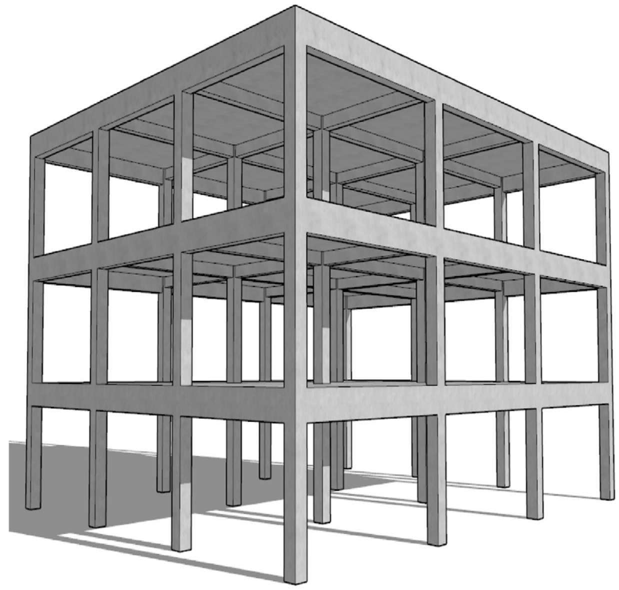
Architecture 324
Structures II

Reinforced Concrete Beams Ultimate Strength Design

(ACI 318-14) – PART II

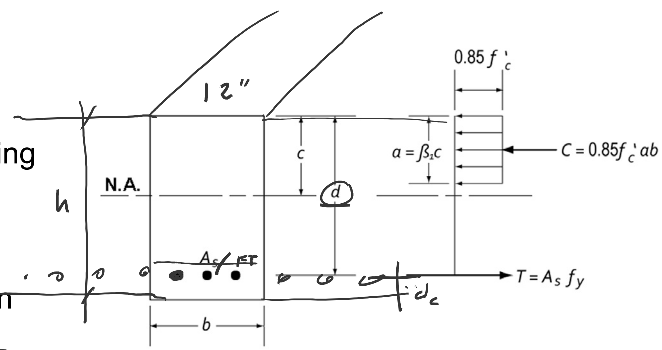


- Rectangular Slab Analysis
- Reinforcement Detailing
- Rectangular Beam Design – Method I

One-way Slab Analysis

Data:
- Section dimensions – b, h, (span)
- Steel area – As , bar diam. b_d , o.c. spacing
- Material properties – f'c, fy

Required:
- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity



1. Calculate d = h – cover – $bar_d/2$
2. Find As/ft. Check As min
3. Calculate a
4. Determine c
5. Check that $\varepsilon_t \geq 0.005$ (tension controlled)
6. Find nominal moment, Mn
7. Calculate required moment, ϕ Mn ≥ Mu (if $\varepsilon_t \geq 0.005$ then ϕ = 0.9)
8. Determine max. loading (or span)

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi		$A_{s,min}$
Deformed bars	< 60,000		$0.0020A_g$
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			$0.0014A_g$

As/ft = As x 12/o.c.

Ag = b h

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_t = \frac{d - c}{c} 0.003 \geq 0.005$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\varphi M_n \geq M_u$$

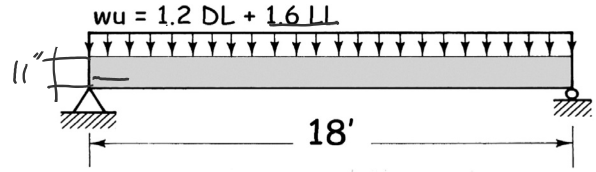
$$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$$

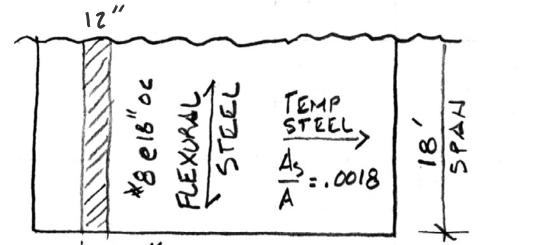
$$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$$

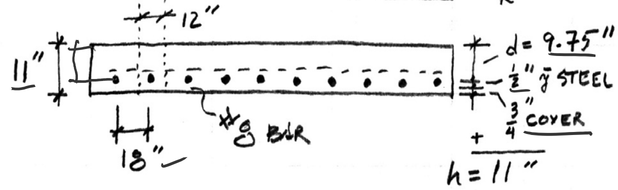
Slab Analysis

Data:
- Span = 18 ft
- h = 11”
  take b = 12”
- Steel #8 @ 18” o.c.
- f'_c = 3000 psi
- f_y = 60 ksi

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257







Required:
- Design moment capacity – M_u
- Maximum LL in PSF

$d = 11" - \frac{1}{2} - \frac{3}{4} = 9.75"$

1. Find d
2. Find A_s
   Check $A_{s,min}$

$A_s = \frac{12"}{18"}(0.79\ in^2) = 0.5267\ in^2/FT$

Ag = 12” x 11” = 132 in²

[0.0018(60)/60] 132 = 0.237 in²

0.0014 (132) = 0.1848 in²

0.527 > 0.237 ok

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi		$A_{s,min}$
Deformed bars	< 60,000		$0.0020A_g$
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60{,}000}{f_y} A_g$
			$0.0014A_g$

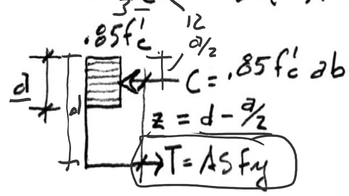
ACI 318-14

Slab Analysis

f'c	β1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

3. Find a

$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.5267(60)}{.85(3)(12)} = 1.033"$



4. Find $c = \beta_1\ a$

$c = \frac{a}{\beta_1} = \frac{1.033"}{0.85} = 1.215"$

5. Check failure mode
   $\varepsilon_t \geq$ 0.005 for tension controlled

$\varepsilon_t = \frac{0.003\ d}{c} - 0.003$

$\varepsilon_t = \frac{0.003(9.75")}{1.215"} - 0.003 = 0.021$ "/"

$\varepsilon_t = 0.021 > 0.005$ ∴ TENSION CONTROLLED

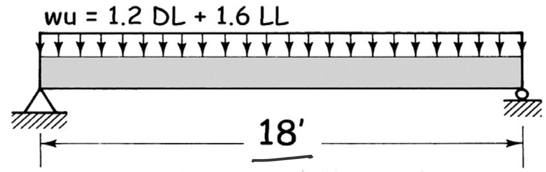
6. Find force T
7. Find moment arm z
8. Find nominal strength moment, M_n

$T = A_s f_y = 0.5267(60) = 31.6^K$

$z = d - \frac{a}{2} = 9.75 - \frac{1.033}{2} = 9.23"$

$M_n = Tz = 31.6(9.23) = 291.8^{K-"}$

Slab Analysis



9. Find required moment, M_u

$$M_u = \phi M_n = 0.9\,(291.8)\,\frac{1000}{12} = 21885 \text{ 1-\#}$$

10. Find slab DL

$$w_{DL} = \gamma_c \frac{h}{12} = 150\,\frac{11}{12} = 137.5 \text{ PSF}$$

(150 (REINF. CONC.) #/FT³)

11. Determine max. loading

$$M_u = 21885 \text{ 1-\#} = \frac{(1.2\,w_{DL} + 1.6\,w_{LL})\,l^2}{8}$$

$$\frac{21885\,(8)}{(18')^2} = 1.2\,(137.5) + 1.6\,(w_{LL})$$

$$540.37 = 165 + 1.6\,(w_{LL})$$

$$w_{LL} = 234.6 \text{ PSF CAPACITY}$$

Details of Reinforcement

Size
- Nominal 1/8" increments

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
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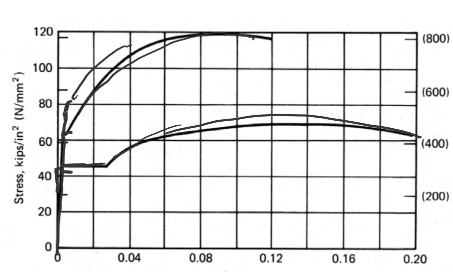
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257

3/8 1/2 5/8 8/8 = 1"

Grade
- 40 (40 ksi)
- 60 (60 ksi)
- 75 (75 ksi)



Details of Reinforcement

ACI 318 Chapter 25.2
Placement of Reinforcement

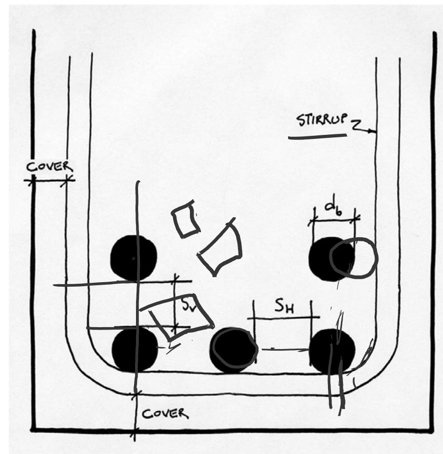
- Cover (ACI 20.6.1)
- Horizontal spacing in beams, s_h (ACI 25.2.1)
  - 1 inch
  - d_b
  - $4/3\ d_{agg,max}$
- Vertical spacing in beams (ACI 25.2.2)
  - Min 1 inch

BAD

https://www.constructioncost.co/honeycombing-in-concrete.html

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

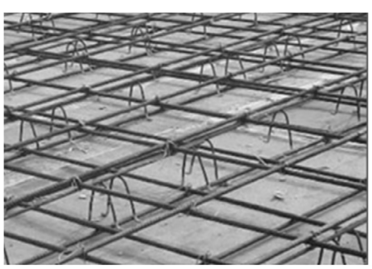
Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2



Details of Reinforcement

ACI 318 Chapter 25
Placement of Reinforcement

- Chairs
- Bolsters



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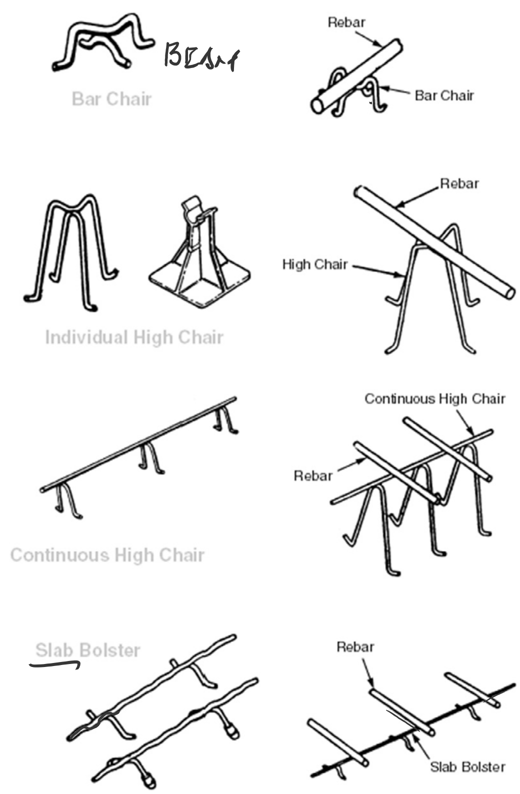


Details of Reinforcement

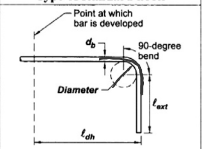
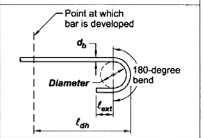
ACI 318 Chapter 25

Minimum bend diameter
- factor x d_b

Hooks for bars in tension
- ACI Table 25.3.1
- Inside diameter

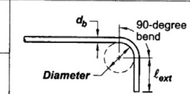
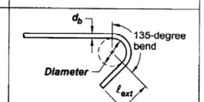
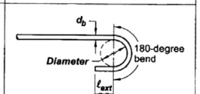
Bends for stirrups
- ACI Table 25.3.2

Table 25.3.1—Standard hook geometry for development of deformed bars in tension

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 8	$6d_b$	$12d_b$	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		
180-degree hook	No. 3 through No. 8	$6d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

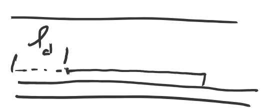
Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

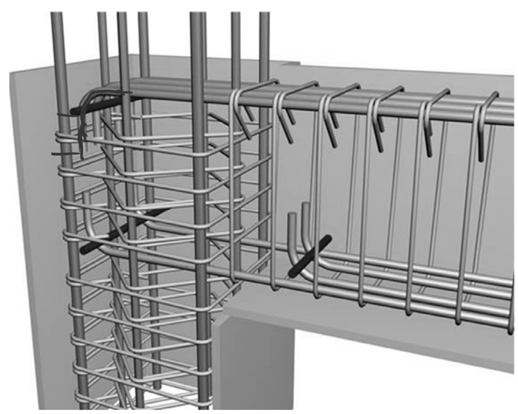
[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Details of Reinforcement

ACI 318 Chapter 25



Development length of bars
- 12" min
- Based on table 25.4.2.2



https://www.buildinghow.com

Table 25.4.2.4—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 7 and larger bars	1.0
	No. 6 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

Table 25.4.2.2—Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\psi_t\psi_e}{25\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{20\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\frac{3f_y\psi_t\psi_e}{50\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{3f_y\psi_t\psi_e}{40\lambda\sqrt{f'_c}}\right)d_b$

Other Useful Tables:

Table A.1 Values of Modulus of Elasticity for Normal-Weight Concrete

Customary Units		SI Units	
f'_c (psi)	E_c (psi)	f'_c (MPa)	E_c (MPa)
3,000	3,140,000	20.7	21 650
3,500	3,390,000	24.1	23 373
4,000	3,620,000	27.6	24 959
4,500	3,850,000	31.0	26 545
5,000	4,050,000	34.5	27 924

Jack C McCormac, 1978, *Design of Reinforced Concrete*,

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

	Customary Units			SI Units		
Bar No.	Diameter (in.)	Cross-sectional Area (in.^{2})	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm^2)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

Table A.4 Areas of Groups of StandardBars (in.^{2})

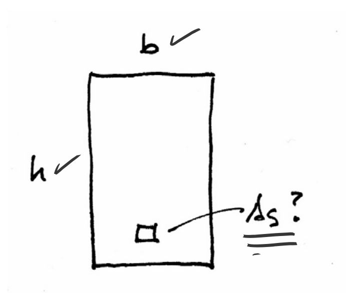
	Number of Bars												
Bar No.	2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

Rectangular Beam Design
Two approaches:

Method 1:

Data:
- Load and Span
- Material properties – f'_c, f_y
- All section dimensions: h and b

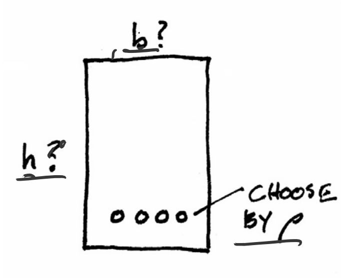
Required:
- Steel area – A_s



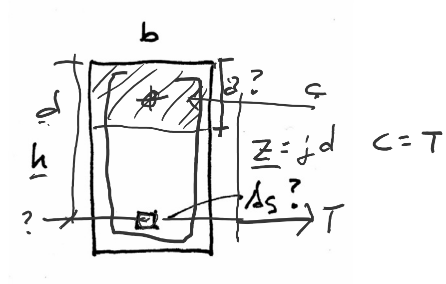
Method 2:

Data:
- Load and Span
- Some section dimensions – h or b
- Material properties – f'_c, f_y
- Choose ρ

Required:
- Steel area – A_s
- Beam dimensions – b or h



Rectangular Beam Design – **Method 1**



Data:

- Load and Span
- Material properties – f'_c, f_y
- All section dimensions – b and h

Required:

- Steel area - A_s

1. Calculate the factored load and find factored required moment, M_u REQUIRED
2. Find d = h – cover – stirrup – $d_b/2$ BEAMS
3. Estimate moment arm z = jd, for beams j ≈ 0.9 for slabs j ≈ 0.95
4. Estimate A_s based on estimate of jd.
5. Use As to find a
6. Use a to find A_s (repeat…until **2%** accuracy)
7. Choose bars for A_s and check A_s max & min
8. Check that $\varepsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)
10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

ESTIMATE

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

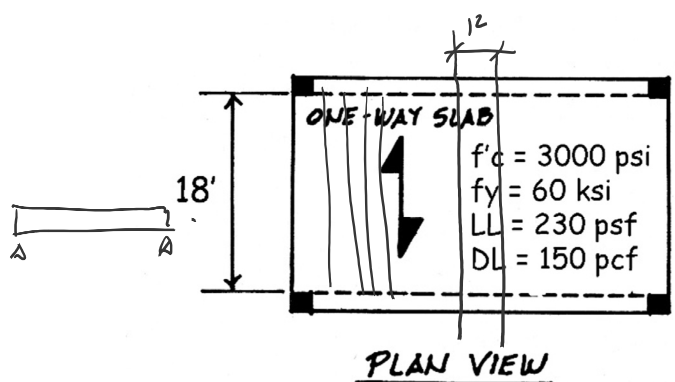
Z = dj (guess)

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

One-way Slab Design

Method 1



Data:

- Load and Span
- Material properties – f'_c, f_y
- All section dimensions:
- h (based on deflection limit)
- b = typical 12" width

Required:

- Steel area – A_s

ACI

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum $h^{[1]}$
Simply supported	ℓ/20
One end continuous	ℓ/24
Both ends continuous	ℓ/28
Cantilever	ℓ/10

First estimate the slab thickness, h.

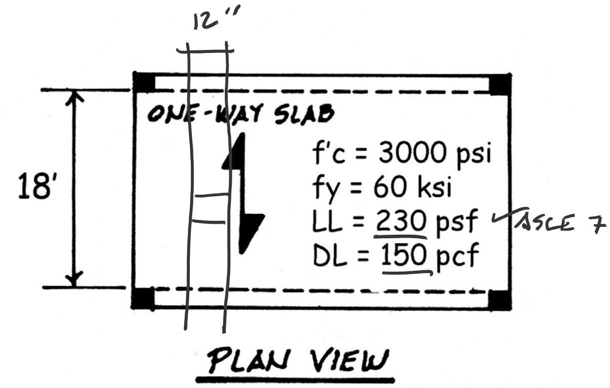
Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

THICKNESS, h, BASED ON DEFLECTION

$h = \ell/20 = \frac{18' \times 12}{20} = 10.8"$ USE 11"

One-way Slab Slab Design



1. Calculate the dead load and find required M_u

FACTOR LOADS

$DL = \frac{11''}{12}(150\ \#/ft^3) = 137.5\ PSF$

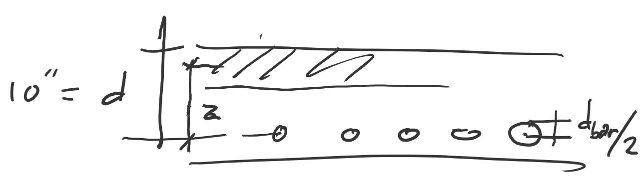
$LL\ (GIVEN) = 230\ PSF$

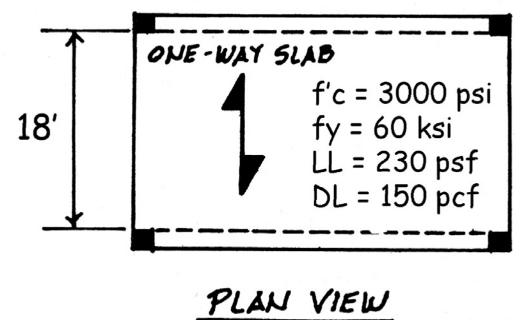
$w_u = 1.2(137.5) + 1.6(230) = 533\ PSF = PLF$

$M_u = \frac{w_u \ell^2}{8} = \frac{533\ PLF\,(18')^2}{8} = 21\,587\ '\text{-}\#$

$\phi M_n = 259\ ''\text{-}K$

One-way Slab Slab Design





2. Find d based on the estimated h and rebar size (guessing #4)

3. Estimate moment arm $z \approx 0.95\ d$

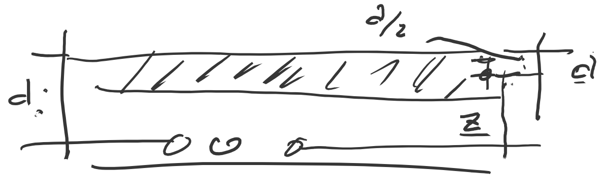
FOR $j \approx 0.95$, $d = h - \text{INSIDE COVER} - \frac{1}{2}\ BAR$

$d = 11'' - \frac{3}{4}'' - \frac{1}{2}\left(\frac{1}{2}''\right)$ #4

$d = 11'' - 1'' = 10''$

$z \approx jd \approx 0.95(10'') = 9.5''$ ESTIMATE

One-way Slab Slab Design



4. Estimate A_s based on estimate of z

5. Use A_s to find a

6. Use a to find A_s (repeat…)

TRIAL 1

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{259''\text{-k}}{0.9(60\text{ ksi})(9.5)''} = 0.505\text{ in}^2$$

REQ. ← GUESS

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.505(60)}{.85(3)(12)} = 0.99''$$

TRIAL 2

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9(60)(10 - \frac{.99}{2})}$$

Z

$A_s = 0.5046\text{ in}^2$ WITHIN 2%

0.505

One-way Slab Slab Design

7. Choose bars for A_s required:

either
choose bars and calculate spacing
or
choose spacing and find bar size
If the bar size changes, re-calculate to find new d. Then, re-calculate A_s…

Check $A_{s,min}$

(for slabs $A_{s,min}$ from ACI Table 7.6.1.1)

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi	$A_{s,min}$	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			$0.0014A_g$

CHOOSE BARS

USING #4

A_s of #4

$$A_s\ \frac{0.505}{12''\text{ o.c.}} : \frac{0.2}{s} \rightarrow s = 4.75'' \rightarrow 4''\text{ o.c.}$$

∴ USE 4" o.c. (always round down)

$A_s = 0.60\text{ in}^2/\text{FT} > 0.505$ ✓✓

ALTERNATE FOR MAX. S = 18" MAX

$$\frac{0.505}{12''} : \frac{A_b?}{18''} \quad A_b = 0.75\text{ in}^2$$

#8 = 0.79 ✓

∴ USE #8 @ 18" o.c.

$A_s = 0.526\text{ in}^2/\text{FT} > 0.505$ ✓

Check As,min

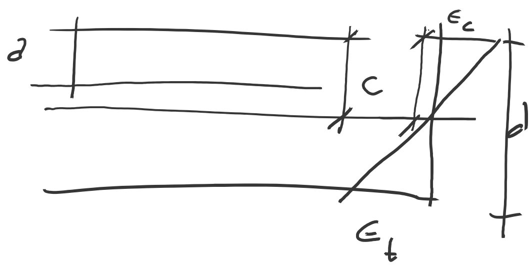
$$A_{s\,min} = 0.0018\ bh = 0.0018(12)(11'')$$

$= 0.24\text{ in}^2 < 0.526\text{ in}^2$ ✓ OK

0.60 ✓

One-way Slab Slab Design

8. Check that $\varepsilon_t \geq 0.005$



RE-CALC a FOR $A_s = 0.6\ in^2/FT$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d-c}{c} 0.003 =$$

$$= \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$$0.01759 > 0.005$$

∴ TENSION CONTROLLED ✓

One-way Slab Slab Design

9. Check $M_u \leq \phi\ M_n$ (final condition)

   $A_s = A_{s,used}$
   $M_n = Tz$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 0.6(60^{KSI})\left(9.5'' - \frac{1.176}{2}\right)$$

$$M_n = 36(8.911'') = 320.8^{K-''}$$

$$\phi M_n = 0.9(320.8) = 288.7^{K-''}$$

$$M_u = 259^{K-''} < 288.7^{K''}$$

$$M_u < \phi M_n \checkmark OK$$

10. Add stirrups (no stirrups in slab)

11. Check deflection, crack control, and rebar development length