Structure II Recitation 2/2

Wood Column Analysis

Before we start …

Today's Tasks

1. Homework Example (Wood Column Analysis) (15 Questions)
2. Lab (Columns)

Reminder

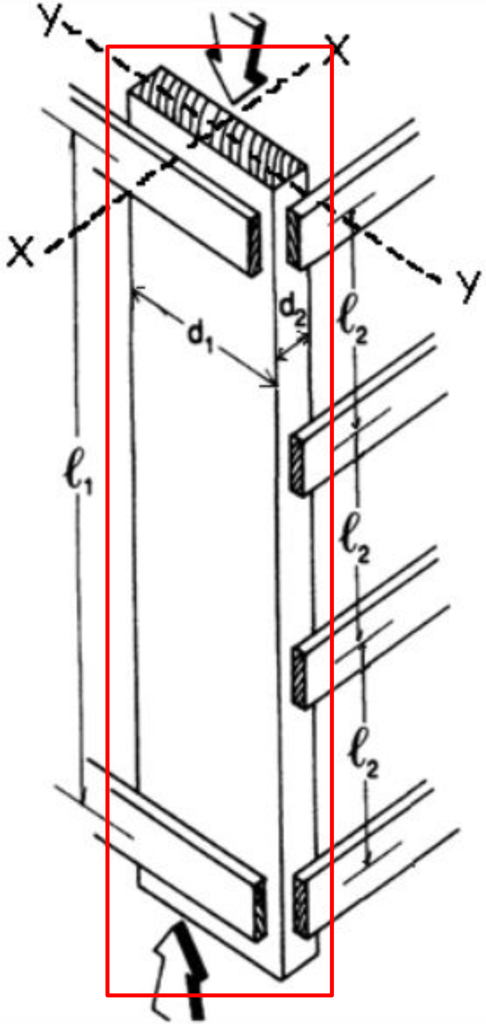
1. Preliminary report due at **2/16**
2. Tower testing at **3/18**

3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

DATASET: 1 -2- -3-

Wood Species	EASTERN HEMLOCK-TAMARACK
Wood Grade	Select Structural
Strong Axis Length, L1	15 FT
Weak Axis Length, L2	5 FT
Narrow Width, d2	4 IN
Wide Width, d1	10 IN
LoadType	Dead Load



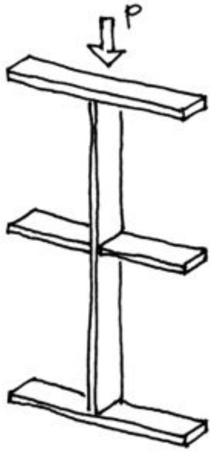
Analysis of Wood Columns

Data:

- Column – size, length
- Support conditions
- Material properties – F_c, E
- Load

Required:

- Pass/Fail or margin of safety



1. Calculate slenderness ratio l_e/d
   largest ratio governs. Must be < 50

2. Find adjustment factors
   C_D C_M C_t C_F C_i

3. Calculate C_P

4. Determine allowable F'c by multiplying the tabulated Fc by all the above factors

5. Calculate the actual stress: fc = P/A
6. Compare Allowable and Actual stress.
   F'c > fc passes

Q1: Tabulated Allowable Compressive Stress (Fc)
Q2: Tabulated Minimum Modulus of Elasticity (Emin)

Given from Question

Wood Species	EASTERN HEMLOCK-TAMARACK
Wood Grade	Select Structural

Check Table 4A:

Fc = **1200 psi,** Emin = **440000 psi**

NDS Supplement, Table 4A, P.41~(PDF)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific Gravity[4]	Grading Rules Agency
		Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	G	
EASTERN HEMLOCK-TAMARACK										
Select Structural	2" & wider	1,250	575	170	555	1,200	1,200,000	440,000	0.41	NELMA NSLB
No. 1	2" & wider	775	350	170	555	1,000	1,100,000	400,000	0.41	NELMA NSLB
No. 2	2" & wider	575	275	170	555	825	1,100,000	400,000	0.41	NELMA NSLB
No. 3	2" & wider	350	150	170	555	475	900,000	330,000	0.41	NELMA NSLB
Stud	2" & wider	450	200	170	555	525	900,000	330,000	0.41	NELMA NSLB
Construction	2" - 4" wide	675	300	170	555	1,050	1,000,000	370,000	0.41	NELMA NSLB
Standard	2" - 4" wide	375	175	170	555	850	900,000	330,000	0.41	NELMA NSLB
Utility	2" - 4" wide	175	75	170	555	550	800,000	290,000	0.41	NELMA NSLB
EASTERN SOFTWOODS										

Q3: Load Duration Factor (C_D)
Look at Table 2.3.2,
Since my load type is Dead Load,
C_D = **0.9**

Q4: Size Factor (C_F)
Look at Table 4A,
C_F = **1.0**

Given from Question

Wood Species	EASTERN HEMLOCK-TAMARACK
Wood Grade	Select Structural
Strong Axis Length, L1	15 FT
Weak Axis Length, L2	5 FT
Narrow Width, d2	4 IN
Wide Width, d1	10 IN
LoadType	Dead Load

Table 2.3.2 Frequently Used Load Duration Factors, C_D[1]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact[2]	2.0	Impact Load

NDS Supplement, Table 4A, P.40 (PDF)

Size Factors, C_F

Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b Thickness (breadth) 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

Q5: Factored Allowable Modulus of Elasticity (E'min)

$E'min = Emin \times (C_M \times C_t \times C_i \times C_T)$

Given from Question:

$C_t = C_i = 1$

(Don't need to consider C_T since its for trusses)

For C_M, Check if M.C. > 19%

If yes, $C_M = 0.9$

If not, $C_M = 1$

Since my M.C. = 15% < 19%, $C_M = 1$

Calculation:

E'min = 440000 x 1 x1 x 1 x 1= **440000 psi**

↑ from Q2

Given from Question

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

$F_{c\perp}' = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_b	K_F	ϕ_c	λ
$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	-	-	-
$E_{min}' = E_{min}$	x	-	C_M	C_t	-	-	-	C_i	-	-	C_T	-	K_F	ϕ_s	-

NDS Code, 4.4.2, P.43 (PDF)

4.4.2 Wood Trusses

4.4.2.1 Increased chord stiffness relative to axial loads where a 2" x 4" or smaller sawn lumber truss compression chord is subjected to combined flexure and axial compression under dry service condition and has 3/8" or thicker plywood sheathing nailed to the narrow face of the chord in accordance with code required roof sheathing fastener schedules (see References 32, 33, and 34), shall be permitted to be accounted for by multiplying the reference modulus of elasticity design value for beam and column stability, E_{min}, by the buckling stiffness factor, C_T, in column stability calculations (see 3.7 and Appendix H). When ℓ_e < 96", C_T shall be calculated as follows:

$$C_T = 1 + \frac{K_M \ell_e}{K_T E} \quad (4.4\text{-}1)$$

NDS Supplement, Table 4A, P.40 (PDF)

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \le 1{,}150$ psi, $C_M = 1.0$

** when $(F_c)(C_F) \le 750$ psi, $C_M = 1.0$

Q6: Strong Axis (x-x) Slenderness Ratio (le_x/d_1)

le_x = K x L1 = 1 x 15 = 15 ft

d_1 = 9.25 in (Check Table 1B to find the actual size)

Slenderness Ratio = 15 / 9.25 x 12 = **19.459**

Convert Unit

Q7: Weak Axis (y-y) Slenderness Ratio (le_y/d_2)

le_y = K x L2 = 1 x 5 = 5 ft

d_2 = 3.5 in (Check Table 1B to find the actual size)

Slenderness Ratio = 5 / 3.5 x 12 = **17.143**

Convert Unit

Q8: Controlling Slenderness Ratio(le/d)

Compare the answer of Q6 & Q7,

The bigger one controls,

For my situation is **19.459**

Given from Question

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

Wood Species	EASTERN HEMLOCK-TAMARACK
Wood Grade	Select Structural
Strong Axis Length, L1	15 FT
Weak Axis Length, L2	5 FT
Narrow Width, d2	4 IN
Wide Width, d1	10 IN
LoadType	Dead Load

NDS Supplement, Table 1B, P.22 (PDF)

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A $in.^2$	X-X AXIS Section Modulus S_{xx} $in.^3$	X-X AXIS Moment of Inertia I_{xx} $in.^4$	Y-Y AXIS Section Modulus S_{yy} $in.^3$	Y-Y AXIS Moment of Inertia I_{yy} $in.^4$	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Boards[1]												
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91	13.24
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53

Q9: Critical Buckling Design Value for Compression (F_{cE})

Formula:

F_{cE} = (0.822 x E'min) / (le/d)^2

from Q2 from Q8

Calculation:

F_{cE} = (0.822 x 440000) / (19.459)^2 = **955.131 psi**

$$F_{cE} = \frac{0.822E'_{min}}{\left(\frac{l_e}{d}\right)^2}$$

Q10: Reference Compression Design Value (Fc*)

Formula:

$$Fc^* = Fc \times (C_D \times C_M \times C_t \times C_F \times C_i)$$

Given from question:

$C_t = C_i = 1$

Get C_F from Q3, C_D from Q4

For C_M, first check if M.C. > 19%

If not, $C_M = 1$

If yes, then check if ($Fc \times C_F$) <= 750 psi.

If yes $C_M = 1$,

If not, $C_M = 0.8$

Since my M.C. = 15% < 19%, $C_M = 1$

Calculation:

$$Fc^* = 1200 \times (0.9 \times 1 \times 1 \times 1 \times 1) = \underline{\mathbf{1080\ psi}}$$

↑ from Q1

Given from Question

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

F_c^* = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_P (see 2.3), psi

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \leq 1{,}150$ psi, $C_M = 1.0$

** when $(F_c)(C_F) \leq 750$ psi, $C_M = 1.0$

$F_t' = F_t$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	-	-	2.70	0.80	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	2.88	0.75	λ
$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	2.40	0.90	λ
$F_{c\perp}' = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_b	1.67	0.90	-
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	-	-	-

Q11: Constant for Saw Lumber (c)

c = <u>0.8</u>

Course Slides P.14

Q12: Column Stability Factor (C_p)

First calculate (FcE/Fc*), then put it into the formula:

FcE/Fc* = 955.131/1080 = 0.884

$$\frac{1+0.884}{1.6} - \sqrt{\left(\frac{1+0.884}{1.6}\right)^2 - \frac{0.884}{0.8}} = 0.646928\ldots$$

$$C_p = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.7\text{-}1)$$

where:

from Q10 — F_c^* = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_P (see 2.3), psi

from Q9 — $F_{cE} = \dfrac{0.822\, E_{min}'}{(\ell_e/d)^2}$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Q13: Factored Allowable Compressive Stress (F'c)

$F'c = Fc^* \times C_p = 1080 \times 0.647 = \underline{\textbf{698.85 psi}}$

(1080 from Q10; 0.647 from Q12)

Q14: Column Area (A)

Check Table 1B for the column (section) area,

$A = 3.5 \times 9.25 = \underline{\textbf{32.375 in}^2}$

Q15: Maximum Allowable Axial Load Capacity (Pmax)

$Pmax = F'c \times A = 698.85 \times 32.375 = \underline{\textbf{22625.3 LBS}}$

(698.85 from Q13; 32.375 from Q14)

NDS Supplement, Table 1B, P.22 (PDF)

Table 1B Section Properties of Stan

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.[2]	X-X AXIS Section Modulus S_{xx} in.[3]	X-X AXIS Moment of Inertia I_{xx} in.[4]
Boards[1]				
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034

Questions?

Lab Session:

Goals:

1. Calculate the slenderness ratio and the critical buckling load (Pcr) for different lengths (L = 6”, 3”, 1”) (Q1~Q4)

2. Calculate the ultimate crushing load (Pmax) (Q5)

3. Locate all the Pcr on the slenderness curve (Q6)

Columns

Description

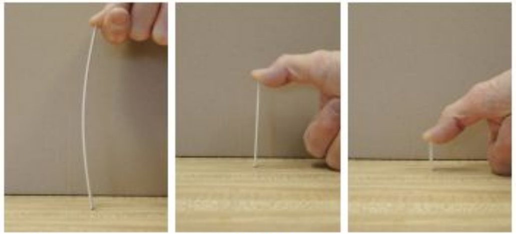
This project uses observation and calculation to understand the effect of slenderness on column capacity.

Goals

To observe the buckling behavior of columns through physical modeling.
To find the controlling slenderness ratio.
To calculate the critical buckling and crushing loads.

Procedure

1. For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and Pcr using the Euler equation. Use K=1.0.
2. Find the actual critical buckling load approximating the load with your finger.
3. Repeat the procedure for L=3" and L=1".
4. Calculate the slenderness and Pcr for both of these lengths.
5. Calculate the ultimate crushing load based on the max compressive stress, F_c.
6. Approximately locate P for each length on the load vs. slenderness curve shown below



Basswood Properties

Emin = 1,650,000. psi

Fc = 4745 psi
Area = 0.015625 in²
d_1 = 0.25 in
d_2 = 0.0625 in

Equations:

$$FcE = \frac{0.822\ E'min}{(le/d)2}$$

$$P_{max} = F_c \times A$$

L = 6"	L/d =	
		P =
L = 3"	L/d =	
		P =
L = 1"	L/d =	
		P =