Wood Beam Analysis and Design

- ASD approach
- NDS criteria
- Wood Beam Analysis
- Wood Beam Design

Allowable Stress Design

Allowable Stress ≥ Actual Stress

Fb from the NDS Supplement
**Allowable Stress Design by NDS**

### Flexure

![Diagram of flexure stresses]

- **Allowable Flexure Stress** $F_b'$
  
  $F_b$ from NDS Supplement tables determined by species and grade
  
  $F_b' = F_b$ (usage factors)
  
  Usage factors for flexure:
  
  - $C_d$: Load Duration Factor
  - $C_m$: Moisture Factor
  - $C_t$: Temperature Factor
  - $C_b$: Beam Stability Factor
  - $C_f$: Size Factor
  - $C_u$: Flat Use
  - $C_i$: Incising Factor
  - $C_r$: Repetitive Member Factor

- **Actual Flexure Stress** $f_b$
  
  $f_b = \frac{M_c}{I} = M/S$
  
  $S = \frac{I}{c} = \frac{bd^2}{6}$

### Shear

![Diagram of shear stresses]

- **Allowable Shear Stress** $F_v'$
  
  $F_v$ from tables determined by species and grade
  
  $F_v' = F_v$ (usage factors)
  
  Usage factors for shear:
  
  - $C_d$: Load Duration Factor
  - $C_m$: Moisture Factor
  - $C_t$: Temperature Factor
  - $C_i$: Incising Factor

- **Actual Shear Stress** $f_v$
  
  $f_v = \frac{VQ}{.1b} = 1.5\ V/A$
  
  Can use $V$ at $d$ from support as maximum

*University of Michigan, TCAUP*
Allowable Stress Design by NDS Compression

\[ F'_c = F'_c \] (usage factors)

usage factors for flexure:
- \( C_D \) Load Duration Factor
- \( C_M \) Moisture Factor
- \( C_T \) Temperature Factor
- \( C_s \) Size Factor
- \( C_i \) Incising Factor
- \( C_c \) Column Stability Factor

\[ f_c = P/A \]

Adjustment Factors

| \( F_b = F_b \) | \( x \) | \( C_D \) | \( C_M \) | \( C_t \) | \( C_L \) | \( C_T \) | \( C_f \) | \( C_i \) | \( C_c \) | \( - \) | \( - \) | \( - \) | \( K_F \) | \( \phi_b \) | \( \lambda \) |
| \( F_i = F_i \) | \( x \) | \( C_D \) | \( C_M \) | \( C_t \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( - \) | \( K_F \) | \( \phi_i \) | \( \lambda \) |
| \( F'_i = F'_i \) | \( x \) | \( C_D \) | \( C_M \) | \( C_t \) | \( - \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( - \) | \( K_F \) | \( \phi_i \) | \( \lambda \) |

| \( F_{cl} = F_{cl} \) | \( x \) | \( - \) | \( C_M \) | \( C_t \) | \( - \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( C_b \) | \( K_F \) | \( \phi_c \) | \( \lambda \) |

| \( F_e = F_e \) | \( x \) | \( C_D \) | \( C_M \) | \( C_t \) | \( - \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( C_T \) | \( K_F \) | \( \phi_e \) | \( \lambda \) |

| \( E'E' = E' \) | \( x \) | \( - \) | \( C_M \) | \( C_t \) | \( - \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( - \) | \( - \) | \( - \) | \( - \) |

| \( E_{min}' = E_{min} \) | \( x \) | \( - \) | \( C_M \) | \( C_t \) | \( - \) | \( - \) | \( C_f \) | \( - \) | \( C_i \) | \( - \) | \( - \) | \( C_T \) | \( K_F \) | \( \phi_{min} \) | \( - \) |
Adjustment Factors

Allowable Flexure Stress $F_b'$

$F_b' = F_b \left( C_D, C_M, C_L, C_F, C_h, C_i, C_r \right)$

Usage factors for flexure:

- $C_D$: Load Duration Factor
- $C_M$: Moisture Factor
- $C_F$: Size Factor

### Table 2.3.2 Frequently Used Load Duration Factors, $C_D$

<table>
<thead>
<tr>
<th>Load Duration</th>
<th>$C_D$</th>
<th>Typical Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
<td>0.9</td>
<td>Dead Load</td>
</tr>
<tr>
<td>Ten years</td>
<td>1.0</td>
<td>Occupancy Live Load</td>
</tr>
<tr>
<td>Two months</td>
<td>1.15</td>
<td>Snow Load</td>
</tr>
<tr>
<td>Seven days</td>
<td>1.25</td>
<td>Construction Load</td>
</tr>
<tr>
<td>Ten minutes-</td>
<td>1.6</td>
<td>Wind/Earthquake Load</td>
</tr>
<tr>
<td>Impact²</td>
<td>2.0</td>
<td>Impact Load</td>
</tr>
</tbody>
</table>

---

### Table 2.3.3 Temperature Factor, $C_t$

<table>
<thead>
<tr>
<th>Reference Design Values</th>
<th>In-Service Moisture Condition</th>
<th>$C_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$\sigma$</td>
<td></td>
</tr>
<tr>
<td>$105^\circ F$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wet or Dry</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Hot (120°F)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Damp (75°F to 125°F)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wet</td>
<td>1.0</td>
<td>0.9</td>
</tr>
</tbody>
</table>

---

Wet Service Factor, $C_M$

When dimension lumber is used where moisture content will exceed 19% for an extended period, design values shall be multiplied by the appropriate wet service factors from the following table:

### Wet Service Factors, $C_M$

```
<table>
<thead>
<tr>
<th>$F_b$</th>
<th>$F_1$</th>
<th>$F_c$</th>
<th>$F_L$</th>
<th>$F_i$</th>
<th>$E$ or $F_{E}$ mini</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>1.0</td>
<td>0.92</td>
<td>0.67</td>
<td>0.8 **</td>
<td>0.9 **</td>
</tr>
</tbody>
</table>
```

* when $(F_a)(C_a) \leq 1,150$ psi, $C_M = 1.0$

** when $(F_a)(C_a) \leq 750$ psi, $C_M = 1.0$

### Table 2.3.4 Size Factors, $C_s$

<table>
<thead>
<tr>
<th>Grades</th>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
<th>$F_s$</th>
<th>$F_s'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select Structural, No.1 &amp; Btr. No.1, No.2, No.3</td>
<td>$2^\prime, 3^\prime, &amp; 4^\prime$</td>
<td>13 &amp; 15</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>$5^\prime$</td>
<td>14</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>$6^\prime$</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>$8^\prime$</td>
<td>1.2</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>$10^\prime$</td>
<td>1.1</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>$12^\prime$</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>14 &amp; wider</td>
<td>0.9</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>$2^\prime, 3^\prime, &amp; 4^\prime$</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Stud</td>
<td>$5^\prime$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$8^\prime$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Use No.3 Grade tabulated design values and size factors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2^\prime, 3^\prime, &amp; 4^\prime$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Utility</td>
<td>$4^\prime$</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>$2^\prime &amp; 3^\prime$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_b'$

$F_b'$ from NDS tables

$F_b' = F_b \left( C_D C_M C_t C_L C_F C_{fu} C_i C_r \right)$

Usage factors for flexure:
- $C_{fu}$ Flat Use
- $C_r$ Repetitive Member Factor

Repellent Member Factor, $C_r$

Bending design values, $F_b$, for dimension lumber 2" to 4" thick shall be multiplied by the repetitive member factor, $C_r = 1.15$, when such members are used as joists, truss chords, rafters, studs, planks, decking, or similar members which are in contact or spaced not more than 24" on center, are not less than 3 in number and are joined by floor, roof, or other load distributing elements adequate to support the design load.

Flat Use Factor, $C_{fu}$

Bending design values adjusted by size factors are based on edgewise use (load applied to narrow face). When dimension lumber is used flatwise (load applied to wide face), the bending design value, $F_b$, shall also be permitted to be multiplied by the following flat use factors:

<table>
<thead>
<tr>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2'' &amp; 3''</td>
<td>1.0</td>
</tr>
<tr>
<td>4''</td>
<td>1.1</td>
</tr>
<tr>
<td>5''</td>
<td>1.1</td>
</tr>
<tr>
<td>6''</td>
<td>1.15</td>
</tr>
<tr>
<td>8''</td>
<td>1.15</td>
</tr>
<tr>
<td>10'' &amp; wider</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Incising Factors, $C_i$

<table>
<thead>
<tr>
<th>Design Value</th>
<th>$C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b$, $F_y$, $F_{fy}$</td>
<td>0.80</td>
</tr>
<tr>
<td>$F_{cu}$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_{b'}$

$F_{b'} = F_b \left( C_D C_M C_t C_r C_u C_i C_s \right)$

Usage factors for flexure:
- $C_L$ Beam Stability Factor

### 3.3.3 Beam Stability Factor, $C_L$

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \leq b$, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1 $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

---

**4.4.1 Stability of Bending Members**

- **2x4**
  - $d/b \leq 2$: no lateral support shall be required.

- **2x8**
  - $2 < d/b \leq 4$: the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

- **2x10**
  - $4 < d/b \leq 5$: the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

- **2x12**
  - $5 < d/b \leq 6$: full depth solid blocking or diagonal cross bracing shall be installed at intervals not exceeding 8 feet, the compression edge of the member shall be held in line as by adequate sheathing or subflooring, and the ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

- **2x14**
  - $6 < d/b \leq 7$: both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

---

$C_L = 1.0$

For depth/width ratio in 4.4.1 $C_L = 1.0$.

Otherwise

$C_L < 1.0$

calculate factor using section 3.3.3
CL Beam Stability Factor

In the case bracing provisions of 4.4.1 cannot be met, CL is calculated using equation 3.3.6

The maximum allowable slenderness, R_B is 50

<table>
<thead>
<tr>
<th>Table 3.3.3 Effective Length, $L_e$ for Bending Members</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Condition</strong></td>
</tr>
<tr>
<td>Uniformly distributed load</td>
</tr>
<tr>
<td>Concentrated load at unsupported end</td>
</tr>
<tr>
<td>Single Span Beam</td>
</tr>
<tr>
<td>Uniformly distributed load</td>
</tr>
<tr>
<td>Concentrated load at center with no intermediate lateral support</td>
</tr>
<tr>
<td>Concentrated load at center with lateral support at center</td>
</tr>
<tr>
<td>Two equal concentrated loads at 0.75 L with lateral support at 0.5 L</td>
</tr>
<tr>
<td>Three equal concentrated loads at 1/4 L with lateral support at 1/3 L</td>
</tr>
<tr>
<td>Four equal concentrated loads at 1/3 L with lateral support at 1/7 L</td>
</tr>
<tr>
<td>Five equal concentrated loads at 1/5 L with lateral support at 1/9 L</td>
</tr>
<tr>
<td>Six equal concentrated loads at 1/7 L with lateral support at 1/11 L</td>
</tr>
<tr>
<td>Seven or more equal concentrated loads, evenly spaced, with lateral support at points of load application</td>
</tr>
</tbody>
</table>

1. For single span or continuous bending members with loading conditions not specified in Table 3.3.3:
   - $L_e=1.54 L$ when $L_e<2.00 L$
   - $L_e=1.80 L$ when $L_e<2.00 L$

2. Multiple span applications shall be based on table values or engineering analysis.

Adjustment Factors for Shear

Allowable Flexure Stress $F_v'$

$F_v$ from tables determined by species and grade

$F_v' = F_v (usage factors)$

Usage factors for shear:

- $C_D$: Load Duration Factor
- $C_M$: Moisture Factor
- $C_t$: Temperature Factor
- $C_i$: Incising Factor

Shear at Supports

Modified V' used to compute reduced shear $f_v'$

$R_B = \frac{L_e d}{B d}$

$C_L = \frac{1 + (6F_v'/F_v)}{1.5} - \frac{1 + (6F_v'/F_v)}{1.9} \cdot \frac{F_v'/F_v}{0.95}$

where:

- $F_v'$: reference bending design value multiplied by all applicable adjustment factors except $C_{D}$, $C_{M}$, $C_{t}$, $C_{i}$ (see 2.3), psi
- $R_B$: slenderness ratio for bending members, shall not exceed 50

$R_B \geq 20 \cdot \frac{1.0 - E_{sec}}{R_B^2}$
Analysis Procedure

Given: loading, member size, material and span.

Req’d: Safe or Unsafe (Pass/Fail)

1. Find Max Shear & Moment
   - Simple case – equations
   - Complex case - diagrams

2. Determine actual stresses
   - \( f_b = \frac{M}{S} \)
   - \( f_v = 1.5 \frac{V}{A} \)

3. Determine allowable stresses
   - \( F_b \) and \( F_v \) (from NDS)
   - \( F'_b = F_b \) (usage factors)
   - \( F'_v = F_v \) (usage factors)

4. Check that actual \( \leq \) allowable
   - \( f_b \leq F'_b \)
   - \( f_v \leq F'_v \)

5. Check deflection

6. Check bearing (\( F_{cl} \geq \frac{\text{Reaction}}{A_{\text{bearing}}} \) from NDS 2012

Analysis Example

Example – Pass / Fail

Given: loading, member size, material and span.

Req’d: Safe or Unsafe?
Analysis Example

1. **Find Max Shear & Moment**
   - Simple cases – equations
   - Complex cases - diagrams

   By equations:
   \[ V = \frac{P}{2} = \frac{145}{2} = 72.5 \]  
   \[ M_e = \frac{PL}{4} = \frac{145(6)}{4} = 217.5 \]  

   By Diagrams:

2. **Determine actual stresses**
   - \( f_b = \frac{M}{S_x} \)
   - \( f_v = 1.5 \frac{V}{A} \)

   \[ f_b = \frac{M}{S_x} = \frac{145}{\frac{1.5(3.5)^2}{4}} = \frac{2 \times 4}{3.063 \text{ in}^3} = 2.952 \text{ psi} \]
   \[ f_v = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \left( \frac{72.5}{5.25 \text{ in}^2} \right) = 20.71 \text{ psi} \]
Species and Grade

S-P-F No.2

$F_b = 875 \text{ psi}$
$F_v = 135 \text{ psi}$

### Table 4A

**Reference Design Values for Visually Graded Dimension Lumber**

(2" - 4" thick)\(^{1,2,3}\)

(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)

<table>
<thead>
<tr>
<th>Species and commercial grade</th>
<th>Size classification</th>
<th>Tension parallel to grain</th>
<th>Shear parallel to grain</th>
<th>Compression perpendicular to grain</th>
<th>Compression parallel to grain</th>
<th>Modulus of Elasticity</th>
<th>Specific Gravity</th>
<th>Grading Rules Agency</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>P</td>
<td>F</td>
<td>2&quot; &amp; wider</td>
<td>1,400</td>
<td>700</td>
<td>135</td>
<td>425</td>
<td>1,400</td>
</tr>
<tr>
<td>No. 3</td>
<td>500</td>
<td>250</td>
<td>135</td>
<td>425</td>
<td>1,150</td>
<td>1,400,000</td>
<td>510,000</td>
<td></td>
</tr>
<tr>
<td>No. 2</td>
<td>675</td>
<td>350</td>
<td>135</td>
<td>425</td>
<td>1,150</td>
<td>1,200,000</td>
<td>440,000</td>
<td></td>
</tr>
<tr>
<td>No. 1</td>
<td>1,000</td>
<td>500</td>
<td>135</td>
<td>425</td>
<td>1,150</td>
<td>1,200,000</td>
<td>440,000</td>
<td></td>
</tr>
<tr>
<td>Slab</td>
<td>2&quot; &amp; wider</td>
<td>675</td>
<td>350</td>
<td>135</td>
<td>425</td>
<td>725</td>
<td>1,200,000</td>
<td>440,000</td>
</tr>
<tr>
<td>Construction</td>
<td>550</td>
<td>275</td>
<td>135</td>
<td>425</td>
<td>1,150</td>
<td>1,200,000</td>
<td>440,000</td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>275</td>
<td>125</td>
<td>135</td>
<td>425</td>
<td>750</td>
<td>1,100,000</td>
<td>400,000</td>
<td></td>
</tr>
</tbody>
</table>

**Analysis Example**

3. **Determine allowable stresses**
   - $F_b = 875 \text{ psi}$
   - $F_v = 135 \text{ psi}$

Determin factors:

- $CD = \ ?$
- $CM = 1$
- $CL = 1$
- $CF = \ ?$
- $Cfu = 1$
- $Ci = 1$
- $Cr = 1$

### Table 4.3.1

**Applicability of Adjustment Factors for Sawn Lumber**

<table>
<thead>
<tr>
<th>Load Duration Factor</th>
<th>ASD only</th>
<th>ASD and LRFD</th>
<th>LRFD only</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASD only</td>
<td>ASD and LRFD</td>
<td>LRFD only</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_b' = F_b x C_D C_M C_I C_L C_F C_{fu} C_i C_r$</td>
<td>$K_F \phi_b \lambda$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_v' = F_v x C_D C_M C_I$</td>
<td>$K_F \phi_v \lambda$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Adjustment Factors

Allowable Flexure Stress $F_{b'}$

$F_{b'} = F_b \left( C_D \ C_M \ C_L \ C_F \ C_{fu} \ C_i \ C_r \right)$

Usage factors for flexure:

$C_L$ Beam Stability Factor

### 3.3.3 Beam Stability Factor, $C_L$

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \leq b$, no lateral support is required and $C_L = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_L = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_L = 1.0$.

2012 NDS

---

### Analysis Example

$C_D$ Load duration factor

Use 1.6 (10 minutes)

$C_F$ Size factor

2 x 4

use 1.5

---

### 4.4.1 Stability of Bending Members

(a) $d/b \leq 2$; no lateral support shall be required.

(b) $2 < d/b \leq 4$; the ends shall be held in position, as by full depth solid blocking, bridging, hangers, nailing, or bolting to other framing members, or other acceptable means.

(c) $4 < d/b \leq 5$; the compression edge of the member shall be held in line for its entire length to prevent lateral displacement, as by adequate sheathing or subflooring, and ends at point of bearing shall be held in position to prevent rotation and/or lateral displacement.

(d) $5 < d/b \leq 6$; both edges of the member shall be held in line for their entire length and ends at points of bearing shall be held in position to prevent rotation and/or lateral displacement.

### Table 2.3.2 Frequently Used Load Duration Factors, $C_D$

<table>
<thead>
<tr>
<th>Load Duration</th>
<th>$C_D$</th>
<th>Typical Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent</td>
<td>0.9</td>
<td>Dead Load</td>
</tr>
<tr>
<td>Ten years</td>
<td>1.0</td>
<td>Occupancy Live Load</td>
</tr>
<tr>
<td>Two months</td>
<td>1.15</td>
<td>Snow Load</td>
</tr>
<tr>
<td>Seven days</td>
<td>1.25</td>
<td>Construction Load</td>
</tr>
<tr>
<td>Ten minutes</td>
<td>1.6</td>
<td>Wind/Earthquake Load</td>
</tr>
<tr>
<td>Impact</td>
<td>2.0</td>
<td>Impact</td>
</tr>
</tbody>
</table>

### Table 2.3.3 Frequently Used Size Factors, $C_F$

<table>
<thead>
<tr>
<th>Grades</th>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$2^\prime$, $3^\prime$, &amp; $4^\prime$</td>
<td>$3^\prime$ &amp; $3^\prime$</td>
</tr>
<tr>
<td>Select</td>
<td>5&quot;</td>
<td>1.4</td>
</tr>
<tr>
<td>Structural</td>
<td>6&quot;</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>8&quot;</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>10&quot;</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>12&quot;</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>14&quot; &amp; wider</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.1</td>
</tr>
<tr>
<td>Stud</td>
<td>5&quot; &amp; 6&quot;</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>8&quot; &amp; wider</td>
<td>Use No.3 Grade tabulated design values and size factors</td>
</tr>
<tr>
<td>Construction</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.0</td>
</tr>
<tr>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>4&quot;</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2&quot; &amp; 3&quot;</td>
<td>0.4</td>
</tr>
</tbody>
</table>

2012 NDS
Analysis Example

3. Determine allowable stresses
   • \( F_{b}' = F_b (C_D)(C_F) \)
   \[ F_{b}' = 875 (1.6)(1.5) = 2100 \text{ psi} \]
   • \( F_{v}' = F_v (C_D) \)
   \[ F_{v}' = 135 (1.6) = 216 \text{ psi} \]

4. Check that actual \( \leq \) allowable
   • \( f_b < F_b' \)
   • \( f_v < F_v' \)

5. Check deflection
6. Check bearing \( (F_{c,l} \geq R/A_b) \)

Analysis Procedure

Given: member size, material and span.
Req’d: Max. Safe Load (capacity)

1. Assume \( f = F \)
   • Maximum actual = allowable stress

2. Solve stress equations for force
   • \( M = F_b S \)
   • \( V = 0.66 F_v A \)

3. Use maximum moment to find loads
   • Back calculate a load from moment
   • Assumes moment controls

4. Check Shear
   • Use load found is step 3 to check shear stress.
   • If it fails \( (f_v > F_v') \), then find load based on shear.

5. Check deflection
6. Check bearing
Analysis Example

Given: member size, material and span.

Req’d: Max. Safe Load (capacity)

1. Assume $f = F'$
   - Maximum actual = allowable stress

2. Solve stress equation for moment
   - $M = F'_b S$ (i.e. moment capacity)

3. Use maximum forces to find loads
   - Back calculate a maximum load from moment capacity

4. Check shear
   - Check shear for load capacity from step 3.
   - Use $P$ from moment to find $V_{max}$
   - Check that $f_v < F_v$

4. Check deflection (serviceability)
5. Check bearing (serviceability)
Analysis Example

Given: loading, member size, material and span.

Req’d: Safe or Unsafe

<table>
<thead>
<tr>
<th>DATASET: 1</th>
<th>Wood Species</th>
<th>HEM-FIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Grade</td>
<td>Select</td>
<td>Structural</td>
</tr>
<tr>
<td>Span A</td>
<td>16 FT</td>
<td></td>
</tr>
<tr>
<td>Span B</td>
<td>12 FT</td>
<td></td>
</tr>
<tr>
<td>Nominal Depth of Beam, d</td>
<td>12 IN</td>
<td></td>
</tr>
<tr>
<td>Moisture Content, m.c.</td>
<td>15 %</td>
<td></td>
</tr>
<tr>
<td>Floor DL</td>
<td>7 PSF</td>
<td></td>
</tr>
<tr>
<td>Floor LL</td>
<td>35 PSF</td>
<td></td>
</tr>
</tbody>
</table>

Analysis Example

Find Specific Gravity for Hem-Fir

• (from NDS)

Table 4A (Cont.)

Reference Design Values for Visually Graded Dimension Lumber

(2" - 4" thick)¹,²,³

(All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)
Analysis Example

Section Properties:

4 x 12 (3.5” x 11.25”)

Area = 39.38 in²

Sx = 73.83 in³
**Analysis Example**

**Selfweight of member:**

Density at 0 m.c. = 62.4 x G (dry)

62.4 x 0.43 = 26.8 PCF

m.c. = 15% (given)

To include m.c. use NDS formula.

\[
D = 62.4 \left[ \frac{0.43}{1 + 0.43(0.009)(15)} \right] \left[ 1 + \frac{15}{100} \right]
\]

\[
= 25.35 \times 1.15 = 29.16 \text{ PCF}
\]

\[
w = \text{PLF} = \frac{\text{AREA}}{144} = \frac{29.16}{144} = 0.2016 \text{ PLF}
\]

\[
w = 7.975 \text{ PLF}
\]

---

**Analysis Example**

**Determine Beam Forces**

by superposition equations

\[
R = \frac{wL^2}{2} + \frac{PZ}{2}
\]

\[
= \frac{7975(12)^2}{2} + \frac{2016}{2} = 1055.8 \text{ kips}
\]

\[
V_{max} = R
\]

\[
M_d = \frac{wL^2}{8} + \frac{PL}{4}
\]

\[
= \frac{7975(12)^2}{8} + \frac{2016(12)}{4} = 6191.5 \text{ kips-feet}
\]

or

by diagrams

- **P = 2016 LBS**
- **w = 7.975 PLF**
- **1055.8 kips**
- **W = 6048 PLF**
- **1055.8 kips**
- **143.5 ft**
- **6191.5 ft**
- **6048 ft**
- **12 ft**
Analysis Example

Determine actual stresses

- \( f_b = \frac{M}{S} \)
- \( f_v = 1.5 \frac{V}{A} \)

<table>
<thead>
<tr>
<th>Board</th>
<th>( f_b )</th>
<th>( f_v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 3</td>
<td>2.882</td>
<td>0.929</td>
</tr>
<tr>
<td>1 x 4</td>
<td>2.882</td>
<td>0.929</td>
</tr>
<tr>
<td>1 x 6</td>
<td>2.882</td>
<td>0.929</td>
</tr>
<tr>
<td>1 x 8</td>
<td>2.882</td>
<td>0.929</td>
</tr>
</tbody>
</table>

Dimension Lumber (see NDS 4.1.2.2) and Dowling (see NDS 4.1.2.3)

<table>
<thead>
<tr>
<th>Board</th>
<th>Actual Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 3</td>
<td>2.882 / 1.5 = 1.9216</td>
</tr>
<tr>
<td>1 x 4</td>
<td>2.882 / 1.5 = 1.9216</td>
</tr>
<tr>
<td>1 x 6</td>
<td>2.882 / 1.5 = 1.9216</td>
</tr>
<tr>
<td>1 x 8</td>
<td>2.882 / 1.5 = 1.9216</td>
</tr>
</tbody>
</table>

Table 4A (Cont.) Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick) (All species except Southern Pine—see Table 4B) (Tabulated design values are for normal load duration and dry service conditions. See NDS 4.3 for a comprehensive description of design value adjustment factors.)
Analysis Example

3. Determine allowable stresses
   - \( F_b = 1400 \text{ psi} \)
   - \( F_v = 150 \text{ psi} \)

Determine factors:

\[
\begin{align*}
CD &= \\
CM &= \\
Ci &= \\
CL &= \\
CF &= \\
Cu &= \\
Cr &= \\

\end{align*}
\]

Table 4.3.1 Applicability of Adjustment Factors for Sawn Lumber

<table>
<thead>
<tr>
<th>Nominal Size b x d</th>
<th>Standard dressed size (548)</th>
<th>Area of section A</th>
<th>Moment of Inertia about X-X axis</th>
<th>Moment of Inertia about Y-Y axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boards</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 3</td>
<td>1-1/2 x 2-1/2</td>
<td>1.750</td>
<td>1.405</td>
<td>0.306</td>
</tr>
<tr>
<td>3 x 4</td>
<td>1-1/2 x 3-1/2</td>
<td>2.870</td>
<td>1.600</td>
<td>0.470</td>
</tr>
<tr>
<td>4 x 6</td>
<td>1-1/2 x 4-1/2</td>
<td>5.970</td>
<td>2.200</td>
<td>0.750</td>
</tr>
<tr>
<td>6 x 8</td>
<td>1-1/2 x 6-1/2</td>
<td>12.800</td>
<td>4.800</td>
<td>1.500</td>
</tr>
<tr>
<td>8 x 12</td>
<td>1-1/2 x 8-1/2</td>
<td>25.600</td>
<td>12.000</td>
<td>2.250</td>
</tr>
<tr>
<td>12 x 14</td>
<td>1-1/2 x 14-1/2</td>
<td>64.800</td>
<td>35.800</td>
<td>4.650</td>
</tr>
<tr>
<td>16 x 20</td>
<td>1-1/2 x 20-1/2</td>
<td>129.600</td>
<td>71.600</td>
<td>7.150</td>
</tr>
<tr>
<td>Dimension lumber (see NDS 4.1.3.2) and decking (see NDS 4.1.3.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 4</td>
<td>1-1/2 x 3-1/2</td>
<td>1.750</td>
<td>1.405</td>
<td>0.306</td>
</tr>
<tr>
<td>3 x 4</td>
<td>1-1/2 x 3-1/2</td>
<td>2.870</td>
<td>1.600</td>
<td>0.470</td>
</tr>
<tr>
<td>4 x 6</td>
<td>1-1/2 x 4-1/2</td>
<td>5.970</td>
<td>2.200</td>
<td>0.750</td>
</tr>
<tr>
<td>6 x 8</td>
<td>1-1/2 x 6-1/2</td>
<td>12.800</td>
<td>4.800</td>
<td>1.500</td>
</tr>
<tr>
<td>8 x 12</td>
<td>1-1/2 x 8-1/2</td>
<td>25.600</td>
<td>12.000</td>
<td>2.250</td>
</tr>
<tr>
<td>12 x 14</td>
<td>1-1/2 x 14-1/2</td>
<td>64.800</td>
<td>35.800</td>
<td>4.650</td>
</tr>
<tr>
<td>16 x 20</td>
<td>1-1/2 x 20-1/2</td>
<td>129.600</td>
<td>71.600</td>
<td>7.150</td>
</tr>
</tbody>
</table>

Analysis Example

Determine allowable stresses

- M.C. = 15%
- size: 4x12

<table>
<thead>
<tr>
<th>Grades</th>
<th>Width (depth)</th>
<th>Thickness (breadth)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.5 1.5</td>
</tr>
<tr>
<td>Structural, No.1 &amp; Btr, No.1, No.2, No.3</td>
<td>5&quot; 1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>No.3</td>
<td>12&quot; 1.0</td>
<td>0.9 1.0</td>
</tr>
<tr>
<td>14&quot; &amp; wider</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.1 1.1</td>
</tr>
<tr>
<td>8&quot; &amp; wider</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.1 1.1</td>
</tr>
<tr>
<td>Construction, Standard</td>
<td>4&quot;, 5&quot;, &amp; 6&quot;</td>
<td>1.0 1.0</td>
</tr>
<tr>
<td>Utility</td>
<td>2&quot;, 3&quot;, &amp; 4&quot;</td>
<td>1.0 1.0</td>
</tr>
</tbody>
</table>

Size Factors, \( C_T \)

- Use No.3 Grade tabulated design values

University of Michigan, TCAUP
Structures II
Slide 36 of 51
Adjustment Factors

Allowable Flexure Stress $F'_{b}$

$F'_{b} = F_{b} \cdot (C_{D} \cdot C_{M} \cdot C_{L} \cdot C_{R} \cdot C_{F} \cdot C_{fu} \cdot C_{i} \cdot C_{r})$

$b/d = 3.5 / 11.25 = 3.11$ (case b)

Assuming ends are braced, $CL = 1.0$

3.3.3 Beam Stability Factor, $C_{L}$

3.3.3.1 When the depth of a bending member does not exceed its breadth, $d \leq b$, no lateral support is required and $C_{L} = 1.0$.

3.3.3.2 When rectangular sawn lumber bending members are laterally supported in accordance with 4.4.1, $C_{L} = 1.0$.

3.3.3.3 When the compression edge of a bending member is supported throughout its length to prevent lateral displacement, and the ends at points of bearing have lateral support to prevent rotation, $C_{L} = 1.0$.

2012 NDS

Analysis Example

3. Determine allowable stresses
   - $F'_{b} = F_{b}$ (usage factors)

   $F_{b} = 1.0$ (L24 L20)
   
   $C_{D} = 1.0$ (No Lumber)
   
   $C_{M} = 1.0$ (15% 19% (NDS Sup. P. 22))
   
   $C_{L} = 1.0$ (100% (NDS Sup. P. 23))
   
   $C_{R} = 1.0$ (NDS Sup. P. 23)
   
   $C_{F} = 1.0$ (NDS Sup. P. 30)
   
   $C_{fu} = 1.0$ (NDS Sup. P. 32)
   
   $C_{i} = 1.0$ (NDS Sup. P. 32)
   
   $C_{r} = 1.0$ (NDS Sup. P. 32)

   $F'_{b} = 1400 (1.1) = 1540 \text{ psi}$

   **Actual Stress:**
   
   $f_{b} = \frac{M}{I_{x}} = \frac{61915 (12)}{73.83} = 10063 \text{ psi}$
Analysis Example

3. Determine allowable stresses
   - $F'_v = F_v$ (usage factors)

Check that actual $\leq$ allowable
- $f_b \leq F'_b$
- $f_v \leq F'_v$

Check deflection
Check bearing ($F_{cd} \geq \text{Reaction} / A_{\text{bearing}}$)
Design Procedure

Given: load, wood, span
Req’d: member size

1. Find Max Shear & Moment
   • Simple case – equations
   • Complex case - diagrams

2. Determine allowable stresses

3. Solve \( S=M/F' \)

4. Choose a section from Table 1B
   • Revise DL and \( F' \)

5. Check shear stress
   • First for \( V_{\text{max}} \) (easier)
   • If that fails try \( V \) at distance from support.
   • If the section still fails, choose a new section with \( A=1.5V/F'_{\text{v}} \)

6. Check deflection

7. Check bearing

---

Design Example

Given: load, wood, span
Req’d: member size

1. Find Max Shear & Moment
   • Simple case – equations
   • Complex case - diagrams
Design Example

2. Determine allowable stresses
   (given in this example)
   \[ F'_b = 1000 \text{ psi} \]
   \[ F'_v = 100 \text{ psi} \]

3. Solve \[ S = M / F'_b \]

4. Choose a section from S table
   - Revise DL and \( F'_b \)

5. Check shear stress
   - First for \( V_{\text{max}} \) (easier)
   - If that fails try \( V \) at \( d \) distance
     (remove load \( d \) from support)
   - If the section still fails, choose a
     new section with \( A = 1.5V / F'_v \)

6. Check deflection
7. Check bearing

---

Design Example

Given: load, wood, span

Req'd: member size

---

4. Wood Beam Design

Design a 2x dimensioned lumber floor joist to carry the
given dead + live floor load. Assume the floor meets
condition of 4.4.1 to CL=1.0. Also CL, Cfu, and CI = 1.0.
Find the short term deflection of your chosen beam under
live load only (100% LL is sustained). Compare your LL
deflection with the code limit of L/360.

<table>
<thead>
<tr>
<th>DATASET: 1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood Species</td>
<td>HEM-FIR</td>
<td></td>
</tr>
<tr>
<td>Wood Grade</td>
<td>No.1</td>
<td></td>
</tr>
<tr>
<td>Span</td>
<td>20 FT</td>
<td></td>
</tr>
<tr>
<td>Joist Spacing, o.c.</td>
<td>12 IN</td>
<td></td>
</tr>
<tr>
<td>Moisture Content, m.c.</td>
<td>15 %</td>
<td></td>
</tr>
<tr>
<td>Floor DL</td>
<td>7 PSF</td>
<td></td>
</tr>
<tr>
<td>Floor LL</td>
<td>35 PSF</td>
<td></td>
</tr>
</tbody>
</table>
Design Example

Determine allowable stresses

- \( F_b \) and \( F_v \) (from NDS)

**Table 4A Reference Design Values for Visually Graded Dimension Lumber (2" - 4" thick)**

<table>
<thead>
<tr>
<th>Species and commercial grade</th>
<th>Design values in pounds per square inch (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bending</td>
</tr>
<tr>
<td></td>
<td>( F_b )</td>
</tr>
<tr>
<td>HEM-FIR</td>
<td></td>
</tr>
<tr>
<td>Select Structural</td>
<td>1,400</td>
</tr>
<tr>
<td>No. 1 &amp; Bit</td>
<td>1,200</td>
</tr>
<tr>
<td>No. 2</td>
<td>850</td>
</tr>
<tr>
<td>No. 3</td>
<td>500</td>
</tr>
<tr>
<td>Std. 12</td>
<td>675</td>
</tr>
<tr>
<td>Construction</td>
<td>975</td>
</tr>
<tr>
<td>Standard</td>
<td>550</td>
</tr>
<tr>
<td>Utility</td>
<td>250</td>
</tr>
</tbody>
</table>

University of Michigan, TCAUP

Structures II

Slide 45 of 51

---

Design Example

Determine allowable stresses

**Table 4A Allowance Factors**

<table>
<thead>
<tr>
<th>Repetitive Member Factor, ( C_r )</th>
<th>When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Use Factor, ( C_u )</td>
<td>Bonding design values adjusted by size factors are based on adhesive use (load applied to narrow face). When dimension lumber is used for fasteners (load applied to wide face), the bonding design value, ( F_b ), shall also be multiplied by the following flat use factors:</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width (in.)</th>
<th>Thickness (in.)</th>
<th>( C_u )</th>
<th>( F_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2&quot; - 3&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>3&quot; - 4&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>4&quot; - 6&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>6&quot; - 8&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>8&quot; - 10&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4A Allowance Factors**

<table>
<thead>
<tr>
<th>Repetitive Member Factor, ( C_r )</th>
<th>Flat Use Factor, ( C_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>When dimension lumber is used where moisture content will exceed 19% for an extended time period, design values shall be multiplied by the appropriate wet service factors from the following table:</td>
<td></td>
</tr>
<tr>
<td>Bonding design values adjusted by size factors are based on adhesive use (load applied to narrow face). When dimension lumber is used for fasteners (load applied to wide face), the bonding design value, ( F_b ), shall also be multiplied by the following flat use factors:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Width (in.)</th>
<th>Thickness (in.)</th>
<th>( C_u )</th>
<th>( F_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2&quot; - 3&quot;</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>3&quot; - 4&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>4&quot; - 6&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>6&quot; - 8&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>8&quot; - 10&quot;</td>
<td>1.1</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>10&quot; - 12&quot;</td>
<td>1.2</td>
<td>1.2</td>
<td></td>
</tr>
</tbody>
</table>

**Copyright**

American Wood Council. Downloading and using the document is governed by a Copyright License Agreement. No reproduction or transfer authorized.

University of Michigan, TCAUP

Structures II

Slide 46 of 51
**Design Example**

Determine allowable stresses.

Since the size is not known you have to skip $C_F$ (or make a guess).

$$F_b' = F_b \left( \frac{P_{EF}}{F_{b0}} \right) = 975 \left( 1.0 \times 1.15 \times 1.0 \times C_F \right) \approx 1121 \text{ psi}$$

$$F_v' = F_v \left( C_D, C_M, C_E, C_I \right) = 150 \left( 1.0 \times 1.0 \times 1.0 \times 1.0 \right) = 150 \text{ psi}$$

**Design Example**

Determine Moment from Loading

First find the uniform beam load, $w$, from the floor loading.

$$w = \left( P_{EF} \right) \frac{C_D \cdot C_M}{12} = \frac{P_{LF}}{12}$$

$$= \left( 7 + 35 \right) \frac{12}{12} = 42 \text{ psi}$$

With the beam loading, calculate the maximum moment.

$$M = \frac{wL^2}{8} = \frac{42^2 (20)^2}{8} = 2100 \text{ in-lb}$$
Design Example

Estimate the Required Section Modulus.

\[ S_x = \frac{H}{F_b} = \frac{2100 (12)}{1121} = 22.47 \text{ in}^3 \]

Compare this required \( S_x \) to the actual \( S_x \) of available sections in NDS Table 1B. Remember CF will be multiplied which may make some pass which at first fail.

From Table 1B (NDS)

\[ \begin{array}{c|c|c}
   \text{FROM} & \text{TABLE 1B (NDS)} & \text{Sx} \\
   \text{2x10} & 21.39 \ (C_F = 1.0) & \text{MIGHT WORK} \\
   \text{2x12} & 31.64 \ (C_F = 1.0) & \\
\end{array} \]

Design Example

Choose a section and test it (by analysis with all factors including \( C_F \))

\[ \begin{align*}
   f_b &= \frac{M}{S_x} = \frac{2100 (12)}{21.39} = 1178 \text{ psi} < 1233 \text{ psi; OK} \\
   f_v &= \frac{3 \sqrt{V}}{2A} = \frac{1.5 \sqrt{480}}{13.88} = 45.39 \text{ psi} < 160 \text{ psi; OK}
\end{align*} \]

\[ \therefore \text{ USE 2x10} \]
Design Example

Check Deflection

In this case LL only against code limit of L/360

\[ \text{LL} = 3.5 \text{ psf} = 3.5 \text{ plf} \]

\[ \Delta_{\text{LL}} = \frac{5wL^4}{384EI} = \frac{5(3.5)(20)^4(1728)}{384(1500000)(98.93)} = 0.849'' \]

\[ \Delta_{\text{LIMIT}} = \frac{L}{360} = \frac{20'(12)}{360} = 0.667'' \]

\[ 0.849 > 0.667 \implies \text{FAIL} \]