Wood Columns 2/2

HW - Wood Column Analysis

Tower Project Intro

Lab - Columns

Structure II
Section 004

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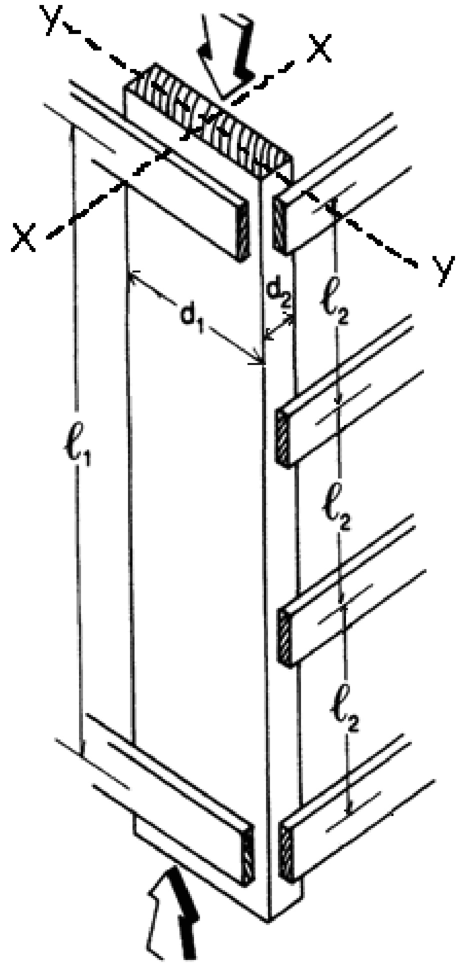
HW - Wood Column Analysis

3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

DATASET: 1 -2- -3-

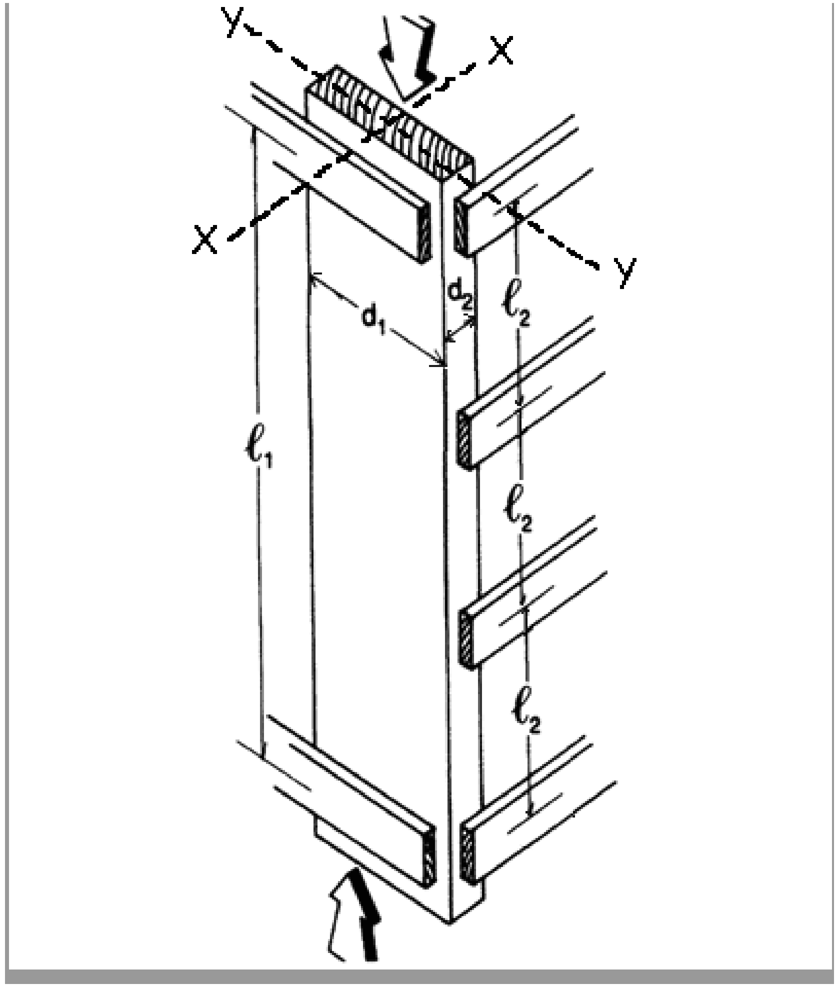
Wood Species	WESTERN CEDARS
Wood Grade	No.1
Strong Axis Length, L1	8 FT
Weak Axis Length, L2	2.666666667 FT
Narrow Width, d2	4 IN
Wide Width, d1	10 IN
LoadType	Wind Load



Given:
certain wood type
bracing type
member size
load type

Goal:
load capacity?

HW - Wood Column Analysis



Given:
certain wood type
bracing type
member size
load type

Goal:
load capacity?

1. Calculate slenderness ratio le/d

   largest ratio governs. Must be < 50

2. Find adjustment factors

   CD CM Ct CF Ci

3. Calculate CP(column stability factor)
4. Determine F'c = tabulated Fc * factors
5. Set actual stress = allowable, fc=F'c (limitation)
6. Find the maximum allowable load Pmax=F'c / A

3. Wood Column Analysis

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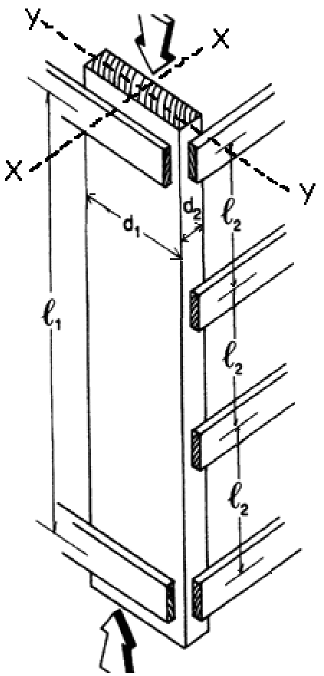

1. Tabulated Allow. Compressive Stress, Fc= 825 psi

2. Tabulated Minimum Modulus of Elasticity, Emin= 370000 psi

NDS Supplement table 4A

Table 4A (Cont.) **Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)[1,2,3]**

(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

USE WITH TABLE 4A ADJUSTMENT FACTORS

Species and commercial grade	Size classification	Bending F_b	Tension parallel to grain F_t	Shear parallel to grain F_v	Compression perpendicular to grain $F_{c\perp}$	Compression parallel to grain F_c	Modulus of Elasticity E	Modulus of Elasticity E_{min}	Specific Gravity[4] G	Grading Rules Agency
WESTERN CEDARS										
Select Structural	2" & wider	1,000	600	155	425	1,000	1,100,000	400,000	0.36	WCLIB WWPA
No. 1		725	425	155	425	825	1,000,000	370,000		
No. 2		700	425	155	425	650	1,000,000	370,000		
No. 3		400	250	155	425	375	900,000	330,000		
Stud	2" & wider	550	325	155	425	400	900,000	330,000		
Construction	2" - 4" wide	800	475	155	425	850	900,000	330,000		
Standard		450	275	155	425	650	800,000	290,000		
Utility		225	125	155	425	425	800,000	290,000		

3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

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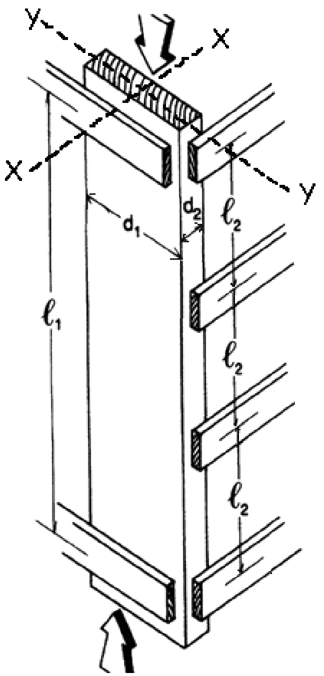

3. Load duration Factor, CD

CD = 1.6

Table 2.3.2 Frequently Used Load Duration Factors, C_D[1]

Load Duration	C_D	Typical Design Loads
Permanent	0.9	Dead Load
Ten years	1.0	Occupancy Live Load
Two months	1.15	Snow Load
Seven days	1.25	Construction Load
Ten minutes	1.6	Wind/Earthquake Load
Impact[2]	2.0	Impact Load

4. Size Factor, CF

CF_c = 1.0

Size Factors, C_F

Grades	Width (depth)	F_b Thickness (breadth) 2" & 3"	F_b Thickness (breadth) 4"	F_t	F_c
Select Structural, No.1 & Btr, No.1, No.2, No.3	2", 3", & 4"	1.5	1.5	1.5	1.15
	5"	1.4	1.4	1.4	1.1
	6"	1.3	1.3	1.3	1.1
	8"	1.2	1.3	1.2	1.05
	10"	1.1	1.2	1.1	1.0
	12"	1.0	1.1	1.0	1.0
	14" & wider	0.9	1.0	0.9	0.9
Stud	2", 3", & 4"	1.1	1.1	1.1	1.05
	5" & 6"	1.0	1.0	1.0	1.0
	8" & wider	Use No.3 Grade tabulated design values and size factors			
Construction, Standard	2", 3", & 4"	1.0	1.0	1.0	1.0
Utility	4"	1.0	1.0	1.0	1.0
	2" & 3"	0.4	—	0.4	0.6

NDS Supplement table 4A

3. Wood Column Analysis

For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

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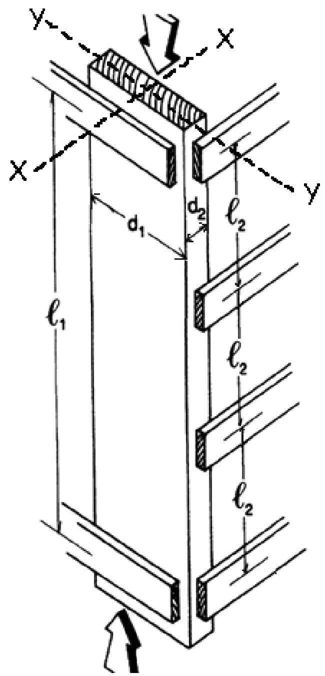

5. Factored Allow. Modulus of Elasticity, E'min

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

		ASD only	ASD and LRFD										LRFD only		
		Load Duration Factor	Wet Service Factor	Temperature Factor	Beam Stability Factor	Size Factor	Flat Use Factor	Incising Factor	Repetitive Member Factor	Column Stability Factor	Buckling Stiffness Factor	Bearing Area Factor	Format Conversion Factor	Resistance Factor	Time Effect Factor
$F_b' = F_b$	x	C_D	C_M	C_t	C_L	C_F	C_{fu}	C_i	C_r	-	-	-	K_F	ϕ_b	λ
$F_t' = F_t$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	-	-	-	K_F	ϕ_t	λ
$F_v' = F_v$	x	C_D	C_M	C_t	-	-	-	C_i	-	-	-	-	K_F	ϕ_v	λ
$F_{c\perp}' = F_{c\perp}$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	C_b	K_F	ϕ_c	λ
$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ
$E' = E$	x	-	C_M	C_t	-	-	-	C_i	-	-	-	-	-	-	-
$E_{min}' = E_{min}$	x	-	C_M	C_t	-	-	-	C_i	-	-	C_T	-	K_F	ϕ_s	-

Wet Service Factor, C_M

When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:

Wet Service Factors, C_M

F_b	F_t	F_v	$F_{c\perp}$	F_c	E and E_{min}
0.85*	1.0	0.97	0.67	0.8**	0.9

* when $(F_b)(C_F) \leq 1{,}150$ psi, $C_M = 1.0$

** when $(F_c)(C_F) \leq 750$ psi, $C_M = 1.0$

4.4.2 Wood Trusses

4.4.2.1 Increased chord stiffness relative to axial loads where a 2" x 4" or smaller sawn lumber truss compression chord is subjected to combined flexure and axial compression under dry service condition and has 3/8" or thicker plywood sheathing nailed to the narrow face of the chord in accordance with code required roof sheathing fastener schedules (see References 32, 33, and 34), shall be permitted to be accounted for by multiplying the reference modulus of elasticity design value for beam and column stability, E_{min}, by the buckling stiffness factor, C_T, in column stability calculations (see 3.7 and Appendix H). When $\ell_e < 96"$, C_T shall be calculated as follows:

$$C_T = 1 + \frac{K_M \ell_e}{K_T E} \qquad (4.4\text{-}1)$$

E'min = Emin * (CM * Ct * Ci * CT) = 370000 x 1= 370000 psi

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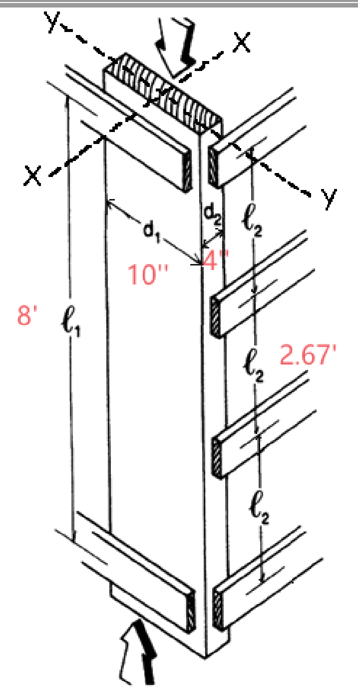

6. Strong Axis(x-x) Slenderness Ratio, lex/d1

use actual member size   lex/d1 = 8*12/9.25 = 10.38

7. Weak Axis(y-y) Slenderness Ratio, ley/d2

use actual member size   ley/d2 = 2.67*12/3.5 = 9.15

8. Controlling Slenderness Ratio, le/d

The larger ratio will govern   10.38 > 9.15   So le/d = 10.38

Table 1B Section Properties of Standard Dressed (S4S) Sawn Lumber

Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Boards[1]												
1 x 3	3/4 x 2-1/2	1.875	0.781	0.977	0.234	0.088	0.326	0.391	0.456	0.521	0.586	0.651
1 x 4	3/4 x 3-1/2	2.625	1.531	2.680	0.328	0.123	0.456	0.547	0.638	0.729	0.820	0.911
1 x 6	3/4 x 5-1/2	4.125	3.781	10.40	0.516	0.193	0.716	0.859	1.003	1.146	1.289	1.432
1 x 8	3/4 x 7-1/4	5.438	6.570	23.82	0.680	0.255	0.944	1.133	1.322	1.510	1.699	1.888
1 x 10	3/4 x 9-1/4	6.938	10.70	49.47	0.867	0.325	1.204	1.445	1.686	1.927	2.168	2.409
1 x 12	3/4 x 11-1/4	8.438	15.82	88.99	1.055	0.396	1.465	1.758	2.051	2.344	2.637	2.930
Dimension Lumber (see NDS 4.1.3.2) and Decking (see NDS 4.1.3.5)												
2 x 3	1-1/2 x 2-1/2	3.750	1.56	1.953	0.938	0.703	0.651	0.781	0.911	1.042	1.172	1.302
2 x 4	1-1/2 x 3-1/2	5.250	3.06	5.359	1.313	0.984	0.911	1.094	1.276	1.458	1.641	1.823
2 x 5	1-1/2 x 4-1/2	6.750	5.06	11.39	1.688	1.266	1.172	1.406	1.641	1.875	2.109	2.344
2 x 6	1-1/2 x 5-1/2	8.250	7.56	20.80	2.063	1.547	1.432	1.719	2.005	2.292	2.578	2.865
2 x 8	1-1/2 x 7-1/4	10.88	13.14	47.63	2.719	2.039	1.888	2.266	2.643	3.021	3.398	3.776
2 x 10	1-1/2 x 9-1/4	13.88	21.39	98.93	3.469	2.602	2.409	2.891	3.372	3.854	4.336	4.818
2 x 12	1-1/2 x 11-1/4	16.88	31.64	178.0	4.219	3.164	2.930	3.516	4.102	4.688	5.273	5.859
2 x 14	1-1/2 x 13-1/4	19.88	43.89	290.8	4.969	3.727	3.451	4.141	4.831	5.521	6.211	6.901
3 x 4	2-1/2 x 3-1/2	8.75	5.10	8.932	3.646	4.557	1.519	1.823	2.127	2.431	2.734	3.038
3 x 5	2-1/2 x 4-1/2	11.25	8.44	18.98	4.688	5.859	1.953	2.344	2.734	3.125	3.516	3.906
3 x 6	2-1/2 x 5-1/2	13.75	12.60	34.66	5.729	7.161	2.387	2.865	3.342	3.819	4.297	4.774
3 x 8	2-1/2 x 7-1/4	18.13	21.90	79.39	7.552	9.440	3.147	3.776	4.405	5.035	5.664	6.293
3 x 10	2-1/2 x 9-1/4	23.13	35.65	164.9	9.635	12.04	4.015	4.818	5.621	6.424	7.227	8.030
3 x 12	2-1/2 x 11-1/4	28.13	52.73	296.6	11.72	14.65	4.883	5.859	6.836	7.813	8.789	9.766
3 x 14	2-1/2 x 13-1/4	33.13	73.15	484.6	13.80	17.25	5.751	6.901	8.051	9.201	10.35	11.50
3 x 16	2-1/2 x 15-1/4	38.13	96.90	738.9	15.89	19.86	6.619	7.943	9.266	10.59	11.91	13.24
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 x 12	3-1/2 x 11-1/4	39.38	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53

9. Critical Buckling Design Value for Compression, FcE

FcE= 0.822*Emin'/(le/d)²
=0.822* 370000/10.38²
=2823.67 psi

10. Reference Compression Design Value, Fc*

Fc* = Fc·(CD·CM·Ct·CF·Ci)
= 825*1.6*1.0
= 1320 psi

11. Constant for Sawn Lumber, c

c = 0.8

12. Column Stability Factor, CP

use precise FcE

$$CP = \frac{1+\left(\frac{2823.67}{1320}\right)}{2*0.8} - \sqrt[2]{\left[\frac{1+\left(\frac{2823.67}{1320}\right)}{2*0.8}\right]^2 - \frac{\frac{2823.67}{1320}}{0.8}} = 0.88$$

$$C_p = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.7\text{-}1)$$

where:

F_c^* = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_p (see 2.3), psi

$F_{cE} = \frac{0.822\, E_{min}'}{(\ell_e/d)^2}$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ

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For the given dimensioned lumber column with 1/3 point weak axis bracing, determine the maximum load capacity of the given load type. Moisture Content = 15%. Ct = Ci = 1.0. Assume pinned end conditions (K=1).

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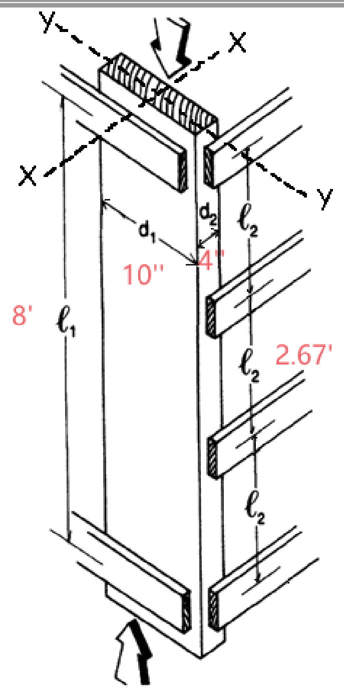

13. Factored Allow. Compressive Stress, F'c(PSI)

F’c = Fc*(CD*CM*Ct*CF*Ci*CP) = 825*1.6*1.0*0.88 = 1161.6 psi

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

$F_c' = F_c$	x	C_D	C_M	C_t	-	C_F	-	C_i	-	C_P	-	-	K_F	ϕ_c	λ

14. Column Area, A

A = 32.38 in²

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Nominal Size b x d	Standard Dressed Size (S4S) b x d in. x in.	Area of Section A in.^{2}	X-X AXIS Section Modulus S_{xx} in.^{3}	X-X AXIS Moment of Inertia I_{xx} in.^{4}	Y-Y AXIS Section Modulus S_{yy} in.^{3}	Y-Y AXIS Moment of Inertia I_{yy} in.^{4}	Approximate weight in pounds per linear foot (lbs/ft) of piece when density of wood equals: 25 lbs/ft^3	30 lbs/ft^3	35 lbs/ft^3	40 lbs/ft^3	45 lbs/ft^3	50 lbs/ft^3
Boards[1]												
4 x 4	3-1/2 x 3-1/2	12.25	7.15	12.51	7.146	12.51	2.127	2.552	2.977	3.403	3.828	4.253
4 x 5	3-1/2 x 4-1/2	15.75	11.81	26.58	9.188	16.08	2.734	3.281	3.828	4.375	4.922	5.469
4 x 6	3-1/2 x 5-1/2	19.25	17.65	48.53	11.23	19.65	3.342	4.010	4.679	5.347	6.016	6.684
4 x 8	3-1/2 x 7-1/4	25.38	30.66	111.1	14.80	25.90	4.405	5.286	6.168	7.049	7.930	8.811
4 x 10	3-1/2 x 9-1/4	32.38	49.91	230.8	18.89	33.05	5.621	6.745	7.869	8.993	10.12	11.24
4 x 12	3-1/2 x 11-1/4	[illegible]	73.83	415.3	22.97	40.20	6.836	8.203	9.570	10.94	12.30	13.67
4 x 14	3-1/2 x 13-1/4	46.38	102.41	678.5	27.05	47.34	8.051	9.661	11.27	12.88	14.49	16.10
4 x 16	3-1/2 x 15-1/4	53.38	135.66	1034	31.14	54.49	9.266	11.12	12.97	14.83	16.68	18.53

15. Maximum Allowable Axial Load Capacity, Pmax

Pmax = F’c * A = 1161.6*32.38 = 37612.61 lbs

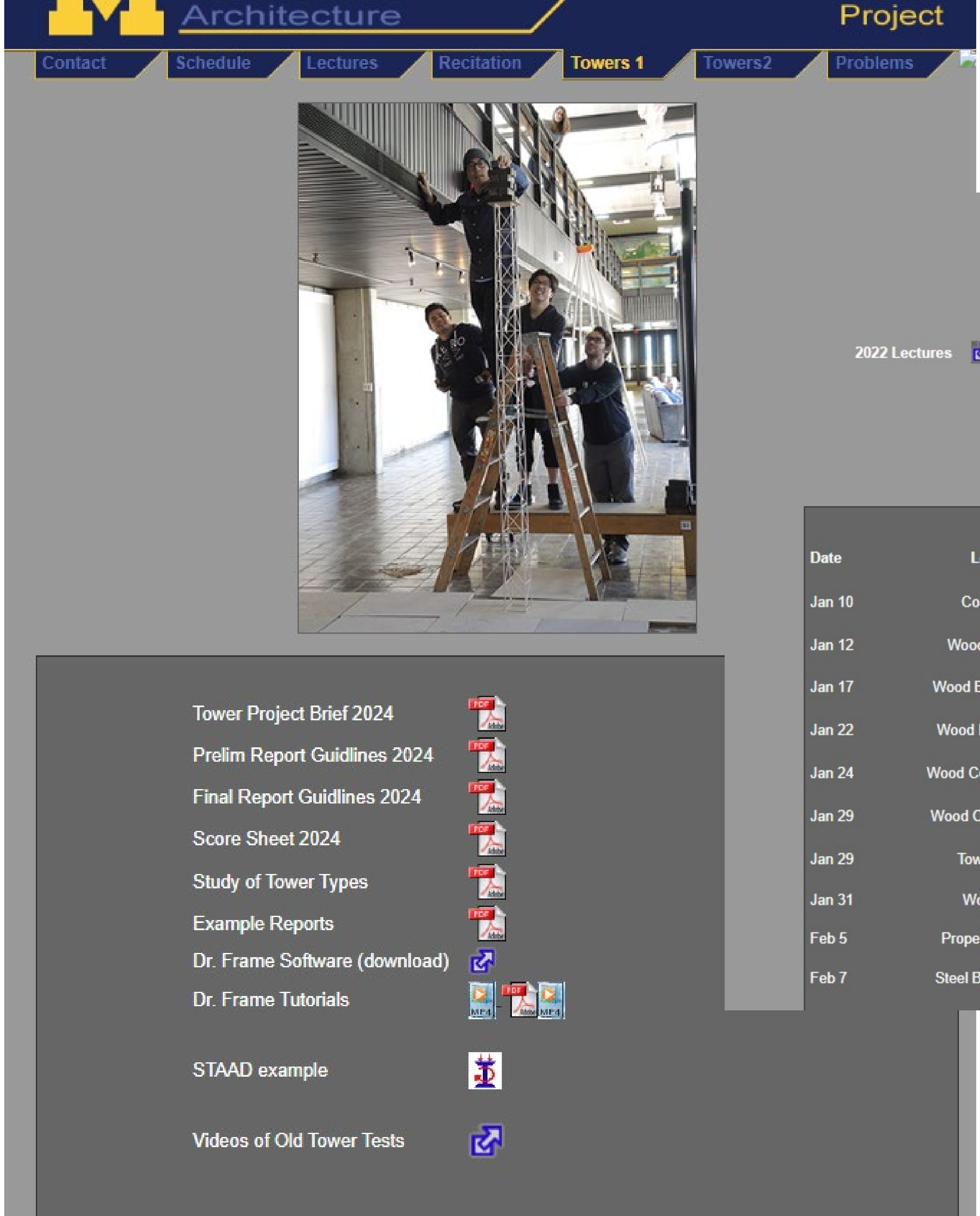
Tower Project Intro

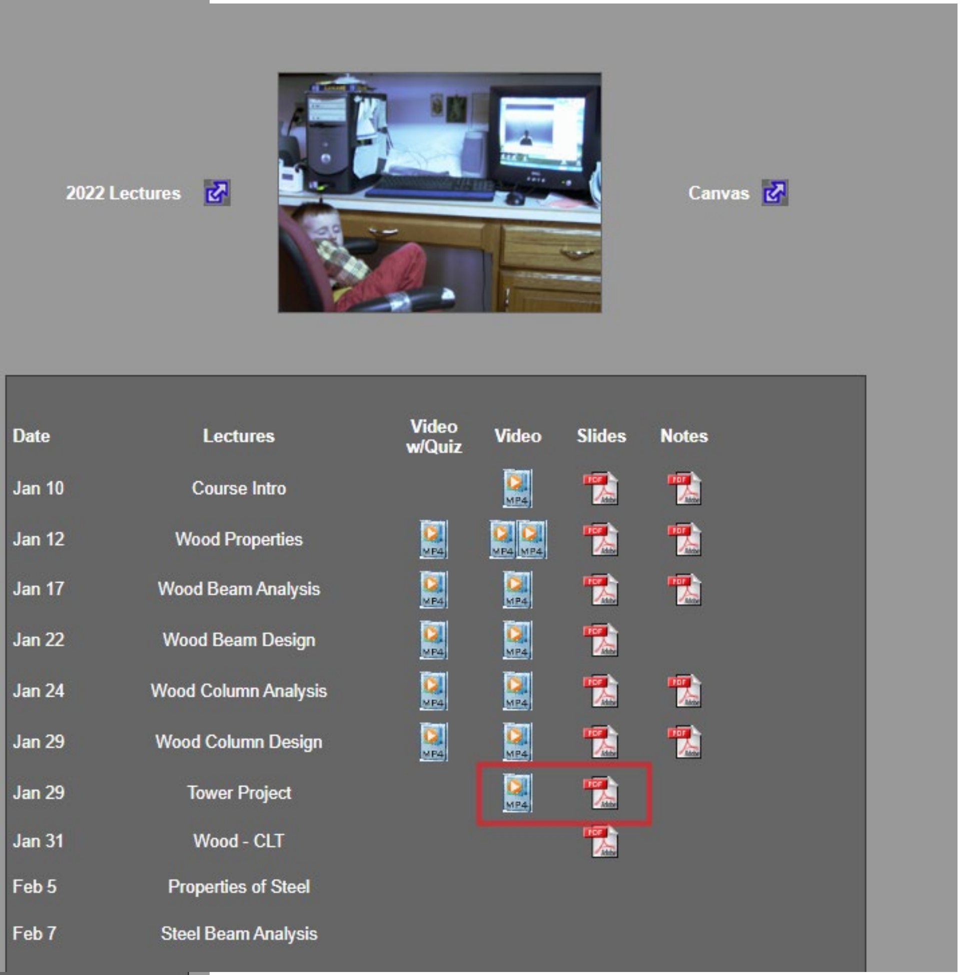
Timeline

DATE	TOPIC	Text Reading	PROBLEMS (due dates online)
JAN 10	Course Intro	Onouye, Schodek	
JAN 12	Wood Properties	NDS	
JAN 15	**Martin Luther King Day **** No Class **** Martin Luther King Day **** No Class**		
JAN 17	Wood Beam Analysis	Schodek 6.4.2	
JAN 19	Recitation **[1-Wood Beams]**		1. Wood Beam Analysis
JAN 22	Wood Beam Design	Onouye 8	
JAN 24	Column Buckling	Onouye 9.1-9.2 & 9.4, Schodek 7.4.3	
JAN 26	Recitation		2. Wood Beam Design
JAN 29	Wood Columns - Tower Intro	NDS	
JAN 31	Cross Laminated Timbers	CLT Handbook	
FEB 2	Recitation **[2-Wood Columns]**		3. Wood Column Analysis
FEB 5	Steel Properties	AISC, Onouye 8.7	
FEB 7	Steel Beam Analysis	Schodek 6.4.3	
FEB 9	Recitation **[3-Steel Beams]**		4 Steel Beam Analysis
FEB 12	Steel Beam Design	Schodek 6.4.3	
FEB 14	Steel Column Analysis	Onouye 9.3, Schodek 7.4.4	
FEB 16	Recitation **[4-Steel Columns]**		**Prelim. Tower Report Due**
			5. Steel Beam Design
FEB 19	Steel Column Design	Onouye 9.3, Schodek 7.4.4	
FEB 21	"Skyscrapers" David Macaulay video		
FEB 32	Recitation		6. Steel Column Analysis
FEB 26	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
FEB 27	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
MAR 1	**WINTER RECESS **** NO CLASS **** WINTER RECESS **** NO CLASS ******		
MAR 4	Continuous Beams	I. Engel Ch. 17, Schodek 8	
MAR 6	Gerber Beams	Schodek 8.4.4	
MAR 8	Recitation **[5-Continuous Beams]**		7. Three Moment Theorem
MAR 11	Intro to Concrete – PCA video.		
MAR 13	Concrete Beams	Schodek 6.4.4 – 6.4.6	
MAR 15	Recitation		
MAR 18	**Tower Testing **** Tower Testing **** Tower Testing **** Tower Testing ******		
MAR 20	Concrete Beams	I. Engel Ch.15	
MAR 22	Recitation **[6-Stress vs Strain]**		8. Concrete Beam Analysis
MAR 25	Concrete Beams		
MAR 27	Concrete Columns	Schodek 7.4.5	
MAR 29	Recitation **[7-Concrete Reinforcing]**		9. Concrete Beam Design
APR 1	Composite Sections	TMS 402	
APR 3	Masonry Walls	TMS 402	
APR 5	Recitation **[8-Composite Sections]**		10. Composite Sections
APR 8	Masonry Walls	TMS 402	
APR 10	Shells and Vaults	Schodek 12	
APR 12	Recitation **[9-Lateral Stability]**	**Final Tower Report Due**	11. Masonry Walls
APR 15	Combined Stress	I. Engel Ch. 19	
APR 17	Combined Stress	I. Engel Ch. 19	
APR 19	Recitation **[10-Combined Stress]**		12. Combined Stress
APR 22	Prestress & Post Tension		

Tower Project Intro

Resources

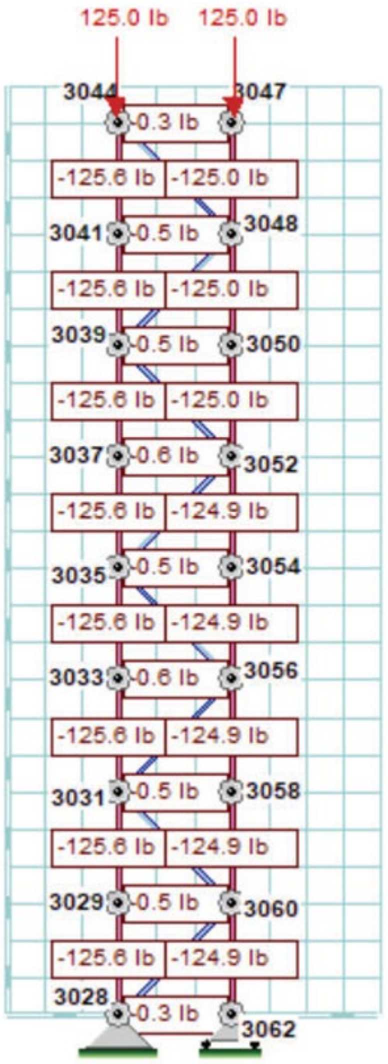




Date	Lectures	Video w/Quiz	Video	Slides	Notes
Jan 10	Course Intro				
Jan 12	Wood Properties				
Jan 17	Wood Beam Analysis				
Jan 22	Wood Beam Design				
Jan 24	Wood Column Analysis				
Jan 29	Wood Column Design				
Jan 29	Tower Project				
Jan 31	Wood - CLT				
Feb 5	Properties of Steel				
Feb 7	Steel Beam Analysis				

Tower Project Intro

Resources



Criteria & Procedure

Criteria

- The tower is to be made of wood. Either linear wood (sticks) or wood panels (sheets) can be used. Glue can be used to connect the elements. Gusset plates at the joints are allowed and can also be glued. But **no steel pins** or fasteners may be used.
- Wood: **any species**. **maximum cross-sectional dimension = 1/4".**
- **NO** paper, mylar or plastic or string or dental floss.
- If a member is made by laminating multiple pieces together, the maximum cross-sectional dimension or thickness still cannot exceed 1/4".
- The **height of the tower = 48"**.
- The tower **must hold at least 50 lbs**.
- The entire tower **can weigh no more than 4 oz**.
- The top of the tower must be loadable. The weights will be stacked on top of the tower, but you may optionally use a loose piece of MDF or plywood as a tray under the weights. (It will not be counted in either weight or load)
- Towers will be graded on their low weight, high load-carrying capacity, and the load/weight ratio. The evaluation formula is:

  (4/weight in OZ) + (load in LBS/50) + (load LBS/weight OZ)x1.5
- The score will be normalized to a range of 50 to 100. It is used together with report scores to assess your project (a detailed evaluation form is given separately).

Procedure

1. Develop a structural concept for a tower meeting the above criteria.
2. Analyze the design concept with **either** hand calculations or a computer program (e.g. Dr. Frame)
3. Determine the capacity of the major members and of the overall tower (total capacity in LBS)
4. Estimate your expected score using the formula above.
5. Write the preliminary report.
6. Construct the structural model.
7. Test the model. 5-pound steel bars will be placed on top of the model, until the model fails. (bar size: 1 ½" x 2" x 5 13/16").
8. Produce final report documenting requirements and process. See also score sheet.

Tower Project Intro

Resources

Analysis

Use NDS approach

Find load P and stress F'c for each member

Use 1.0 for all factors except C_P

Analysis – the report should include the following:

- **Choose wood type and stress properties**. Either use values below for typical model grade Basswood or use values in the NDS or find test values online. Indicate in the report which values you choose.
- **Determine the cross-sectional area of each member**. Find the axial force P and the allowable stress F'c. The force P can be determined either by a hand calculated truss analysis or as a second order analysis in Dr. Frame or STAAD.Pro. The stress F'c should be found using the NDS equations for C_P and F'c. Other NDS stress adjustment factors (C_D, C_M, C_t, C_F and C_i) can be taken equal to 1.0. Size members based on the predicted load, P and the allowable stress F'c. Target (or predict) some total capacity load for the tower. A minimum of 50 LBS is required. Then size the members based on the force in each member.
- **Predict the total weight of the tower**. Provide a table with each member type showing, length, section and weight for each. Make an estimate of the weight added by glue joints and/or gusset plates. The total weight should be under 4 OZ.
- **Predict Capacity**. Predict the ultimate capacity in pounds that the entire tower can carry based on the actual cross-sections chosen. Produce a utilization table to show for each member type (e.g. main vertical, horizontal tie, diagonal brace) the utilization ratio fc/F'c based on the predicted total capacity load. This ratio should be below 1.0 for all members.
- **Calculate the buckling capacity of the tower as a whole**. This is done by treating the tower as one column loaded at the top, made up in cross section of multiple columns. Show the moment of inertia of the tower cross-section, and use it to calculate the critical buckling load using the Euler equation. An example of this calculation is given in the slides from the class lecture. The ultimate capacity is the lower of the two capacities (critical member or tower as a whole).

$$C_p = \frac{1+(F_{cE}/F_c^*)}{2c} - \sqrt{\left[\frac{1+(F_{cE}/F_c^*)}{2c}\right]^2 - \frac{F_{cE}/F_c^*}{c}} \quad (3.7\text{-}1)$$

where:

F_c^* = reference compression design value parallel to grain multiplied by all applicable adjustment factors except C_p (see 2.3), psi

$$F_{cE} = \frac{0.822\,E_{min}'}{(\ell_e/d)^2}$$

c = 0.8 for sawn lumber

c = 0.85 for round timber poles and piles

c = 0.9 for structural glued laminated timber or structural composite lumber

Analysis	Capacity	Design
$f_c = \frac{P}{A} \le F'_c$	$P = F'_c\,A$	$A = \frac{P}{F'_c}$

Properties of Basswood:

Density (oven dry)	20 pcf *
E (buckling)	1,650,000 psi **
F (Compression ∥ to grain)	4745 psi *
F (Compression ⊥ to grain)	377 psi *
F (Tension ∥ to grain)	4500 psi (estimate)
F (Tension ⊥ to grain)	348 psi *
F (Shear ∥ to grain)	986 psi *
F (Flexure)	5900 psi *

* www.matweb.com
** tested by PvB

University of Michigan, TCAUP   Structures II   Slide 4 of 17

PREDICATE CAPACITY

1. Vertical Member Buckling Capacity:

If $K = 1$ then $\frac{Le}{d} = \frac{L.K}{d} = \frac{(6)''}{0.25''} = 24 < 50$

$Fce = \frac{0.822\,Emin}{(Le/d)} = \frac{0.822\times1650000}{24^2} = 2355\ PSI$

$Fc^* = Fc = 4745\ PSI,\ \frac{Fce}{Fc^*} = 0.496$

$$Cp = \frac{1+(\frac{Fce}{F^*c})}{2c} - \sqrt{\left[\frac{1+(\frac{Fce}{F^*c})}{2c}\right]^2 - \frac{(\frac{Fce}{F^*c})}{c}} = \frac{1+0.496}{2\times0.8} - \sqrt{\left[\frac{1+0.496}{2\times0.8}\right]^2 - \frac{0.496}{0.8}} = 0.43$$

$F'c = Fc.(CD.Cm.Ct.CF.Ci.Cp) = 4745 \times 0.43 = 2040PSI$

$P = F'c.A = 2040 \times 0.25^2 = 127\#$

2. Buckling Capacity of the Tower as a whole:

$I = \Sigma I + \Sigma Ad^2 = 4 \times \frac{0.25\times0.25^3}{12} + 4 \times (0.25 \times 0.5) \times (3 - 0.125)^2 = 2.07\ in^4$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.07}{0.25^2\times4}} = 2.88''$

$\frac{Kl}{r} = \frac{1\times48}{2.88} = 16.67$

$Pcr = \frac{\pi^2 IE}{(\frac{KL}{r})^2} = \frac{\pi^2\times2.07\times1650000}{(16.67)^2} = 121306\#$

Each column: $\frac{Pcr}{4} = 30326.5\#$

3. Crushing Capacity of Vertical Members:

$P = Fc.A = 4745 \times 0.25^2 = 296\#$

Tower Project Intro

Resources

Explanation:

We started by looking at precedents - both experimental and real-life implementations. Previous winning groups - Beam Me Up Scotty, Tower 2015, Take a Pisa My Heart - all seemed to use the same method: four long members supported by diagonal bracing and (except for Beam Me Up Scotty) horizontal bracing members. These members serve an important purpose: they shorten the effective buckling length of the four vertical members.

Radio towers use a similar method: vertical supporting masts, supported by diagonal bracing (and guy wires). While these do not experience compression outside of weight, they do experience a lot of wind - lateral forces - that can lead to buckling if not designed correctly.

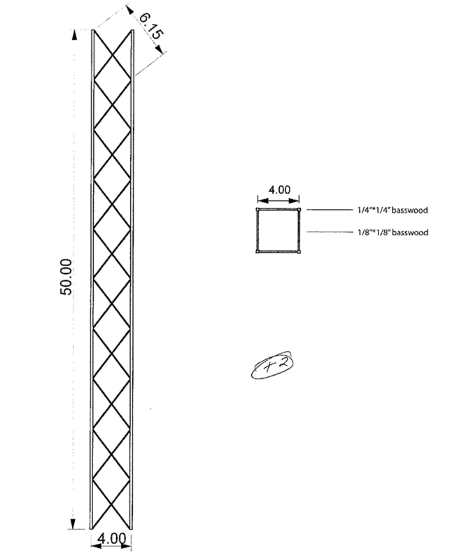
Finally, the optimization sheet given to us also shows similar structures with diagonal bracing. So it was clear that this was the way to go.

We chose to divide the tower into 10 "levels," one for every 5 inches. We believe (based on the equations) that this should effectively combat buckling, and in an ideal situation with perfect craft, the tower may be able to support up to **296** pounds!

+2

overall clarity +1

Illustration (Elevation/Cross-Section):



Analysis:

Derivation of cross-sectional areas of each member:

We began our design by aiming at the capacity of 200 lb, *not* 50 lb. Using the Euler buckling equation, we can solve for a required slenderness ratio:

KL/r=√(π^2EA/P)= √(π^2*1,650,000*0.625/50)=142.67

This is a very high slenderness ratio, and it is very attainable.
Assuming K=1.0, we can get the radius of gyration, r:

r=KL/142.67=1*5/142.67=0.035

Checking whether the member we are using is appropriate:

r=h/√12 h=0.035* √12=0.12 in.

.12 in is smaller than .71268 in (the r for a 1/4"*1/4" basswood column), which confirms the viability of our choice of the 1/4"*1/4" basswood column.

Comparing crushing:

P=F*A=4745*0.0625=296.5625lbs

Compared to our load of 50lbs for each vertical member, this crushing capacity is larger. Thus, our colums should be able to hold up.

+2

Predicted weight estimate of entire tower:

Basswood: 20 lb/ft^3, divided by (12^3) = .0116 lb/in^3, multipled by 16 =

.185 oz/in^3

Total weight: [.185*({4 vertical members * 50in * .25in * .25 in}
+ {80 diagonal members * 6.4in * .125in * .125in})] + [.25 oz of glue)] =

4.04 oz. (can be adjusted) +1

Just keep an eye on it and you should be ok.

Predicted Capacity...

...Of vertical members:

Length = 50"/10 spaces = 5"; Area = .0625"

r = √I/A = ... width/√12 <== Based off of JY recitation notes on wood columns
= .25/√12
= .07217

Vertical crushing: $P = F_c A$ = (4745 *psi*)(.0625*in^2*) = **296.56 lb**

Vertical buckling: P = (π^2)AE/(KL/r)^2
= 1017802.95/((1*5)/.07217)^2
= 1017802.95/(65.252)^2
= **212.05 lb**

*This is spread out over 4 members, making the buckling capacity a whopping **848** lbs (of course, assuming perfect craft and materials, and no other factors). Thus, crushing will probably happen first.

...Of the tower as whole:

Moment of interia:

Using the subtractive method (subtracting void of "column" from 4" * 4" occupied area of "column":

$I = \Sigma I_{total} - I_{void}$

I = [((4in)(4in)^3)/12] - [2*((.25in)(3.5in)^3)/12] - [((3.5in)(4in)^3)/12]

I = .88 in^4

Critical buckling load: +2

Assume K = 1

r = √(.88in^4/(4*.0625in^2)) = 1.876 in

KL/R = 1(50in)/1.876 = 26.6524

F_{cr} = (π^2)(1,650,000psi)/(26.6542)^2 = 22,921.95 psi

$P_{cr} = F_{cr}A$ = (22,921.95 psi)(4*.0625in^2) = **5,730.4875 lb**

Of course, this is assuming that the tower is one large column with perfect craft. One can only dream...

LAB - Columns

Description

This project uses observation and calculation to understand the effect of slenderness on column capacity.

Goals

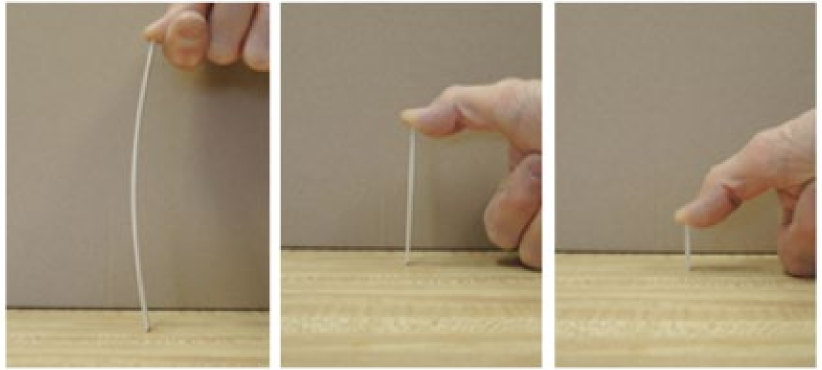
To observe the buckling behavior of columns through physical modeling.

To find the controlling slenderness ratio.

To calculate the critical buckling and crushing loads.

Procedure

1. For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and Pcr using the Euler equation. Use K=1.0.
2. Find the actual critical buckling load approximating the load with your finger.
3. Repeat the procedure for L=3" and L=1".
4. Calculate the slenderness and Pcr for both of these lengths.
5. Calculate the ultimate crushing load based on the max compressive stress, F_c.
6. Approximately locate P for each length on the load vs. slenderness curve shown below



Basswood Properties

Emin = 1,650,000. psi

Fc = 4745 psi
Area = 0.015625 in^2
d_1 = 0.25 in
d_2 = 0.0625 in

L = 6"   L/d =
P =

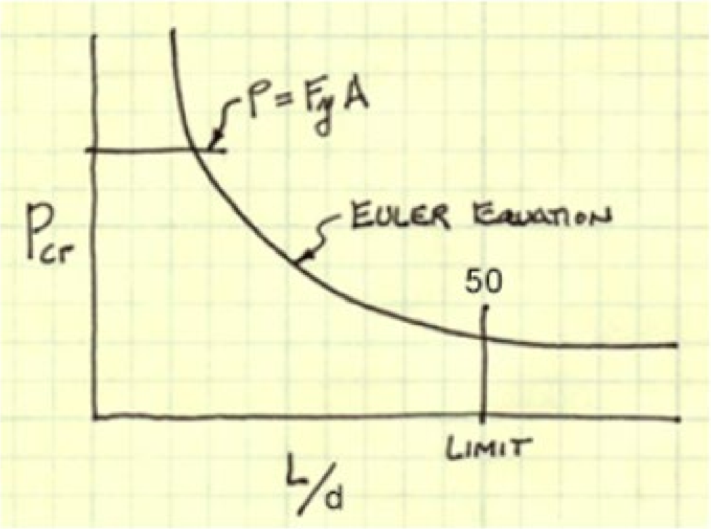
L = 3"   L/d =
P =

L = 1"   L/d =
P =

Equations:

$$F_cE = \frac{0.822\ E'min}{(le/d)2}$$

$$P_{max} = F_c \times A$$



Failure Modes – Stability

Long Columns – fail by buckling

Traditional Euler

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

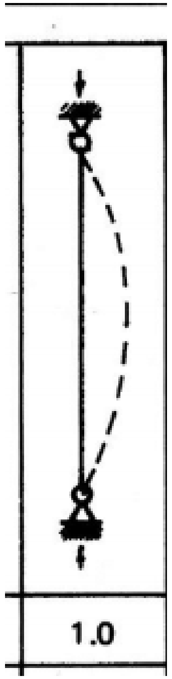
- E = Modulus of elasticity of the column material (psi)
- K = Stiffness (curvature mode) factor
- L = Column length between ends (inches)
- r = radius of gyration = $\sqrt{I/A}$ (inches)

$$r = d/\sqrt{12}$$

NDS Equation

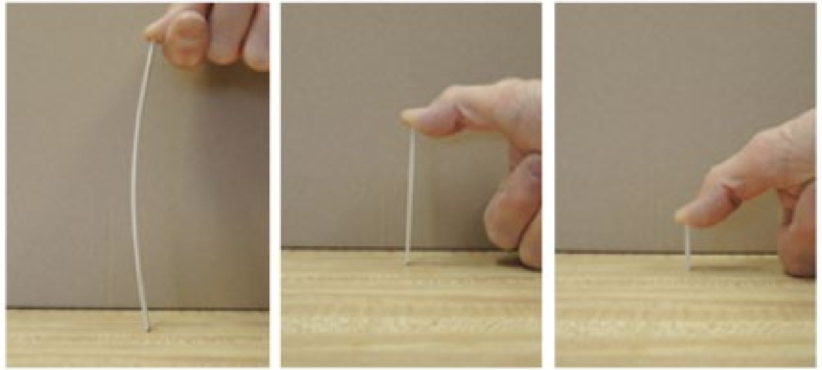
$$F_{cE} = \frac{0.822 E'_{min}}{\left(\frac{l_e}{d}\right)^2}$$

- E'min = reduced E modulus (psi)
- le = Ke l_u (inches)
- d (inches)
- $0.822 = \pi^2/12$



Procedure

1. For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and Pcr using the Euler equation. Use K=1.0.
2. Find the actual critical buckling load approximating the load with your finger.
3. Repeat the procedure for L=3" and L=1". Slenderness ratio? Pcr?
4. Calculate the slenderness and Pcr for both of these lengths.
5. Calculate the ultimate crushing load based on the max compressive stress, F_c.
6. Approximately locate P for each length on the load vs. slenderness curve shown below



Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

$$I = Ar^2$$

- A = Cross sectional area (in^2)
- E = Modulus of elasticity of the material (lb/in^2)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad F_{cE} = \frac{0.822\ E'_{min}}{\left(\frac{le}{d}\right)^2}$$

Buckling Force

Pcr = FcE * A

le = L*K = L * 1.0 = L

Basswood Properties

Emin = 1,650,000. psi

Fc = 4745 psi
Area = 0.015625 in^2
d_1 = 0.25 in
d_2 = 0.0625 in

L = 6" L/d =
P =

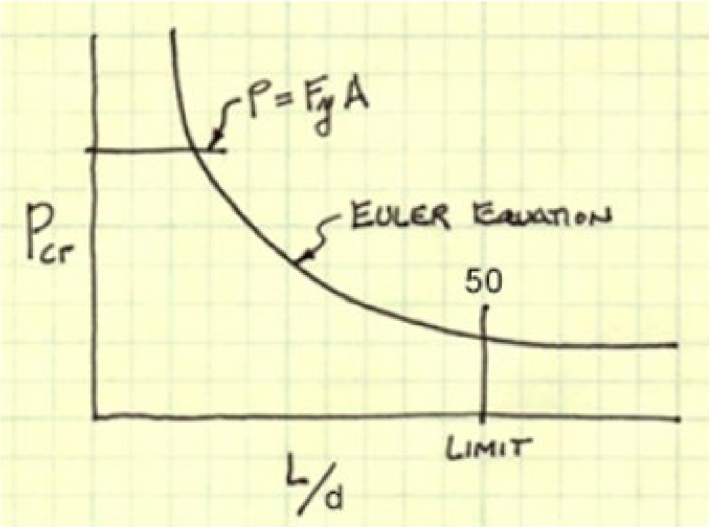
L = 3" L/d =
P =

L = 1" L/d =
P =

Equations:

$$FcE = \frac{0.822\ E'min}{(le/d)2}$$

$$P_{max} = F_c \times A$$



Failure Mode - Strength

Short Columns – fail by crushing

$$f_c = \frac{P}{A} \le F_c$$

$$A = \frac{P}{F_c}$$

- f_c = Actual compressive stress
- A = Cross-sectional area of column (in^2)
- P = Load on the column
- F_c = Allowable compressive stress per codes

Crushing Force:

Pmax = Fc * A

Guess:
Which stick would fail by buckling?
Which stick would fail by crushing?

Procedure

1. For the 1/16"x1/4" basswood column provided, with L=6" calculate the controlling (weak axis) slenderness ratio and Pcr using the Euler equation. Use K=1.0.
2. Find the actual critical buckling load approximating the load with your finger.
3. Repeat the procedure for L=3" and L=1".
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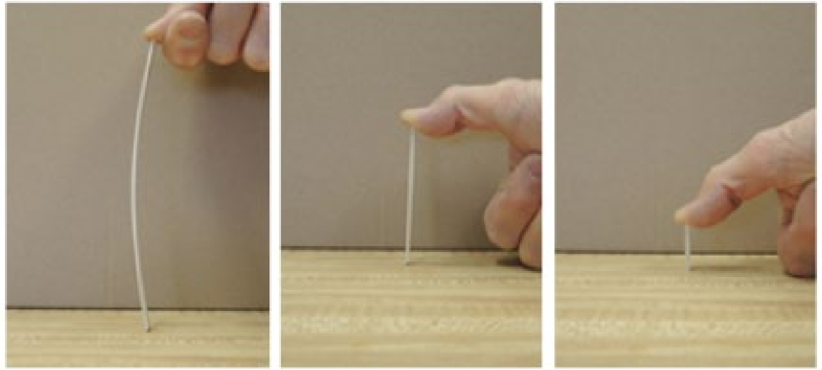

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P =

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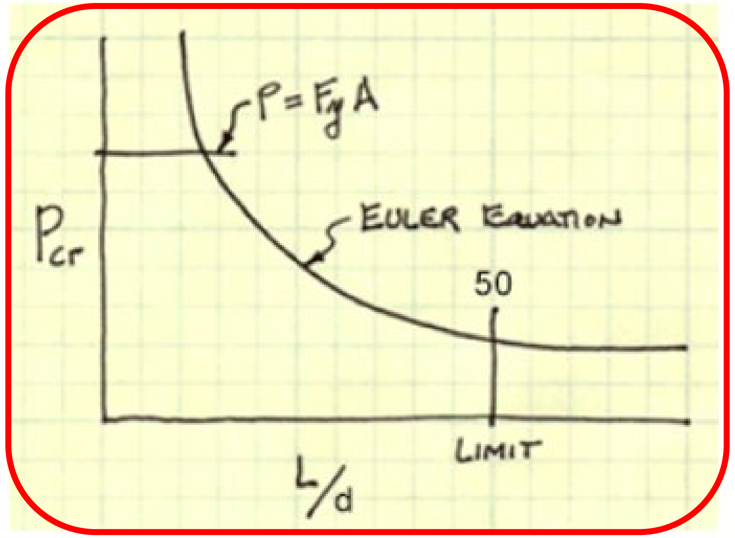
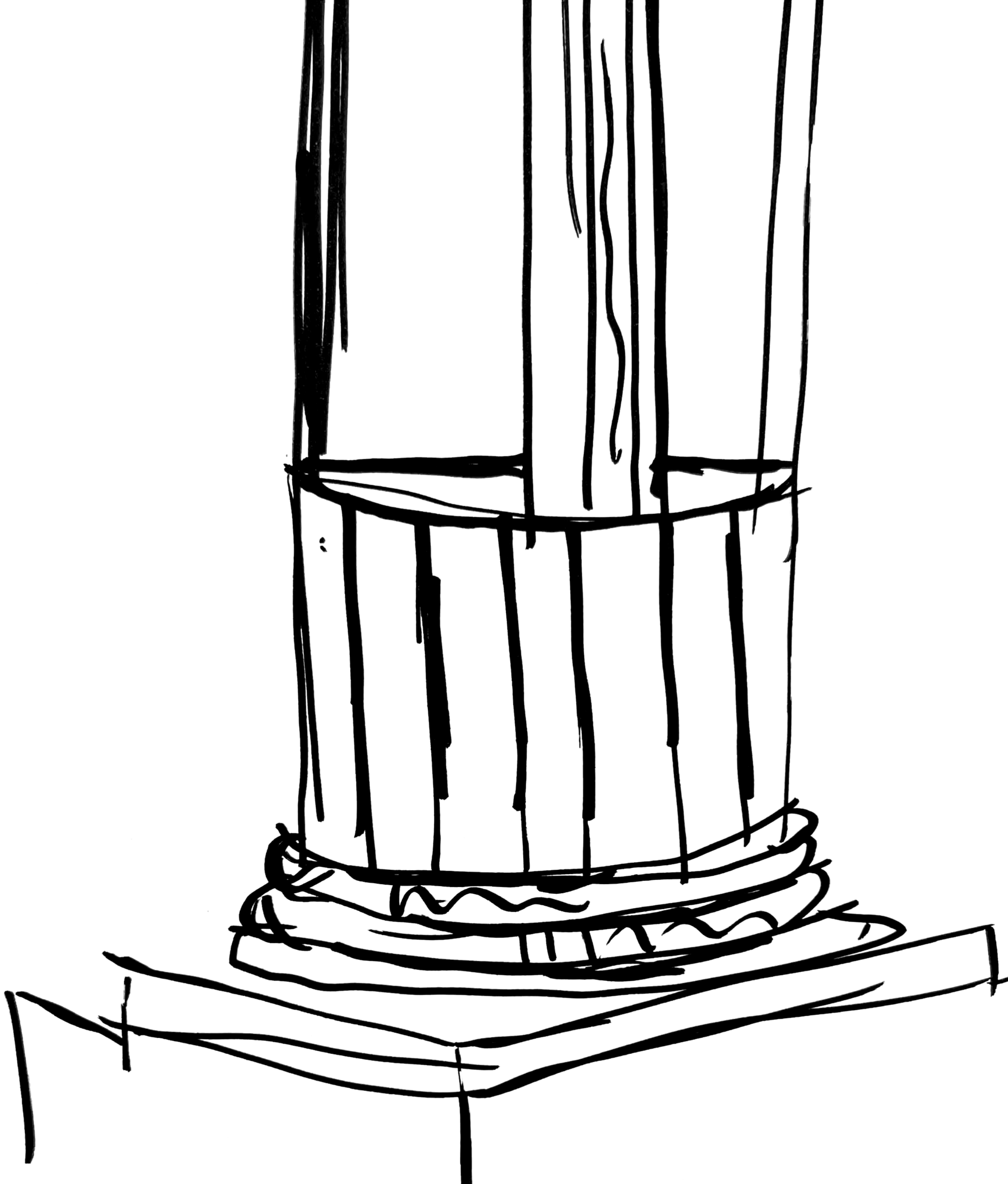
$$P_{max} = F_c \times A$$



Any Questions?

yifanma@umich.edu



Thank You!