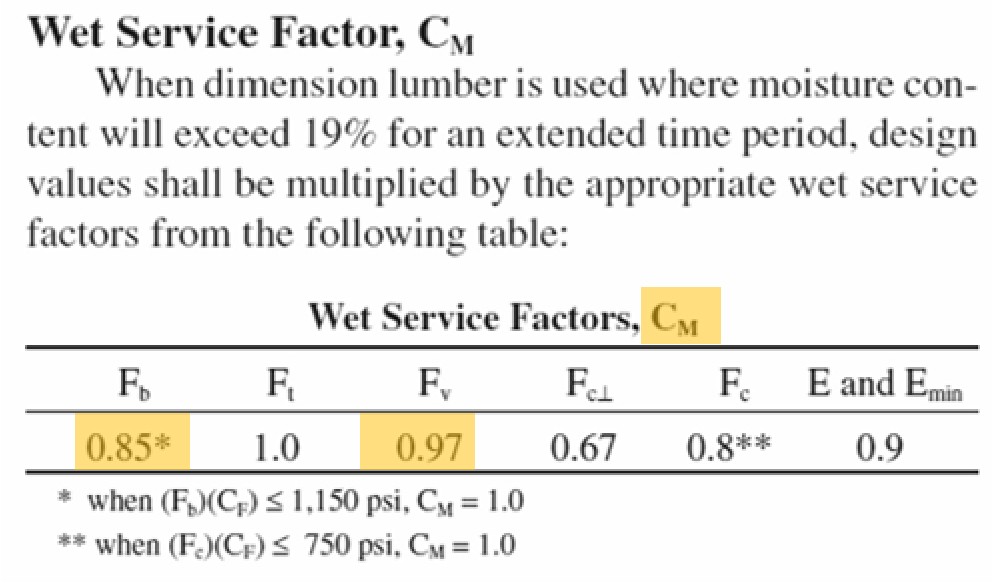


Table 4.3.8 Incising Factors, C_i

Design Value	C_i
E, E_{min}	0.95
F_b, F_t, F_c, F_v	0.80
$F_{c\perp}$	1.00



Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

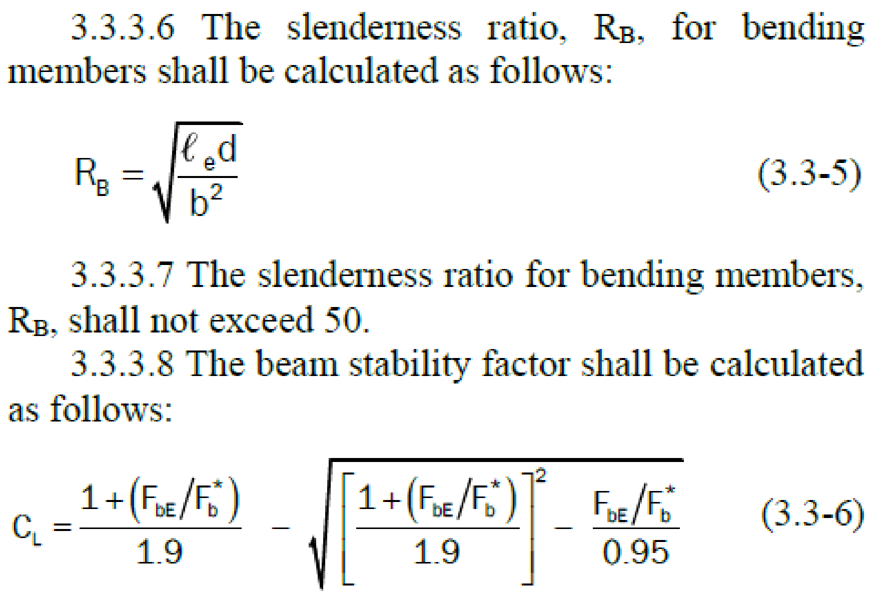
* when $(F_b)(C_F) \leq 1{,}150$ psi, $C_M = 1.0$

** when $(F_c)(C_F) \leq 750$ psi, $C_M = 1.0$

no incising, $C_i = 1.0$

Wet Service Factors, C_M

Provide the solution for the assignment – **HW2**



3.3.3.6 The slenderness ratio, R_B, for bending members shall be calculated as follows:

$$R_B = \sqrt{\frac{\ell_e d}{b^2}} \qquad (3.3\text{-}5)$$

3.3.3.7 The slenderness ratio for bending members, R_B, shall not exceed 50.

3.3.3.8 The beam stability factor shall be calculated as follows:

$$C_L = \frac{1+\left(F_{bE}/F_b^*\right)}{1.9} - \sqrt{\left[\frac{1+\left(F_{bE}/F_b^*\right)}{1.9}\right]^2 - \frac{F_{bE}/F_b^*}{0.95}} \qquad (3.3\text{-}6)$$

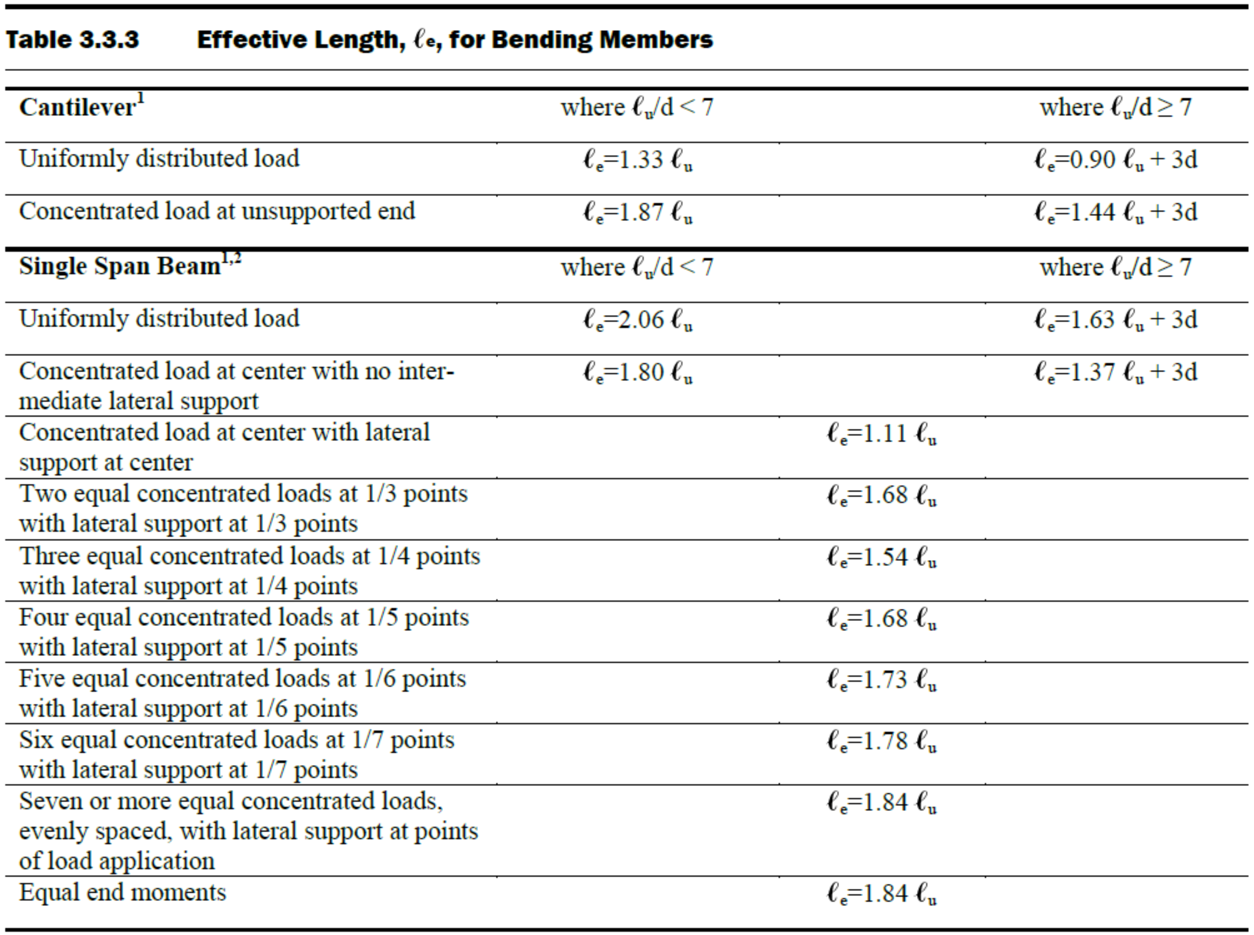


Table 3.3.3 Effective Length, ℓ_e, for Bending Members

Cantilever[1]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	$\ell_e = 1.33\,\ell_u$		$\ell_e = 0.90\,\ell_u + 3d$
Concentrated load at unsupported end	$\ell_e = 1.87\,\ell_u$		$\ell_e = 1.44\,\ell_u + 3d$
Single Span Beam[1,2]	where $\ell_u/d < 7$		where $\ell_u/d \geq 7$
Uniformly distributed load	$\ell_e = 2.06\,\ell_u$		$\ell_e = 1.63\,\ell_u + 3d$
Concentrated load at center with no intermediate lateral support	$\ell_e = 1.80\,\ell_u$		$\ell_e = 1.37\,\ell_u + 3d$
Concentrated load at center with lateral support at center		$\ell_e = 1.11\,\ell_u$	
Two equal concentrated loads at 1/3 points with lateral support at 1/3 points		$\ell_e = 1.68\,\ell_u$	
Three equal concentrated loads at 1/4 points with lateral support at 1/4 points		$\ell_e = 1.54\,\ell_u$	
Four equal concentrated loads at 1/5 points with lateral support at 1/5 points		$\ell_e = 1.68\,\ell_u$	
Five equal concentrated loads at 1/6 points with lateral support at 1/6 points		$\ell_e = 1.73\,\ell_u$	
Six equal concentrated loads at 1/7 points with lateral support at 1/7 points		$\ell_e = 1.78\,\ell_u$	
Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application		$\ell_e = 1.84\,\ell_u$	
Equal end moments		$\ell_e = 1.84\,\ell_u$	

Beam stability factor, C_L

The beam is braced at the ends and the C.L. (meets criteria in 4.4.1) so $C_L = 1.0$.

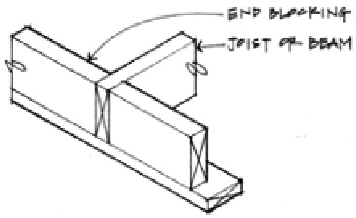
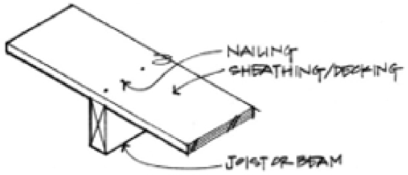
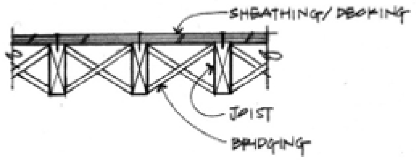
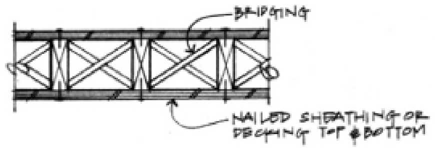
Provide the solution for the assignment – **HW2**

C_L

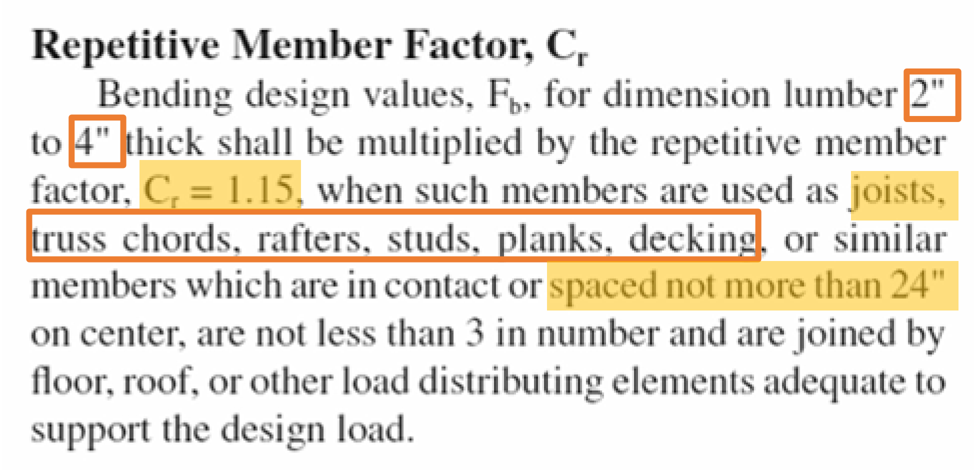
$C_L = 1.0$
when bracing meets 4.4.1
for the depth/width ratio

Otherwise

$C_L < 1.0$
calculate factor using
section 3.3.3

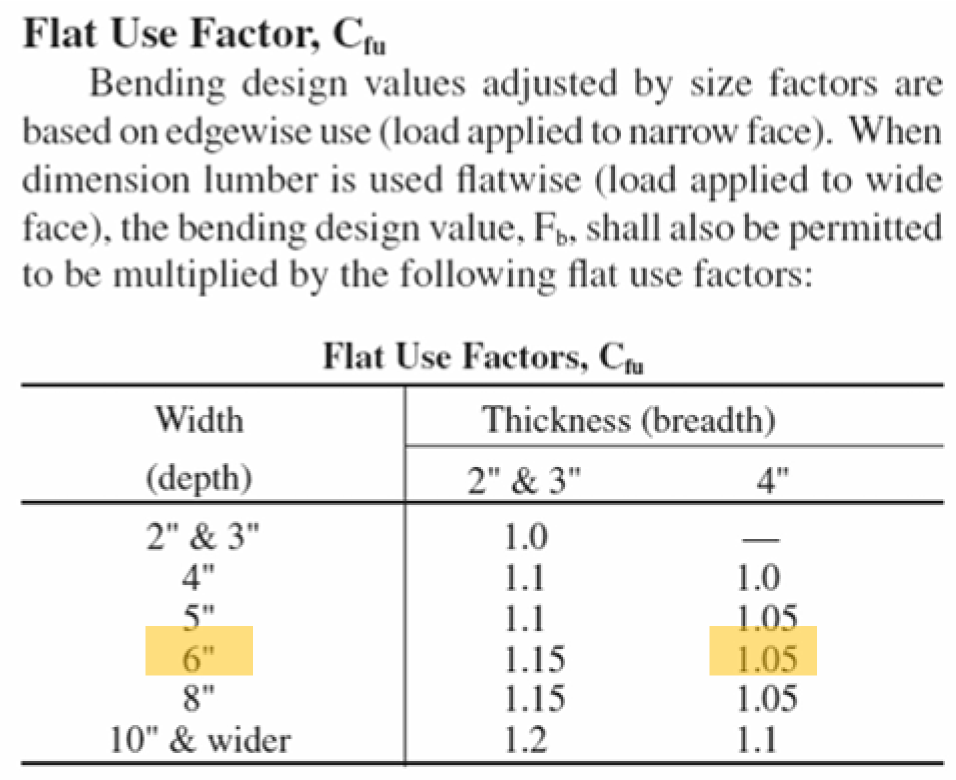
Beam Depth/ Width Ratio	Type of Lateral Bracing Required	Example
2 to 1	None	
3 to 1 2x6 2x8	The ends of the beam should be held in position	
5 to 1 2x10	Hold compression edge in line (continuously)	
6 to 1 2x12	Diagonal bridging should be used	
7 to 1 2x14	Both edges of the beam should be held in line	

Provide the solution for the assignment – **HW2**



Repetitive Member Factor, C_r

Bending design values, F_b, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r = 1.15$, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.



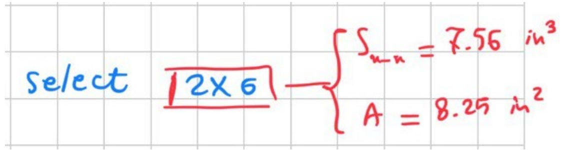
Flat Use Factor, C_{fu}

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, F_b, shall also be permitted to be multiplied by the following flat use factors:

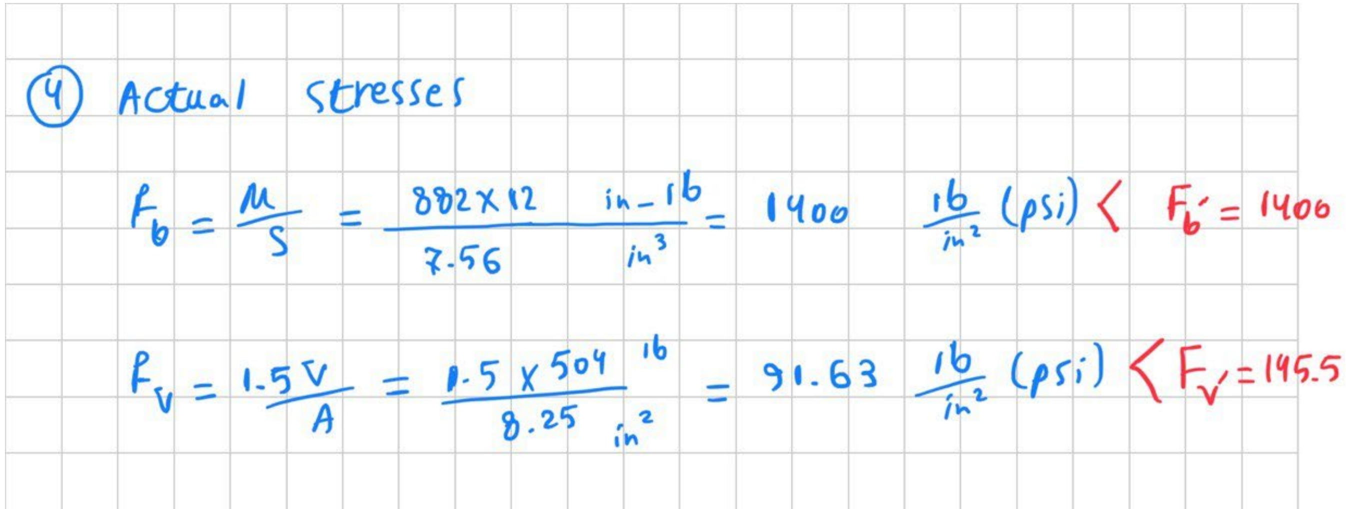
Flat Use Factors, C_{fu}

Width (depth)	Thickness (breadth) 2" & 3"	Thickness (breadth) 4"
2" & 3"	1.0	—
4"	1.1	1.0
5"	1.1	1.05
6"	1.15	1.05
8"	1.15	1.05
10" & wider	1.2	1.1

Provide the solution for the assignment – **HW2**



select $\boxed{2X6}$ — $\begin{cases} S_{x-x} = 7.56 \ in^3 \\ A = 8.25 \ in^2 \end{cases}$

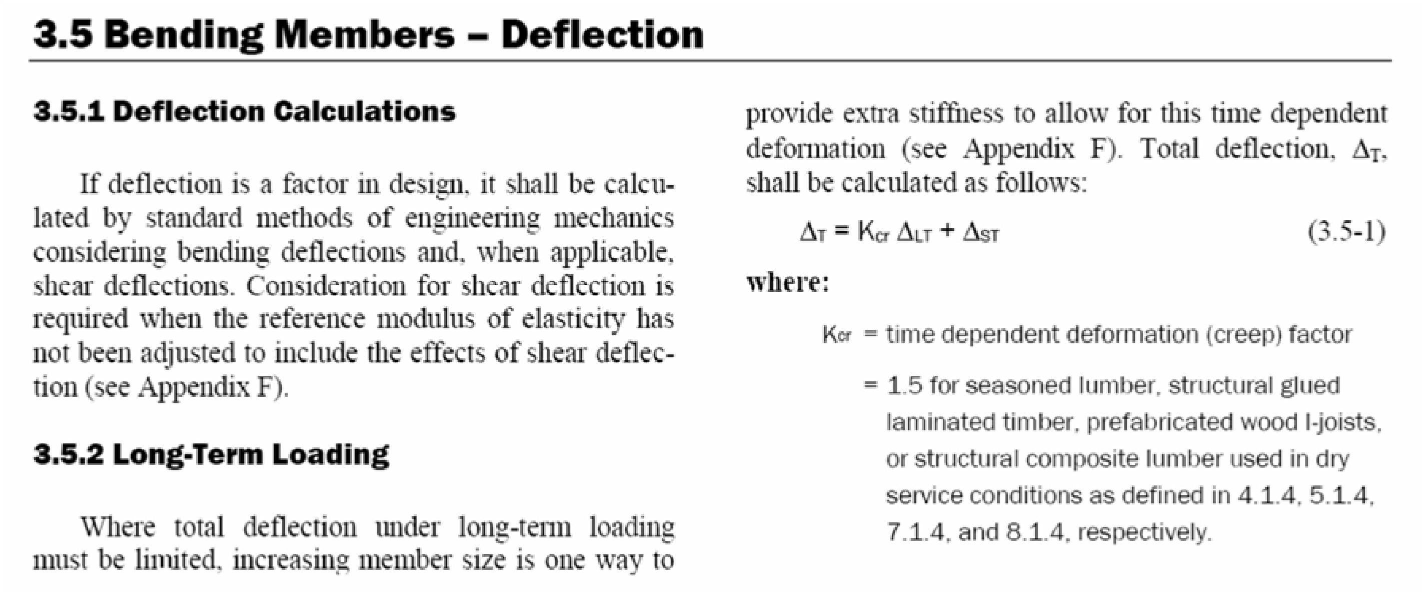


④ Actual stresses

$$f_b = \frac{M}{S} = \frac{882 X 12 \quad in-lb}{7.56 \quad in^3} = 1400 \ \frac{lb}{in^2} (psi) < F_b' = 1400$$

$$f_v = \frac{1.5V}{A} = \frac{1.5 \ x \ 504 \quad lb}{8.25 \quad in^2} = 91.63 \ \frac{lb}{in^2} (psi) < F_v' = 145.5$$

Provide the solution for the assignment – **HW2**



3.5 Bending Members – Deflection

3.5.1 Deflection Calculations

If deflection is a factor in design, it shall be calculated by standard methods of engineering mechanics considering bending deflections and, when applicable, shear deflections. Consideration for shear deflection is required when the reference modulus of elasticity has not been adjusted to include the effects of shear deflection (see Appendix F).

3.5.2 Long-Term Loading

Where total deflection under long-term loading must be limited, increasing member size is one way to provide extra stiffness to allow for this time dependent deformation (see Appendix F). Total deflection, Δ_T, shall be calculated as follows:

$$\Delta_T = K_{cr}\,\Delta_{LT} + \Delta_{ST} \qquad (3.5\text{-}1)$$

where:

K_{cr} = time dependent deformation (creep) factor

= 1.5 for seasoned lumber, structural glued laminated timber, prefabricated wood I-joists, or structural composite lumber used in dry service conditions as defined in 4.1.4, 5.1.4, 7.1.4, and 8.1.4, respectively.

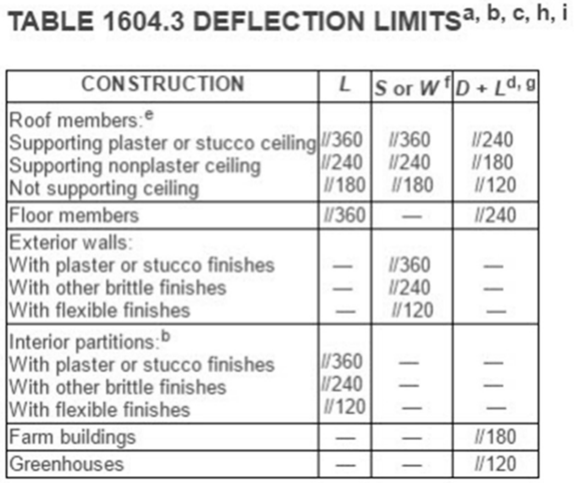
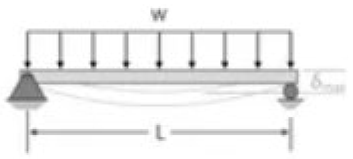


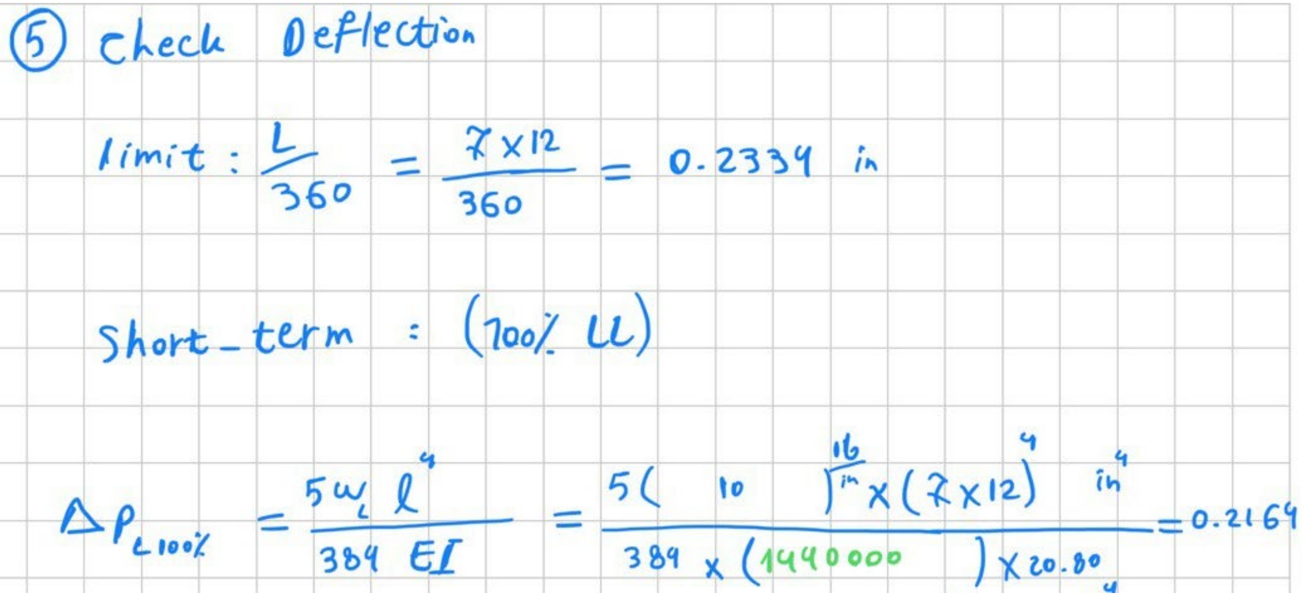
TABLE 1604.3 DEFLECTION LIMITS[a, b, c, h, i]

CONSTRUCTION	L	S or W [f]	$D + L$ [d, g]
Roof members:[e]			
Supporting plaster or stucco ceiling	l/360	l/360	l/240
Supporting nonplaster ceiling	l/240	l/240	l/180
Not supporting ceiling	l/180	l/180	l/120
Floor members	l/360	—	l/240
Exterior walls:			
With plaster or stucco finishes	—	l/360	—
With other brittle finishes	—	l/240	—
With flexible finishes	—	l/120	—
Interior partitions:[b]			
With plaster or stucco finishes	l/360	—	—
With other brittle finishes	l/240	—	—
With flexible finishes	l/120	—	—
Farm buildings	—	—	l/180
Greenhouses	—	—	l/120



$$\delta_{max} = \frac{5wL^4}{384EI}$$

Provide the solution for the assignment – **HW2**

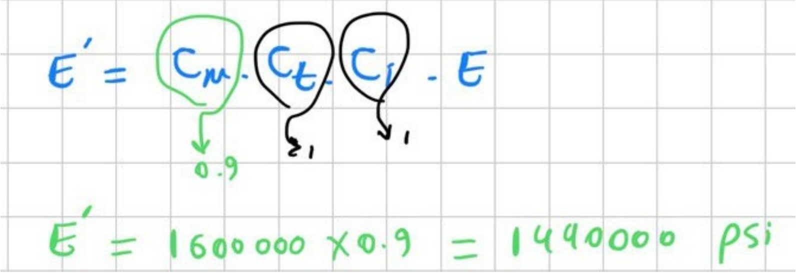


⑤ Check Deflection

$$\text{limit}: \frac{L}{360} = \frac{7\times 12}{360} = 0.2334 \text{ in}$$

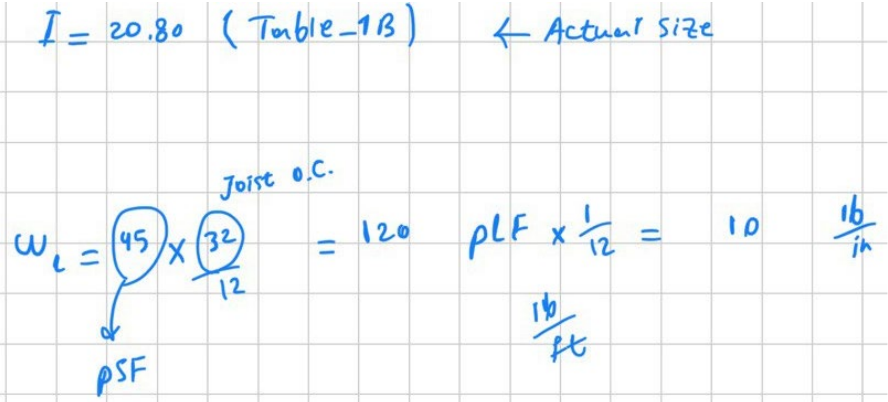
Short_term : (100% LL)

$$\Delta P_{L100\%} = \frac{5 w_L l^4}{384\ EI} = \frac{5(10 \overset{lb}{/in}) \times (7\times 12)^4 \text{ in}^4}{384 \times (1440000) \times 20.80 \text{ in}^4} = 0.2164$$



$$E' = \underset{\downarrow\, 0.9}{C_M} \cdot \underset{\downarrow\, 1}{C_t} \cdot \underset{\downarrow\, 1}{C_i} \cdot E$$

$$E' = 1600000 \times 0.9 = 1440000 \text{ psi}$$



$I = 20.80$ (Table_1B) ← Actual Size

$$w_L = \underset{\downarrow\, psF}{45} \times \overset{\text{Joist o.C.}}{\frac{32}{12}} = \underset{lb/ft}{120 \text{ plF}} \times \frac{1}{12} = 10 \ \frac{lb}{in}$$

Thank you.

Any question?

Please feel free to ask questions.

Contact: Mohsenv@umich.edu