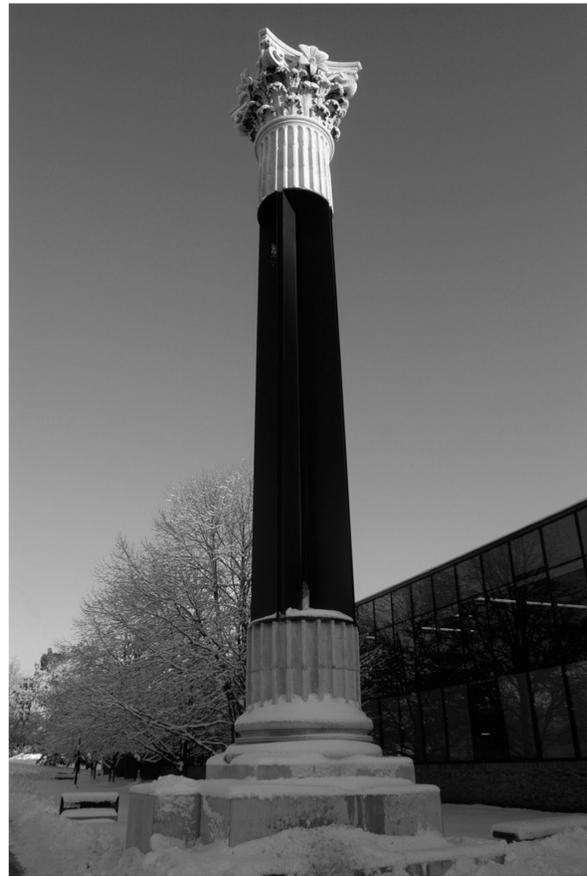


## Steel Column Analysis

- Failure Modes
- Effects of Slenderness
- Stress Analysis of Steel Columns



## Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 IE}{(KL)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

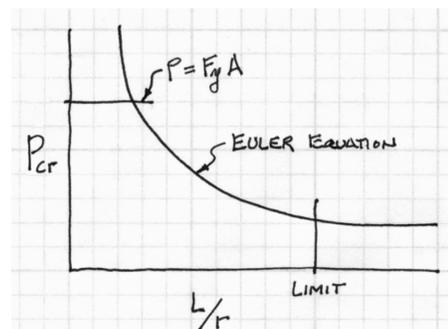
$$I = Ar^2$$

- A = Cross sectional area (in<sup>2</sup>)
- E = Modulus of elasticity of the material (lb/in<sup>2</sup>)
- K = Stiffness (curvature mode) factor
- L = Column length between braced ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$



portrait by Emanuel Handmann, 1753



# Analysis of Steel Columns

## Conditions of an Ideal Column

- initially straight
- axially loaded
- uniform stress (no residual stress)
- uniform material (no holes)
- no transverse load
- pinned (or defined) end conditions



# Analysis of Steel Columns

## Short columns

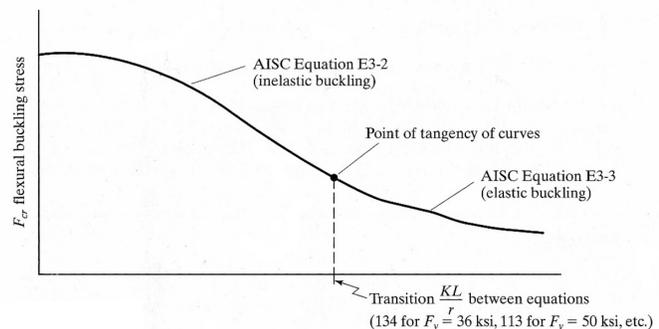
Fail by material crushing  
Plastic behavior

## Intermediate columns

Crush partially and then buckle  
Inelastic behavior  
Local buckling – flange or web  
Flexural torsional buckling - twisting

## Long columns

Fail in Euler buckling  
Elastic behavior



$$\text{slenderness} = \frac{KL}{r}$$

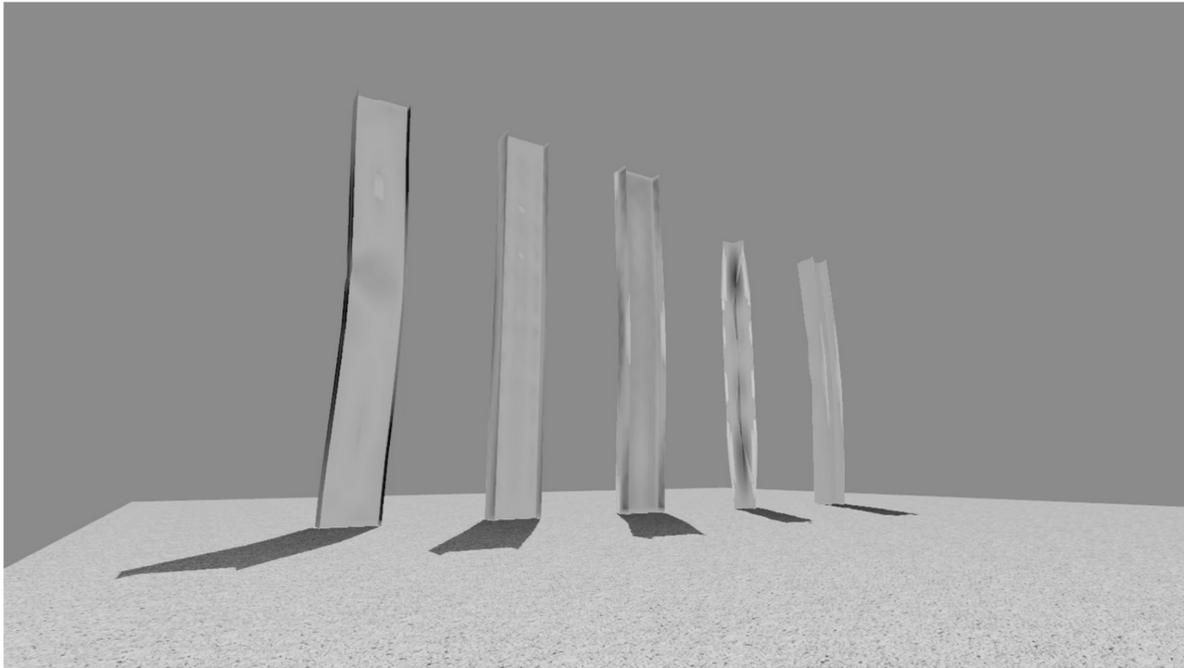
← short intermediate long →

Transition Slenderness  $4.71\sqrt{\frac{E}{F_y}}$

# Failure Modes

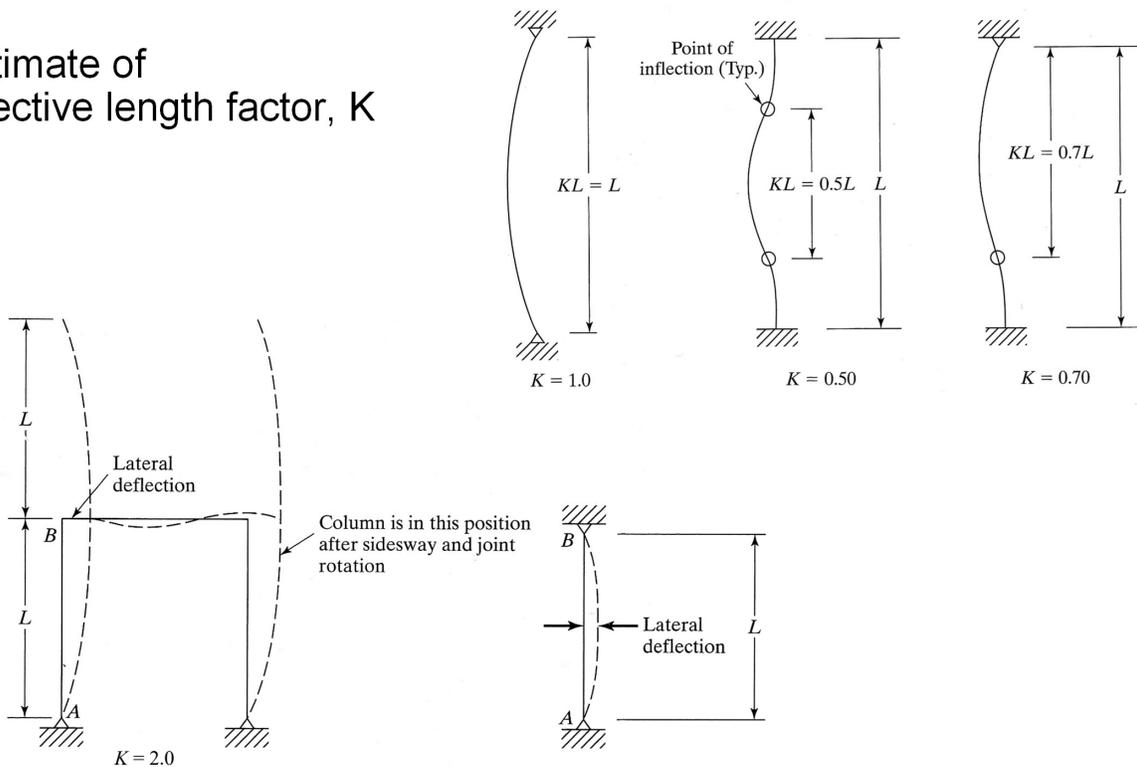
- Column 1: Strong axis flexural buckling
- Column 2: Web local buckling
- Column 3: Weak axis flexural buckling
- Column 4: Torsional buckling
- Column 5: Flange local buckling

“Dancing Columns”  
Sherif El-Tawil



# Analysis of Steel Columns

Estimate of effective length factor,  $K$



# Analysis of Steel Columns

Estimate of K:

TABLE C-A-7.1 Approximate Values of Effective Length Factor, K						
Buckled shape of column is shown by dashed line	(a)	(b)	(c)	(d)	(e)	(f)
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	<ul style="list-style-type: none"> <li> Rotation fixed and translation fixed</li> <li> Rotation free and translation fixed</li> <li> Rotation fixed and translation free</li> <li> Rotation free and translation free</li> </ul>					

## Determining K factors by Alignment Charts

Sidesway Inhibited:  
Braced frame  
 $1.0 > K > 0.5$

Sidesway Uninhibited:  
Un-braced frame  
unstable  $> K > 1.0$

More Pinned:  
If  $I_c/L_c$  is large  
and  $I_g/L_g$  is small  
The connection is more pinned

More Fixed:  
If  $I_c/L_c$  is small  
and  $I_g/L_g$  is large  
The connection is more fixed

## Sidesway inhibited

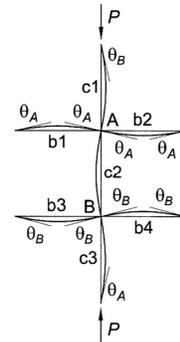
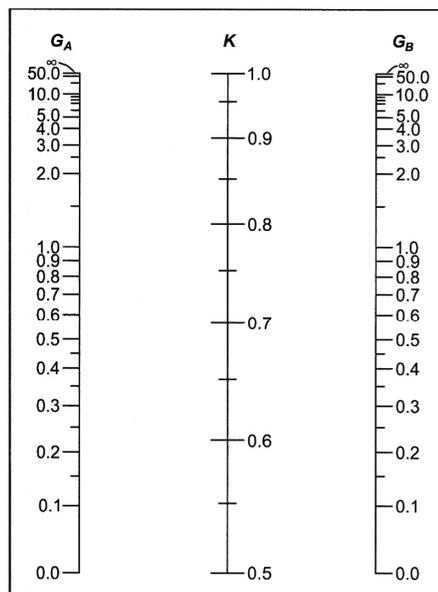


Fig. C-A-7.1. Alignment chart—sidesway inhibited (braced frame).

$$G = \frac{\sum \left( \frac{EI}{L} \right)_{column}}{\sum \left( \frac{EI}{L} \right)_{beam}}$$

# Determining K factors by Alignment Charts

Sidesway Inhibited:  
Braced frame  
 $1.0 > K > 0.5$

Sidesway Uninhibited:  
Un-braced frame  
unstable  $> K > 1.0$

More Pinned:  
If  $I_c/L_c$  is large  
and  $I_g/L_g$  is small  
The connection is more pinned  
and in this case unstable

More Fixed:  
If  $I_c/L_c$  is small  
and  $I_g/L_g$  is large  
The connection is more fixed

# Sidesway uninhibited

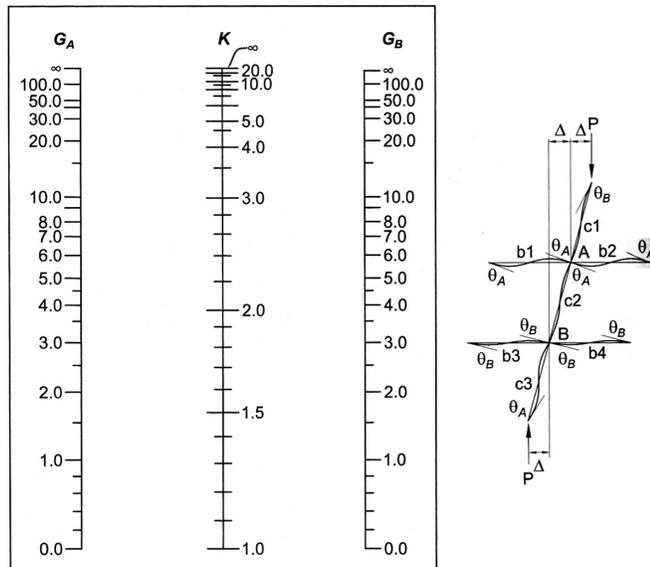


Fig. C-A-7.2. Alignment chart—sidesway uninhibited (moment frame).

$$G = \frac{\sum \left( \frac{EI}{L} \right)_{column}}{\sum \left( \frac{EI}{L} \right)_{beam}}$$

# Analysis of Steel Columns - LRFD

Euler equation:

$$F_e = \frac{\pi^2 E}{\left( \frac{KL}{r} \right)^2}$$

Short & Intermediate Columns:

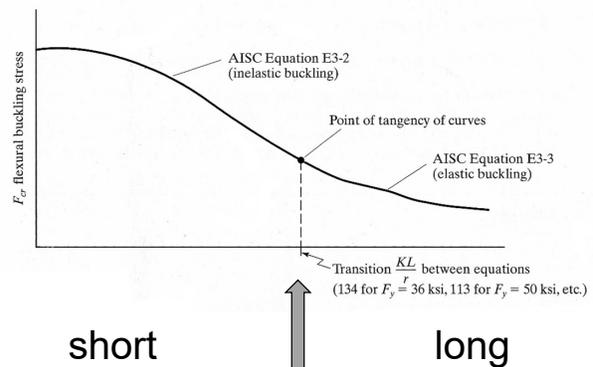
$$F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y$$

Equation E3-2

Long Columns:

$$F_{cr} = 0.877 F_e$$

Equation E3-3



Transition Slenderness  $4.71 \sqrt{\frac{E}{F_y}}$

$$P_n = F_{cr} A_g$$

$$\phi_c P_n = \phi_c F_{cr} A_g$$

$$(\phi_c = 0.90)$$

# Analysis of Steel Columns pass / fail by LRFD



Data:

- Column – size, length
- Support conditions
- Material properties –  $F_y$
- Factored load –  $P_u$

Required:

- $P_u \leq \phi P_n$  (pass)

1. Calculate slenderness ratios:  $L_c/r_x$  and  $L_c/r_y$  ( $L_c = KL$ )  
The largest ratio governs.

2. Check slenderness ratio against upper limit of 200 (recommended)

3. Calculate transition slenderness  $4.71\sqrt{E/F_y}$   
and determine column type (short or long)

$$F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y \quad \text{Short}$$

4. Calculate  $F_{cr}$  based on slenderness

5. Determine  $\phi P_n$  and compare to  $P_u$

$$P_n = F_{cr} A_g \quad \phi = 0.9$$

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

6. If  $P_u \leq \phi P_n$ , then OK

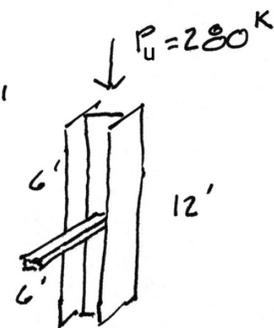
# Example - Analysis of Steel Columns pass / fail by ASD

Data:

- Column – size, length
- Support conditions
- Material properties –  $F_y$
- Factored Load –  $P_u$

DATA :

W 8x35  
 $r_x = 3.51$  "  
 $r_y = 2.03$  "  
 $A = 10.3$  in<sup>2</sup>  
 $l_x = 12'$   $l_y = 6'$   
 $K_x = K_y = 1.0$   
 A-36  
 $F_y = 36$  ksi



Required:

- $P_u \leq \phi P_n$  (pass)

1. Calculate slenderness ratios:  $L_c/r_x$  and  $L_c/r_y$  ( $L_c = KL$ )  
The largest ratio governs.

Table 1-1 (continued)  
W-Shapes  
Dimensions

Shape	Area, A		Depth, d		Web		Flange		Distance						
	in. <sup>2</sup>	in.	in.	in.	Thickness, t <sub>w</sub>	t <sub>w</sub> /2	Width, b <sub>f</sub>	Thickness, t <sub>f</sub>	k		k <sub>1</sub>	T	Work-able Gage		
									k <sub>des</sub>	k <sub>det</sub>					
W8x67	19.7	9.00	9	0.570	9/16	5/16	8.28	8 1/4	0.935	1 5/16	1.33	1 5/8	1 5/16	5 3/4	5 1/2
x58	17.1	8.75	8 3/4	0.510	1/2	1/4	8.22	8 1/4	0.810	1 3/16	1.20	1 1/2	7/8		
x48	14.1	8.50	8 1/2	0.400	3/8	3/16	8.11	8 1/8	0.685	1 1/16	1.08	1 3/8	1 3/16		
x40	11.7	8.25	8 1/4	0.360	3/8	3/16	8.07	8 1/8	0.560	9/16	0.954	1 1/4	1 3/16		
x35	10.3	8.12	8 1/8	0.310	3/8	3/16	8.02	8	0.495	1/2	0.889	1 3/16	1 3/16		
x31	9.13	8.00	8	0.285	5/16	3/16	8.00	8	0.435	7/16	0.829	1 1/8	3/4		

Table 1-1 (continued)  
W-Shapes  
Properties

Nominal Wt.	Compact Section Criteria		Axis X-X				Axis Y-Y				r <sub>ts</sub>	h <sub>o</sub>	Torsional Properties	
	b <sub>t</sub> /2t <sub>w</sub>	h/t <sub>w</sub>	I	S	r	Z	I	S	r	Z			J	C <sub>w</sub>
67	4.43	11.1	272	60.4	3.72	70.1	88.6	21.4	2.12	32.7	2.43	8.07	5.05	1440
58	5.07	12.4	228	52.0	3.65	59.8	75.1	18.3	2.10	27.9	2.39	7.94	3.33	1180
48	5.92	15.9	184	43.2	3.61	49.0	60.9	15.0	2.08	22.9	2.35	7.82	1.96	931
40	7.21	17.6	146	35.5	3.53	39.8	49.1	12.2	2.04	18.5	2.31	7.69	1.12	726
35	8.10	20.5	127	31.2	3.51	34.7	42.6	10.6	2.03	16.1	2.28	7.63	0.769	619
31	9.19	22.3	110	27.5	3.47	30.4	37.1	9.27	2.02	14.1	2.26	7.57	0.536	530

# Example - Analysis of Steel Columns

pass / fail by ASD

Data:

- Column – size, length
- Support conditions
- Material properties –  $F_y$
- Factored Load –  $P_u$

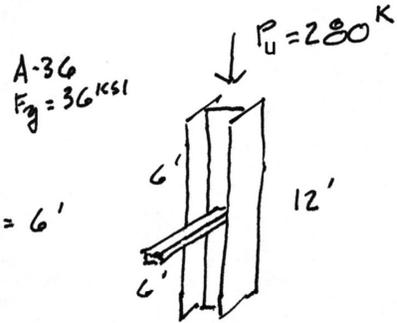
Required:

- $P_u \leq \phi P_n$  (pass)

1. Calculate slenderness ratios.  
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)

DATA :

$W 8 \times 35$   
 $r_x = 3.51''$   
 $r_y = 2.03''$   
 $A = 10.3 \text{ in}^2$   
 $l_x = 12'$     $l_y = 6'$   
 $K_x = K_y = 1.0$



X - X AXIS

$$\frac{K_x l_x}{r_x} = \frac{144''}{3.51''}$$

$$\underline{41.03} < 200$$

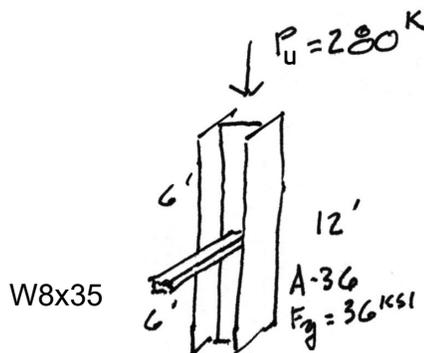
Y - Y AXIS

$$\frac{K_y l_y}{r_y} = \frac{72''}{2.03''}$$

$$35.47$$

# Example - Analysis of Steel Columns

pass / fail by ASD



$$4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000}{36}} = 134$$

$$41 < 134 \therefore \text{SHORT}$$

Euler Equation

$$F_e = \frac{\pi^2 E}{\left(\frac{K L}{r}\right)^2} = \frac{\pi^2 29000 \text{ ksi}}{41^2} = 170.2 \text{ ksi}$$

Short Column Equation

$$F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y = 0.9153 (36) = 32.95 \text{ ksi}$$

Column Strength

$$P_n = F_{cr} A_g = 32.95 \text{ ksi} \times 10.3 \text{ in}^2 = 339.39 \text{ k}$$

$$\phi P_n = 0.9 P_n = 0.9 (339.39) = 305.4 \text{ k}$$

$$P_u = 280 \text{ k} < 305.4 \text{ k} = \phi P_n \quad \checkmark \text{OK}$$

3. Calculate transition slenderness  $4.71 \sqrt{E/F_y}$  and determine column type (short or long)
4. Calculate  $F_{cr}$  based on slenderness
5. Determine  $\phi P_n$  and compare to  $P_u$
6. If  $P_u \leq \phi P_n$ , then OK

# Analysis of Steel Columns capacity by LRFD

Data:

- Column – size, length
- Support conditions
- Material properties –  $F_y$

Required:

- Max load capacity

1. Calculate slenderness ratios.  
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness  $4.71\sqrt{E/F_y}$  and determine column type (short or long)
4. Calculate  $F_{cr}$  based on slenderness
5. Determine  $\phi P_n$  and Compute allowable capacity:  
 $P_n = F_{cr} A_g$      $P_u = \phi P_n$

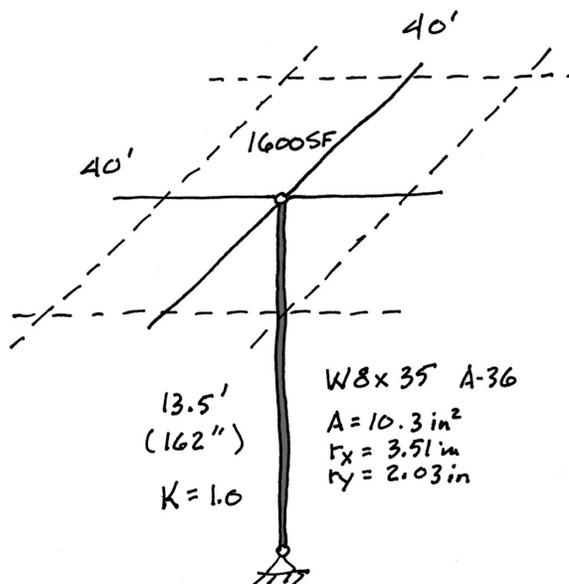


$$F_{cr} = \left[ 0.658 \frac{F_y}{F_e} \right] F_y \quad \text{Short}$$

$$F_{cr} = 0.877 F_e \quad \text{Long}$$

## Capacity Example 1

Free standing column  
Third floor studio space  
Supports roof load = 20 psf DL + SL  
snow  $\approx$  15lbs / FT depth



## Capacity Example 1

1. Calculate slenderness ratios.  
The largest ratio governs.
2. Check slenderness ratio against upper limit of 200 (recommended)
3. Calculate transition slenderness  $4.71\sqrt{E/F_y}$  and determine column type (short or long)
4. Calculate  $F_{cr}$  based on slenderness

y-y Axis (CONTROLS)

$$\frac{K_y L_y}{r_y} = \frac{1(162'')}{2.03''} = 79.8 < 200 \checkmark$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29000}{36}} = 134$$

$$79.8 < 134 \therefore \text{SHORT}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{(K L/r)^2} = \frac{\pi^2 29000}{79.8^2} = 44.94 \text{ ksi}$$

Short Column Equation

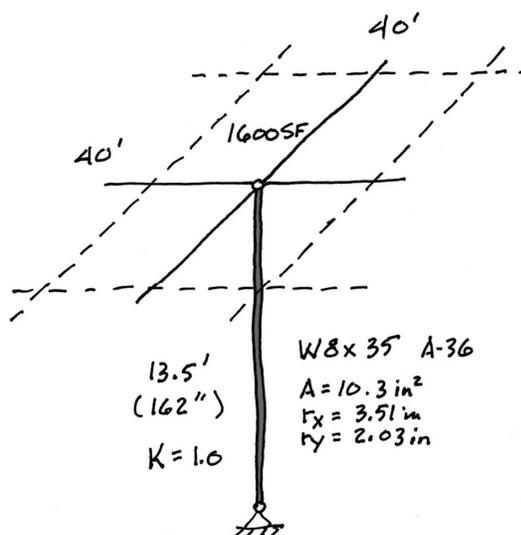
$$F_{cr} = \left[ 0.658^{\frac{F_y}{F_e}} \right] F_y = \left[ 0.7151 \right] 36 = 25.74 \text{ ksi}$$

## Capacity Example 1

5. Determine  $\phi P_n$  and Compute allowable capacity:  $P_u = \phi P_n$

DL = 20 psf

20 psf (1600 sf) = 32k on column



Column nominal strength

$$P_n = F_{cr} A_g = 25.74 \text{ ksi} \cdot 10.3 \text{ in}^2 = 265.1 \text{ k}$$

$$\phi P_n = 0.9(265) = 238.6 \text{ k} = P_u$$

Load capacity

$$P_u = 1.2(32) + 1.6(SL) = 238.6 \text{ k}$$

$$SL = 125.1 \text{ k}$$

$$\text{For } A_r = 40 \times 40 = 1600 \text{ SF}$$

$$SL = \frac{125100^*}{1600 \text{ SF}} = 78.2 \text{ PSF}$$

$$78.2 \text{ lbs} / 15 \text{ lbs/ft} = 5.21 \text{ ft}$$

# Capacity Example 2

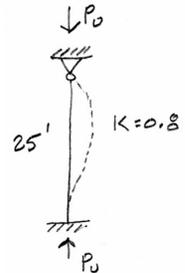
## long column – using equations

Find the capacity for the  
25 ft. column shown.

$$r_x = 3.51 \text{ in.}$$

$$r_y = 2.03 \text{ in.}$$

W8 x 35  
 $F_y = 50 \text{ ksi}$   
 $E = 29000 \text{ ksi}$   
 $L = 25' \text{ (No BRACINGS)}$



Slenderness y-y

$$\frac{KL_y}{r_y} = \frac{0.8(25)12}{2.03} = 118.2$$

$$4.71\sqrt{\frac{E}{F_y}} = 113 < 118.2 \therefore \text{LONG}$$

Euler Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 29000}{118.2^2} = 20.47 \text{ ksi}$$

Long Column Equation  $F_{cr} = 0.877 F_e$

$$F_{cr} = 0.877 (20.47) = 17.95 \text{ ksi}$$

Column strength

$$\phi P_n = \phi F_{cr} A_g = 0.9 (17.95) (10.3) = 166.4 \text{ k}$$

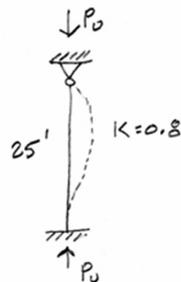
**Table G1 Buckling Length Coefficients,  $K_e$**

Buckling modes						
Theoretical $K_e$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design $K_e$ when ideal conditions approximated	0.65	<b>0.80</b>	1.2	1.0	2.10	2.4
End condition code		Rotation fixed, translation fixed			Rotation free, translation fixed	
		Rotation fixed, translation free			Rotation free, translation free	

# Capacity Example 2

## long column – using table

W8 x 35  
 $F_y = 50 \text{ ksi}$   
 $E = 29000 \text{ ksi}$   
 $L = 25' \text{ (No BRACINGS)}$



$r_y$  CONTROLS

$$KL = 0.8(25') = 20'$$

Shape	Table 4-1a (continued)												$F_y = 50 \text{ ksi}$	
	Available Strength in Axial Compression, kips													
	W-Shapes													
Shape	WBx													
lb/ft	67		98		48		40		35		31			
Design	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, $L_e$ (ft), with respect to least radius of gyration, $r_y$	0	590	886	512	769	422	634	350	526	308	463	273	411	
	6	542	815	470	706	387	581	320	481	281	423	249	374	
	7	526	790	455	685	375	563	309	465	272	409	241	362	
	8	508	763	439	660	361	543	298	448	262	394	232	346	
	9	488	733	422	634	347	521	285	429	251	377	222	333	
	10	467	701	403	606	331	497	272	409	239	359	211	317	
	11	444	668	384	576	314	473	258	388	226	340	200	301	
	12	421	633	363	546	297	447	243	366	213	321	189	283	
	13	397	597	342	514	280	421	228	343	200	301	177	266	
	14	373	560	321	482	262	394	213	321	187	281	165	248	
	15	348	523	299	450	244	367	198	296	174	261	153	230	
	16	324	487	278	418	226	340	183	275	160	241	141	212	
	17	300	450	257	386	209	314	169	253	147	221	130	195	
	18	276	415	236	355	192	288	154	232	135	203	118	178	
	19	253	381	216	325	175	264	141	211	123	184	108	162	
	20	231	347	197	296	159	239	127	191	111	166	97.2	146	
	22	191	287	163	244	132	198	105	158	91.5	136	80.3	121	
	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101	
	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5	
	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5	
	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9	
	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1	
	34	79.9	120	68.1	102	55.1	82.8	44.0	66.1					
Properties														
$P_{no}$ , kips	126	190	102	153	72.0	108	57.2	85.9	45.9	68.9	39.4	59.1		
$P_{ni}$ , kip/in.	19.0	28.5	17.0	25.5	13.3	20.0	12.0	18.0	10.3	15.5	9.50	14.3		
$P_{nb}$ , kips	507	761	363	546	174	262	127	192	81.1	122	63.0	94.7		
$P_{ns}$ , kips	164	246	123	185	87.8	132	68.7	45.9	68.9	35.4	53.2			
$L_p$ , ft	7.49		7.42		7.35		7.21		7.17		7.18			
$L_r$ , ft	47.6		41.6		35.2		29.9		27.0		24.8			
$A_g$ , in. <sup>2</sup>	19.7		17.1		14.1		11.7		10.3		9.13			
$I_x$ , in. <sup>4</sup>	272		228		184		146		127		110			
$I_y$ , in. <sup>4</sup>	88.6		75.1		60.9		49.1		42.6		37.1			
$r_x$ , in.	2.12		2.10		2.08		2.04		2.03		2.02			
$r_y$ , in.	1.75		1.74		1.74		1.73		1.78		1.72			
$P_n/L^2/10^4$ , k-in. <sup>2</sup>	7790		6530		5270		4190		3630		3150			
$P_y/L^2/10^4$ , k-in. <sup>2</sup>	2540		2150		1740		1410		1220		1060			
ASD	LRFD													
$\Omega_c = 1.67$	$\phi_c = 0.90$													
Note: Heavy line indicates $L_e/r_y$ equal to or greater than 200.														