Architecture 324

Structures II

Composite Sections
(Steel Beam + Conc. Slab)

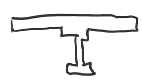
- Composite Sections by LRFD
- Analysis Methods

Photo by Mike Greenwood, 2009. Used with permission

Composite Design

Steel W section with concrete slab "attached" by shear studs.

The concrete slab acts as a wider and thicker compression flange.



Strength increase by 33% to 50%

Deflection reduced by 70% to 80%

Can attain either longer spans or smaller members – more economical in long spans

Smaller floor depth, therefore reduced overall building heights and weights

Reduced DL of system, reduction of other material vertically (façade, walls, plumbing, wiring, etc.)

Shear Studs

Also called Nelson studs after the company that originated them.

From AISC DigiLib

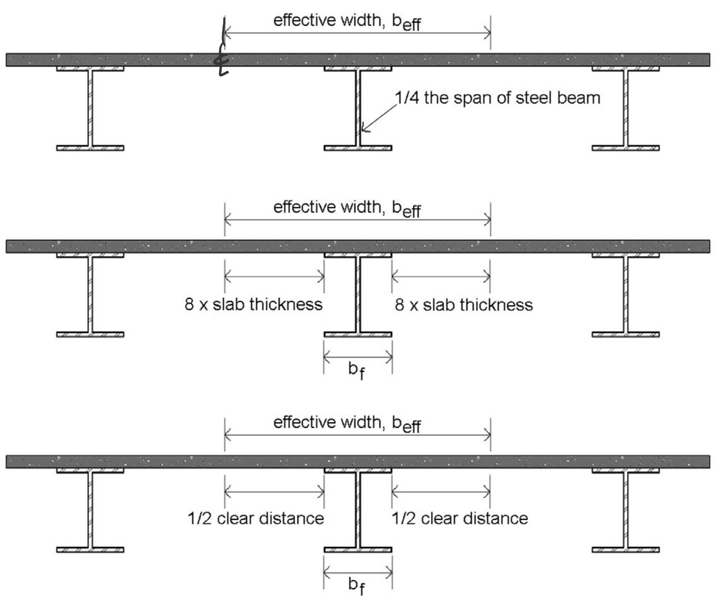
Can be spot welded through light gage decking onto W section

Effective Flange Width, b_e

Slab on both sides:

$\mathbf{b_e}$ is the **least** total width :

- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. b_e is the web on center spacing)

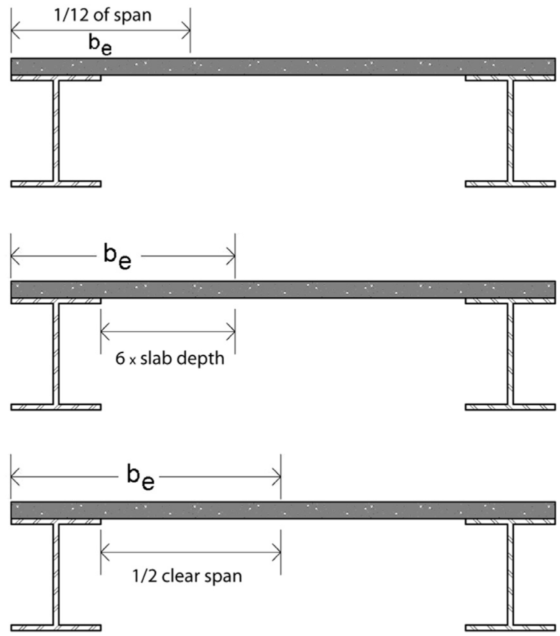


Effective Flange Width, b_e

Slab on one side:

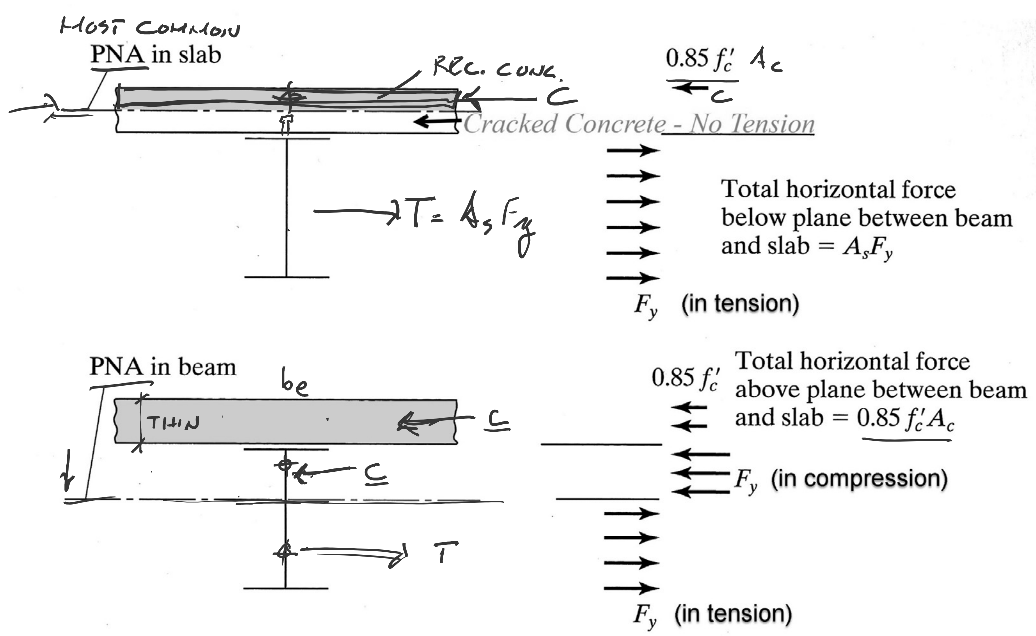
b_e is the **least** total width (i.e. overhang + steel flange) :

- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam



Analysis Procedure (LRFD)

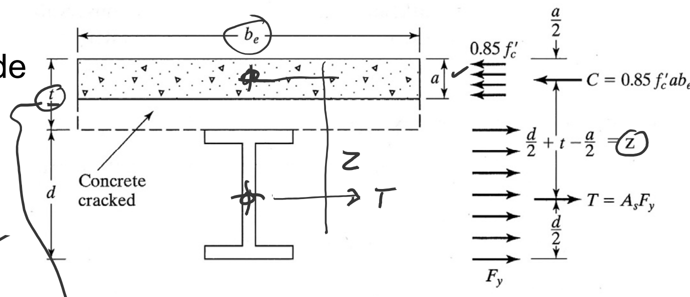
Case 1 – Plastic Neutral Axis (PNA) within slab
Case 2 – PNA within steel section



Analysis Procedure (LRFD)

Case1 – PNA within slab

Given: Slab and beam geometry
W-section size and steel grade
(floor loads)
Find: pass/fail or capacities



1. Define effective flange width, b_e
2. Calculate the effective depth of the concrete stress block, a IF $a < t$
3. If a is within concrete slab, the full steel section is in tension and:
   Mp = T z
   Mn = Mp = As Fy (d/2 + t - a/2)
4. Mu ≤ ϕ Mn

$$T = C$$

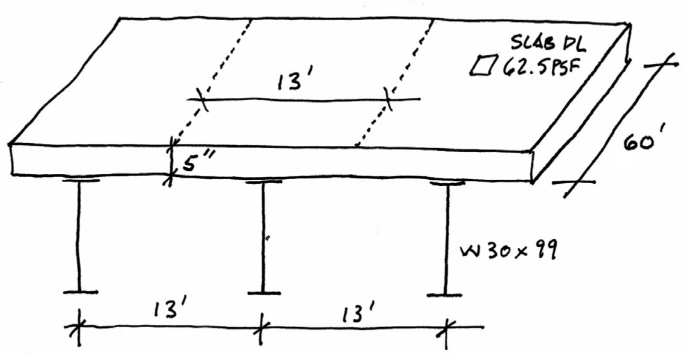
$$A_s f_y = 0.85 f_c' \, a \, b_e$$

$$a = \frac{A_s f_y}{0.85 f_c' \, b_e}$$

Non-composite vs. Composite Sections

Given:

- DL_{slab} = 62.5 psf = 812.5 plf
- DL_{beam} = 99 plf
- LL = ?
- W 30x99
- F_y = 50 ksi
- f'c $_{conc}$ = 4 ksi



LIVE
Find: Load Capacity

For this example, floor capacity is found for two different floor systems:

1. Find capacity of steel section independent from slab

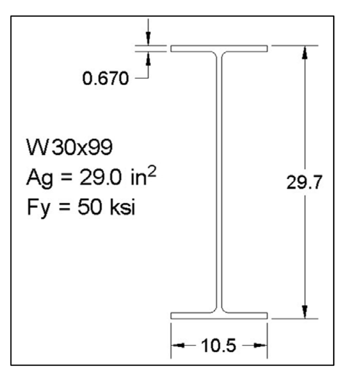
vs.

2. Find capacity of steel and slab as a composite section

WEIGHT of SLAB

$$\frac{5''}{12} \, 150\,PCF = 62.5\,PSF$$

$$13' \times 62.5\,PSF = 812.5\,PLF$$

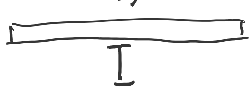


Part 1 Non-composite Capacity Analysis
(Example part 1 - steel beam alone - LRFD)

Given:
- DL_{slab} = 62.5 psf = 812.5 plf
- DL_{beam} = 99 plf
- W 30x99

1. Find section modulus, Z_x in the steel W-section chart.
2. Calculate Mn = Fy Zx.
3. Mu ≤ ϕ Mn
4. Find wu from moment equation
5. Subtract the DL to find the remaining LL.
6. Calculate LL capacity in PSF.

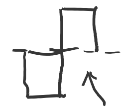
13'



WEIGHT of SLAB

$\frac{5''}{12}$ 150 PCF = 62.5 PSF

13' × 62.5 PSF = 812.5 PLF



PLASTIC

W30×99 $Z_x = 312 \text{ in}^2$ AISC

$M_n = F_y Z_x = \underset{KSI}{50}(\underset{in^2}{312}) = 15600 \text{ ''-K}$

$= 1300 \text{ '-K}$

$\phi M_n = M_U = 0.9(1300) = 1170 \text{ '-K}$

$\phi M_n = M_U = \frac{w_U l^2}{8} = \frac{w_U (60)^2}{8} = 1170 \text{ '-K}$

$w_U = 2.6 \text{ KLF} = 2600 \text{ PLF}$

$w_U = 1.2(w_{DL}) + 1.6(w_{LL}) = 2600 \text{ PLF}$ (γ DL, γ LL ?)

$w_U = 1.2(\underset{SLAB}{812.5} + \underset{I}{99}) + 1.6(w_{LL}) = 2600 \text{ PLF}$

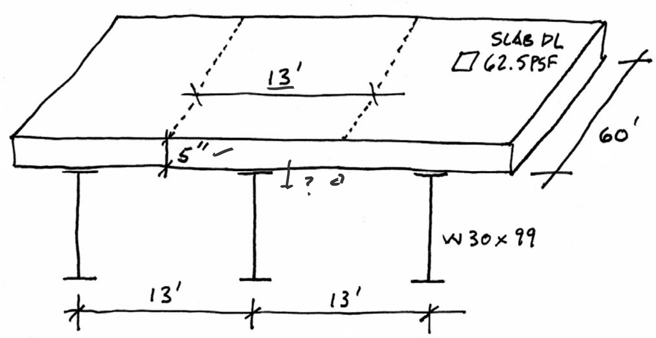
$w_{LL} = 941.3 \text{ PLF}$

$LL = 941.3 \text{ PLF} / 13' = 72.4 \text{ PSF}$

Composite Analysis Procedure
(Case1 – **PNA within slab**)

Given: Slab and beam geometry
W-section size and steel grade
(floor loads)

Find: pass/fail or capacities

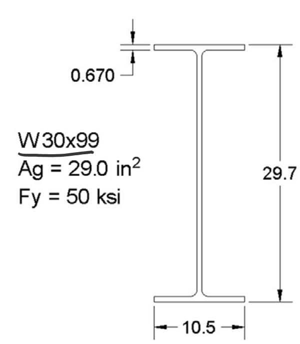


1. Determine effective flange width, b_e
2. Calculate the effective depth of the concrete stress block, a
3. If a is within concrete slab, the full steel section is in tension and:
   Mn = Mp = As Fy (d/2 + t - a/2)
4. Mu ≤ ϕ Mn
5. Use Mu to calculate factored loads with appropriate beam moment equation.

WEIGHT of SLAB

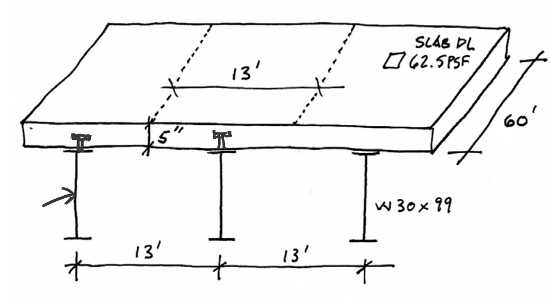
$\frac{5''}{12}$ 150 PCF = 62.5 PSF

13' × 62.5 PSF = 812.5 PLF



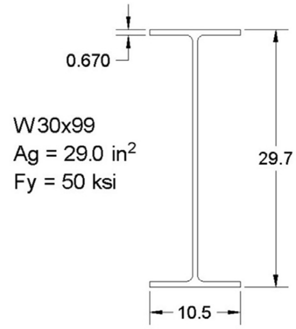
Composite Capacity Analysis
(Example part 2 - composite steel beam and slab)

Given:
- DL_{slab} = 62.5 psf = 812.5 plf
- DL_{beam} = 99 plf
- LL = ?
- W 30x99
- F_y = 50 ksi
- $f'c_{\,conc}$ = 4 ksi



Find:

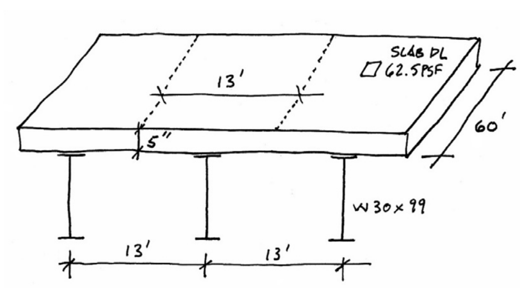
capacity of steel and slab as a composite section

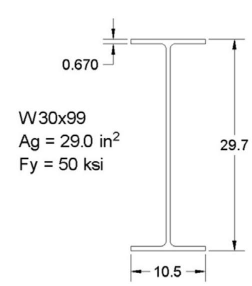


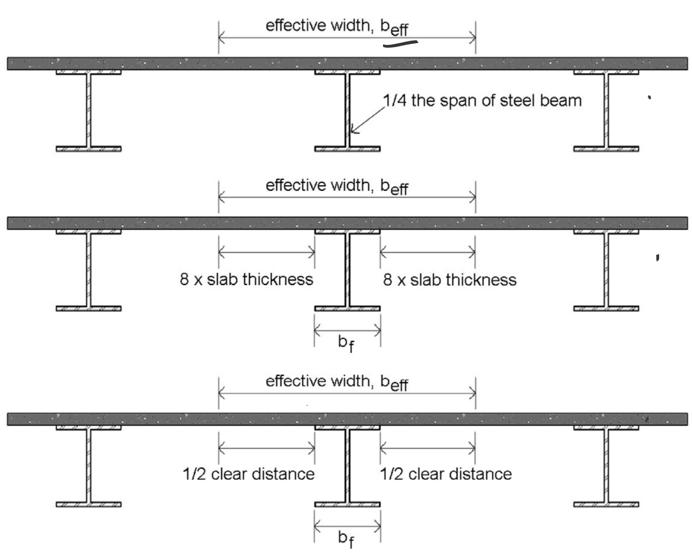
Part 2 Composite Capacity Analysis

1. Determine effective flange width, b_e

b_e is the **least** total width :
- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. b_e is the web on center spacing)







b_e is the **least** total width :

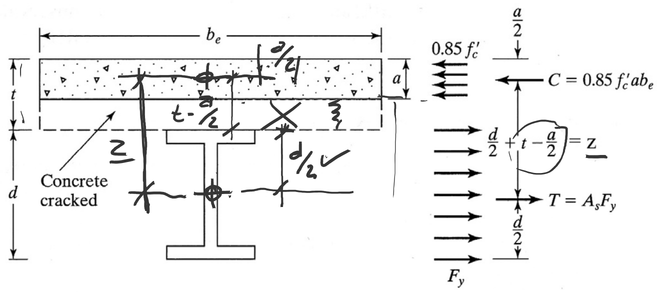
b_e ① = ¼ 60' = 15' = 180"

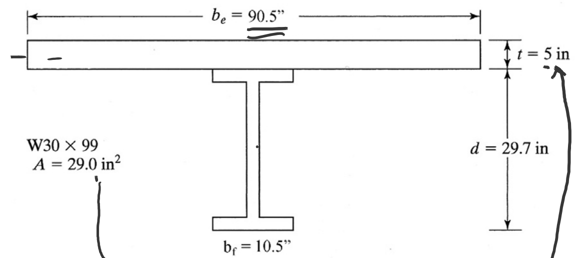
b_e ② = 8(5")×2 + 10.5 = 90.5"

b_e ③ = 13' = 156"

$\therefore b_e = 90.5"$

Part 2 - Composite Capacity Analysis cont.





2. Calculate the effective depth of the concrete stress block, a

$$a = \frac{A_s F_y}{0.85 f'_c b_e} = \frac{29.0(50^{\text{KSI}})}{0.85(4^{\text{KSI}})(90.5)} = 4.712''$$

$4.712'' < 5''$ ∴ WITHIN SLAB

3. If a is within concrete slab, the full steel section is in tension and:

   Mn = Mp = As Fy (d/2 + t - a/2)

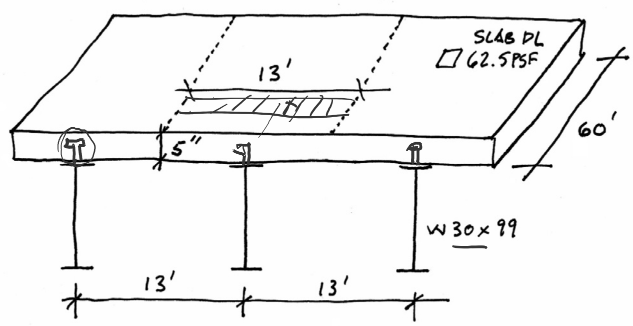
$$M_n = M_p = T \times z$$
$$M_n = A_s F_y \left(\frac{d}{2} + t - \frac{a}{2}\right)$$
$$M_n = 29.0\,\text{in}^2 (50\,\text{KSI}) \left(\frac{29.7''}{2} + 5'' - \frac{4.712''}{2}\right)$$
$$M_n = 25366\ ''\text{-K} = 2114\ '\text{-K}$$

4. Mu ≤ Φ Mn

$$\phi M_n = 0.9(2114) = 1902\ '\text{-K} = M_u$$

Composite Analysis cont.

4. Mu ≤ Φ Mn
5. Find total factored w_u.
6. Subtract the factored w_{DL} to find w_{LL}
7. Calculate the LL in PSF based on the w_{LL}.



WEIGHT of SLAB

$\frac{5''}{12}$ 150 PCF = 62.5 PSF

13' × 62.5 PSF = 812.5 PLF

$$M_u = 1902^{\text{K-'}} = \frac{w_u l^2}{8} = \frac{w_u\, 60^2}{8}$$
$$w_u = 4.227\,\text{KLF} = 1.2(w_{DL}) + 1.6(w_{LL})$$
$$w_u = 4227\,\text{PLF} = 1.2(\underbrace{911.5}_{\text{SLAB}+\text{I}}) + 1.6(w_{LL})$$
$$w_{LL} = 1958\,\text{PLF}$$
$$LL = 1958\,\text{PLF}/13' = 150\,\text{PSF}$$

Conclusion:

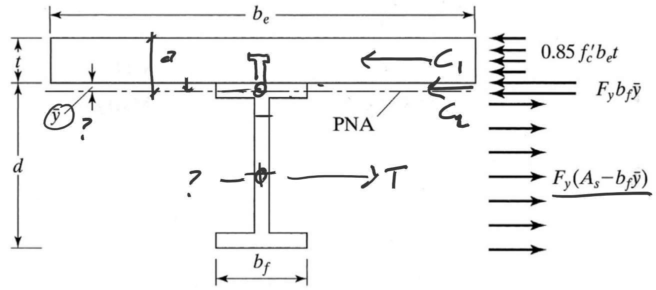
Non-composite LL = 72.4 PSF

Composite LL = 150 PSF

Composite Analysis Procedure

(Case 2 – **PNA within W-section**)

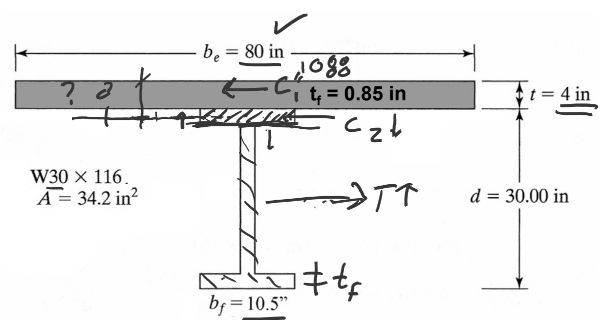
Given: Slab and beam geometry
W-section size and steel grade
(floor loads)
Find: pass/fail or capacities



1. Determine effective flange width, b_e
2. Calculate the effective depth of the concrete stress block, a.
3. If a is within steel section, the part below the Plastic Neutral Axis (PNA) is in tension and everything above the PNA is in compression (the steel and the concrete)
4. Check if PNA falls within flange or web of the W-section
5. Find $\bar{y}$ by equating T = C
6. Mn = Mp = $C_1(z_1) + C_2(z_2) + T(z_3)$
7. Mu = Φ Mn

Composite Analysis Example

(Case 2 – PNA within W-section)



Given: Slab and beam geometry
W-section size and steel grade
(floor loads)
Find: moment capacity

1. Determine effective flange width, b_e = 80"
2. Calculate the effective depth of the concrete stress block, a.
3. Check if PNA is within upper flange. Assume PNA is at top of web. Check C and T. If C is greater than T, then PNA is within the top flange.

Rec.

$$a = \frac{A_s f_y}{0.85 f'_c b_e} = \frac{34.2\text{ in}^2 (50\text{ ksi})}{0.85(4\text{ ksi})(80)} = 6.29''$$

$6.29'' > 4$ ∴ BELOW SLAB

$C_1 = 0.85 f'_c b_e t$

$C_1 = 0.85(4\text{ ksi})(80'')(4'') = 1088^K$

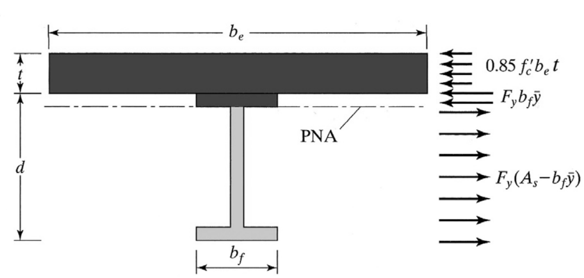
ASSUME $C_2 = F_y b_f t_f = 50\text{ ksi}(10.5'')(0.85'') = 446.25^K$

$C = C_1 + C_2 = 1088^K + 446.25^K = 1534^K$

$T = F_y(A_s - b_f t_f) = 50(34.2 - 8.925) = 1263.7^K$

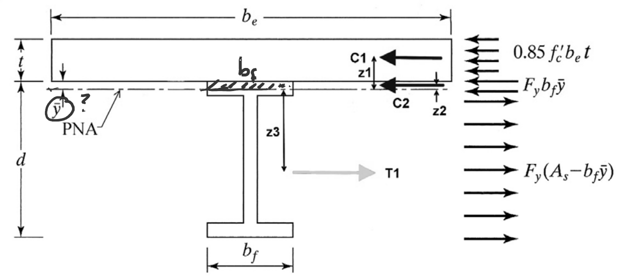
$\Sigma F_H = 0 = T - C$ ∴ $T = C$

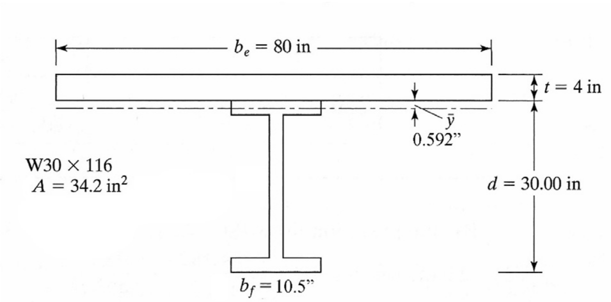
Since horizontal forces should sum to zero, T should equal C. So C should be less than 1534 and T greater than 1263. Therefore, the PNA must be higher and within the flange.



Composite Analysis Procedure
(Case 2 – PNA within W-section)

3. If a is within steel section, only the part below the PNA is in tension and the top is in compression with all concrete
4. Check if PNA falls within flange or web of the W-section
5. Find $\bar{y}$ by equating T = C





T = C (SLAB)

$$(A_s - b_f \bar{y}) F_y = 0.85 f'_c b_e t + b_f \bar{y} \times F_y$$

$$A_s F_y - 0.85 f'_c b_e t = 2 (b_f \bar{y} F_y)$$

$$\bar{y} = \frac{A_s F_y - 0.85 f'_c b_e t}{2 b_f F_y}$$

Composite Analysis Procedure
(Case 2 – PNA within W-section)

5. Find $\bar{y}$ by equating T = C
6. $M_n = M_p = C_1(z_1) + C_2(z_2) + T(z)$
7. $M_u = \phi M_n$

$$\bar{y} = \frac{34.2\,in^2 (50\,ksi) - 0.85(4\,ksi)(80")(4")}{2(10.5")(50\,ksi)}$$

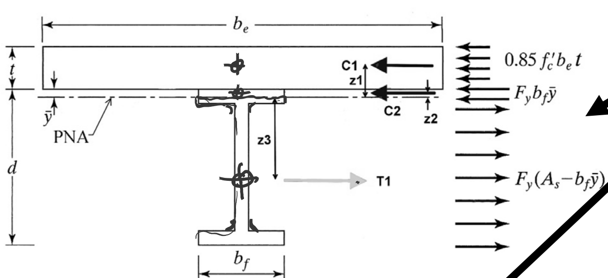
$\bar{y} = 0.592" < t_f = 0.85"$ for W30x116

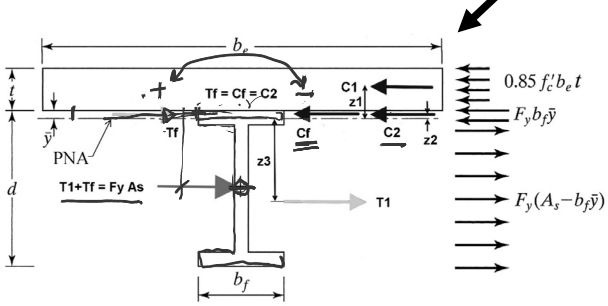
$$\Sigma M_{@PNA} = C_1(z_1) + C_2(z_2) + T_1(z_3) \; ?$$

$$= C_1(z_1) + 2C_2(z_2) + A_s F_y \left(\frac{d}{2} - \bar{y}\right)$$

$$M_n = M_p = \overbrace{0.85 f'_c b_e t}^{C_1} \overbrace{\left(\frac{t}{2} + \bar{y}\right)}^{z_1} + 2 \overbrace{F_y b_f \bar{y}}^{C_2} \overbrace{\left(\frac{\bar{y}}{2}\right)}^{z_2} + \overbrace{F_y A_s}^{T} \overbrace{\left(\frac{d}{2} - \bar{y}\right)}^{z}$$

$$M_n = 0.85(4\,ksi)(80")4"\left(\frac{4"}{2} + 0.592"\right) + 2(50\,ksi)(10.5")(0.592")\frac{0.592"}{2} + (50\,ksi)(34.2\,in^2)\left(\frac{30"}{2} - 0.592"\right) = 27650"\text{-k} = 2304'\text{-k}$$





MOMENT CAPACITY:

$$M_u = \phi M_n = 0.9(2304'\text{-k}) = 2074'\text{-k}$$