

Structural Continuity

- Continuity in Beams
- Deflection Method
- Slope Method



Millennium Bridge, London
Foster and Partners + Arup

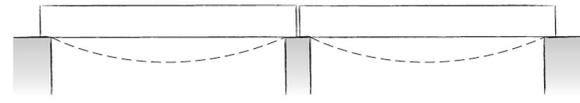
Photo by Ryan Donaghy

Millennium Bridge, London Foster and Partners + Arup



Continuous Beams

- Continuous over one or more supports
 - Most common in monolithic concrete
 - Steel: continuous or with moment connections
 - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
 - Cannot be solved by the three equations of statics alone
 - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

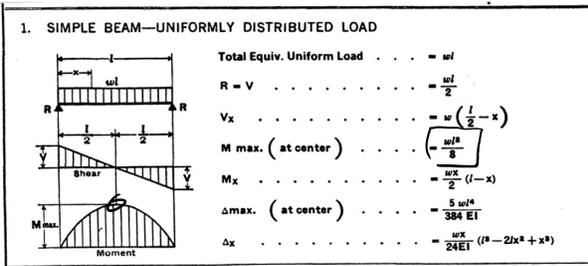


two spans - simply supported

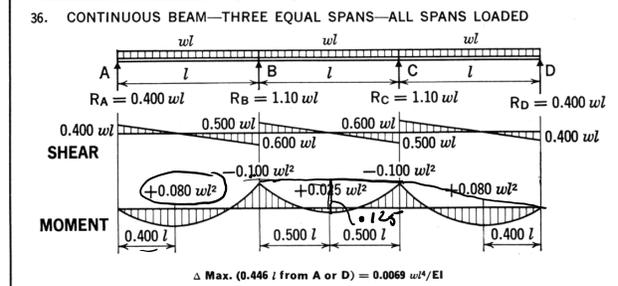
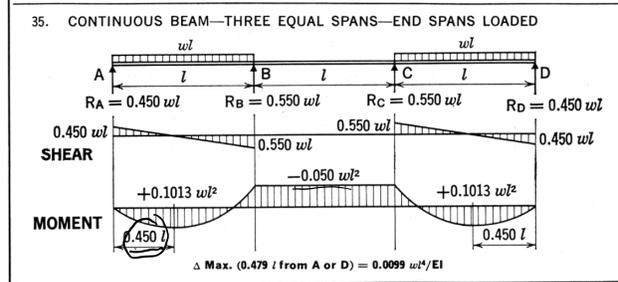
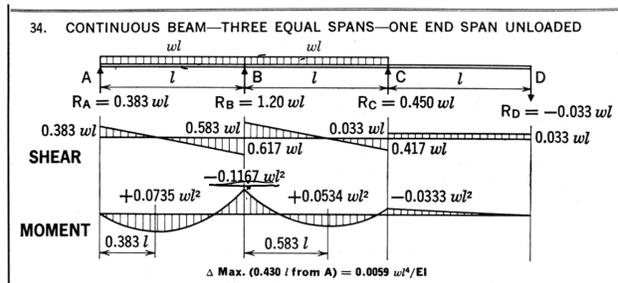


two spans - continuous

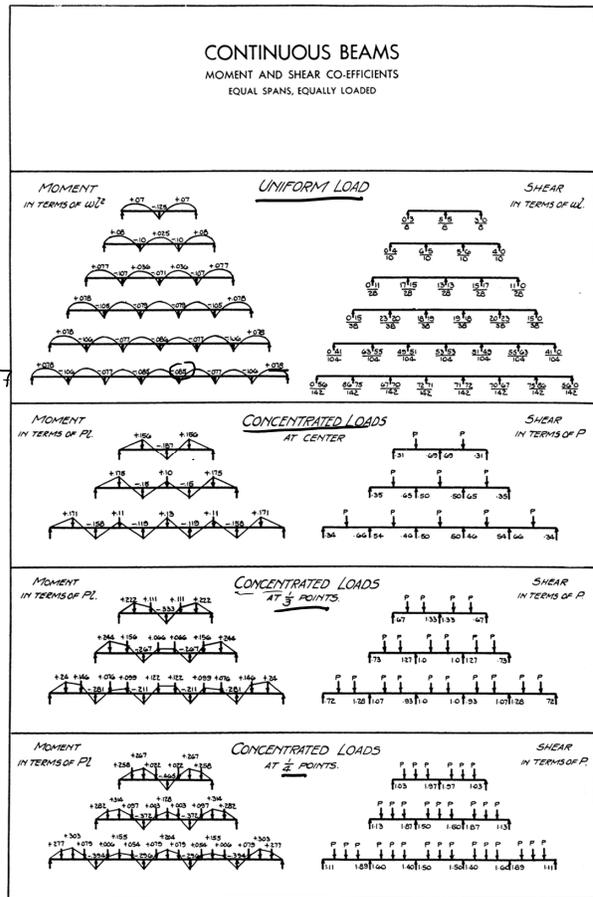
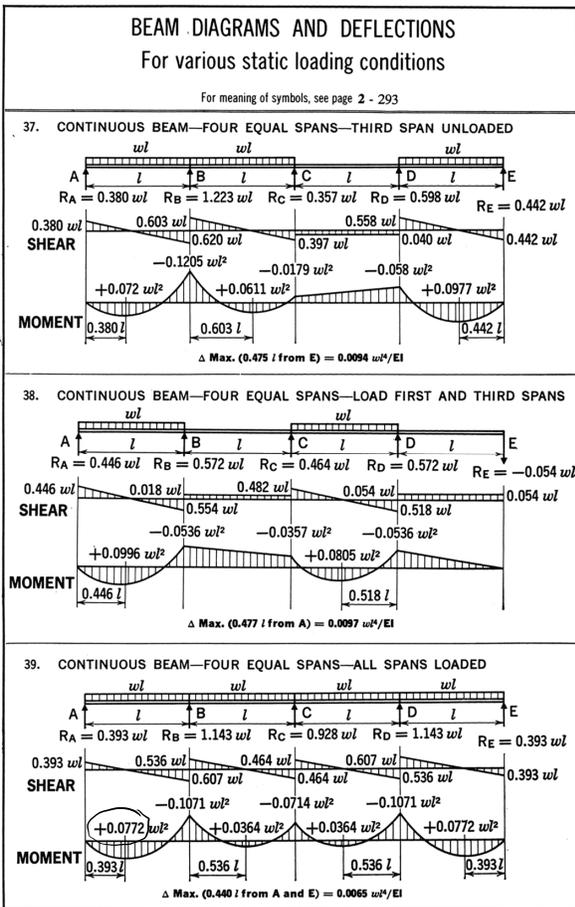
Simple vs. Continuous Beams



- Simple Beam
 - End moments = 0
 - M_{max} at C.L. = $wL^2/8 = 0.125wL^2$
- Continuous Beam
 - Exterior end moments = 0
 - Interior support moments are usually negative
 - Mid-span moments are usually positive



Note: Moment diagrams are drawn upside down

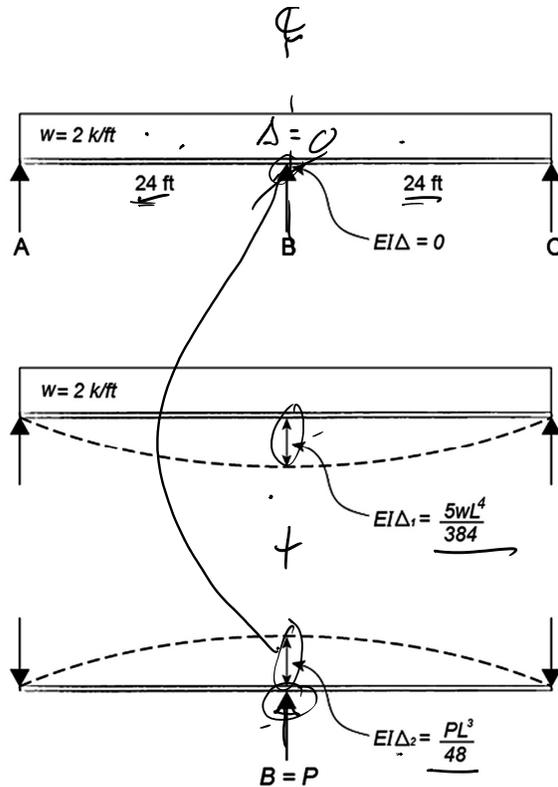


Deflection Method

- Two continuous, symmetric spans
- Symmetric Load

Procedure:

1. Remove the center support and calculate the center deflection for each load case as a simple span.
2. Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.
3. Solve the resulting equation for the center reaction force. (upward point load)
4. Calculate the remaining two end reactions.
5. Draw shear and moment diagrams as usual.

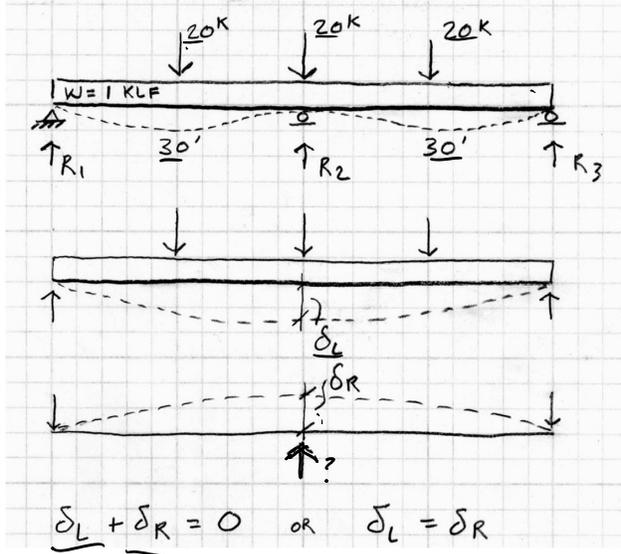


$$EI\Delta_1 + EI\Delta_2 = 0$$

Deflection Method - Example:

Given: Two symmetric spans with symmetric loading as shown.

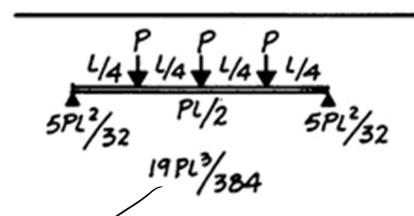
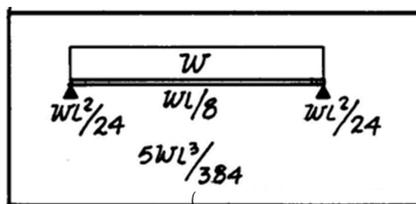
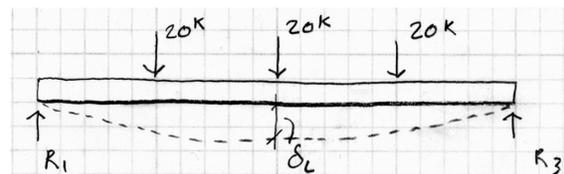
Find: All three reactions



MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT
NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"

Deflection Method

1. Remove the center support and calculate the center deflection for each load case as a simple span.

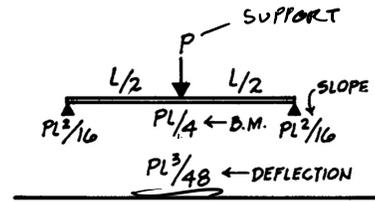


$$\delta_L = \frac{5wl^4}{384 EI} + \frac{19Pl^3}{384 EI} = \frac{5(1)(60)^4 + 19(20)(60)^3}{384 EI}$$

$$\delta_L = 382500/EI$$

Deflection Method – Example

- Remove the applied loads and replace the center support. Set the deflection equation for this case (center point load) equal to the deflection from step 1.



- Solve the resulting equation for the center reaction force. (upward point load)

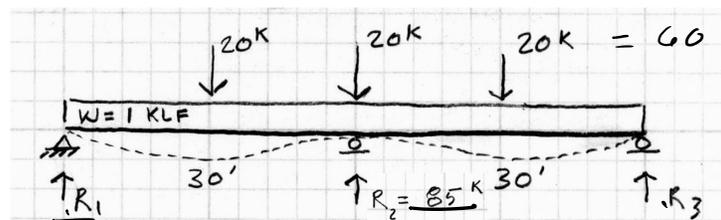
$$\delta_L = \delta_R$$

$$\frac{382500}{EI} = \frac{R_2 L^3}{48 EI}$$

$$R_2 = \underline{85 \text{ K}}$$

Deflection Method – Example

- Calculate the remaining two end reactions.

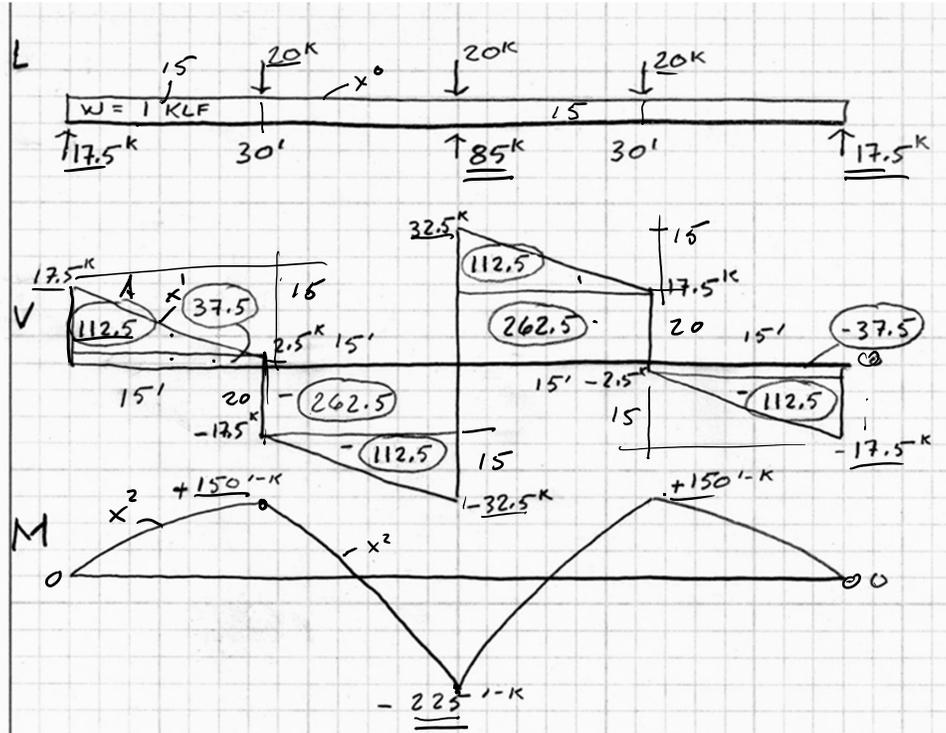


$$\sum F_v = 0 = R_1 + R_3 + 85 - 60 - 60 = 0$$

$$R_1 = R_3 = \underline{17.5 \text{ K}}$$

Deflection Method - Example cont.:

5. Draw shear and moment diagrams as usual.

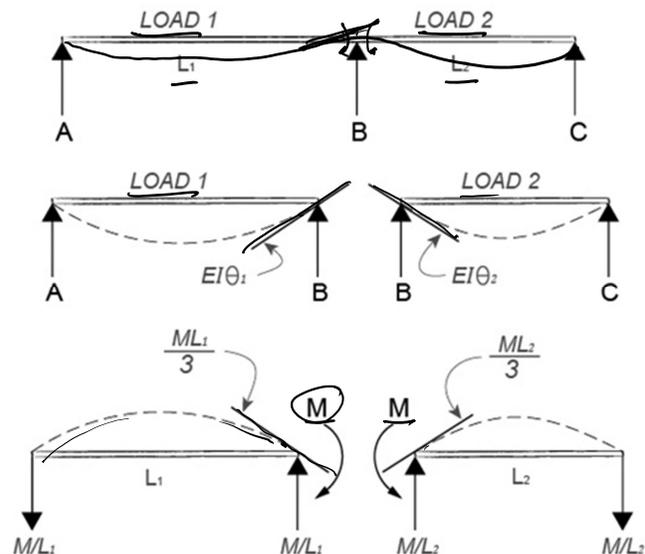


Slope Method

- Two continuous spans
- Non-symmetric loads and spans

Procedure:

1. Break the beam into two halves at the interior support and calculate the interior slopes of the two simple spans.
2. Use the Slope Equation to solve for the negative interior moment.
3. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
4. Add all the reactions by superposition.
5. Draw the shear and moment diagrams as usual.

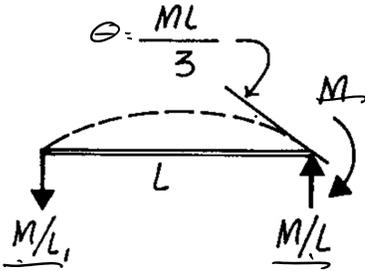


$$\underline{M} = \frac{3}{L_1 + L_2} \left[\underline{EI}\Theta_1 + \underline{EI}\Theta_2 \right]$$

Slope Method

Slope equations:

$$M = \frac{3}{L_1 + L_2} [EI\theta_1 + EI\theta_2]$$

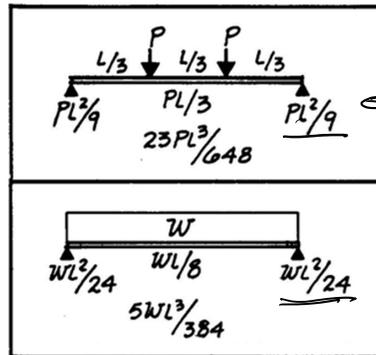


MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT
 NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"

Slope Method - Example

Given: Two non-symmetric spans with loading as shown.

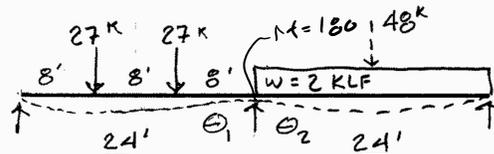
Find: All three reactions



1. Break the beam into two halves at the interior support and calculate the interior slopes of the two simple spans.

2. Use the Slope Equation to solve for the negative interior moment.

$$M = \frac{3}{L_1 + L_2} [EI\theta_1 + EI\theta_2]$$



$$EI\theta_1 = \frac{PL^2}{9} = \frac{27(24)^2}{9} = 1728$$

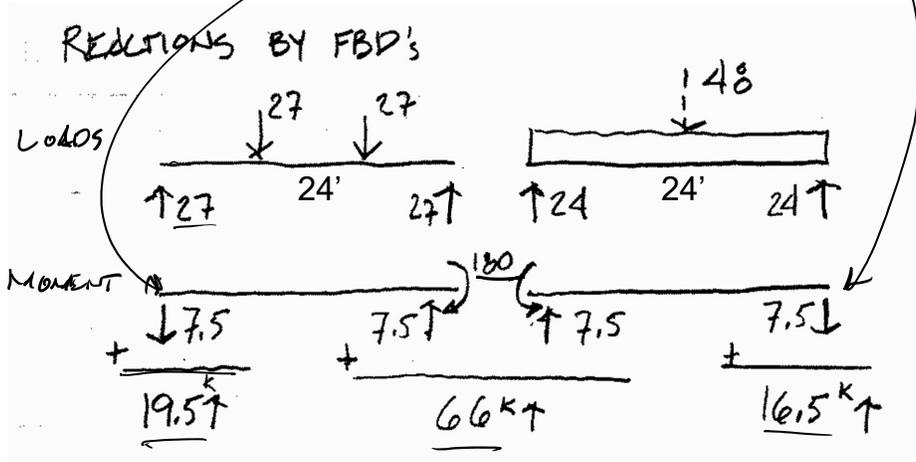
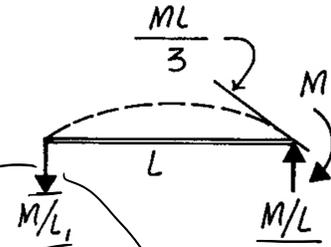
$$EI\theta_2 = \frac{WL^2}{24} = \frac{48(24)^2}{24} = 1152$$

$$M = \frac{3}{L_1 + L_2} [EI\theta_1 + EI\theta_2] = \frac{3}{48} [2880]$$

$$M = 180^k$$

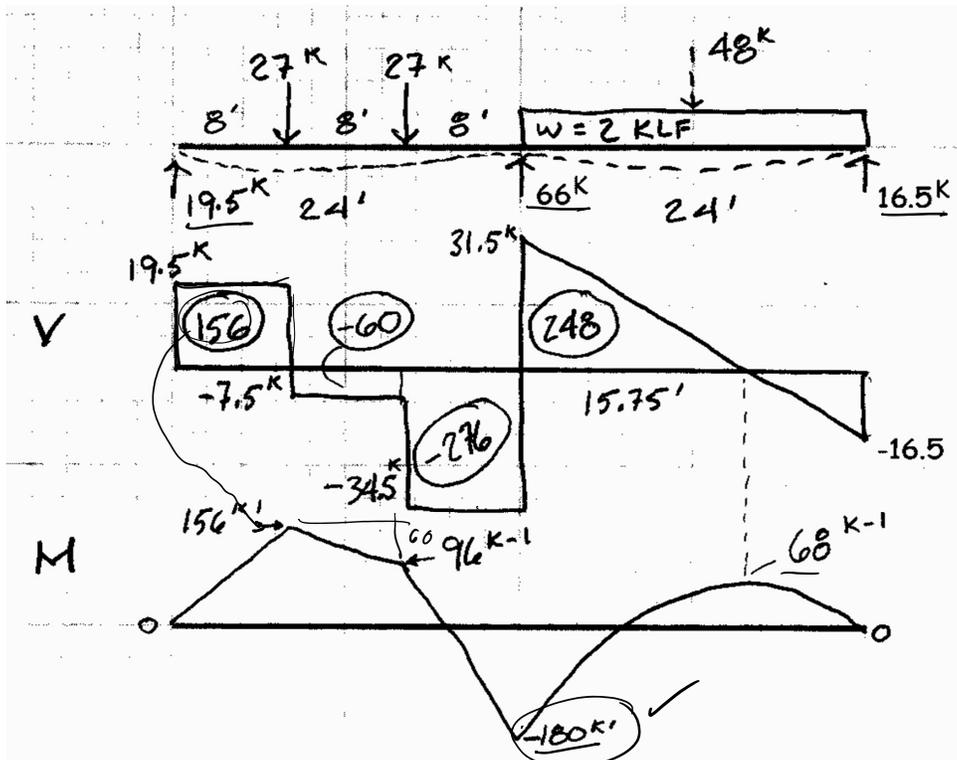
Example of Slope Method cont.:

- Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
- Add all the reactions by superposition.



Example of Slope Method cont.:

- Draw the shear and moment diagrams as usual.



2-span Bridges



Most Uniwersytecki, Poland



Trois-Puits Railroad Overpass, France