Architecture 324
Structures II

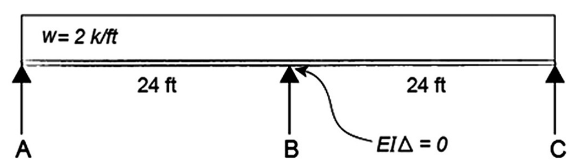
3-Moment Theorem

- Continuity in Beams
- Three-Moment Theorem
- 2-Hinged Frames
- 3-Hinged Frames
- 3-Hinged Arches

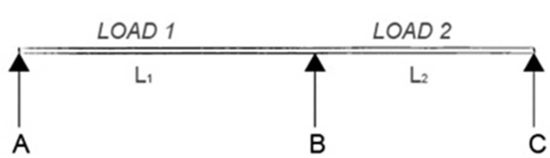
Salginatobel Bridge
Switzerland, 1930
300 ft span 42 ft rise
Robert Maillart

Continuous Beam Analysis

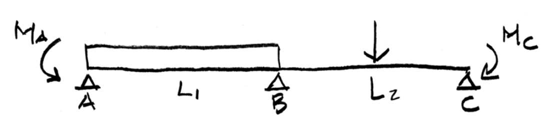
Deflection Method
- Two continuous, symmetric spans
- Symmetric Load



Slope Method
- Two continuous spans
- Non-symmetric loads and spans



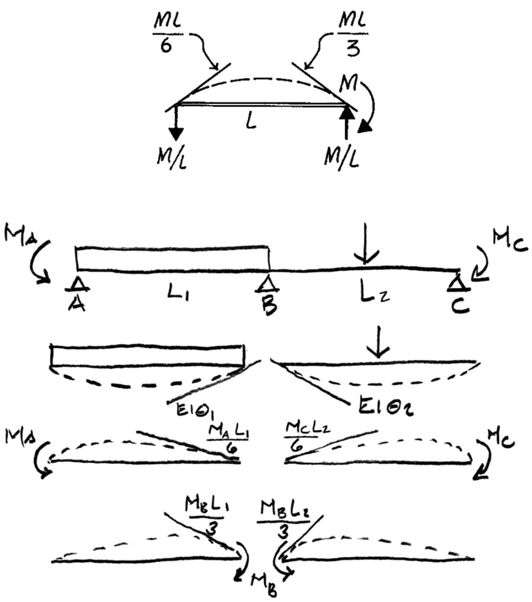
3-Moment Theorem
- Any number of continuous spans
- Non-Symmetric Load and Spans



$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

3-Moment Theorem

- Any number of continuous spans
- Non-Symmetric Load and Spans

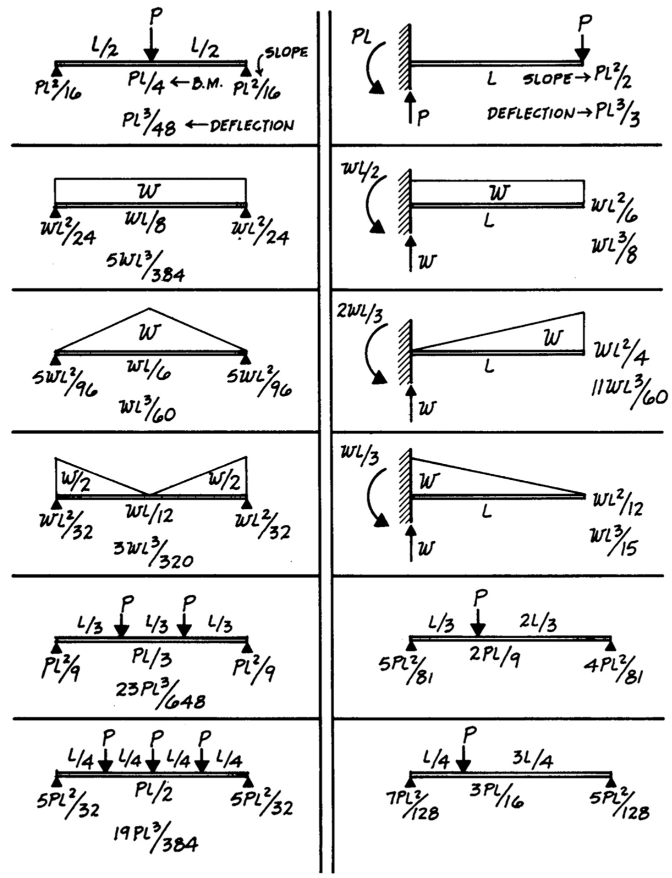


$$+EI\theta_1 - \frac{M_A L_1}{6} - \frac{M_B L_1}{3} = -EI\theta_2 + \frac{M_C L_2}{6} + \frac{M_B L_2}{3}$$

$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

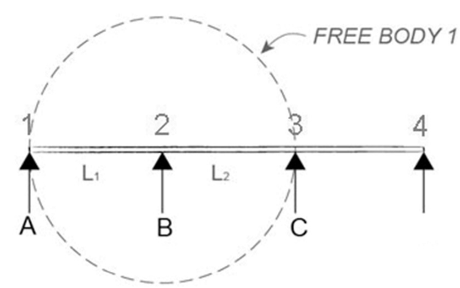
MAXIMUM VALUES: SLOPE, DEFLECTION, AND BENDING MOMENT

NOTE: VALUES OF SLOPE AND DEFLECTION TO BE DIVIDED BY "EI"



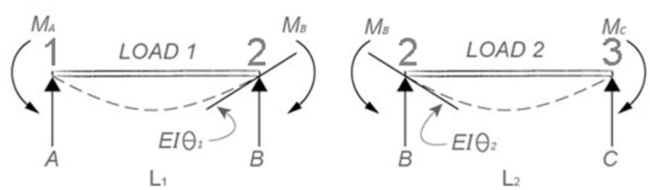
Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric



Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.

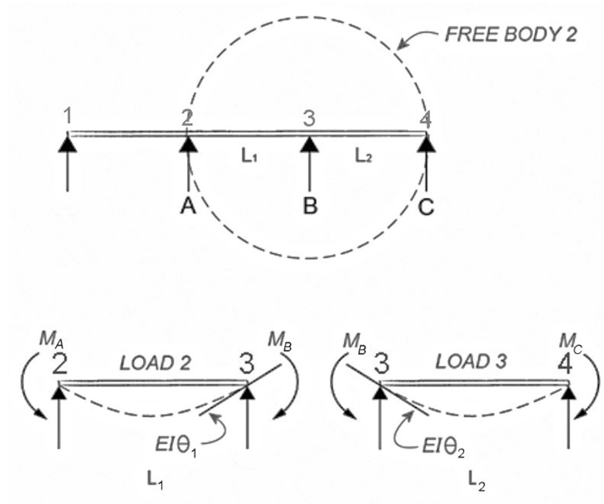


$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

Three-Moment Theorem

Procedure (continued):

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), can now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.

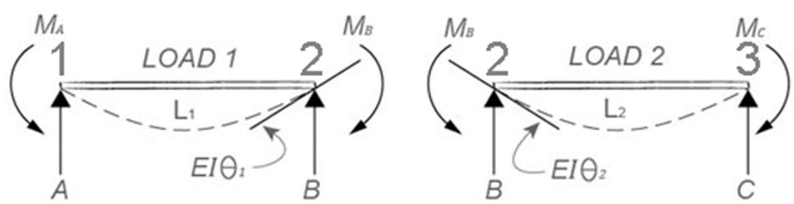


$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

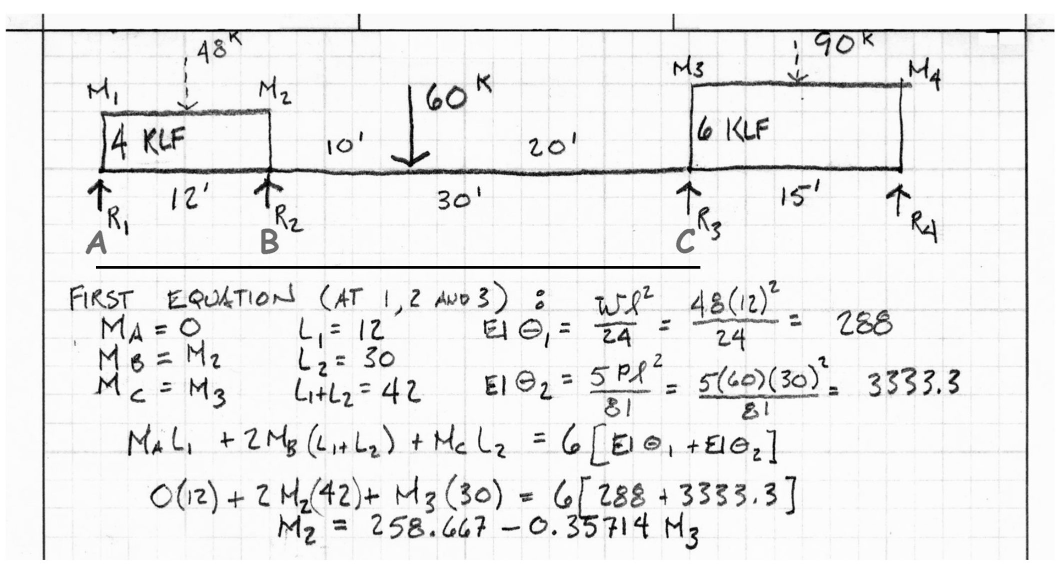
Three-Moment Theorem Example

Given: Three non-symmetric spans with loading as shown.

Find: All four reactions



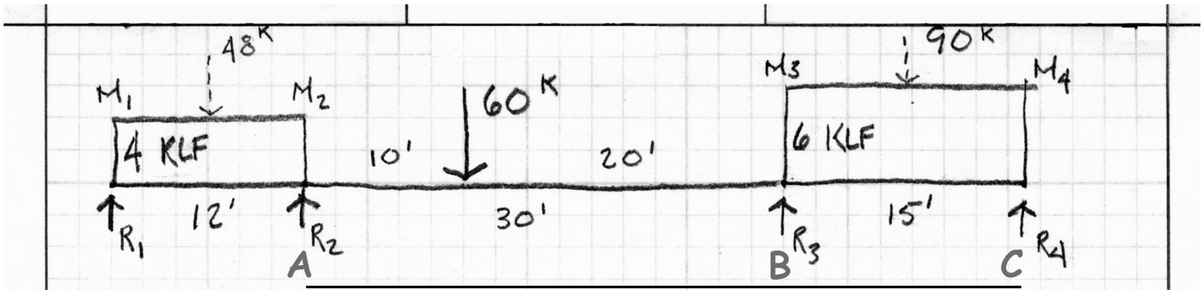
1. Draw FBD
2. Label
3. Solve 3-moment equation



Three-Moment Theorem Example (cont.)

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

4. Move one span further and repeat the procedure.



SECOND EQUATION (AT 2, 3 AND 4):

$M_A = M_2$ $L_1 = 30$ $EI\Theta_1 = \frac{4P\ell^2}{81} = \frac{4(60)(30)^2}{81} = 2666.67$

$M_B = M_3$ $L_2 = 15$

$M_C = 0$ $L_1+L_2 = 45$ $EI\Theta_2 = \frac{w\ell^2}{24} = \frac{90(15^2)}{24} = 843.75$

$$M_2(30) + 2M_3(45) + 0(15) = 6[2666.67 + 843.75]$$
$$M_2 = 702.084 - 3M_3$$

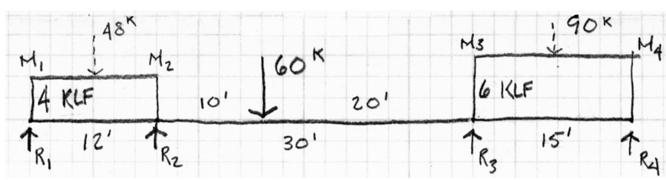
SIMULTANIOUS SOLUTION:

$$258.667 - .35714\,M_3 = 702.084 - 3M_3$$
$$M_3 = 443.417/2.6428 = 167.779^{K\text{-}'}$$
$$M_2 = 702.084 - 3(167.779) = 198.746^{K\text{-}'}$$

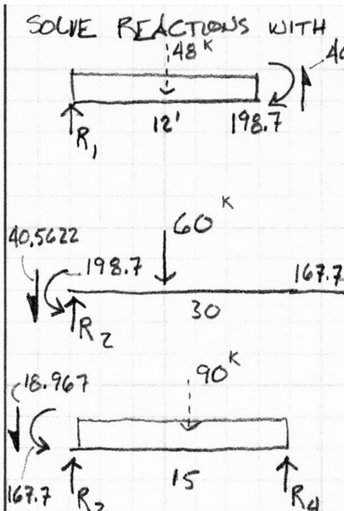
Three-Moment Theorem Example (cont.)

Sign convention

8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.



SOLVE REACTIONS WITH FBD'S



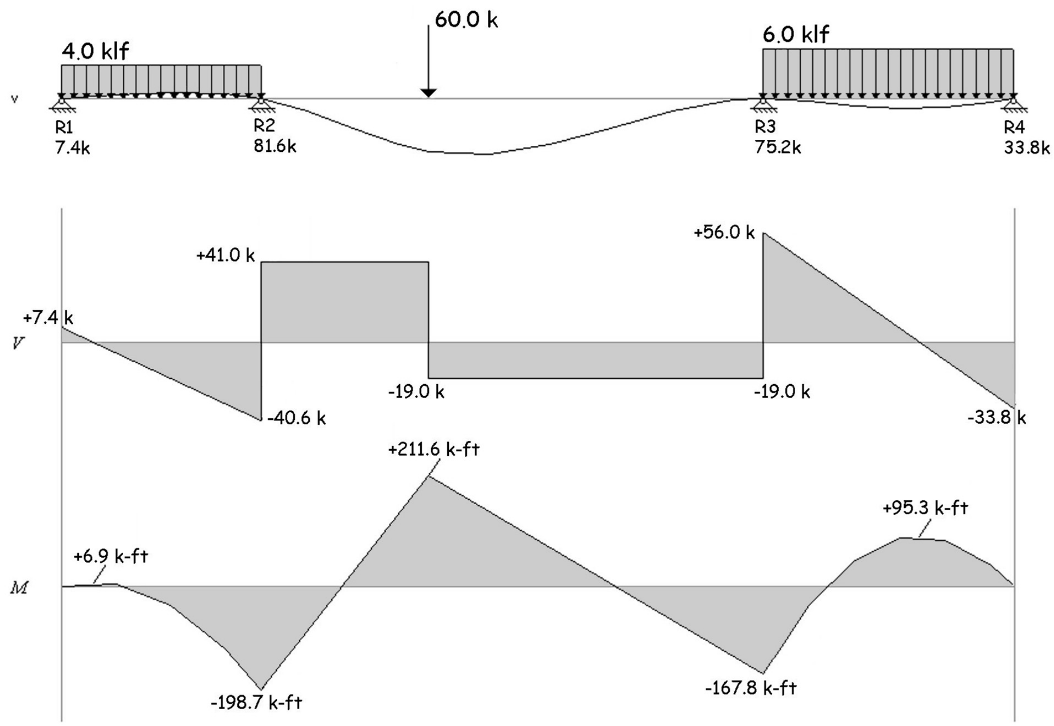
$$\Sigma M_{@R_2} = R_1(12) - 48(6) + 198.7 = 0$$
$$R_1 = 7.4378^K$$
$$\Sigma F_V = 7.4378 - 48 + V = 0$$
$$V = 40.5622$$

$$\Sigma M_{@R_3} = R_2(30) - 40.5622(30) - 60(20) - 198.75 + 167.78 = 0$$
$$R_2 = 81.594^K$$
$$\Sigma F_V = -40.5622 + 81.594 - 60 + V = 0$$
$$V = 18.967$$

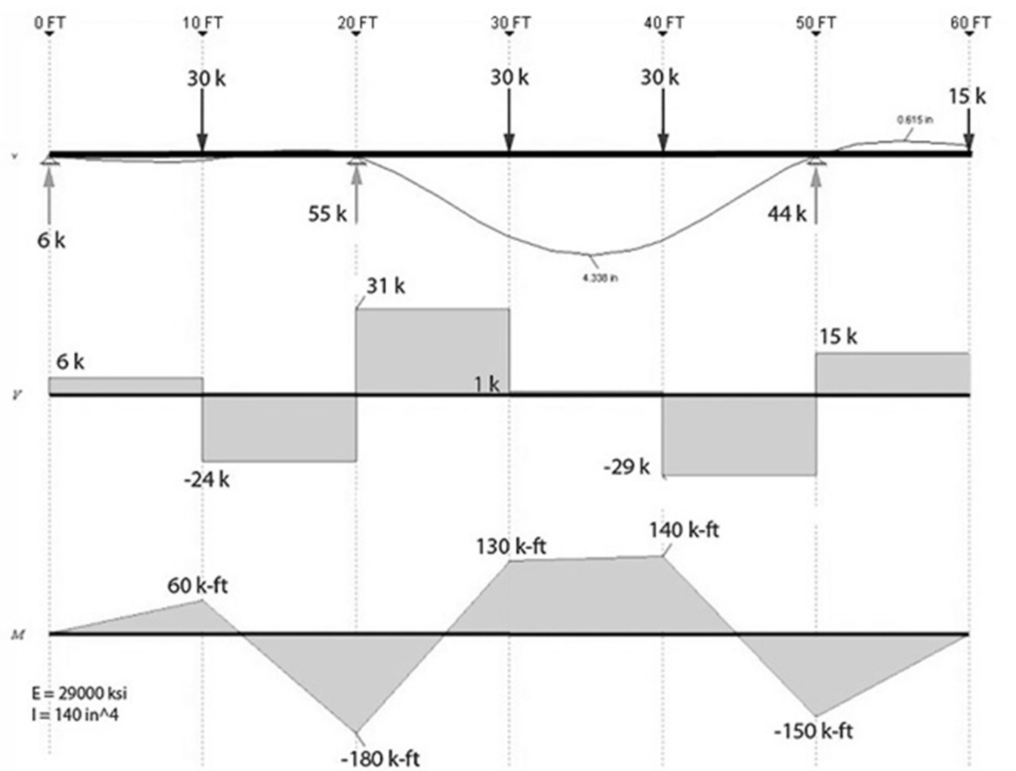
$$\Sigma M_{@R_4} = -167.78 - 18.967(15) + R_3(15) - 90(7.5) = 0$$
$$R_3 = 75.1531^K$$
$$\Sigma M_{@R_3} = -167.78 + 90(7.5) - R_4(15) = 0$$
$$R_4 = 33.8147^K$$

Three-Moment Theorem Example (cont.)

9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.

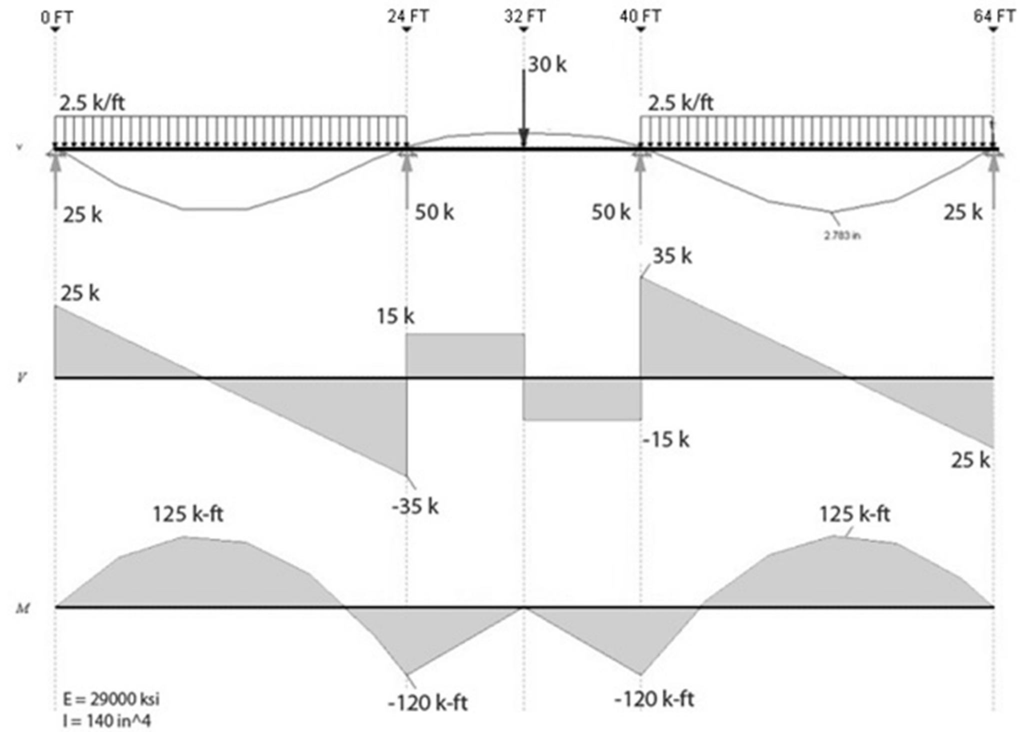


Three-Moment Theorem – 2 Spans



$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

Three-Moment Theorem – 3 Spans

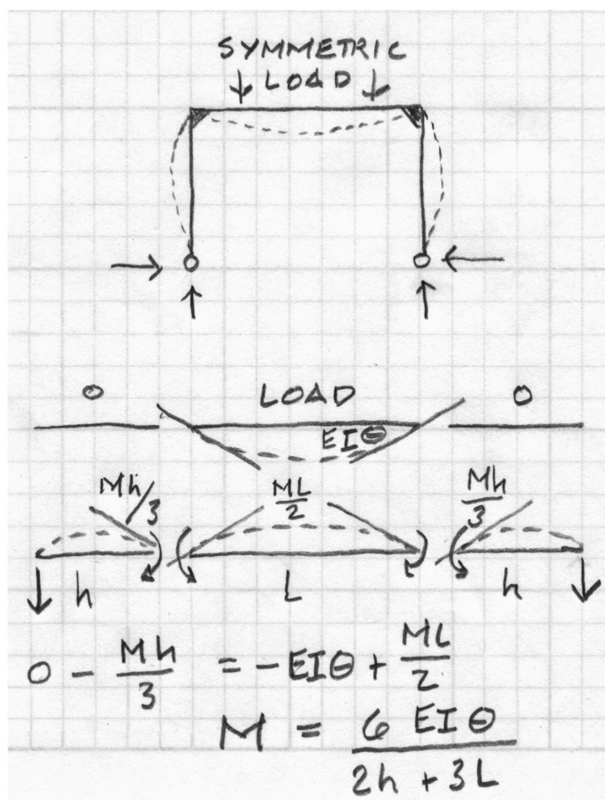


$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

2-Hinge Frame

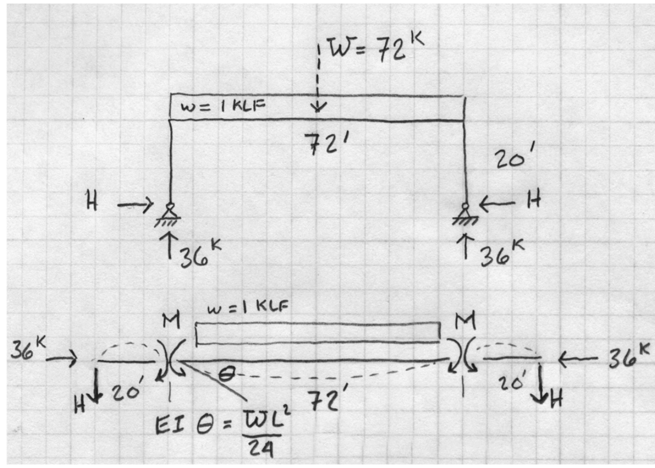
- Statically indeterminate
- Find negative moment at knee
- Sum of Θ's left and right of knee are equal
- Symmetric case solution is:

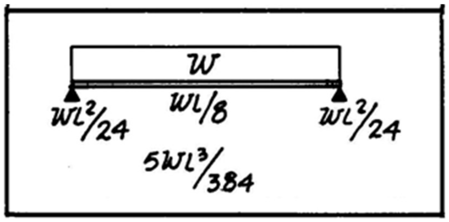
$$M = \frac{6\ EI\Theta}{2h + 3L}$$



2-Hinge Frame example

- Symmetric case solution
- Vertical reactions by symmetry
- Find moment at knee
- With FBD of one leg find H



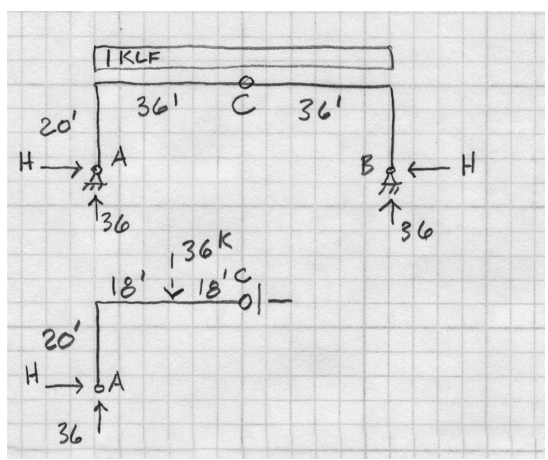


$$M = \frac{6\,EI\theta}{2h + 3L} = \frac{6\,\frac{72(72)^2}{24}}{2(20) + 3(72)} = 364.5^{K\text{-}1}$$

$$M = H(20), \quad H = M/20 = 364.5/20 = 18.2^{K}$$

3-Hinge Frame comparison

- Statically determinate
- Solve with statics
- FBD of half from hinge
- Solve for H
- Use FBD of leg to solve M



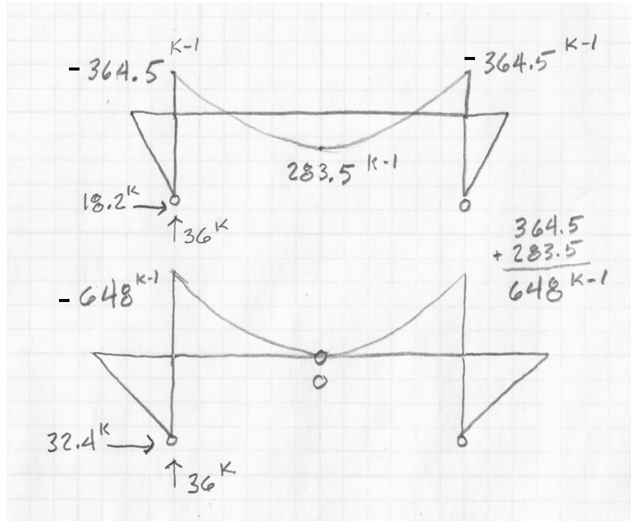
$$\Sigma M_C = 0 = -36(18) + 36(36) - H(20)$$

$$H = 32.4^{K}$$

$$M = H(20) = 32.4(20) = 648^{K\text{-}1}$$

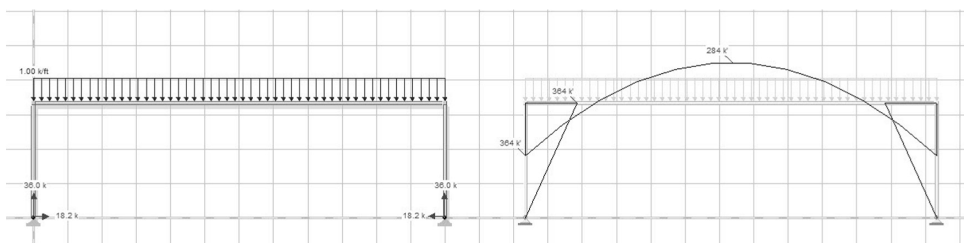
Comparison of moments

- 2-hinge frame
- 3-hinge frame



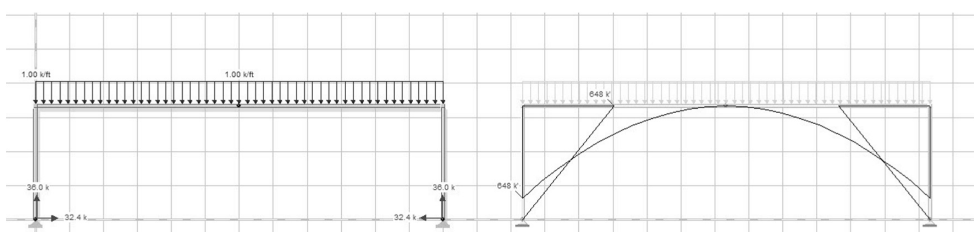
The effect of shape and hinges

Moment:
knee: -364 ft-lbs
center: +284 ft-lbs
horz. react. = 18.2 k



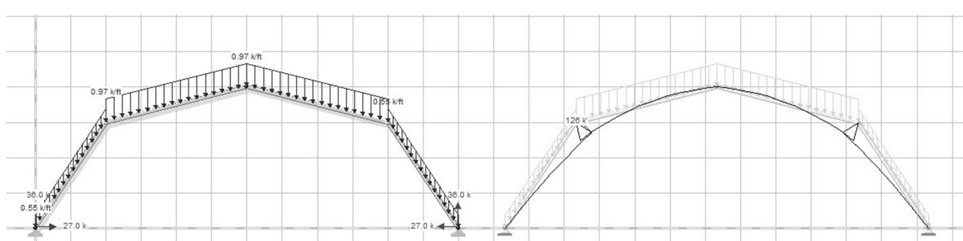
Continuous Beam

Moment:
knee: -648 ft-lbs
center: 0 ft-lbs
horz. react. = 32.4 k



Beam with center hinge

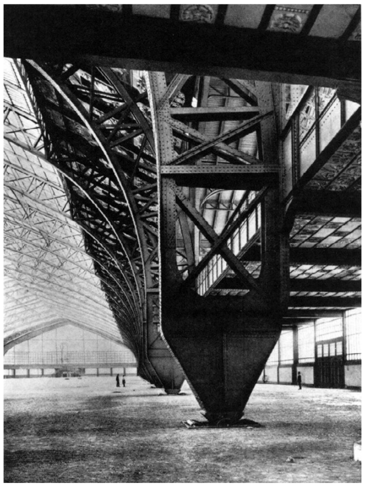
Moment:
knee: -126 ft-lbs
center: 0 ft-lbs
horz. react. = 27.0 k

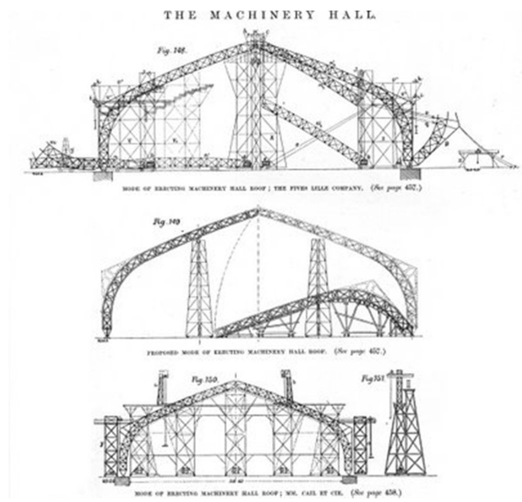


3 Hinged Arch

Characteristics of a 3-Hinged Arch

- Statically determinate – can be calculated with statics
- Movement or settling of foundations will not alter member stresses
- Small fabrication errors in length do not affect internal stresses
- Hinge placement can reduce internal stresses

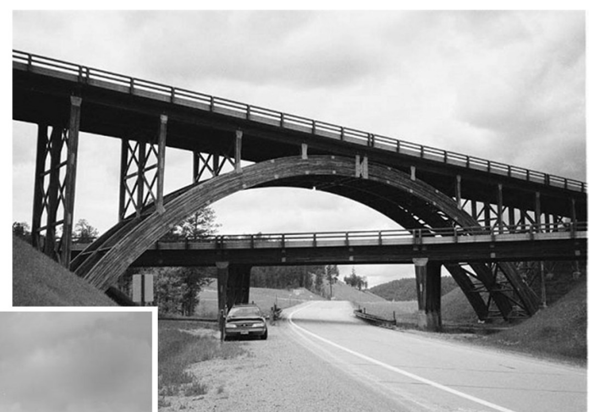


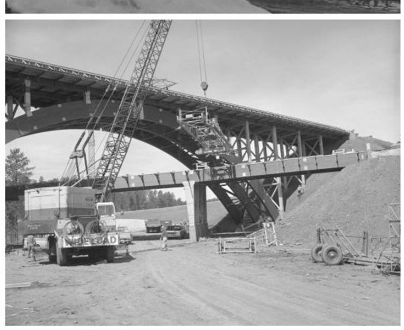


Gallery of the Machines, 1889 Paris
Architect: Ferdinand Dutert
Engineer: Victor Contamin

Keystone Wye Bridge

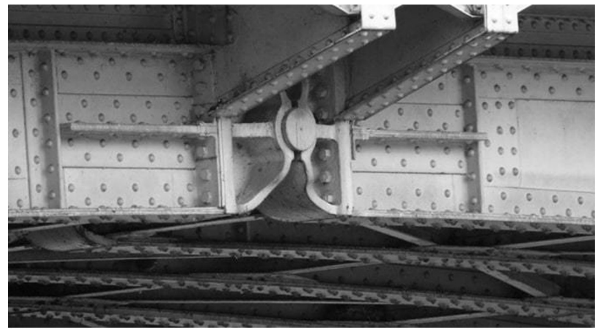
Glulam arches, spanning 160 ft.
built 1967-68 in South Dakota



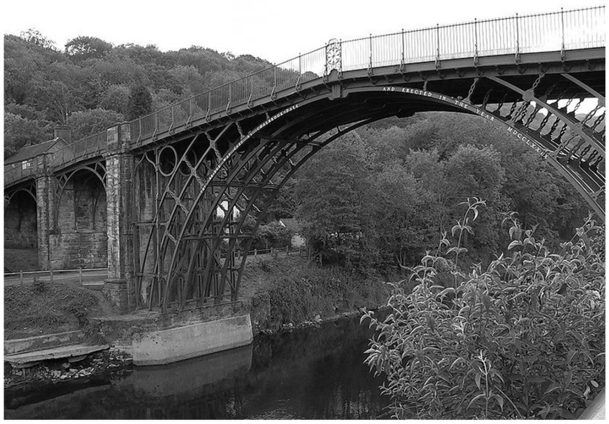


Examples and Details

Sydney Harbour Bridge



Center Hinge



The Iron Bridge
Telford England

Hinged Glulam Timbers