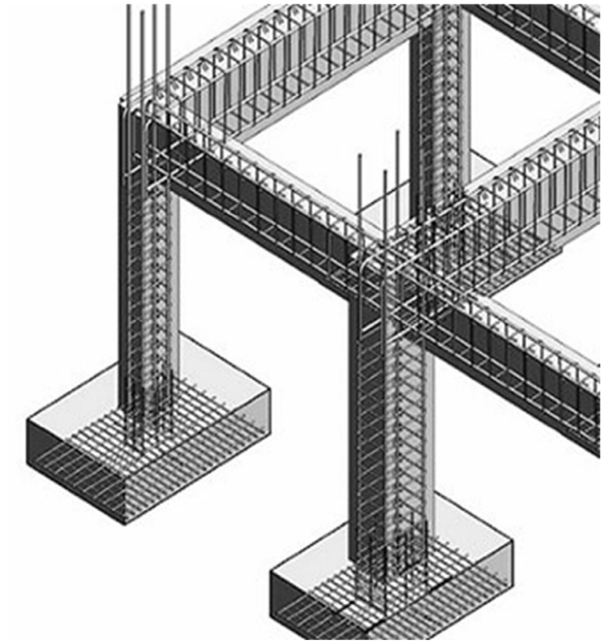
Architecture 324
Structures II

Reinforced Concrete Beams Ultimate Strength Design
(ACI 318-14) – PART III

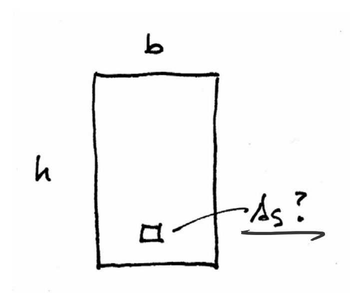


- Rectangular Beam Design – Method 2
- Non-Rectangular Beam Analysis
- Reinforced Concrete Examples
- 3D-Print Evolution (Video)

Rectangular Beam Design
Two approaches:

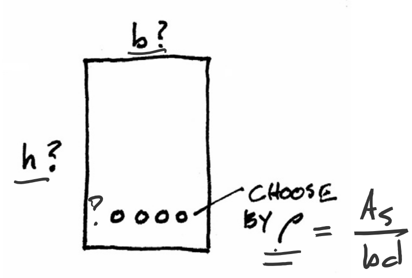
Method 1:
Data:
- Load and Span
- Material properties – f'_c, f_y
- All section dimensions: h and b

Required:
- Steel area – A_s



Method 2:
Data:
- Load and Span
- Some section dimensions – h or b
- Material properties – f'_c, f_y
- Choose ρ

Required:
- Steel area – A_s
- Beam dimensions – b and h



Rectangular Beam Design – **Method 2**

Data:

- Load and Span ✓
- Some section dimensions – b or h ✓
- Material properties – f'_c, f_y

Required:

- Steel area - A_s
- Beam dimensions – b and h

OR

1. Estimate the dead load (estimate h and b) DL
   ($L/21 \le h \le L/8$, $h \approx L/12$ and $b:h \approx 1:2$ to $2:3$), find M_u
2. Choose ρ (equation assumes $\varepsilon_t = 0.0075$)
3. Calculate bd^2
4. Choose b and solve for d (or d and solve b)
5. Revise h, weight, M_u, and bd^2
6. Find $A_s = \rho bd$
7. Choose bars for A_s, determine spacing and cover, and revise d ✓
8. Check that $\varepsilon_t \ge 0.005$ (if not, increase h and reduce A_s)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

$$\rho = \frac{\beta_1 f'_c}{4 f_y}$$

$$bd^2 = \frac{M_u}{\varphi \rho f_y \left(1 - 0.59\rho (fy/f'_c)\right)}$$

PICK ON

ASSUME

STEEL

CONC

$$A_s = \rho bd$$

$$a = \frac{\rho f_y\, d}{0.85 f'_c}$$

Rectangular Beam Design

Data:

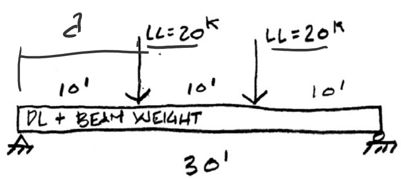
- Load and Span
- Material properties – f'_c, f_y

Required:

- Steel area - A_s
- Beam dimensions – b and d

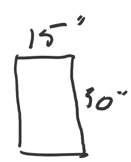
f'c = 3000 psi
fy = 60 ksi

DL = 2 klf + beam ?
LL = 2 x 20 k



ASSUME $h \approx \frac{L}{12} = \frac{360''}{12} = 30''$

ASSUME $b:h \approx 1:2 \quad \therefore b \approx 15''$



BEAM DL $= 150 \; \frac{15 \times 30}{144} = 469$ PLF

0.469 KLF

ESTIMATE M_U

$M_U = Pa + \frac{w l^2}{8}$

2 KLF

$= 1.6(20^k)(10') + \frac{1.2(2.469\text{ KLF})(30')^2}{8}$

$= 320 + 333.3 = 653.3$ K-'

1. Estimate the dead load (self-weight), and find M_u ($h \approx L/12$ and $b:h \approx 1:2$ to $2:3$)

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and f_y = 60,000 psi. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

2. Choose ρ (equation assumes $\varepsilon_t = 0.0075$)

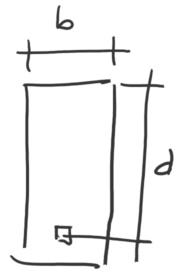
CHOOSE ρ

$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$

Rectangular Beam Design cont.

3. Calculate bd^2

$$bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - 0.59\rho\left(\frac{f_y}{f'_c}\right)\right)}$$



$$bd^2 = \frac{653.3\,(12)}{0.01(0.9)60 \left[1 - 0.59(0.01)\left(\frac{60\ \text{ksi}}{3\ \text{ksi}}\right)\right]}$$

$\underbrace{0.01}_{\rho}$, 60 ksi

$$bd^2 = \frac{7840}{0.573(0.882)} = 15492\ in^3$$

4. Choose b and solve for d
   (or d and solve for b)
   b is based on form size – matches column size
   h ≈ L/12, b:h ≈ 1:2 to 2:3

TRY

b	d	h ≈ 1.12 d	A_g
14"	33.27"	38"	532
→ 15"	32.14"	36"	540
16"	31.11"	35"	560

↓ w_c

CHOOSE 15 × 36

5. Revise h, weight, M_u, and bd^2

Rectangular Beam Design cont.

5. Revise h, weight, M_u, and bd^2

USE 15 × 36

REVISE DL $= 150 \frac{540}{144} = 563$ PLF

CHECK M_u

$$M_u = 320 + \frac{1.2(2{,}563)\,30^2}{8} = 666\ ^{k\text{-}1}$$

REVISE bd

$$bd^2 = \frac{666\,(12)}{0.505} = 15814\ in^3$$

FOR b = 15" d = 32.5"

6. Find $A_s = \rho bd$

$$A_s = \rho\, bd = (0.01)(\underset{b}{15''})(\underset{d}{32.5''})$$

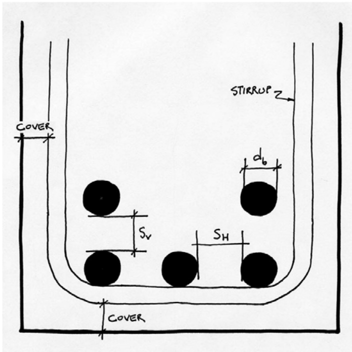
$$A_s = 4.87\ in^2$$

Rectangular Beam Design

7. Choose bars for A_s, determine spacing and cover, and revise d

CHOOSE BARS (SEE TABLE A.4)

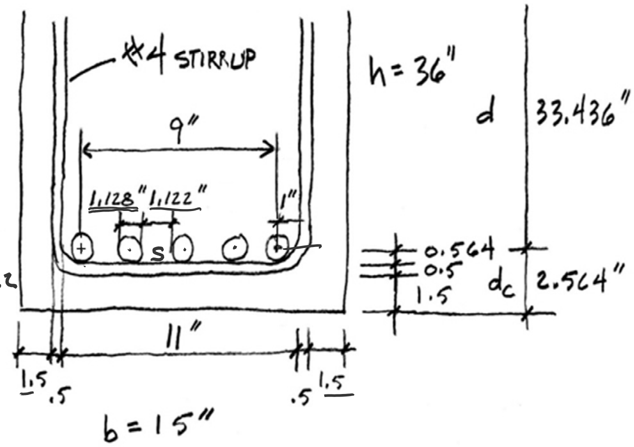
TRY 5 x #9 BARS $A_s = 5.0\ in^2 > 4.87\ in^2$ ✓



S 1"

$\frac{4}{3}$ Agg

d_b 1.128 ≈ 1.122



If bars do not fit in one layer, d is measured to the centroid of the pattern.

$$\bar{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}}$$

Table A.4 Areas of Groups of StandardBars (in.[2])

Bar No.	Number of Bars 2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

Jack C McCormac, 1978
Design of Reinforced Concrete,

Rectangular Beam Design

7. Choose bars for A_s and determine spacing and cover, recheck h and weight

$d = 33.436"$ REV. 5x #9

$$a = \frac{A_s f_y}{.85 f'_c\, b} = \frac{5(60)}{.85(3)15} = 7.843"$$

Make final check of M_n using final d, and check that $M_u \le \phi M_n$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5(60)\left(33.436 - \frac{7.843}{2}\right)$$

$$M_n = 8854 \text{ K-''} = 737.8 \text{ K-'}$$

$$\phi M_n = 0.9(737.8) = 664 \text{ K-'}$$

$$\underset{M_u}{M_u = 653.3} < \underset{\phi M_n}{664}$$ ✓ OK

8. Check that $\varepsilon_t \ge 0.005$ (if not, increase h and reduce A_s)

$$c = \frac{a}{\beta_1} = \frac{7.843"}{0.85} = 9.227"$$

9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

$$\epsilon_t = \frac{d - c}{c}(0.003)$$

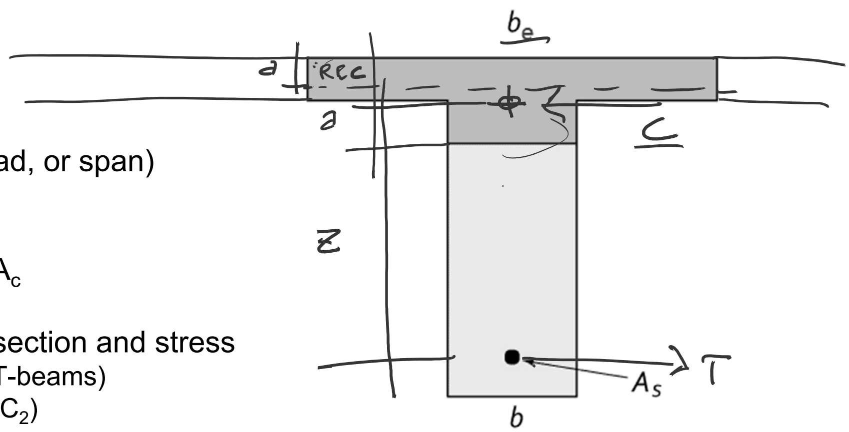
$$\epsilon_t = \frac{33.436 - 9.227}{9.227}(0.003)$$

✓ $\epsilon_t = 0.00787 > 0.005$ ✓ ok

Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, b_e, h, (span)
- Steel area - A_s
- Material properties – f'_c, f_y

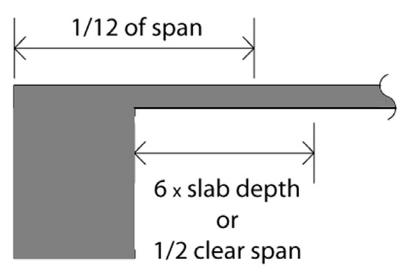
Required:
- Required Moment – M_u (or load, or span)



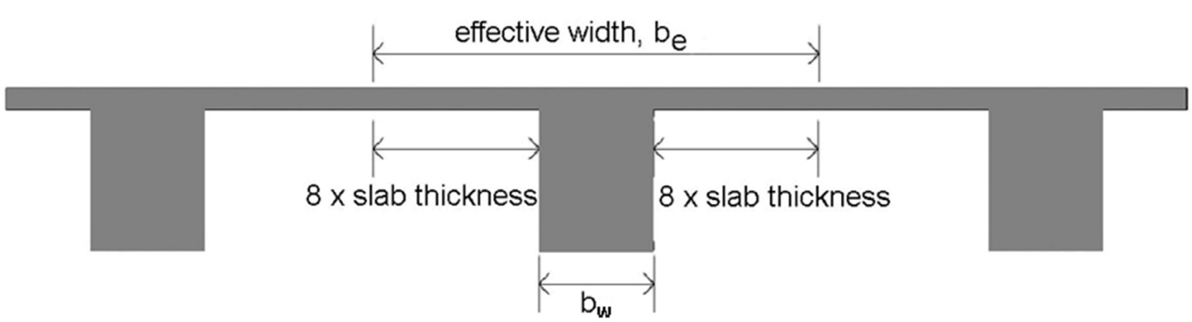
1. Find $\underline{T} = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set T = C and solve for A_c
3. Draw and label diagrams for section and stress
    1. Determine b effective (for T-beams)
    2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
    Working from the top down,
    add up area to make A_c
5. Find moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$
8. Check $A_{s,min} < A_s < A_{s,max}$
9. Check that $\varepsilon_t \geq 0.005$

Effective Flange Width, b_e

Slab on one side:
b_e least of either (total width) or (overhang + stem)
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam



Slab on both sides:
b_e least of either (total width) or (2 x overhang + stem)
- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)



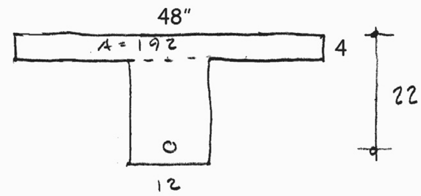
T-beam Design – Method 1

Given: f'_c = 3000 psi
f_y = 60 ksi
Applied load, M_u

Req'd: Reinforcement, As

1. Find b_{eff} for flange
2. Find d (estimate bar size)
3. Estimate moment arm, z
   0.9 d
4. Estimate As
   Check d (iterate)
4. Calculate Ac (total)

DATA: f'_c = 3000 psi
f_y = 60000 psi
M_u = 800 ᶠ-K



REQUIRED: A_s

CHOOSE z

$$.9d = .9 \times 22 = 19.8$$

$$d - \frac{h_f}{2} = 22 - \frac{4}{2} = 20 \leftarrow \text{TRY}$$

TRIAL A_s

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{800 \times 12}{.9(60)(20)} = 8.89 \text{ in}^2$$

COMPUTE A_c

$$A_c = \frac{A_s f_y}{.85 f'_c} = \frac{8.89(60)}{.85(3)} = 209.2$$

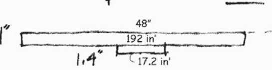
Non-Rectangular Section Analysis (cont.)

6. Check if A_c is within flange
   If yes:
   same as rectangular beam
   If no:
   find area below flange
7. Find a
8. Find centroid of compression
   Area, y-bar
9. Calculate z
10. Calculate As
    Check within 2%
    If not – find new a
11. Choose bars
    Check spacing

FIND a

① IS A_c WITHIN FLANGE? A_c = 209.2 A_f = 192 ∴ NO

② WHAT IS a?
209.2 − 192 = 17.2
17.2 ÷ 12 = 1.43



FIND $\bar{y}$

$$\bar{y} = \frac{\Sigma A d}{\Sigma A} = \frac{(192 \times 2) + (17.2 \times 4.7)}{209} = 2.22''$$

CALCULATE z

$$z = d - \bar{y} = 22 - 2.2 = 19.78''$$

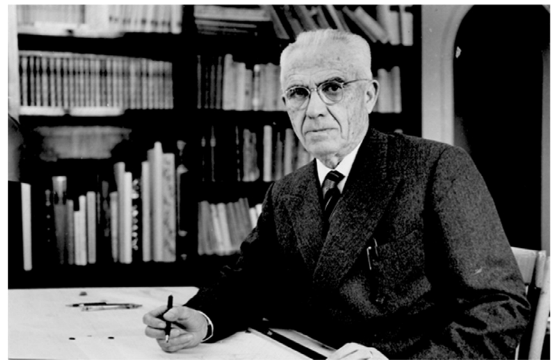
CALCULATE A_s

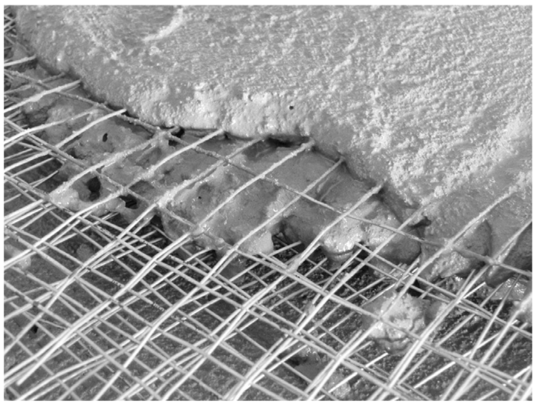
$$A_s = \frac{M_u}{\phi f_y z} = \frac{800 \times 12}{.9(60)(19.78)} = 8.99 \text{ in}^2$$

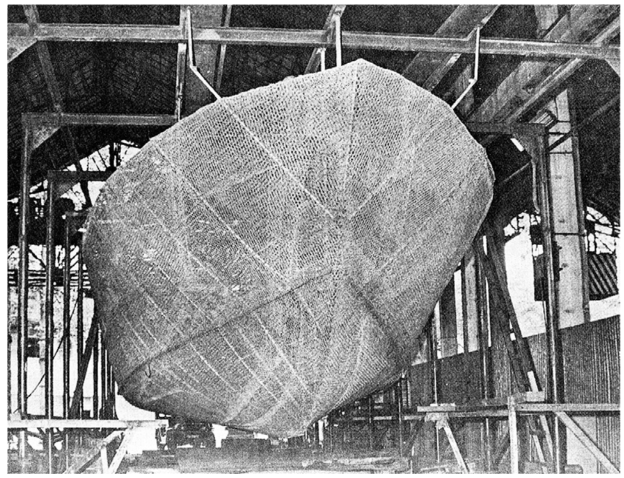
USE 9 × #9 or 6 × #11

Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking







Ferrocement

- Pioneered by Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking
- Low-tech applications

Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

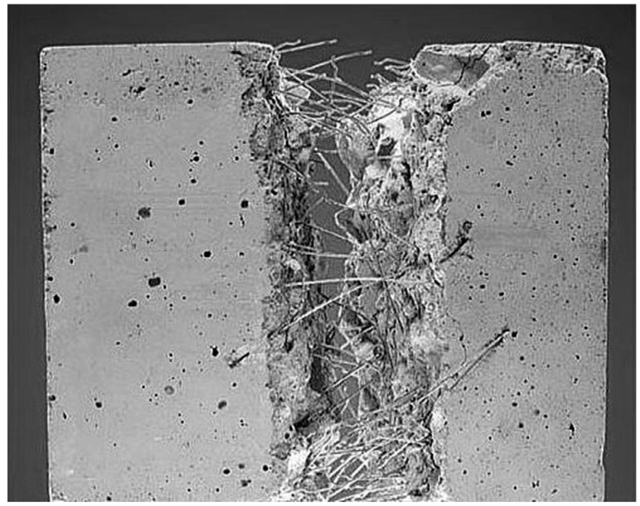
Palazetto dello Sport, Rome, 1957. P.L. Nervi

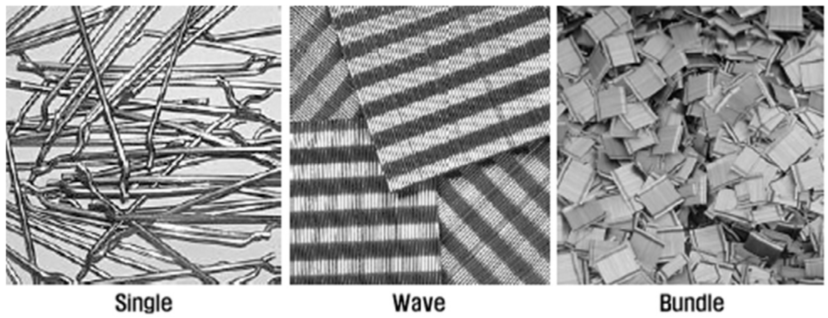
Fiber Reinforced Concrete

Several different fiber types:

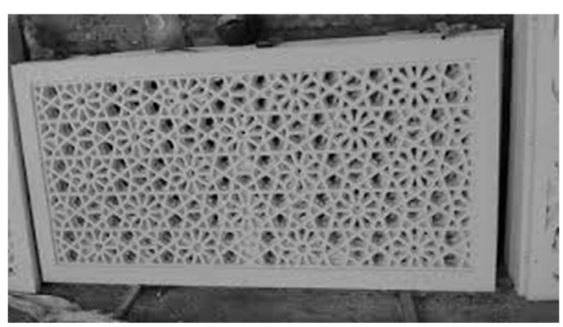
- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

Better crack control
Secondary reinforcement





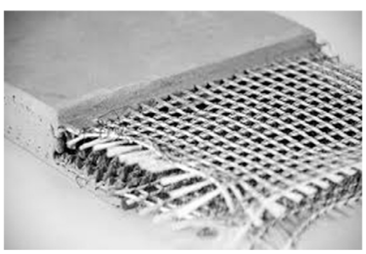
Glass Fiber Reinforced Concrete - GFRC



Carbon Fiber

Bamboo

Textile Reinforced Concrete (TRC)



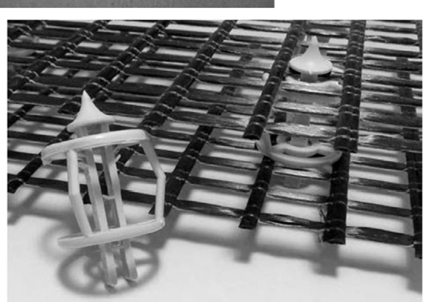


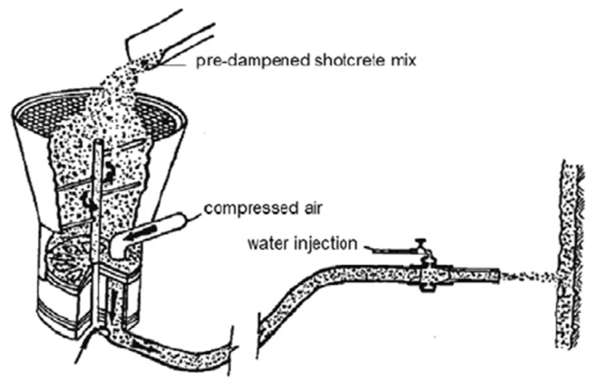
Figure 12: *distTEX: special spacers for textile grids [photo: Frank Schladitz, TU Dresden]*

Figure 13: *Manufacturing of the TRC hypar-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]*

Figure 10: *Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]*

Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels



3D-Print Evolution

https://www.youtube.com/watch?v=awpmJriWcEw