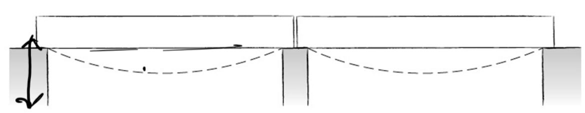
Architecture 324
Structures II

Gerber Beams

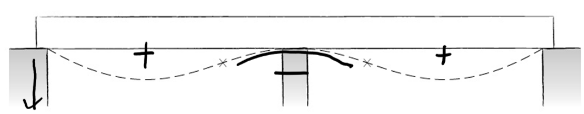
- Continuity in Beams
- Gerber Beams
- Optimization

Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports



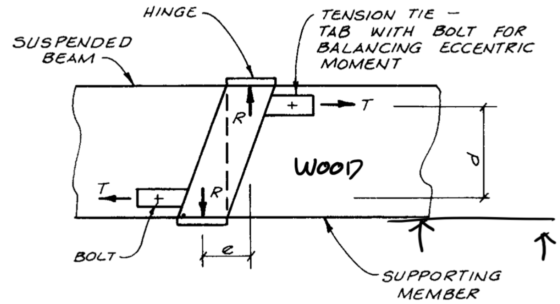
two spans - simply supported

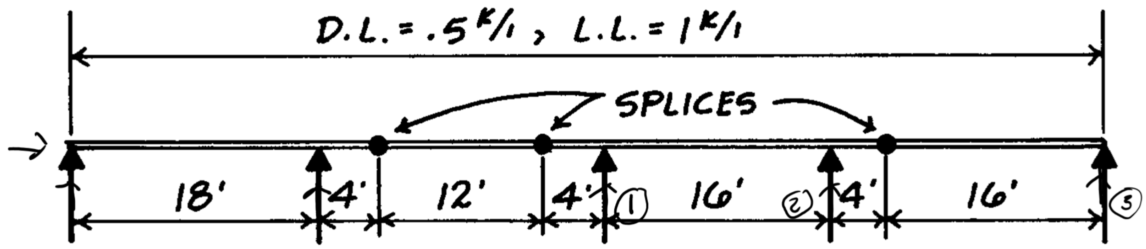


two spans - continuous

Splice or Hinge

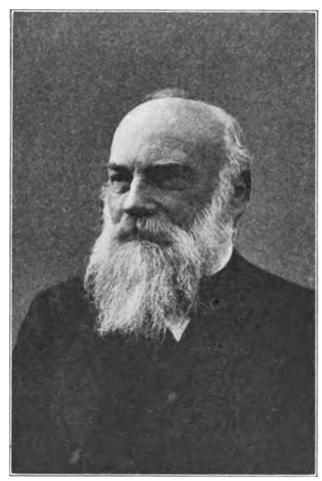
- Can add one hinge for each redundant reaction
- Reduces length for transport
- Moment = 0 at hinge
- Can be used to balance – and + moments for optimization



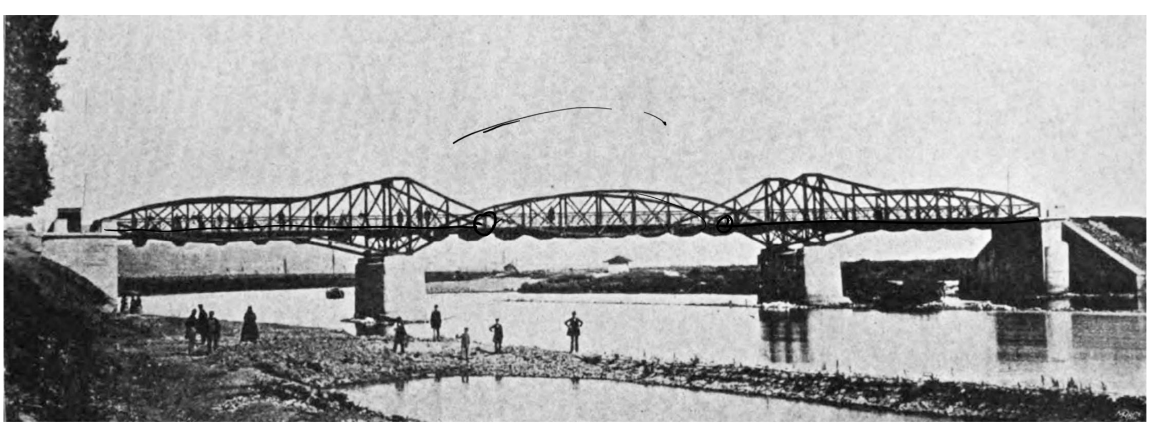


Gottfried Heinrich Gerber
(1832-1912)

Developed a cantilever bridge spanning system used in many bridges worldwide. The system became know as the “Gerber Beam” and uses cantilever segments to support a simple span.

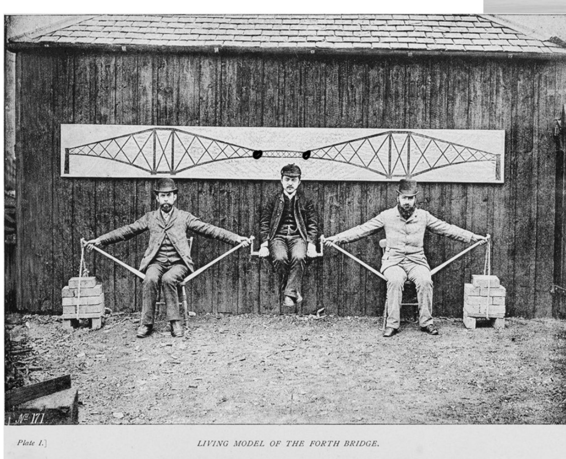


Haßfurter Brücke, 1864. Span of 38 m over the Main River.



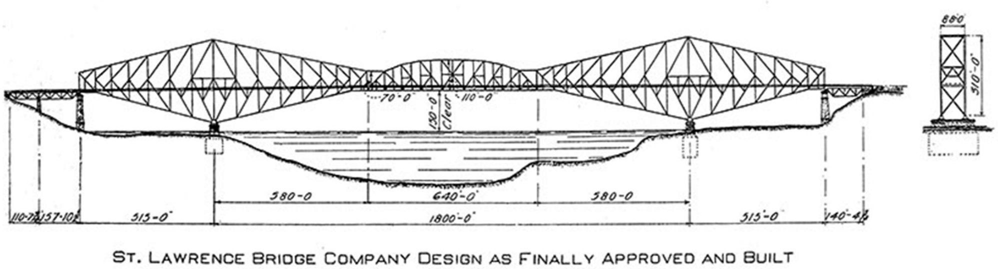
Examples of the Gerber system

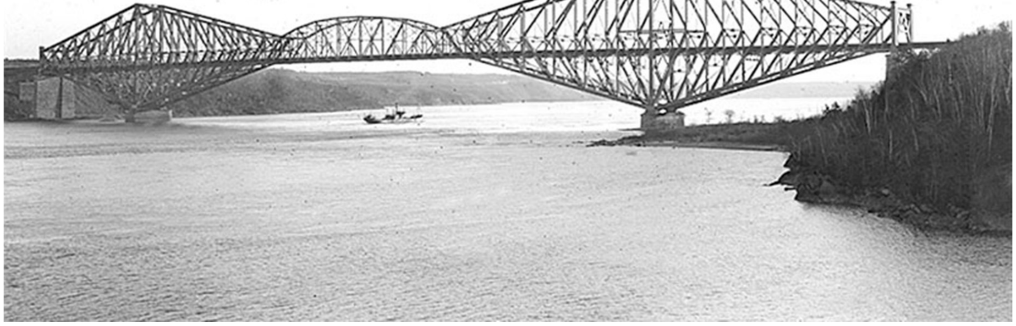
Firth of Fourth Bridge, 1890
- total length 8094 ft.
- central span 1700 ft.
- Design Fowler & Baker
- Construction 1882 - 1889



Static modeling of the Firth of Forth Bridge
by Fowler & Baker

Quebec Bridge Final Completion 1917

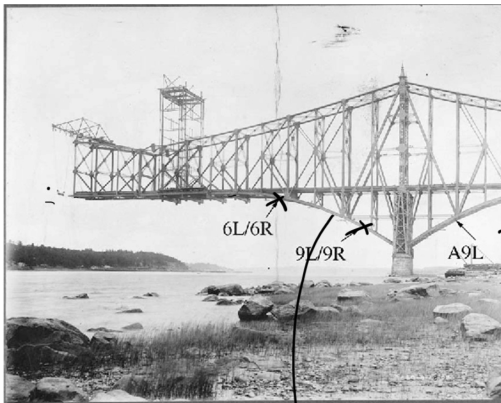


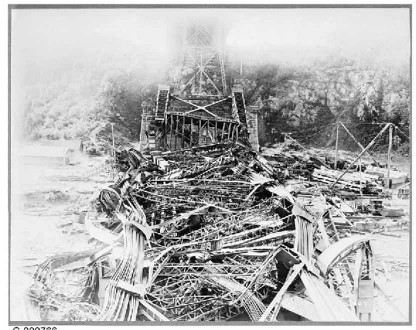


Final successful completion 1917

Quebec Bridge failure – 1907 and 1916

Compression members that failed in 1907





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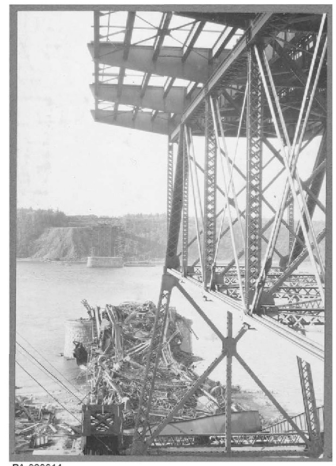
1916 hoisting failure



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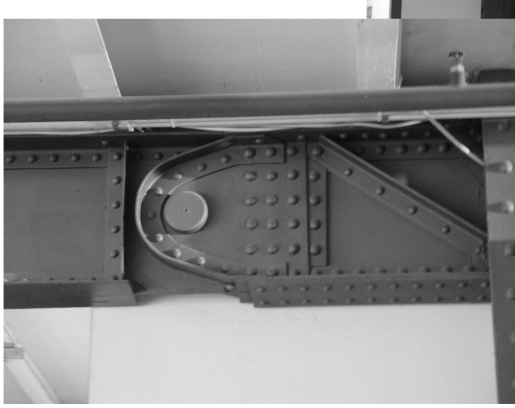
1907 failure due to miscalculation of the steel strength and dead load.

Gerber system in building frames

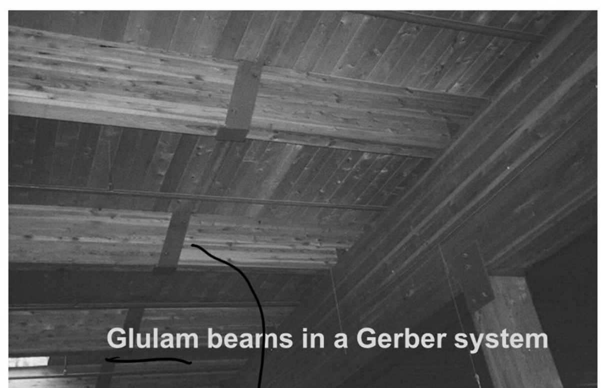


Speicherstadt Hamburg Kaffeerösterei 1888

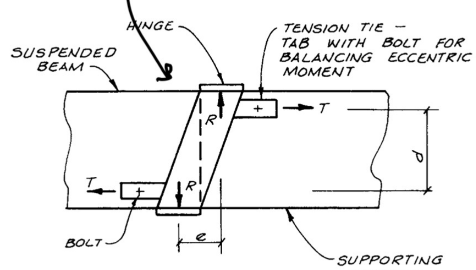
Gerber Beams in Detroit

Example Gerber Beams

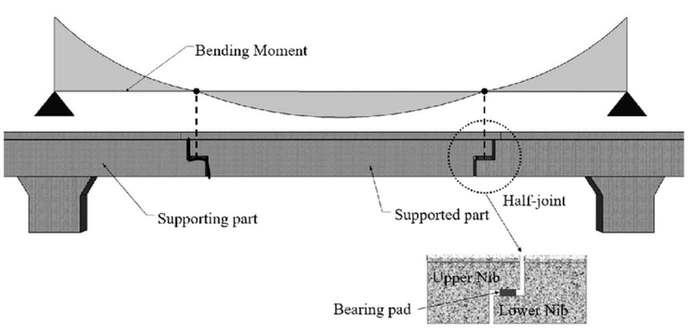
Steel



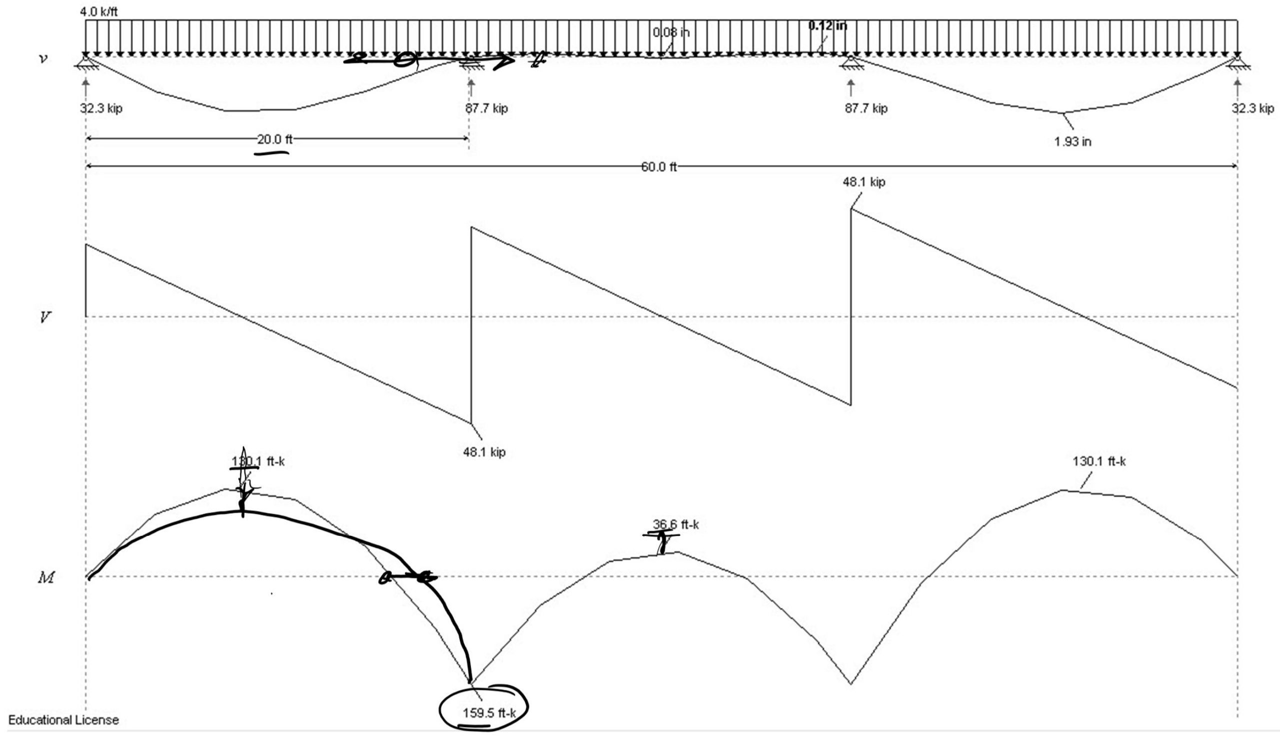
Wood



Concrete

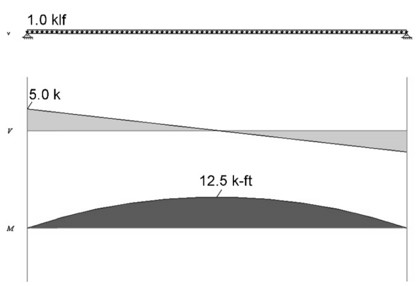


Moment control in beams

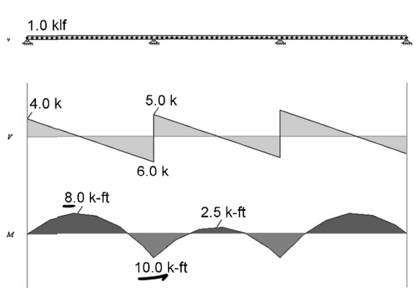


Moment control in beams

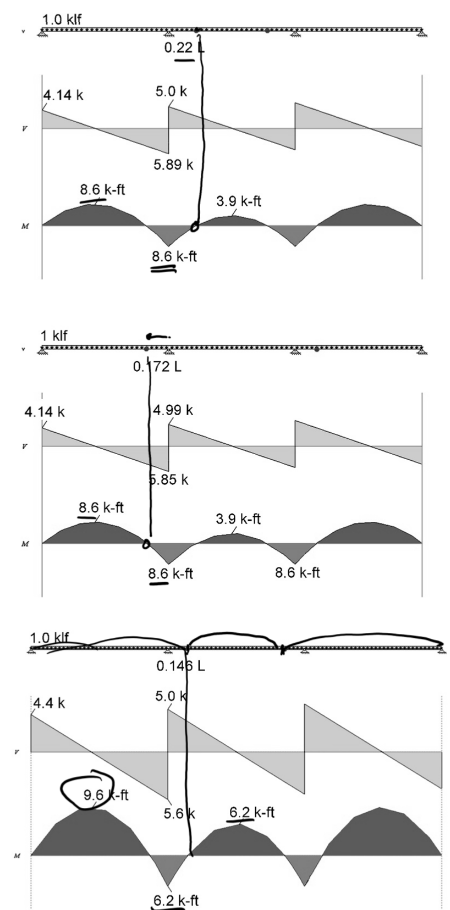
Spans = 10 ft



simple span



three spans – without hinges



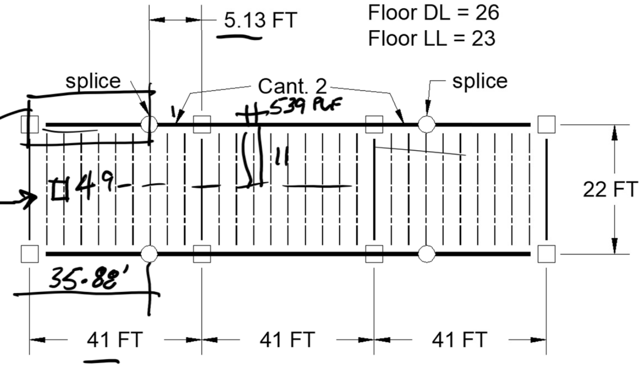
three spans – with hinges

Example Problem

Given:

Span and loading

- D + L = 49 psf
- 49 psf x 11 ft = 539 plf



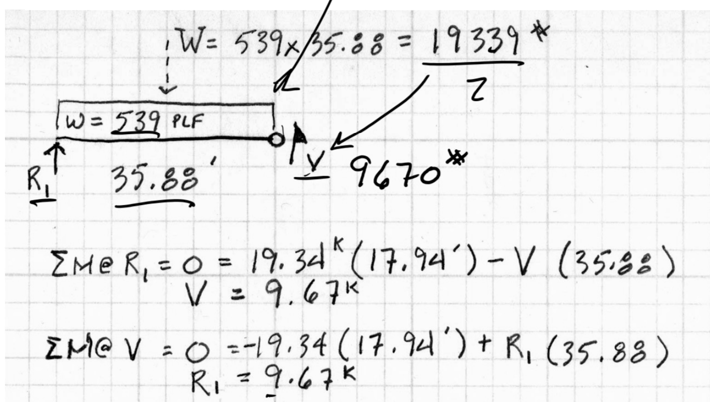
Find:

shear and moment

beam section

FBD 1

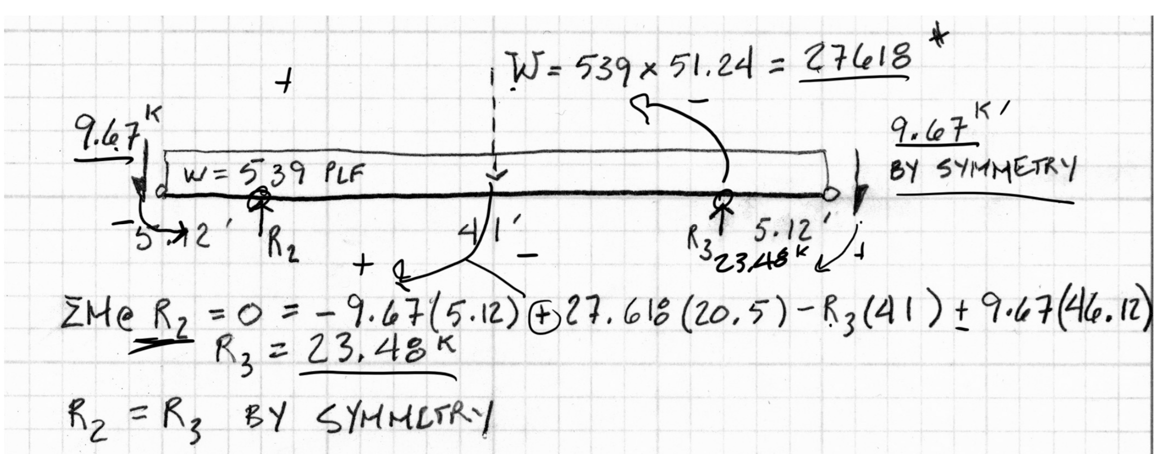
Reactions



Example Problem cont.

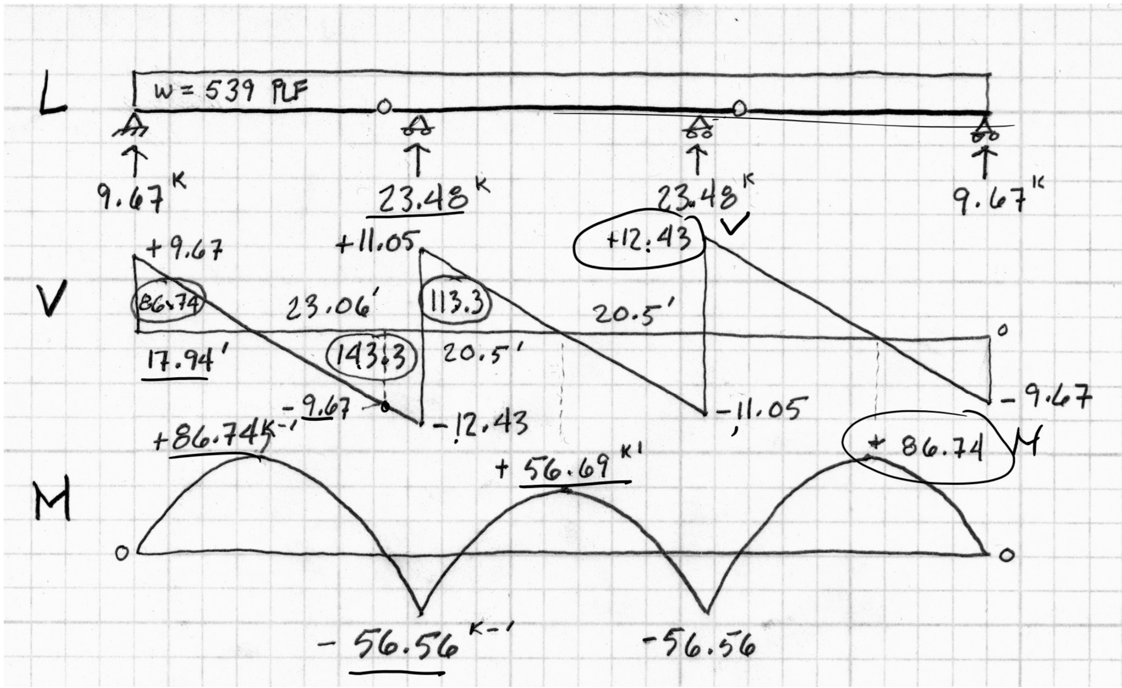
FBD 2

Reactions



Example Problem cont.

Force Diagrams



Example Problem cont.

STEEL BEAM DESIGN

$M_U = 86.74$ K-FT $V_U = 12.43$ K

$M_U = \phi M_n$

$M_n = \frac{M_U}{\phi} = \frac{86.74}{0.9} = 96.38$ K-FT

$M_n = F_y Z_x$

$Z_x = \frac{M_n}{F_y} = \frac{96.38 (12)}{50 \text{ KSI}} = 23.13$ in³

CHECK SHEAR

$h/t_w = 46.2 < 59.$ ✓

$A_w = t_w d = 0.235 (12.5) = 2.87$ in² ✓

$\phi V_n = (1.0)\, 0.6 F_y A_w = 0.6(50)(2.87)$

$\phi V_n = 86.01 > 12.43 = V_U$ ✓ ok

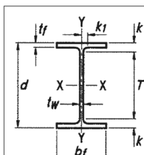


Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A (in.²)	Depth, d (in.)		Web Thickness, t_w (in.)		Web $\frac{t_w}{2}$ (in.)	Flange Width, b_f (in.)		Flange Thickness, t_f (in.)		Distance k, k_{des} (in.)	Distance k, k_{det} (in.)	k_1 (in.)	T (in.)	Workable Gage (in.)
W12×58	17.0	12.2	12¼	0.360	3/8	3/16	10.0	10	0.640	5/8	1.24	1½	15/16	9¼	5½
×53	15.6	12.1	12	0.345	3/8	3/16	10.0	10	0.575	9/16	1.18	1⅜	15/16	9¼	5½
W12×50	14.6	12.2	12¼	0.370	3/8	3/16	8.08	8⅛	0.640	5/8	1.14	1½	15/16	9¼	5½
×45	13.1	12.1	12	0.335	5/16	3/16	8.05	8	0.575	9/16	1.08	1⅜	15/16	↓	↓
×40	11.7	11.9	12	0.295	5/16	3/16	8.01	8	0.515	1/2	1.02	1⅜	7/8	↓	↓
W12×35[c]	10.3	12.5	12½	0.300	5/16	3/16	6.56	6½	0.520	1/2	0.820	1 3/16	3/4	10⅛	3½
×30[c]	8.79	12.3	12⅜	0.260	1/4	1/8	6.52	6½	0.440	7/16	0.740	1⅛	3/4	↓	↓
×26[c]	7.65	12.2	12¼	0.230	1/4	1/8	6.49	6½	0.380	3/8	0.680	1 1/16	3/4	↓	↓
W12×22[c]	6.48	12.3	12¼	0.260	1/4	1/8	4.03	4	0.425	7/16	0.725	15/16	5/8	10⅜	2¼[g]
×19[c]	5.57	12.2	12¼	0.235	1/4	1/8	4.01	4	0.350	3/8	0.650	7/8	9/16	↓	↓
×16[c]	4.71	12.0	12	0.220	1/4	1/8	3.99	4	0.265	1/4	0.565	13/16	9/16	↓	↓
×14[c,v]	4.16	11.9	11⅞	0.200	3/16	1/8	3.97	4	0.225	1/4	0.525	3/4	9/16	↓	↓

Table 1-1 (continued)
W-Shapes
Properties

W12-W10

Nominal Wt. (lb/ft)	Compact Section Criteria $\frac{b_f}{2t_f}$	Compact Section Criteria $\frac{h}{t_w}$	Axis X-X I (in.⁴)	Axis X-X S (in.³)	Axis X-X r (in.)	Axis X-X Z (in.³)	Axis Y-Y I (in.⁴)	Axis Y-Y S (in.³)	Axis Y-Y r (in.)	Axis Y-Y Z (in.³)	r_{ts} (in.)	h_o (in.)	Torsional Properties J (in.⁴)	Torsional Properties C_w (in.⁶)
58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.81	11.6	2.10	3570
53	8.69	28.1	425	70.6	5.23	77.9	95.8	19.2	2.48	29.1	2.79	11.5	1.58	3160
50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1880
45	7.00	29.6	348	57.7	5.15	64.2	50.0	12.4	1.95	19.0	2.23	11.5	1.26	1650
40	7.77	33.6	307	51.5	5.13	57.0	44.1	11.0	1.94	16.8	2.21	11.4	0.906	1440
35	6.31	36.2	285	45.6	5.25	51.2	24.5	7.47	1.54	11.5	1.79	12.0	0.741	879
30	7.41	41.8	238	38.6	5.21	43.1	20.3	6.24	1.52	9.56	1.77	11.9	0.457	720
26	8.54	47.2	204	33.4	5.17	37.2	17.3	5.34	1.51	8.17	1.75	11.8	0.300	607
22	4.74	41.8	156	25.4	4.91	29.3	4.66	2.31	0.848	3.66	1.04	11.9	0.293	164
19	5.72	46.2	130	21.3	4.82	24.7	3.76	1.88	0.822	2.98	1.02	11.9	0.180	131
16	7.53	49.4	103	17.1	4.67	20.1	2.82	1.41	0.773	2.26	0.983	11.7	0.103	96.9
14	8.82	54.3	88.6	14.9	4.62	17.4	2.36	1.19	0.753	1.90	0.961	11.7	0.0704	80.4

W12x19 →

Example Problem cont.

LOOK UP SECTION IN Z_x TABLE

CHOOSE W12×19

$Z_x = 24.7 > 23.13$ ✓

$\phi M_n = 92.6 > 86.74$ ✓

Table 3-2 (continued)

W-Shapes

Selection by Z_x

$F_y = 50$ ksi

Z_x

Shape	Z_x	M_{px}/Ω_b	$\phi_b M_{px}$	M_{rx}/Ω_b	$\phi_b M_{rx}$	BF/Ω_b	$\phi_b BF$	L_p	L_r	I_x	V_{nx}/Ω_v	$\phi_v V_{nx}$
		kip-ft	kip-ft	kip-ft	kip-ft	kips	kips				kips	kips
	in.^{3}	ASD	LRFD	ASD	LRFD	ASD	LRFD	ft	ft	in.^{4}	ASD	LRFD
W14×26	40.2	100	151	61.7	92.7	5.33	8.11	3.81	11.0	245	70.9	106
W8×40	39.8	99.3	149	62.0	93.2	1.64	2.46	7.21	29.9	146	59.4	89.1
W10×33	38.8	96.8	146	61.1	91.9	2.39	3.62	6.85	21.8	171	56.4	84.7
W12×26	37.2	92.8	140	58.3	87.7	3.61	5.46	5.33	14.9	204	56.1	84.2
W10×30	36.6	91.3	137	56.6	85.1	3.08	4.61	4.84	16.1	170	63.0	94.5
W8×35	34.7	86.6	130	54.5	81.9	1.62	2.43	7.17	27.0	127	50.3	75.5
W14×22	33.2	82.8	125	50.6	76.1	4.78	7.27	3.67	10.4	199	63.0	94.5
W10×26	31.3	78.1	117	48.7	73.2	2.91	4.34	4.80	14.9	144	53.6	80.3
W8×31[f]	30.4	75.8	114	48.0	72.2	1.58	2.37	7.18	24.8	110	45.6	68.4
W12×22	29.3	73.1	110	44.4	66.7	4.68	7.06	3.00	9.13	156	64.0	95.9
W8×28	27.2	67.9	102	42.4	63.8	1.67	2.50	5.72	21.0	98.0	45.9	68.9
W10×22	26.0	64.9	97.5	40.5	60.9	2.68	4.02	4.70	13.8	118	49.0	73.4
W12×19	24.7	61.6	92.6	37.2	55.9	4.27	6.43	2.90	8.61	130	57.3	86.0
W8×24	23.1	57.6	86.6	36.5	54.9	1.60	2.40	5.69	18.9	82.7	38.9	58.3
W10×19	21.6	53.9	81.0	32.8	49.4	3.18	4.76	3.09	9.73	96.3	51.0	76.5
W8×21	20.4	50.9	76.5	31.8	47.8	1.85	2.77	4.45	14.8	75.3	41.4	62.1
W12×16	20.1	50.1	75.4	29.9	44.9	3.80	5.73	2.73	8.05	103	52.8	79.2
W10×17	18.7	46.7	70.1	28.3	42.5	2.98	4.47	2.98	9.16	81.9	48.5	72.7
W12×14[v]	17.4	43.4	65.3	26.0	39.1	3.43	5.17	2.66	7.73	88.6	42.8	64.3
W8×18	17.0	42.4	63.8	26.5	39.9	1.74	2.61	4.34	13.5	61.9	37.4	56.2
W10×15	16.0	39.9	60.0	24.1	36.2	2.75	4.14	2.86	8.61	68.9	46.0	68.9
W8×15	13.6	33.9	51.0	20.6	31.0	1.90	2.85	3.09	10.1	48.0	39.7	59.6
W10×12[f]	12.6	31.2	46.9	19.0	28.6	2.36	3.53	2.87	8.05	53.8	37.5	56.3
W8×13	11.4	28.4	42.8	17.3	26.0	1.76	2.67	2.98	9.27	39.6	36.8	55.1
W8×10[f]	8.87	21.9	32.9	13.6	20.5	1.54	2.30	3.14	8.52	30.8	26.8	40.2

ASD	LRFD
$\Omega_b = 1.67$ $\Omega_v = 1.50$	$\phi_b = 0.90$ $\phi_v = 1.00$

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi; tabulated values have been adjusted accordingly.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with $F_y = 50$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

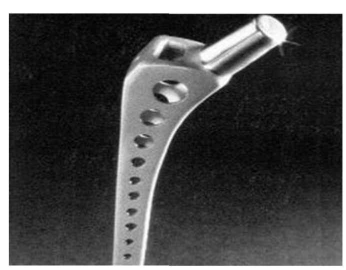
Structural Optimization

Optimization procedure: Find the “best” solution for a given problem.

- Describe the goal – objectives (single vs. multiple)
- Determine limitations – constraints
- Describe the parameters – variables

Optimization type: What to optimize

- Material
- Member (section)
- Geometry SHAPE
- Topology



Truss: 1 weight = 25484 lb
16 joints 35 members

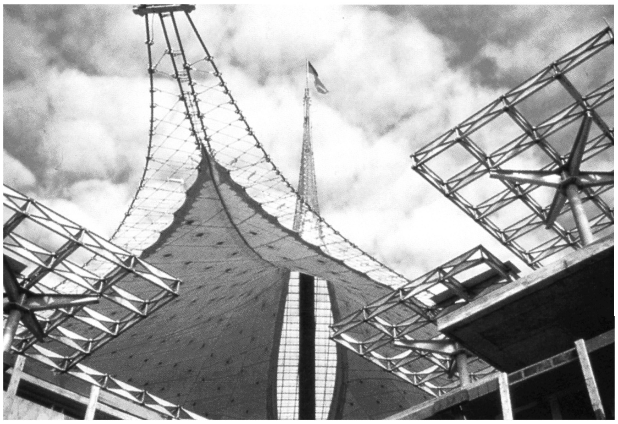
Truss: 2 weight = 25050 lb
16 joints 35 members

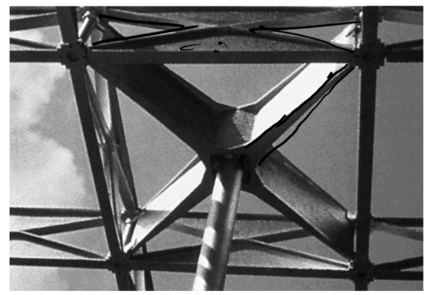
Truss: 3 weight = 24529 lb
16 joints 35 members

Optimization

- Material
  - Composites
  - Steel vs. Aluminum
- Member and Geometry
  - Variable Depth or Width
  - Holes and Cut-outs

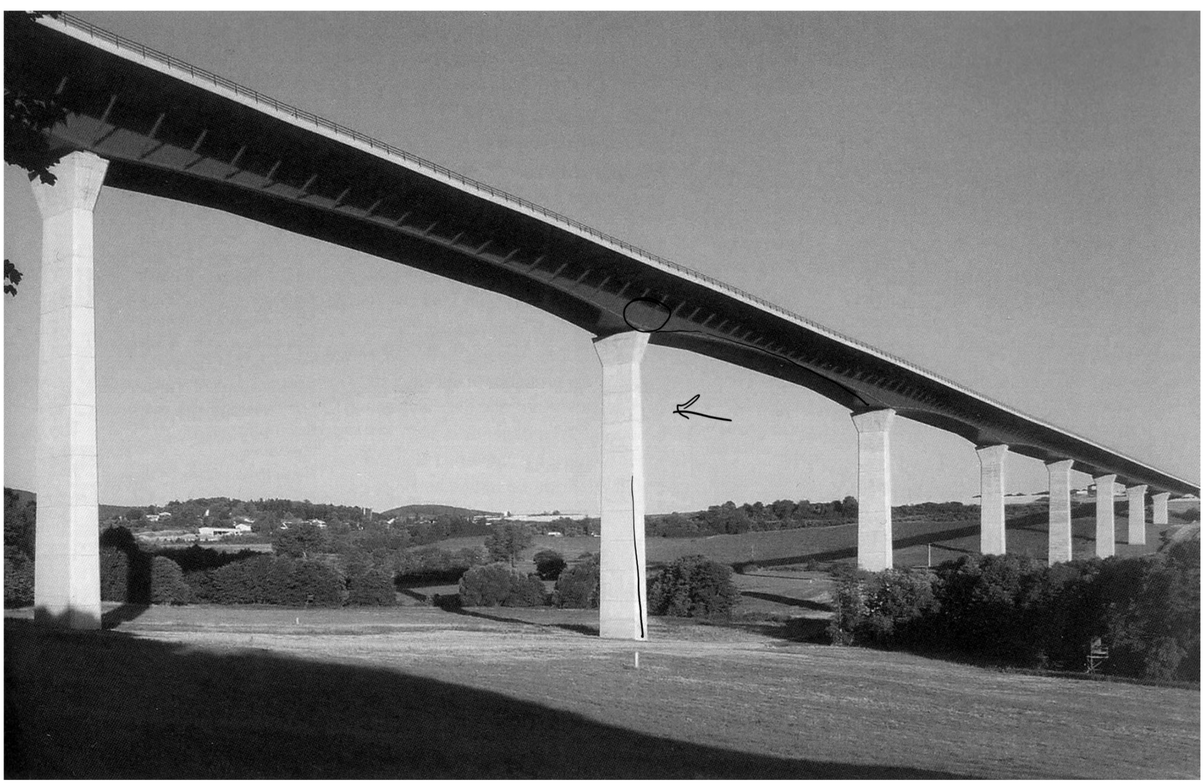
Biesenbach Viaduct, Blumberg Wutachtal Railroad, 1890
Eng. von Würthenau, Kräuter, Gebhard & Gernet





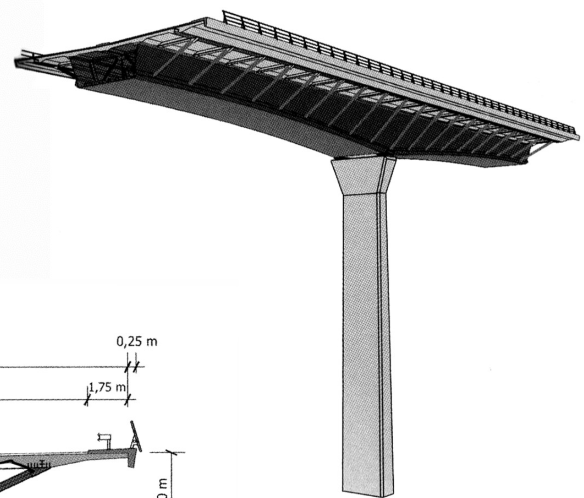
German Pavillion at Expo 1967, Montreal
Eng. Frei Otto Arch. Rolf Gutbrot

Section Optimization



Reichenbach Valley Bridge, 2003
Eng. Büro Peter + Lochner

Section Optimization



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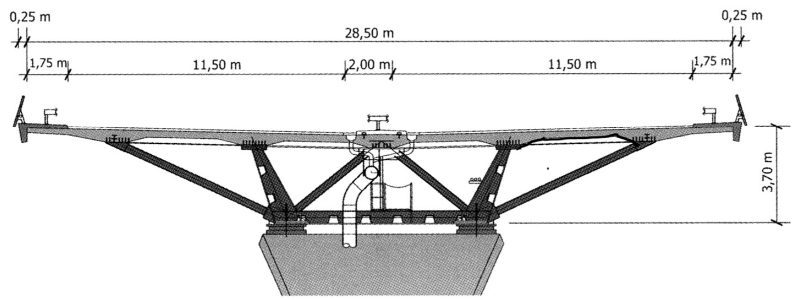
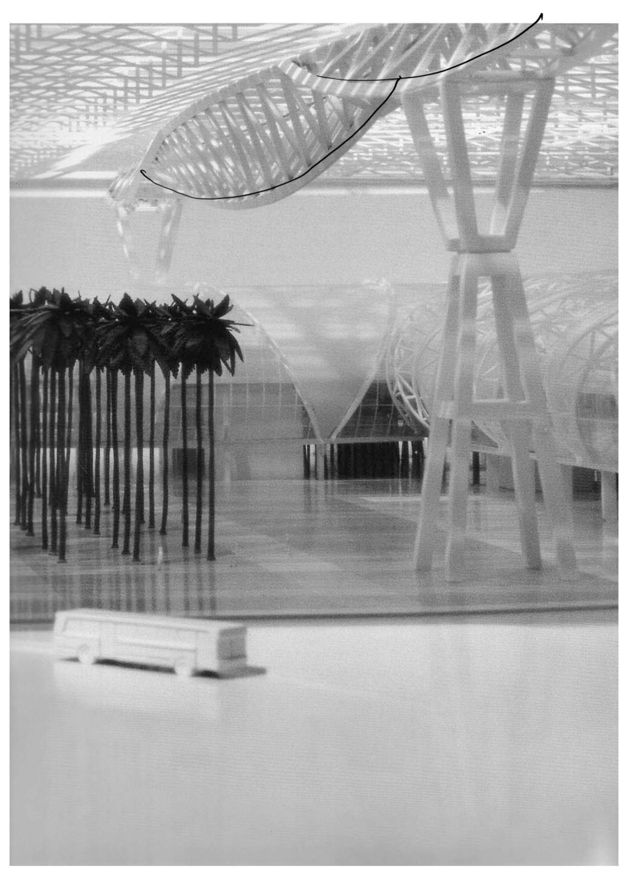
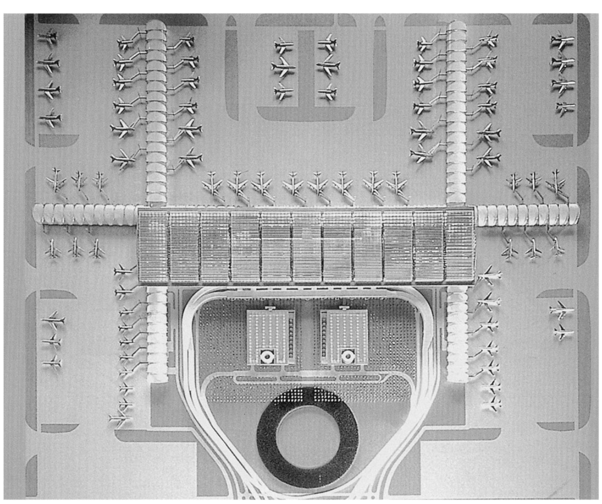


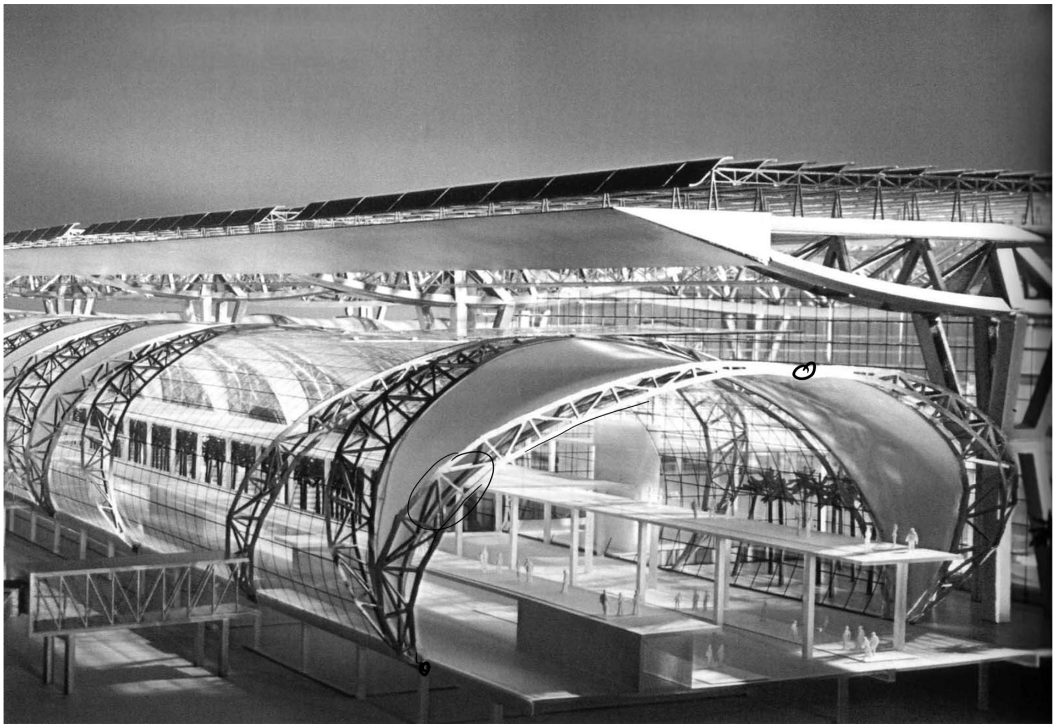
Geometry Optimization





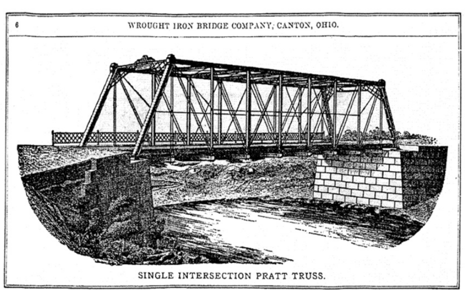
New Bangkok International Airport, 2003
Eng. Werner Sobek Arch. Murphy Jahn

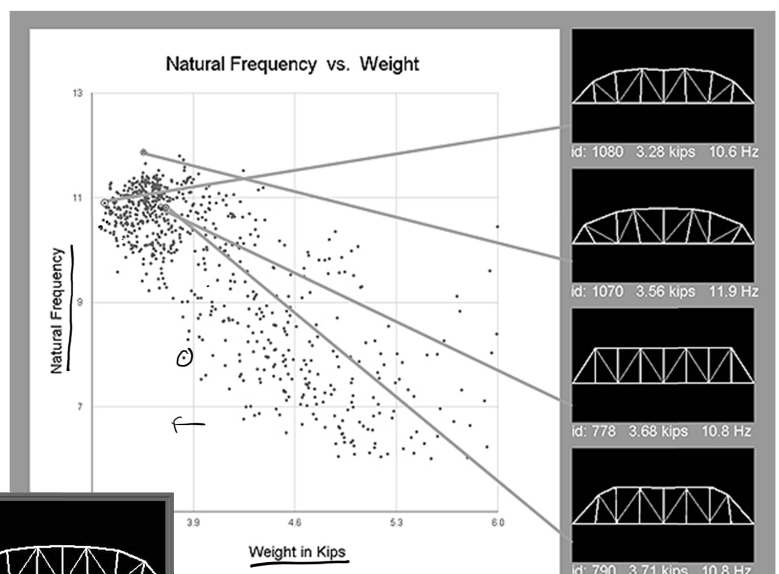
Geometry Optimization



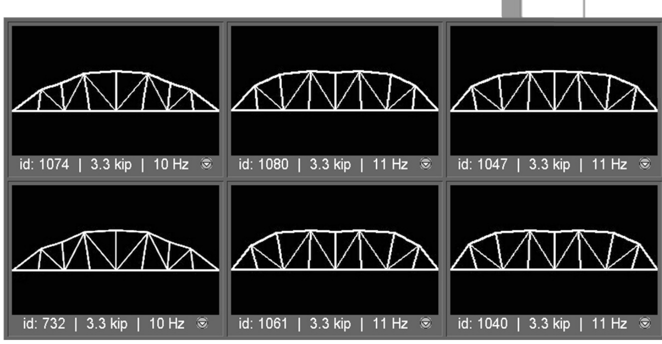
New Bangkok International Airport, 2003
Eng. Werner Sobek Arch. Murphy Jahn

Geometry Optimization - Bridges



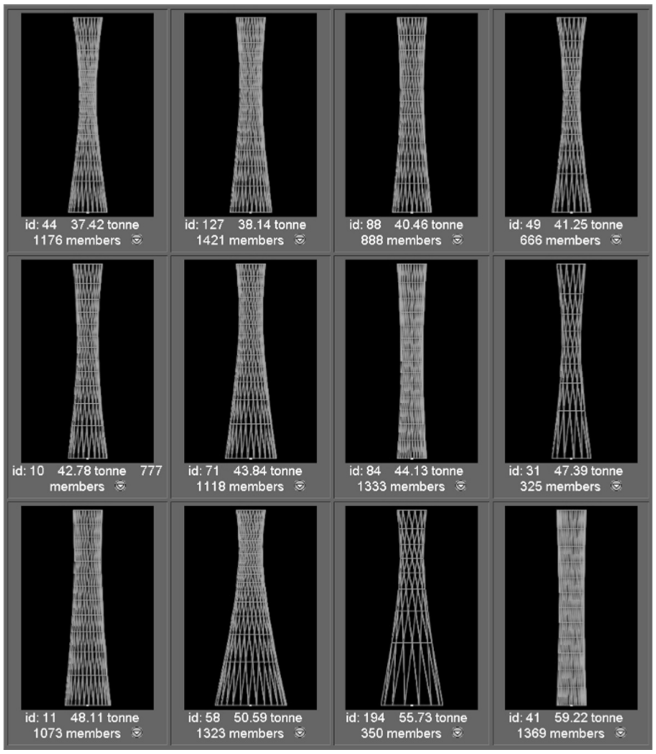


plot of weight vs natural frequency



lightest solutions

Topology Optimization - Shukhov towers



Nizhny Novgorod, 1896