

# Steel Beam Analysis

- Steel Codes: ASD vs. LRFD
- Analysis Methods



## Steel Beams by LRFD

### Yield Stress Values

- A36 Carbon Steel  $F_y = 36$  ksi
- A992 High Strength  $F_y = 50$  ksi

### Elastic Analysis for Bending

#### • Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z$$

- Braced against LTB ( $L_b < L_p$ )

#### • Inelastic Buckling “Decreased” (zone 2)

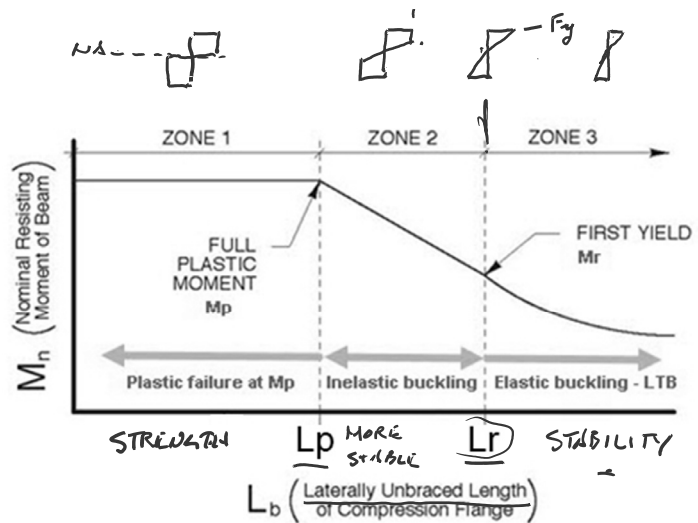
$$M_n = C_b(M_p - (M_p - M_r)[(L_b - L_p)/(L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

#### • Elastic Buckling “Decreased Further” (zone 3)

$$M_{cr} = C_b * \pi/L_b \sqrt{(E * I_y * G * J + (\pi * E / L_b)^2 * I_y * C_w)}$$

- $L_b > L_r$



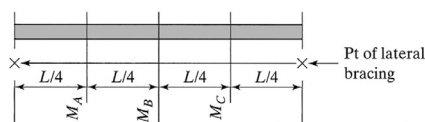
$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$M_p = F_y Z_x$$

$$M_r = 0.7 F_y S_x$$

$C_b$  is LTB modification factor

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3 M_A + 4 M_B + 3 M_C}$$



# Steel Beams by LRFD

## Analysis for Bending

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$$M_n = M_p = F_y Z_x$$

- Braced against LTB ( $L_b < L_p$ )

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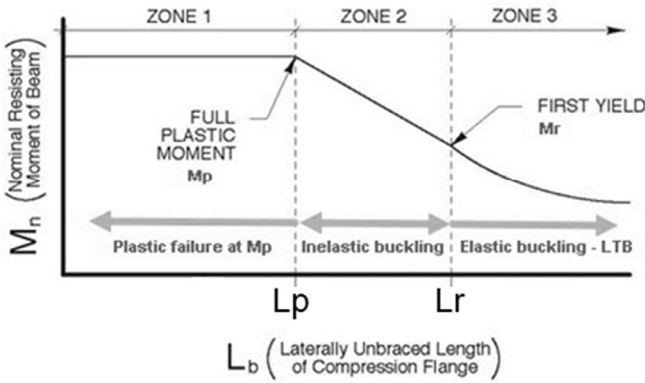
$$M_n = C_b (M_p - (M_p - M_r) [(L_b - L_p) / (L_r - L_p)]) < M_p$$

- $L_p < L_b < L_r$

- Elastic Buckling "Decreased Further" (zone 3)

$$M_{cr} = C_b * \pi / L_b \sqrt{(E * I_y * G * J + (\pi * E / L_b)^2 * I_y * C_w)}$$

- $L_b > L_r$



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Table 3-2 (continued)  
 $F_y = 50$  ksi  
**W-Shapes**  
 Selection by  $Z_x$

$Z_x$

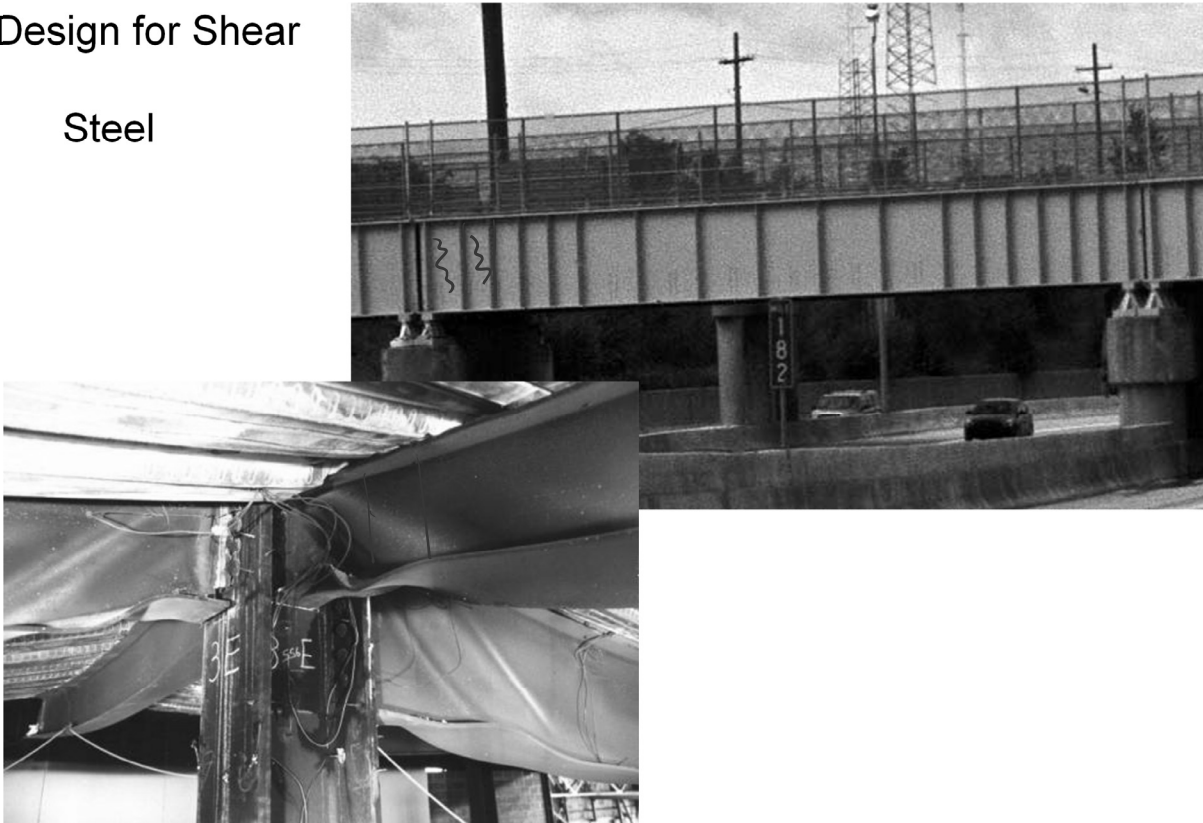
Shape	$Z_x$ in. <sup>3</sup>	$M_{px}/\Omega_b$		$\phi_b M_{px}$		$M_{rx}/\Omega_b$		$\phi_b M_{rx}$		$BF/\Omega_b$		$\phi_b BF$		$L_p$ ft	$L_r$ ft	$I_x$ in. <sup>4</sup>	$V_{nx}/\Omega_v$		$\phi_v V_{nx}$	
		kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft	kip-ft				ASD	LRFD	ASD	LRFD
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234								
W14x74	126	314	473	196	294	5.31	8.05	6.76	31.0	795	128	192								
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227								
W12x79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175								
W14x68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174								
W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196								
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212								
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237								
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159								
W21x48 <sup>(1)</sup>	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216								
W16x57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212								
W14x61	102	254	383	161	242	4.93	7.48	8.65	27.5	640	104	156								
W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192								
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169								
W12x65 <sup>(1)</sup>	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142								
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217								
W16x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186								
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195								
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154								
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132								
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	40.6	394	97.8	147								
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167								
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169								
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141								
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	129								
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	125								
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146								
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135								
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154								
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125								
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112								

ASD      LRFD      <sup>(1)</sup>Shape exceeds compact limit for flexure with  $F_y = 50$  ksi; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$        $\phi_b = 0.90$   
 $\Omega_v = 1.50$        $\phi_v = 1.00$

## Design for Shear

### Steel



# Design for Shear

Shear stress in steel sections is approximated by averaging the stress in the web:

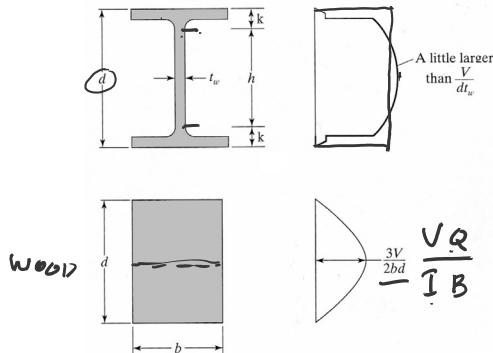
$$F_v = V / A_w$$

$$A_w = d * t_w$$

To adjust the stress a reduction factor of 0.6 is applied to  $F_y$

$$F_v = 0.6 F_y$$

so,  $V_n = 0.6 F_y A_w$  (Zone 1)



The equations for the 3 stress zones:  
( $\phi$  in all cases = 1.0)

## Zone 1:

**WEB YIELDING (Most beam sections fall into this category)**

if  $\left(\frac{h}{t_w}\right) \leq 2.45 \sqrt{E/F_y} = 59$  (for 50 ksi steel)

then:  $V_n = 0.6 F_y A_w$

$$P A = F$$

## Zone 2:

**INELASTIC WEB BUCKLING**

if  $2.45 \sqrt{E/F_y} < \frac{h}{t_w} \leq 3.07 \sqrt{E/F_y} = 74$  (for 50 ksi steel)

then:  $V_n = 0.6 F_y A_w (2.45 \sqrt{E/F_y}) / \frac{h}{t_w}$

## Zone 3:

**ELASTIC WEB BUCKLING**

if  $3.07 \sqrt{E/F_y} < \frac{h}{t_w} \leq 260$

then:  $V_n = A_w \left[ \frac{4.25 E}{\left(\frac{h}{t_w}\right)^2} \right]$

# Procedure - Analysis of Steel Beams – for Zone 1 $L_b < L_p$

## Pass/Fail

Given: yield stress, steel section, loading, bracing ( $L_b$ )

Find: pass/fail of section

DESIGN

1. Calculate the factored design load  $w_u$

$$w_u = 1.2 W_{DL} + 1.6 W_{LL} \quad \text{ACI 318 - 7}$$

2. Determine the design moment  $M_u$ .

$M_u$  will be the maximum beam moment using the factored loads

STRENGTH

3. Insure that  $L_b < L_p$  (zone 1)

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

4. Determine the nominal moment,  $M_n$

$$M_n = F_y (Z_x) \quad (\text{look up } Z_x \text{ for section})$$

5. Factor the nominal moment

$$\phi M_n = 0.90 M_n$$

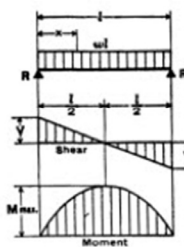
6. Check that  $M_u < \phi M_n$  PASS

DESIGN < STRENGTH

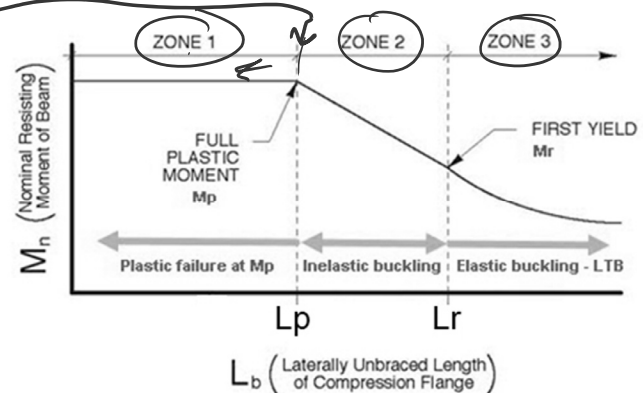
7. Check shear ✓

8. Check deflection ✓

## 1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load	...	=	$wl$
$R = V$	...	=	$\frac{wl}{2}$
$V_x$	...	=	$w\left(\frac{l}{2} - x\right)$
$M$ max. (at center)	...	=	$\frac{wl^2}{8}$
$M_x$	...	=	$\frac{wx}{2}(l-x)$
$\Delta$ max. (at center)	...	=	$\frac{5wl^4}{384EI}$
$\Delta x$	...	=	$\frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$



# Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

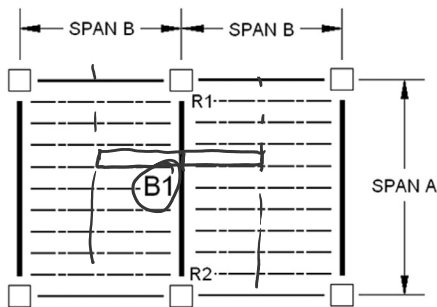
Given: yield stress, steel section, loading, braced 24" o.c.

Find: pass/fail of section

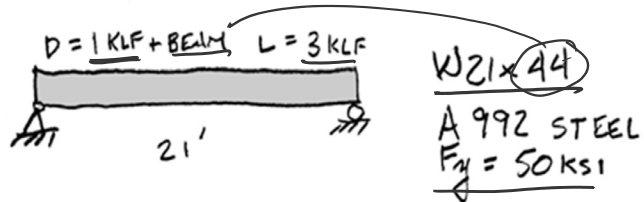
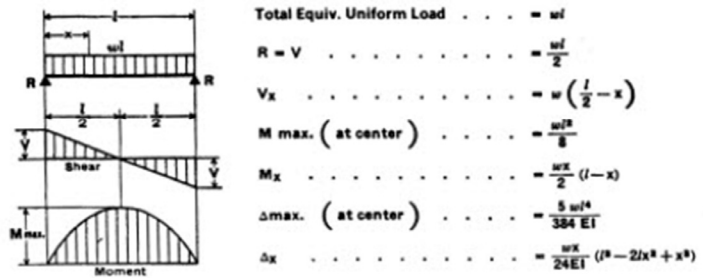
1. Calculate the factored design load  $w_u$

$$w_u = 1.2W_{DL} + 1.6W_{LL}$$

2. Determine the design moment  $M_u$ .  $M_u$  will be the maximum beam moment using the factored loads.



## 1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



FROM TABLE 1-1 AISC  $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21'^2}{8} = 333.5 \text{ K-ft}$$

# Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

3. Insure that  $L_b < L_p$  (zone 1)

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

$$L_p = 1.76 (1.26) \sqrt{29000/50}$$

$$L_p = 53.4 \text{ in.} > 24 \text{ in.} \text{ ok}$$

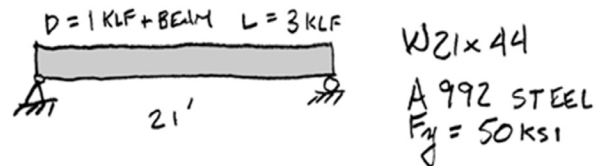
4. Determine the nominal moment,  $M_n$

$$M_n = M_p = F_y Z_x \text{ (for zone 1)}$$

(look up  $Z_x$  for section)

5. Factor the nominal moment  $\phi M_n = 0.90 M_n$

6. Check that  $M_u < \phi M_n$



FROM TABLE 1-1 AISC  $Z_x = 95.4 \text{ in}^3$

$$M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 \text{ in}^3 = 4770 \text{ K-in}$$

$$M_n = 4770 \text{ K-in} / 12 = 397.5 \text{ K-ft}$$

$$\phi M_n = 0.9 (397.5) = 357.7 \text{ K-ft}$$

DESIGN < STRENGTH

$$M_u = 333.5 \text{ K-ft} < 357.7 \text{ K-ft} = \phi M_n$$

$\therefore$  PASS

# Steel Beams by LRFD

## Analysis for Bending

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- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z_x$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.} > 24 \text{ in. o.c.}$

- $\phi_b M_{px} = 358 \text{ k-ft} > M_u = 333.5 \text{ k-ft}$  ✓

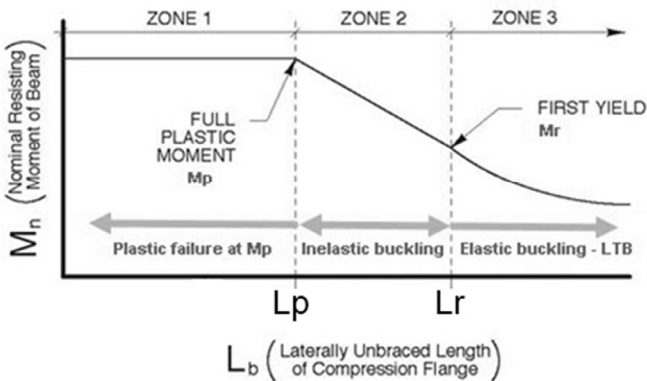


Table 3-2 (continued)  
W-Shapes  
Selection by  $Z_x$

$F_y = 50 \text{ ksi}$

Shape	$Z_x$ in. <sup>3</sup>	$M_{px}/\Omega_b$		$M_{rx}/\Omega_b$		$\phi_b M_{px}$		$BF/\Omega_b$		$\phi_b BF$	$L_p$ ft	$L_r$ ft	$I_x$ in. <sup>4</sup>	$V_{nx}/\Omega_v$		$\phi_v V_{nx}$ kips
		kip-ft	kip-ft	kip-ft	kip-ft	ASD	LRFD	ASD	LRFD					ASD	LRFD	
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234				
W14x74	126	314	473	196	294	5.31	8.05	6.76	31.0	795	128	102				
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227				
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W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196				
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212				
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237				
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159				
W21x48 <sup>(1)</sup>	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216				
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W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192				
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169				
W12x65 <sup>(1)</sup>	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142				
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W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169				
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141				
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125				
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W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154				
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125				
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112				

ASD LRFD <sup>(1)</sup>Shape exceeds compact limit for flexure with  $F_y = 50 \text{ ksi}$ ; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$   $\phi_b = 0.90$   
 $\Omega_v = 1.50$   $\phi_v = 1.00$

## Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

### 7. Check shear for W21x44

CHECK SHEAR:

#### Zone 1:

WEB YIELDING (Most beam sections fall into this category)

if  $\left(\frac{h}{t_w}\right) \leq 2.45 \sqrt{E/F_y} = 59$  (for 50 ksi steel)

then:  $V_n = 0.6 F_y A_w$   $\leftarrow dt_w$

FROM AISC TABLE 1-1

$\frac{h}{t_w} = 53.6 < 59$  (zone 1)

Table 1-1 (continued)  
W-Shapes  
Dimensions

Shape	Area, A in. <sup>2</sup>	Depth, d in.	Web		Flange		Distance				Work-able Gage				
			Thickness, t <sub>w</sub> in.	t <sub>w</sub> <sup>2</sup> / in.	Width, b <sub>f</sub> in.	Thickness, t <sub>f</sub> in.	k	k <sub>des</sub>	k <sub>set</sub>	T					
W21x93	27.3	21.6	2 1/8	0.580	3/16	5/16	8.42	8 3/8	0.930	1 5/16	1.43	1 3/8	1 3/16	18 3/8	5 1/2
>83°	24.4	21.4	2 1/8	0.515	1/2	1/4	8.36	8 3/8	0.835	1 3/16	1.34	1 1/2	7/8		
>73°	21.5	21.2	2 1/8	0.455	7/16	1/4	8.30	8 3/4	0.740	3/4	1.24	1 1/16	7/8		
>68°	20.0	21.1	2 1/8	0.430	7/16	1/4	8.27	8 3/4	0.685	1 1/16	1.19	1 3/8	7/8		
>62°	18.3	21.0	2 1/8	0.400	3/8	3/16	8.24	8 3/4	0.615	5/8	1.12	1 3/16	1 3/16		
>55°	16.2	20.8	2 3/8	0.375	3/8	3/16	8.22	8 3/4	0.522	1/2	1.02	1 3/16	1 3/16		
>48° <sup>(1)</sup>	14.1	20.6	2 3/8	0.350	3/8	3/16	8.14	8 3/8	0.430	7/16	0.930	1 1/8	1 3/16		
W21x57°	16.7	21.1	2 1/8	0.405	3/8	3/16	6.56	6 1/2	0.650	5/8	1.15	1 3/16	1 3/16	18 3/8	3 1/2
>50°	14.7	20.8	2 3/8	0.380	3/8	3/16	6.53	6 1/2	0.535	9/16	1.04	1 1/4	1 3/16		
>44°	13.0	20.7	2 3/8	0.350	3/8	3/16	6.50	6 1/2	0.450	7/16	0.950	1 1/8	1 3/16		

Table 1-1 (continued)  
W-Shapes  
Properties

Nominal wt. lb/ft	Compact Section Criteria		Axis X-X			Axis Y-Y				Torsional Properties				
	b <sub>f</sub> /2t <sub>f</sub>	h/t <sub>w</sub>	I	S	r	J	S	r	Z	r <sub>ts</sub>	h <sub>o</sub>	J	C <sub>w</sub>	
93	4.53	32.3	2070	192	8.70	221	92.9	22.1	1.84	34.7	2.24	20.7	6.03	9940
83	5.00	36.4	1830	171	8.67	196	81.4	19.5	1.83	30.5	2.21	20.6	4.34	8630
73	5.60	41.2	1600	151	8.64	172	70.6	17.0	1.81	26.6	2.19	20.5	3.02	7410
68	6.04	43.6	1480	140	8.60	160	64.7	15.7	1.80	24.4	2.17	20.4	2.45	6760
62	6.70	46.9	1330	127	8.54	144	57.5	14.0	1.77	21.7	2.15	20.4	1.83	5960
55	7.87	50.0	1140	110	8.40	126	48.4	11.8	1.73	18.4	2.11	20.3	1.24	4980
48	9.47	53.6	959	93.0	8.24	107	38.7	9.52	1.66	14.9	2.05	20.2	0.803	3950
57	5.04	46.3	1170	111	8.36	129	30.6	9.35	1.35	14.8	1.68	20.5	1.77	3190
50	6.10	49.4	984	94.5	8.18	110	24.9	7.64	1.30	12.2	1.64	20.3	1.14	2570
44	7.22	53.6	843	81.6	8.06	95.4	20.7	6.37	1.26	10.2	1.60	20.3	0.770	2110

# Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

7. Check shear (zone 1)

FROM AISC TABLE 1-1

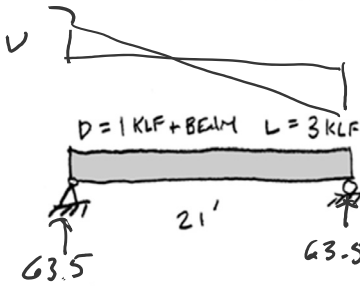
$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

Zone 1: ✓

WEB YIELDING (Most beam sections fall into this category)

$$\text{if } \frac{h}{t_w} \leq 2.45 \sqrt{E/F_y} = 59 \text{ (for 50 ksi steel)}$$

$$\text{then: } V_n = 0.6 F_y A_w$$



W21x44  
A 992 STEEL  
 $F_y = 50 \text{ ksi}$

$$\text{FROM TABLE 1-1 AISC } Z_x = 95.4 \text{ in}^3$$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

CHECK SHEAR:

$$V_u = \frac{w_u L}{2} = \frac{6.05(21)}{2} = 63.5 \text{ K}$$

FROM AISC TABLE 1-1

$$\frac{h}{t_w} = 53.6 < 59 \text{ (zone 1)}$$

$$V_n = 0.6 F_y A_w = 0.6(50)(20.7 \times 0.35)$$

$$V_n = 217.35 \text{ K}$$

$$\phi V_n = 1.0(217.35) = 217.35 \text{ K}$$

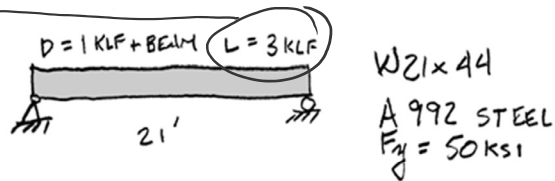
$$V_u = 63.5 \text{ K} < 217.3 \text{ K} = \phi V_n \checkmark$$

DESIGN < STRENGTH

Therefore, pass.

# Example: Pass/Fail Analysis of Steel Beams – for Zone 1 $L_b < L_p$

8. Check deflection



FROM TABLE 1-1 AISC  $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$\Delta_{\text{MAX}} = \frac{5 w_u L^4}{384 EI} = \frac{5(3000) 21^4 (1728)}{384(29000000)(843)}$$

$$= 0.535 \text{ in}$$

$$\frac{L}{360} = \frac{21(12)}{360} = 0.7 \text{ in}$$

$$\Delta_{\text{ACTUAL}} = 0.535 \text{ in} < 0.7 \text{ in} = \Delta_{\text{ALLOWABLE}} \checkmark$$

TABLE 1604.3 DEFLECTION LIMITS<sup>a, b, c, h, i</sup>

CONSTRUCTION	(L)	S or W	D + L <sup>d, g</sup>
Roof members: <sup>e</sup>			
Supporting plaster or stucco ceiling	//360	//360	//240
Supporting nonplaster ceiling	//240	//240	//180
Not supporting ceiling	//180	//180	//120
Floor members	(//360)	—	//240
Exterior walls:			
With plaster or stucco finishes	—	//360	—
With other brittle finishes	—	//240	—
With flexible finishes	—	//120	—
Interior partitions: <sup>b</sup>			
With plaster or stucco finishes	//360	—	—
With other brittle finishes	//240	—	—
With flexible finishes	//120	—	—
Farm buildings	—	—	//180
Greenhouses	—	—	//120

# Procedure - Analysis of Steel Beam - Capacity

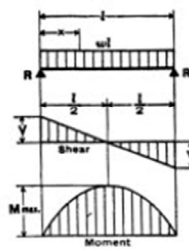
Given: yield stress, steel section, bracing

Find: moment or load capacity

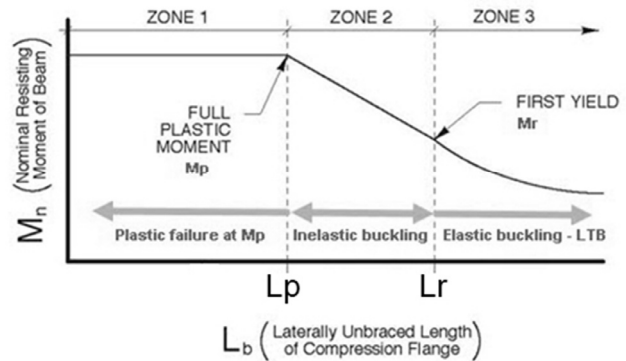
1. Determine the unbraced length of the compression flange ( $L_b$ ).
2. Find the  $L_p$  and  $L_r$  values from the AISC  $Z_x$  Table 3-2
3. Compare  $L_b$  to  $L_p$  and  $L_r$  and determine which equation for  $M_n$  or  $M_{cr}$  to be used.
4. Determine the beam load equation for maximum moment in the beam.
5. Calculate load based on maximum moment.  $M_u = \phi_b M_n$

SOLVE FOR LOAD

## 1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load . . . . . =  $wl$   
 $R = V$  . . . . . =  $\frac{wl}{2}$   
 $V_x$  . . . . . =  $w(\frac{l}{2} - x)$   
 $M_{max. (at center)}$  . . . . . =  $\frac{wl^2}{8}$   
 $M_x$  . . . . . =  $\frac{wx}{2}(l-x)$   
 $\Delta_{max. (at center)}$  . . . . . =  $\frac{5wl^4}{384EI}$   
 $\Delta_x$  . . . . . =  $\frac{wx}{24EI}(l^3 - 2lx^2 + x^3)$



# Example – Analysis of Steel Beam - Capacity

Find applied live load capacity,  $w_{LL}$  in KLF

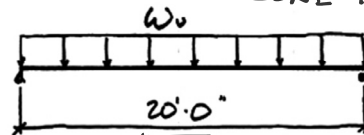
$w_u = 1.2w_{DL} + 1.6w_{LL}$   
 $w_{DL} = \text{beam} + \text{floor} = 44\text{plf} + 1500\text{plf}$

$F_y = 50$  ksi, Fully Braced

1. Find the Plastic Modulus ( $Z_x$ ) for the given section from the AISC table 1-1
2. Check that  $L_b < L_p$  (fully braced – ok) ✓
3. Determine  $M_n = M_p = F_y (Z_x)$

4. Set  $M_u = \phi_b M_n$   
 $\phi_b = 0.90$

GIVEN:  $F_y = 50$  ksi  
 W21x44  
 FULLY BRACED ✓  
 ZONE 1



FOR A W21x44 FROM TABLE  
 $Z_x = 95.4 \text{ in}^3$

$M_p = M_n = F_y Z_x = 50 \text{ ksi} \times 95.4 = 4,770 \text{ k-in}$

$M_u = \phi_b M_n = 0.9 \times 4,770 \text{ k-in}$

$M_u = 4,293 \text{ k-in} = 357.75 \text{ k-ft}$

# Steel Beams by LRFD

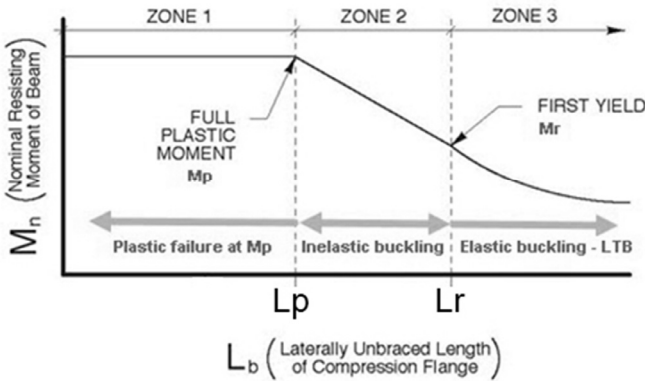
## Analysis for Bending

AISC 16<sup>th</sup> ed.

- Plastic Behavior (zone 1)

$$M_n = M_p = F_y Z$$

- $L_p = 4.45 \text{ ft} = 53.4 \text{ in.}$
- $\phi_b M_{px} = 358 \text{ k-ft} > M_u = 333.5 \text{ k-ft}$



**Table 3-2 (continued)**  
**W-Shapes**  
**Selection by  $Z_x$**

$F_y = 50 \text{ ksi}$

*CAPACITY* ↓

Shape	$Z_x$ in. <sup>3</sup>	$M_{px}/\Omega_b$		$M_{rx}/\Omega_b$		$\phi_b M_{rx}$		$BF/\Omega_b$		$\phi_b BF$		$L_p$ ft	$L_r$ ft	$I_x$ in. <sup>4</sup>	$V_{nx}/\Omega_v$		$\phi_v V_{nx}$	
		kip-ft	kip-ft	kip-ft	kip-ft	ASD	LRFD	ASD	LRFD	ASD	LRFD				kip	kip		
W21x55	126	314	473	192	289	10.8	16.3	6.11	17.4	1140	156	234						
W14x74	126	314	473	196	294	5.31	8.05	6.76	31.0	795	128	192						
W18x60	123	307	461	189	284	9.62	14.4	5.93	18.2	984	151	227						
W12x79	119	297	446	187	281	3.78	5.67	10.8	39.9	662	117	175						
W14x68	115	287	431	180	270	5.19	7.81	8.69	29.3	722	116	174						
W10x88	113	282	424	172	259	2.62	3.94	9.29	51.2	534	131	196						
W18x55	112	279	420	172	258	9.15	13.8	5.90	17.6	890	141	212						
W21x50	110	274	413	165	248	12.1	18.3	4.59	13.6	984	158	237						
W12x72	108	269	405	170	256	3.69	5.56	10.7	37.5	597	106	159						
W21x48 <sup>(1)</sup>	107	265	398	162	244	9.89	14.8	6.09	16.5	959	144	216						
W16x57	105	262	394	161	242	7.98	12.0	5.65	18.3	758	141	212						
W14x61	102	254	383	161	242	4.93	7.48	8.65	27.5	640	104	156						
W18x50	101	252	379	155	233	8.76	13.2	5.83	16.9	800	128	192						
W10x77	97.6	244	366	150	225	2.60	3.90	9.18	45.3	455	112	169						
W12x65 <sup>(1)</sup>	96.8	237	356	154	231	3.58	5.39	11.9	35.1	533	94.4	142						
W21x44	95.4	238	358	143	214	11.1	16.8	4.45	13.0	843	145	217						
W18x50	92.0	230	345	141	213	7.69	11.4	5.62	17.2	659	124	186						
W18x46	90.7	226	340	138	207	9.63	14.6	4.56	13.7	712	130	195						
W14x53	87.1	217	327	136	204	5.22	7.93	6.78	22.3	541	103	154						
W12x58	86.4	216	324	136	205	3.82	5.69	8.87	29.8	475	87.8	132						
W10x68	85.3	213	320	132	199	2.58	3.85	9.15	40.6	394	87.8	147						
W16x45	82.3	205	309	127	191	7.12	10.8	5.55	16.5	586	111	167						
W18x40	78.4	196	294	119	180	8.94	13.2	4.49	13.1	612	113	169						
W14x48	78.4	196	294	123	184	5.09	7.67	6.75	21.1	484	93.8	141						
W12x53	77.9	194	292	123	185	3.65	5.50	8.76	28.2	425	83.5	125						
W10x60	74.6	186	280	116	175	2.54	3.82	9.08	36.6	341	85.7	129						
W16x40	73.0	182	274	113	170	6.67	10.0	5.55	15.9	518	97.6	146						
W12x50	71.9	179	270	112	169	3.97	5.98	6.92	23.8	391	90.3	135						
W8x67	70.1	175	263	105	159	1.75	2.59	7.49	47.6	272	103	154						
W14x43	69.6	174	261	109	164	4.88	7.28	6.68	20.0	428	83.6	125						
W10x54	66.6	166	250	105	158	2.48	3.75	9.04	33.6	303	74.7	112						

ASD      LRFD      <sup>(1)</sup>Shape exceeds compact limit for flexure with  $F_y = 50 \text{ ksi}$ ; tabulated values have been adjusted accordingly.

$\Omega_b = 1.67$        $\phi_b = 0.90$   
 $\Omega_v = 1.50$        $\phi_v = 1.00$

## Example – Analysis of Steel Beam - Capacity

- Using the maximum moment equation, solve for the factored distributed loading,  $w_u$

$$M_u = \frac{w_u l^2}{8} \Rightarrow w_u = \frac{8 M_u}{l^2}$$

$$w_u = \frac{8 \times 357.75 \text{ k-ft}}{20 \text{ ft}^2}$$

$$w_u = 7.155 \text{ k/ft}$$

- The applied (unfactored) load  $w = w_u / (\gamma \text{ factors})$   
 $w_u = 1.2w_{DL} + 1.6w_{LL}$

$$w_u = 7.155 \text{ k/ft} = 1.2(0.044 + 1.5) + 1.6(w_{LL})$$

$$w_u = 1.853 + 1.6w_{LL} = 7.155 \text{ k/ft}$$

$$w_{LL} = 3.31 \text{ k/ft}$$



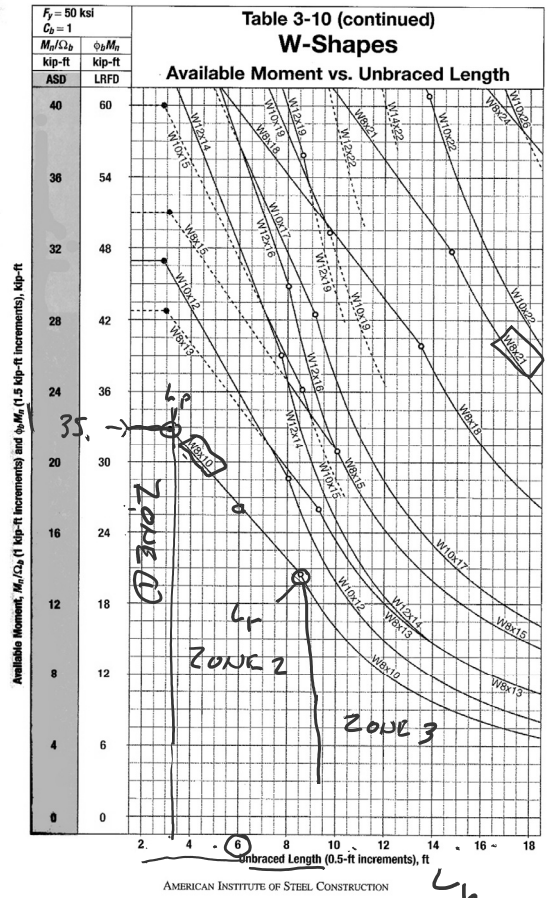
# Steel Beams by LRFD

## Moment Capacity vs. $L_b$ Graphs

### Analysis for Bending

- Plastic Behavior (zone 1)
  - $M_n = M_p$
  - Braced against LTB ( $L_b < L_p$ )
- Inelastic Buckling "Decreased" (zone 2)
  - $M_n < M_p$
  - $L_p < L_b < L_r$
- Elastic Buckling "Decreased Further" (zone 3)
  - $M_n = M_{cr}$
  - $L_b > L_r$

$\phi M_n$



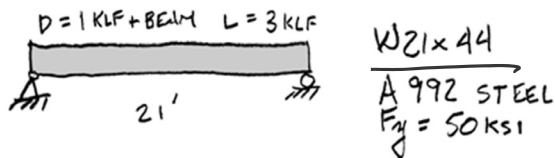
AISC 15<sup>th</sup> ed.

## Pass/Fail Analysis of Steel Beams for Zone 1 $L_b < L_p$

Example:

Given: yield stress, steel section, loading, braced @ 24" o.c.

Find: pass/fail of section



FROM TABLE 1-1 AISC  $Z_x = 95.4 \text{ in}^3$

$$w_u = 1.2(1 + 0.044) + 1.6(3) = 6.05 \text{ KLF}$$

$$M_u = \frac{w_u l^2}{8} = \frac{6.05 \text{ KLF} \times 21^2}{8} = 333.5 \text{ K-ft} \checkmark$$

