Architecture 324
Structures II

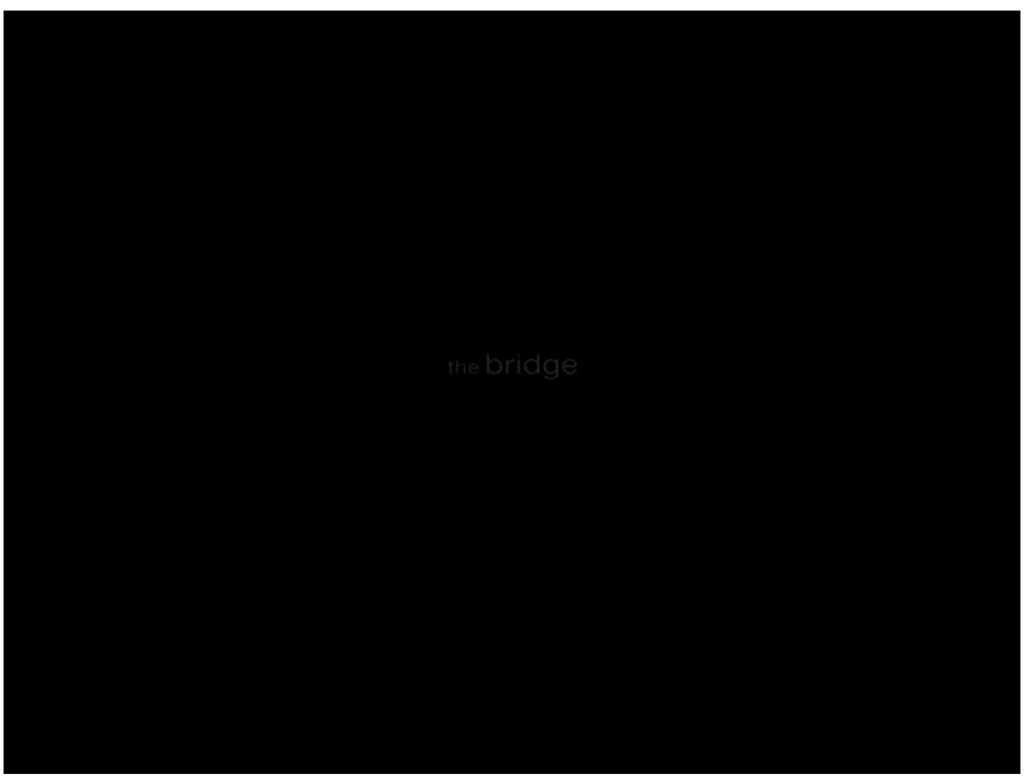
Structural Continuity

- Continuity in Beams
- Deflection Method
- Slope Method

Millennium Bridge, London
Foster and Partners + Arup

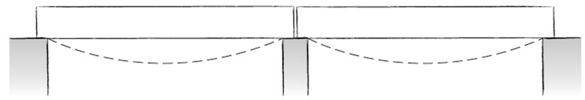
Photo by Ryan Donaghy

Millennium Bridge, London
Foster and Partners + Arup

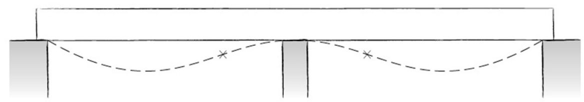


Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, affected long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

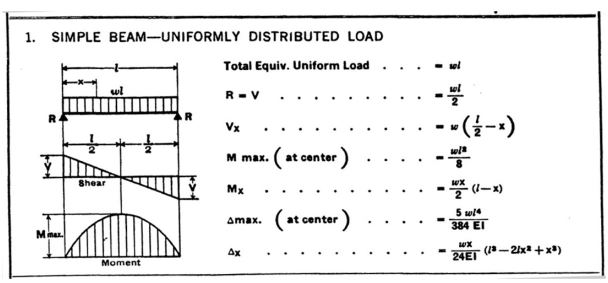


two spans - simply supported



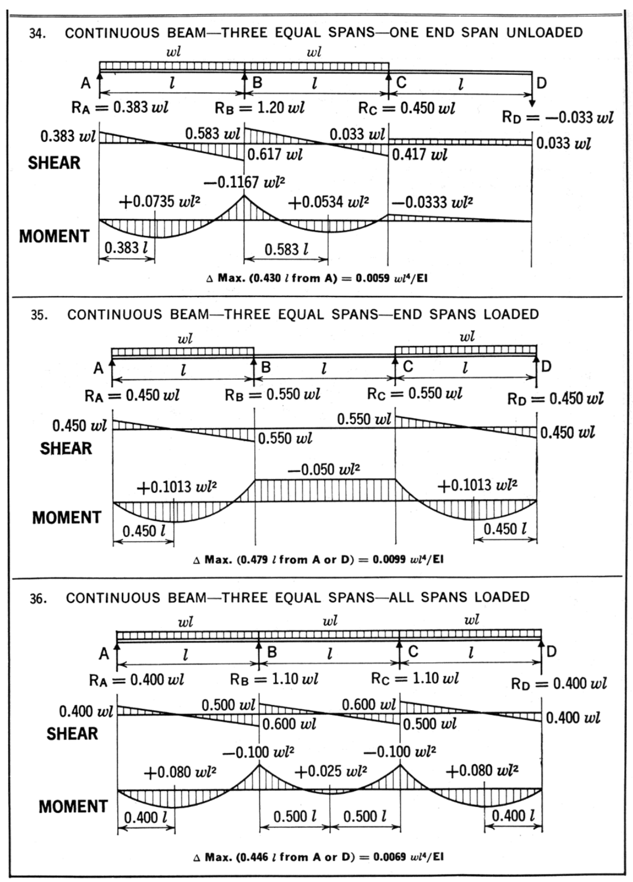
two spans - continuous

Simple vs. Continuous Beams



- Simple Beam
  - End moments = 0
  - Mmax at C.L = $wL^2/8 = 0.125wL^2$
- Continuous Beam
  - Exterior end moments = 0
  - Interior support moments are usually negative
  - Mid-span moments are usually positive

Note: moments shown reversed



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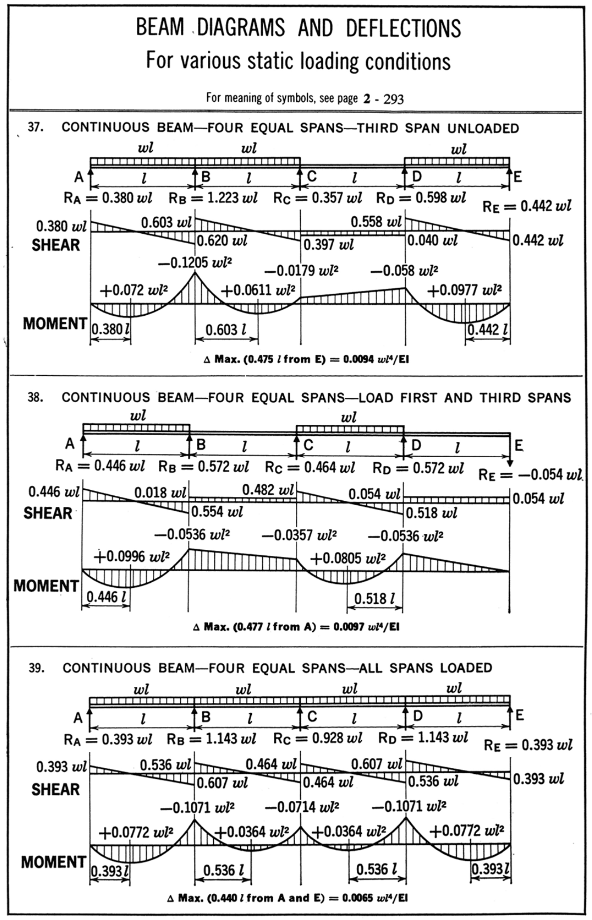
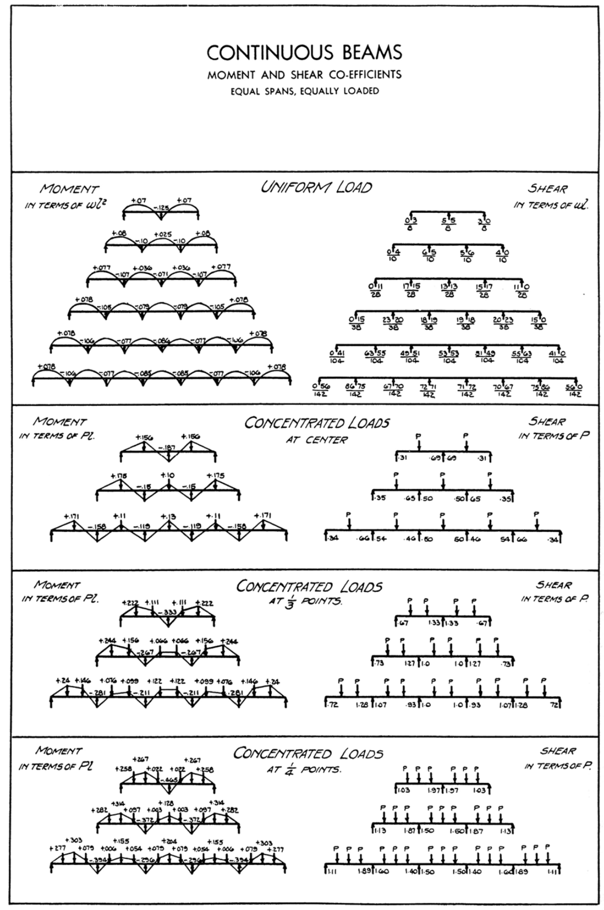
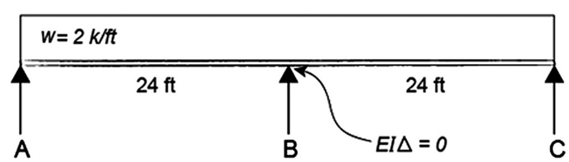




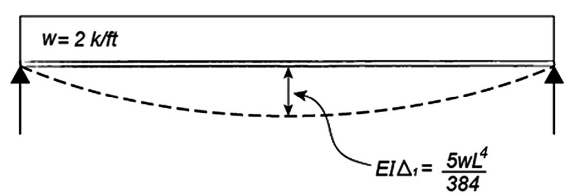
Deflection Method

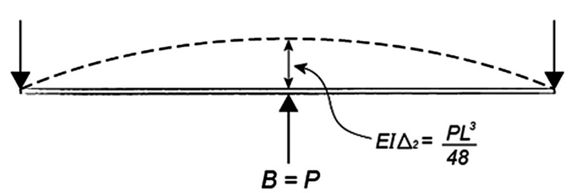
- Two continuous, symmetric spans
- Symmetric Load



Procedure:

1. Remove the center support and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.



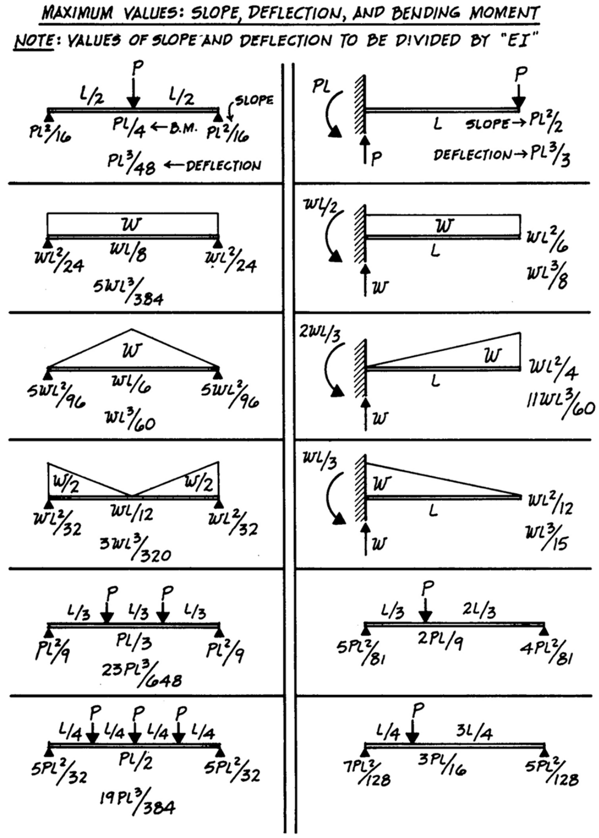


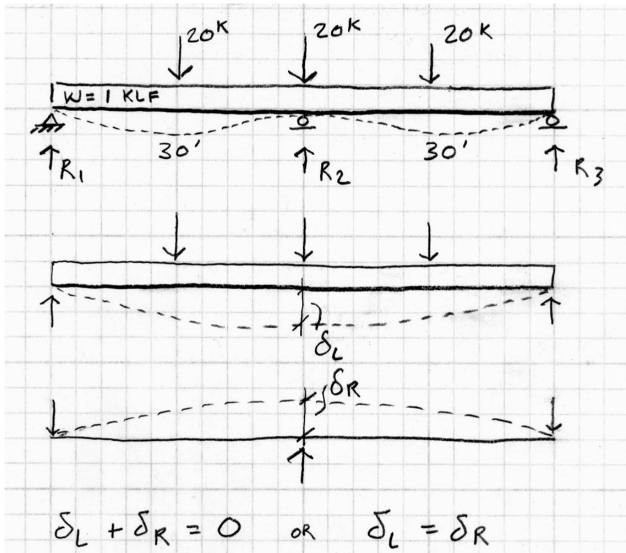
$$EI\Delta_1 + EI\Delta_2 = 0$$

Deflection Method - Example:

Given: Two symmetric spans with symmetric loading as shown.

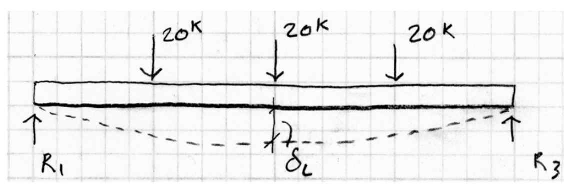
Find: All three reactions

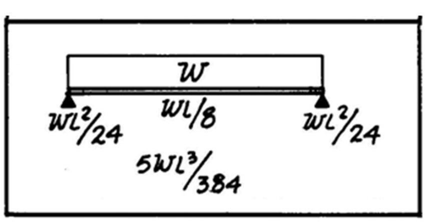


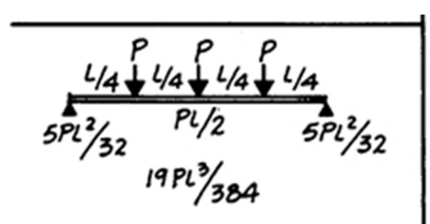


Deflection Method

1. Remove the center support and calculate the center deflection for each load case as a simple span.





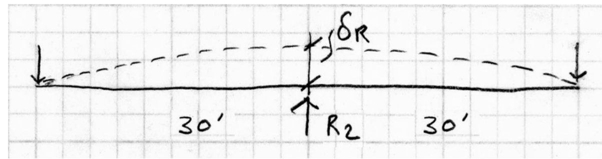


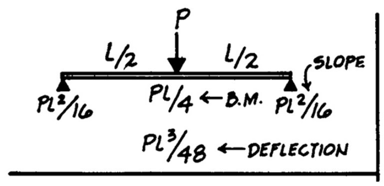
$$\delta_L = \frac{5 w \ell^4}{384\,EI} + \frac{19 P \ell^3}{384\,EI} = \frac{5(1)(60)^4 + 19(20)(60)^3}{384\,EI}$$

$$\delta_L = 382500/EI$$

Deflection Method – Example

2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.





3. Solve the resulting equation for the center reaction force. (upward point load)

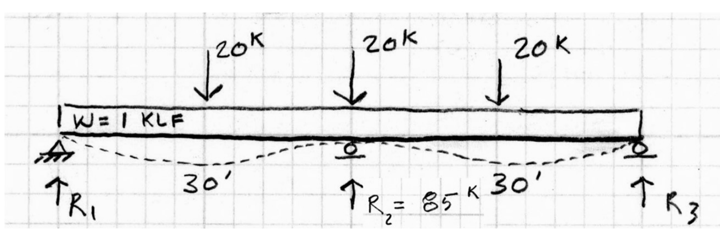
$$\delta_L = \delta_R$$

$$382500/EI = \frac{R_2 \ell^3}{48\,EI}$$

$$R_2 = 85^K$$

Deflection Method – Example

4. Calculate the remaining two end reactions.

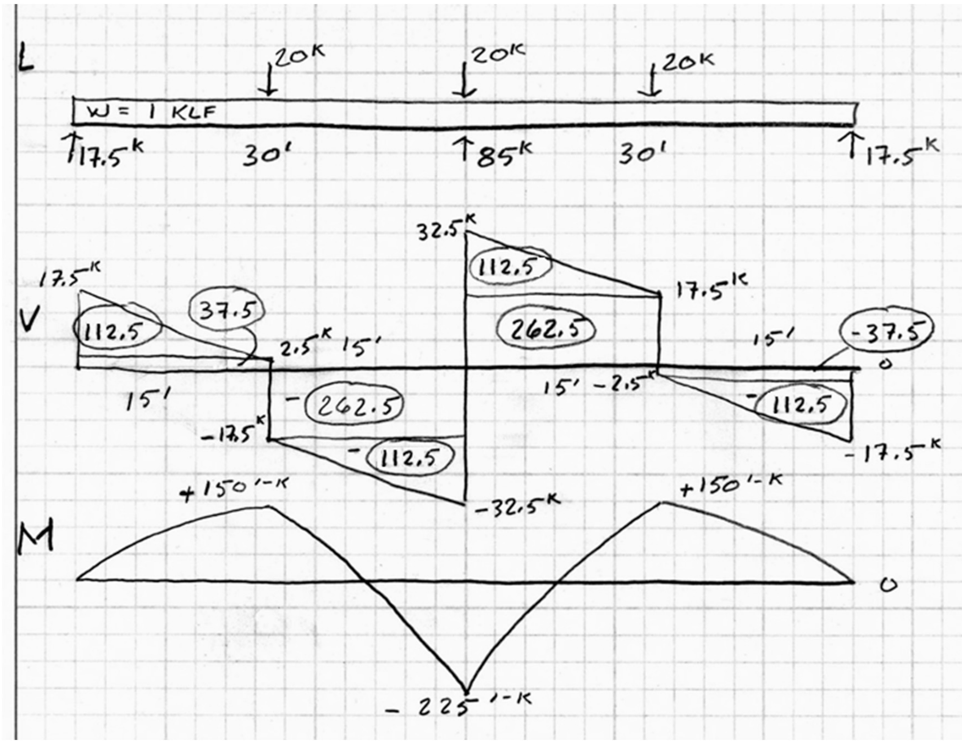


$$\Sigma F_V = 0 = R_1 + R_3 + 85 - 60 - 60 = 0$$

$$R_1 = R_3 = 17.5^K$$

Deflection Method - Example cont.:

5. Draw shear and moment diagrams as usual.



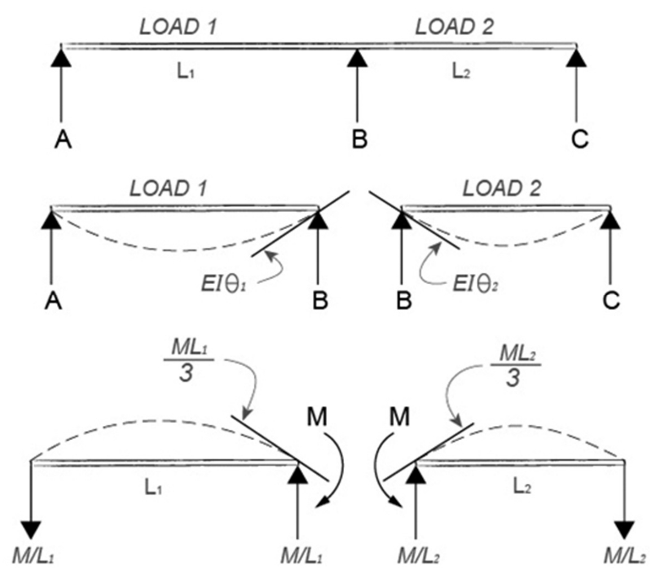
Slope Method

- Two continuous spans
- Non-symmetric loads and spans

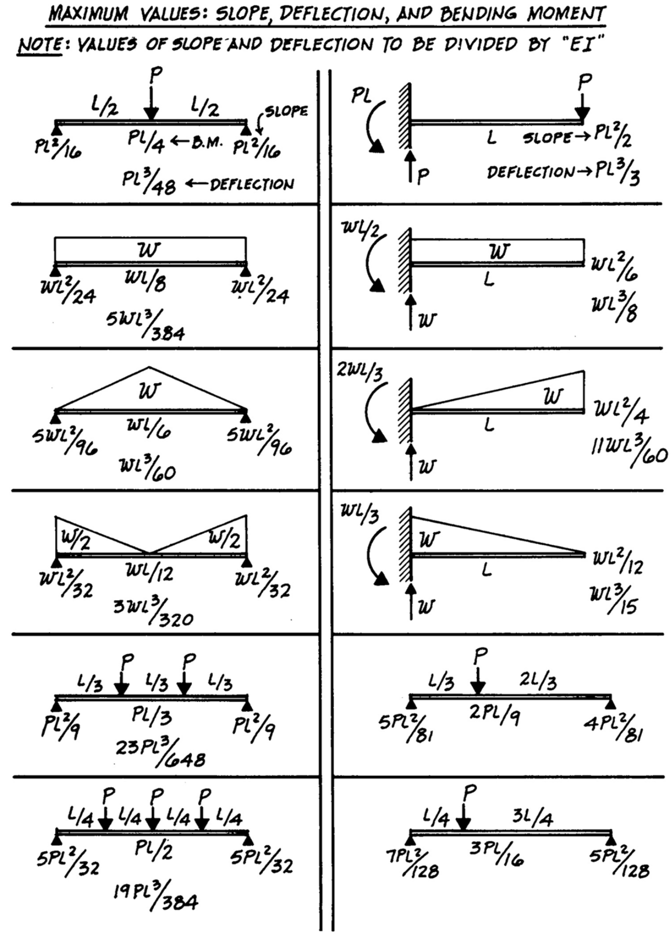
Procedure:

1. Break the beam into two halves at the interior support and calculate the interior slopes of the two simple spans.
2. Use the Slope Equation to solve for the negative interior moment.
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.
5. Draw the shear and moment diagrams as usual.

$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$

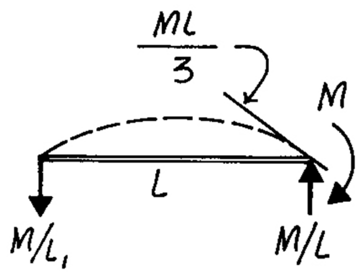


Slope Method



Slope equations:

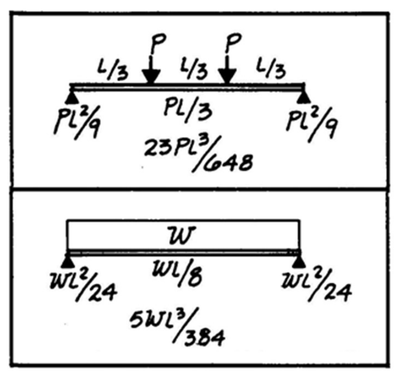
$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$



Slope Method - Example

Given: Two non-symmetric spans with loading as shown.

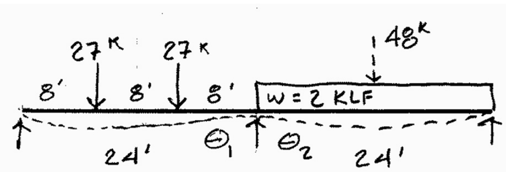
Find: All three reactions



1. Break the beam into two halves at the interior support and calculate the interior slopes of the two simple spans.

2. Use the Slope Equation to solve for the negative interior moment.

$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$



$$EI\Theta_1 = \frac{PL^2}{9} = \frac{27(24)^2}{9} = 1728$$

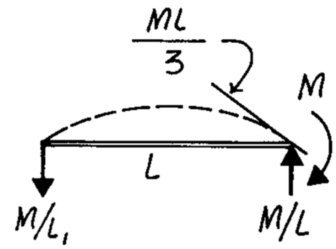
$$EI\Theta_2 = \frac{W\ell^2}{24} = \frac{48(24)^2}{24} = 1152$$

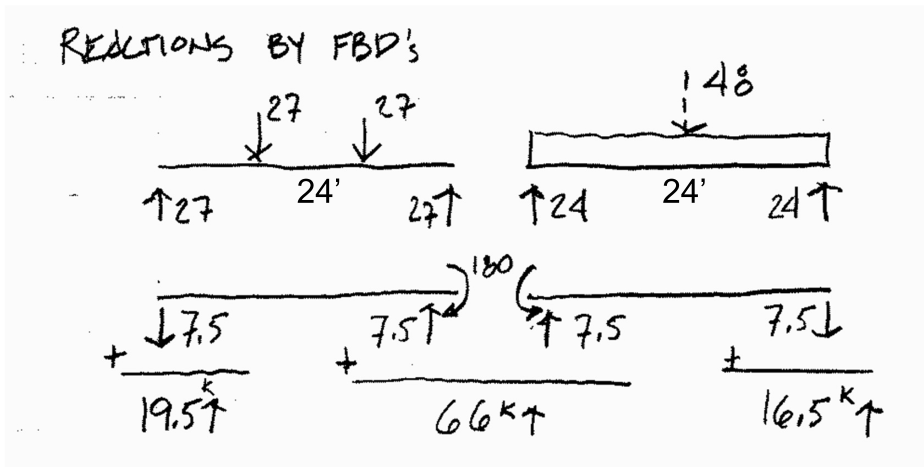
$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right] = \frac{3}{48}\left[2880\right]$$

$$M = 180^{K'}$$

Example of Slope Method cont.:

3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.





Example of Slope Method cont.:

5. Draw the shear and moment diagrams as usual.