

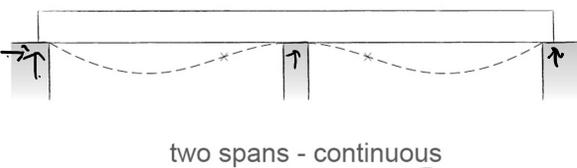
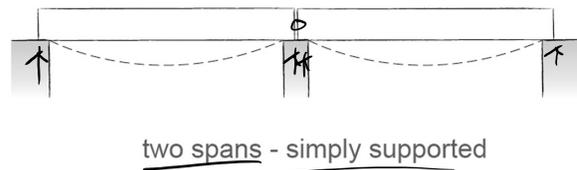
# Gerber Beams

- Continuity in Beams
- Gerber Beams
- Optimization



# Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are affected by movement or settlement of the supports

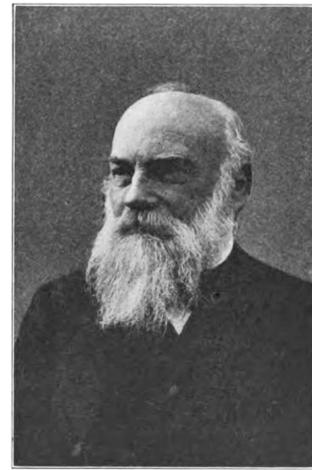


$\sum M = 0$   
 $\sum F_x = 0$   
 $\sum F_y = 0$

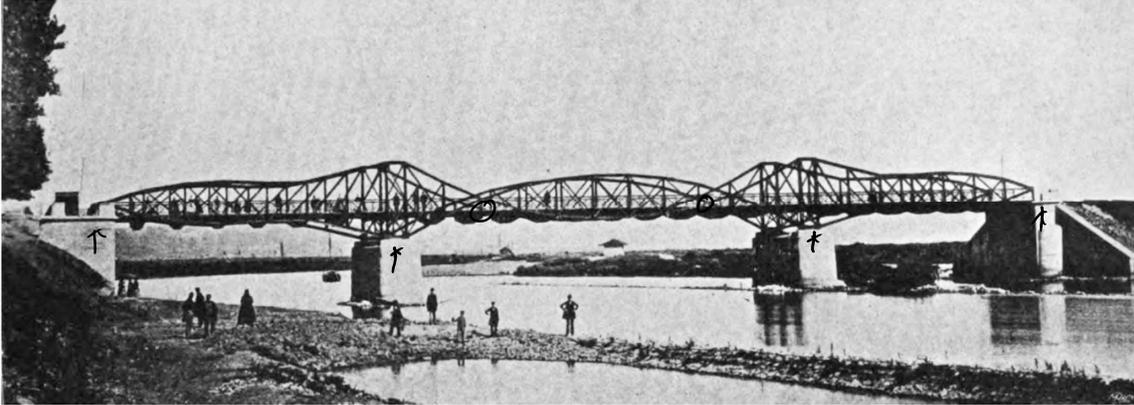
SOLVE w/ 3-MOMENT TH.

# Gottfried Heinrich Gerber (1832-1912)

Developed a cantilever bridge spanning system used in many bridges worldwide. The system became known as the "Gerber Beam" and uses cantilever segments to support a simple span.

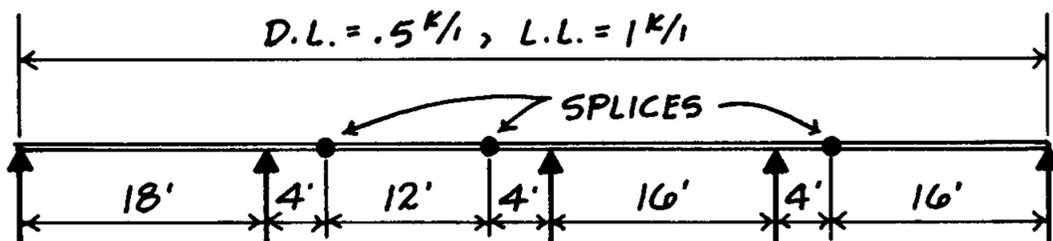
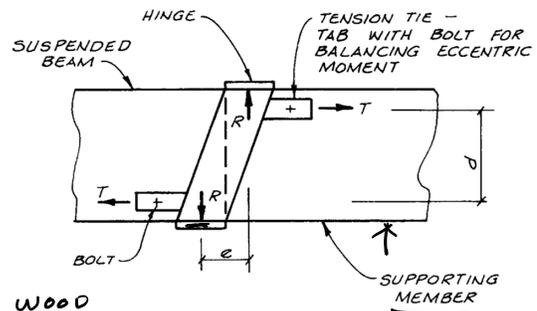


Haßfurter Brücke, 1864. Span of 38 m over the Main River.



## Splice or Hinge – makes Determinate

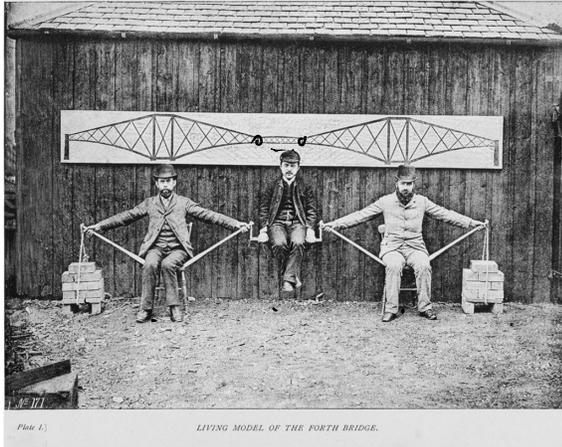
- Can add one hinge for each redundant reaction
- Reduces length for transport
- Moment = 0 at hinge
- Can be used to balance – and + moments for optimization



# Examples of the Gerber system

## Firth of Forth Bridge, 1890

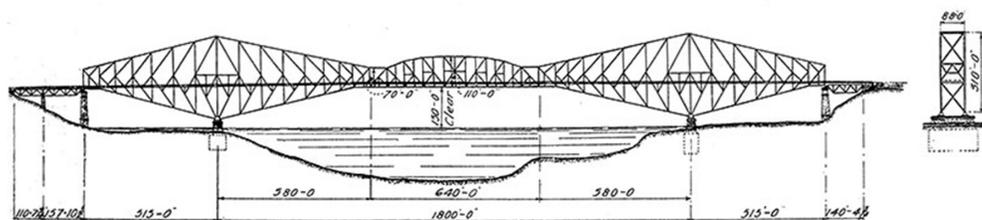
- total length 8094 ft.
- central span 1700 ft.
- Design Fowler & Baker
- Construction 1882 – 1889
- After Tay Bridge failure - 1879



Static modeling of the Firth of Forth Bridge by Fowler & Baker



## Quebec Bridge Final Completion 1917



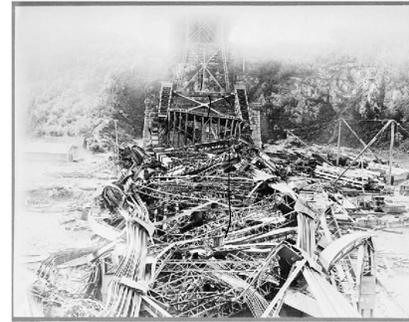
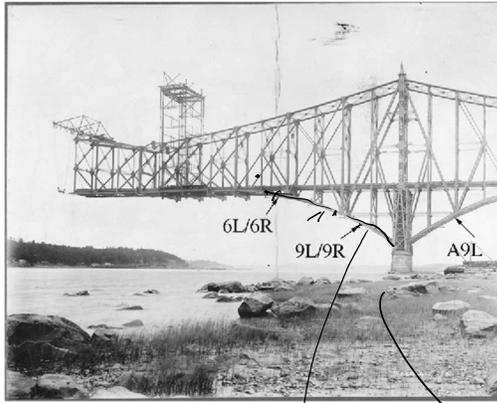
ST. LAWRENCE BRIDGE COMPANY DESIGN AS FINALLY APPROVED AND BUILT



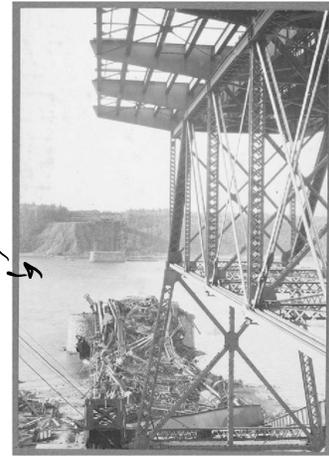
Final successful completion 1917

# Quebec Bridge failure – 1907 and 1916

Compression members that failed in 1907



1916 hoisting failure



1907 failure due to miscalculation of the steel strength and dead load.

# Gerber system in building frames



Speicherstadt Hamburg Kaffeerösterei  
1888

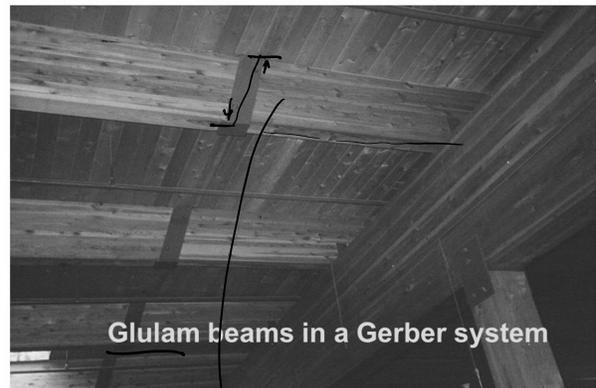
# Gerber Beams in Detroit



## Example Gerber Beams

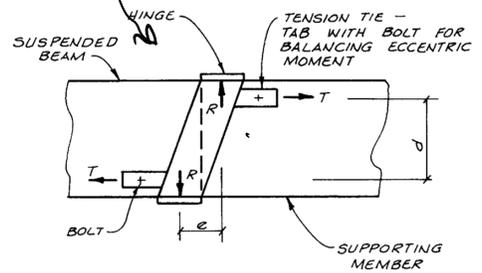
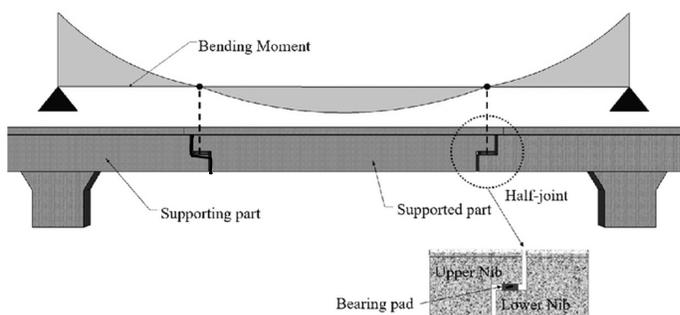


Steel

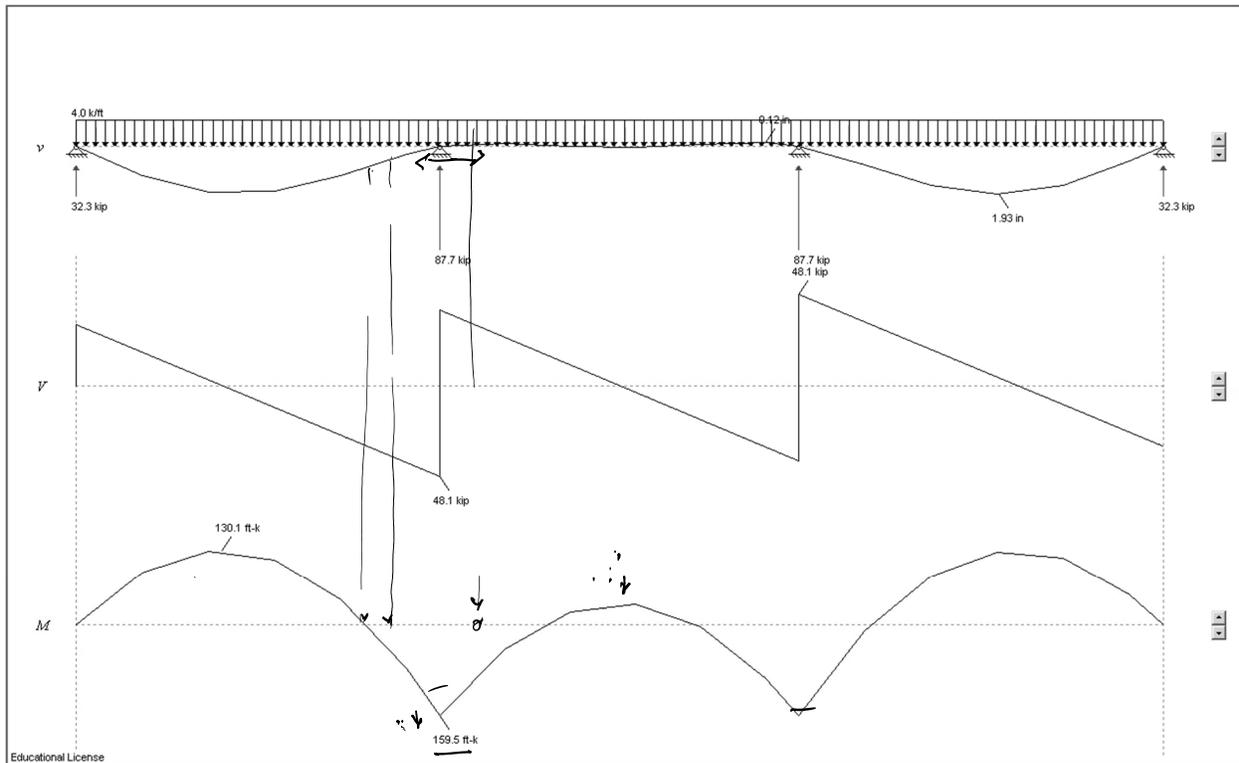


Wood

### Concrete

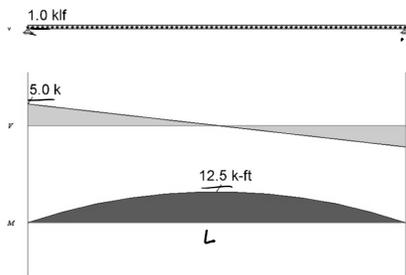


# Moment control in beams

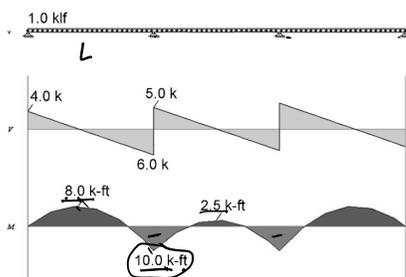


# Moment control in beams

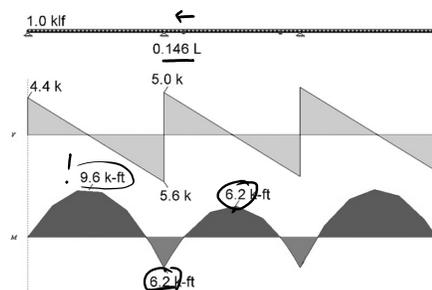
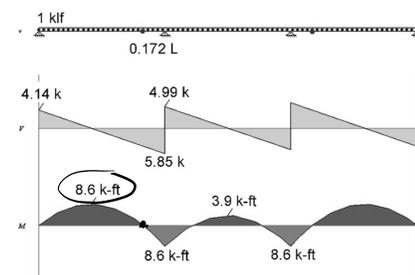
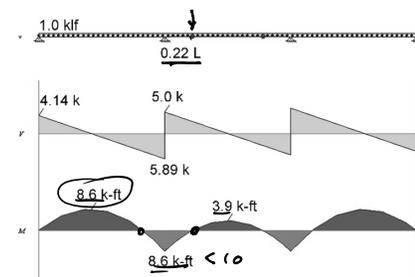
Spans = 10 ft



simple span



three spans – without hinges



three spans – with hinges

# Example Problem

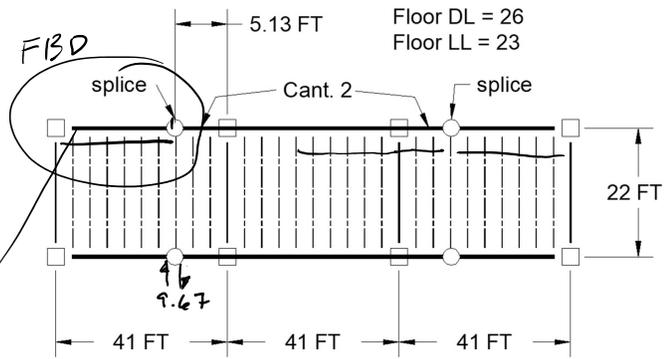
Given:

Span and loading

- D + L = 49 psf
- 49 psf x 11 ft = 539 plf

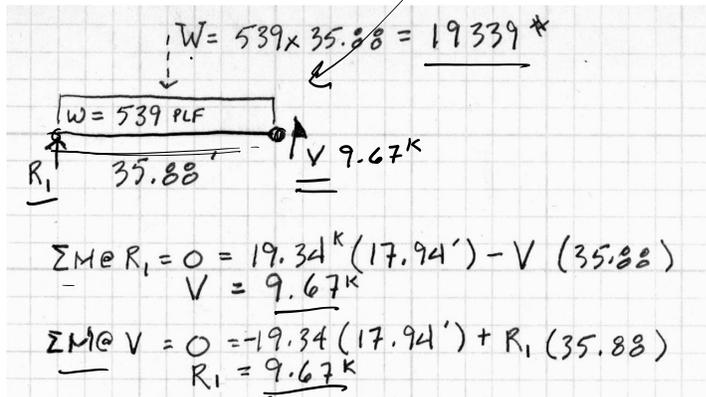
Find:

For each beam section  
find shear and moment



FBD 1

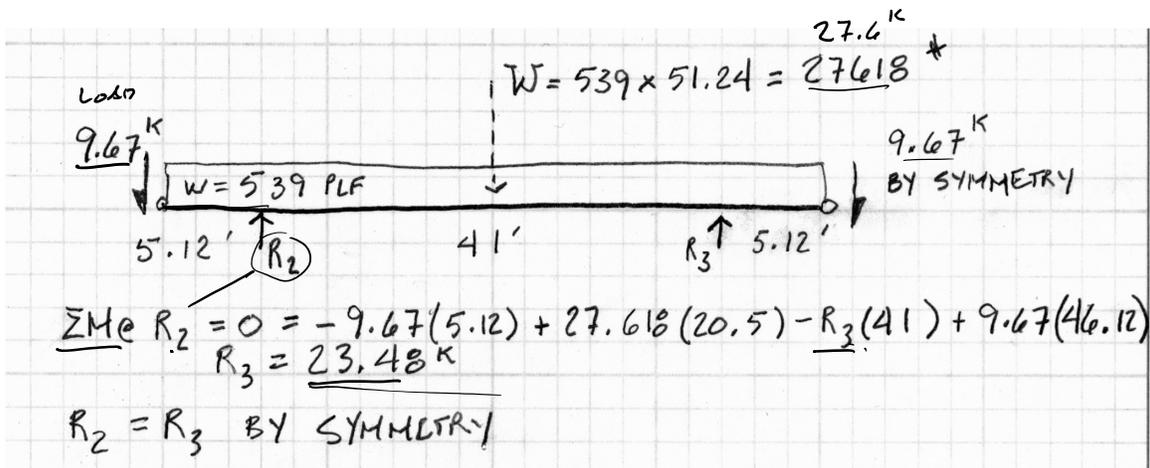
Reactions



# Example Problem cont.

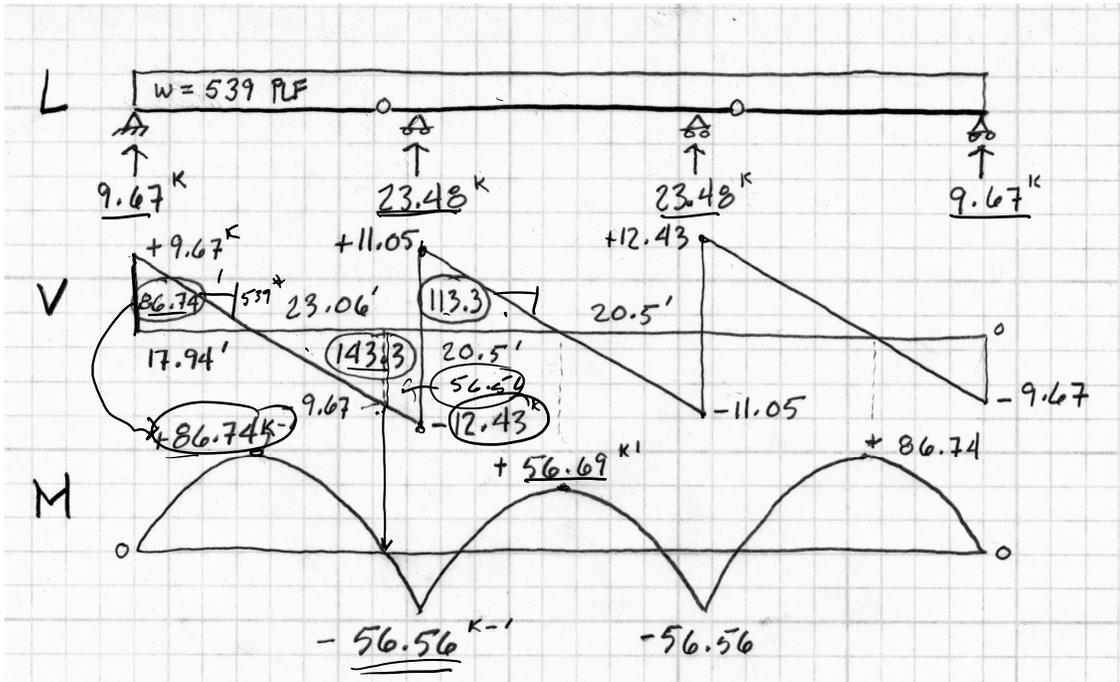
FBD 2

Reactions



# Example Problem cont.

## Force Diagrams



# Example Problem cont.

**STEEL BEAM DESIGN**

$M_u = 86.74 \text{ K-FT}$      $V_u = 12.43 \text{ K}$

$M_u = \phi M_n$

$M_n = \frac{M_u}{\phi} = \frac{86.74}{0.9} = 96.38 \text{ K-FT}$

$M_n = F_y Z_x$

$Z_x = \frac{M_n}{F_y} = \frac{96.38(12)}{50 \text{ ksi}} = 23.13 \text{ in}^3$

**W12x19**

**CHECK SHEAR**

$\frac{h}{t_w} = 46.2 < 59$  ✓

$A_w = t_w d = 0.235(12.5) = 2.94 \text{ in}^2$

$\phi V_n = (1.0) 0.6 F_y A_w = 0.6(50)(2.94)$

$\phi V_n = 86.01 > 12.43 = V_u$  ✓ OK

**Slend**

**Table 1-1 (continued)**  
**W-Shapes**  
**Dimensions**

Shape	Area, A in. <sup>2</sup>	Depth, d in.	Web		Flange		Distance				Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y		Torsional Properties										
			Thickness, t_w in.	t_w/2 in.	Width, b_f in.	Thickness, t_f in.	K <sub>des</sub>	K <sub>det</sub>	k <sub>1</sub>	T		Workable Gage	b <sub>f</sub> /2t <sub>f</sub>	h/t <sub>w</sub>	I	S	r	Z	I	S	r	Z	r <sub>s</sub>	h <sub>s</sub>	J	C <sub>w</sub>				
W12x58	17.0	12.2	12 1/4	3/8	3/8	10.0	10	0.640	5/8	1.24	1 1/2	15/16	9/4	5 1/2	58	7.82	27.0	475	78.0	5.28	86.4	107	21.4	2.51	32.5	2.81	11.5	2.10	3570	
x53	15.6	12.1	12	0.345	3/8	3/8	10.0	10	0.575	9/16	1.18	1 3/8	15/16	9/4	5 1/2	53	8.69	28.1	425	70.6	5.23	77.9	95.8	19.2	2.48	29.1	2.79	11.5	1.58	3160
W12x50	14.6	12.2	12 1/4	0.370	3/8	3/8	8.08	8 3/8	0.640	5/8	1.14	1 1/2	15/16	9/4	5 1/2	50	6.31	26.8	391	64.2	5.18	71.9	56.3	13.9	1.96	21.3	2.25	11.6	1.71	1880
x45	13.1	12.1	12	0.335	3/8	3/8	8.05	8	0.575	9/16	1.08	1 3/8	15/16	9/4	5 1/2	45	7.00	29.6	348	57.7	5.15	64.2	50.0	12.4	1.95	19.0	2.23	11.5	1.26	1650
x40	11.7	11.9	12	0.295	3/8	3/8	8.01	8	0.515	1/2	1.02	1 3/8	7/8	9/4	5 1/2	40	7.77	33.6	307	51.5	5.13	57.0	44.1	11.0	1.94	16.8	2.21	11.4	0.906	1440
W12x35	10.3	12.5	12 1/8	0.300	3/8	3/8	6.56	6 1/2	0.520	1/2	0.820	1 3/8	3/4	10 3/8	3 1/2	35	6.31	36.2	285	45.6	5.25	51.2	24.5	7.47	1.54	11.5	1.79	12.0	0.741	879
x30	8.79	12.3	12 3/8	0.260	1/4	1/4	6.52	6 1/2	0.440	7/16	0.740	1 1/8	3/4	9/4	3 1/2	30	7.41	41.8	238	38.6	5.21	43.1	20.3	6.24	1.52	9.56	1.77	11.9	0.457	720
x26	7.65	12.2	12 1/4	0.230	1/4	1/4	6.49	6 1/2	0.380	3/8	0.680	1 1/8	3/4	9/4	3 1/2	26	8.54	47.2	204	33.4	5.17	37.2	17.3	5.34	1.51	8.17	1.75	11.8	0.300	607
W12x22	6.48	12.3	12 1/4	0.260	1/4	1/4	4.03	4	0.425	7/16	0.725	1 5/8	3/4	10 3/8	2 1/4	22	4.74	41.8	156	25.4	4.91	29.3	4.66	2.31	0.848	3.66	1.04	11.9	0.293	164
x19	5.57	12.2	12 1/4	0.235	1/4	1/4	4.01	4	0.350	3/8	0.650	7/8	9/16	9/4	2 1/2	19	5.72	46.2	130	21.3	4.82	24.7	3.76	1.88	0.822	2.98	1.02	11.9	0.180	131
x16	4.71	12.0	12	0.220	1/4	1/4	3.99	4	0.265	1/4	0.655	1 3/8	3/4	9/4	2 1/2	16	7.53	49.4	103	17.1	4.67	20.1	2.82	1.41	0.773	2.26	0.983	11.7	0.103	96.9
x14	4.16	11.9	11 7/8	0.200	3/16	3/16	3.97	4	0.225	1/4	0.525	3/4	9/16	9/4	2 1/2	14	8.82	54.3	88.6	14.9	4.62	17.4	2.36	1.19	0.753	1.90	0.961	11.7	0.0704	80.4

**Table 1-1 (continued)**  
**W-Shapes**  
**Properties**

Shape	Area, A in. <sup>2</sup>	Depth, d in.	Web		Flange		Distance				Nominal Wt. lb/ft	Compact Section Criteria		Axis X-X				Axis Y-Y		Torsional Properties										
			Thickness, t_w in.	t_w/2 in.	Width, b_f in.	Thickness, t_f in.	K <sub>des</sub>	K <sub>det</sub>	k <sub>1</sub>	T		Workable Gage	b <sub>f</sub> /2t <sub>f</sub>	h/t <sub>w</sub>	I	S	r	Z	I	S	r	Z	r <sub>s</sub>	h <sub>s</sub>	J	C <sub>w</sub>				
W12x10	6.88	12.3	12 1/4	0.260	1/4	1/4	4.03	4	0.425	7/16	0.725	1 5/8	3/4	10 3/8	2 1/4	10	4.74	41.8	156	25.4	4.91	29.3	4.66	2.31	0.848	3.66	1.04	11.9	0.293	164
x10	5.57	12.2	12 1/4	0.235	1/4	1/4	4.01	4	0.350	3/8	0.650	7/8	9/16	9/4	2 1/2	19	5.72	46.2	130	21.3	4.82	24.7	3.76	1.88	0.822	2.98	1.02	11.9	0.180	131
x8	4.71	12.0	12	0.220	1/4	1/4	3.99	4	0.265	1/4	0.655	1 3/8	3/4	9/4	2 1/2	16	7.53	49.4	103	17.1	4.67	20.1	2.82	1.41	0.773	2.26	0.983	11.7	0.103	96.9
x6	4.16	11.9	11 7/8	0.200	3/16	3/16	3.97	4	0.225	1/4	0.525	3/4	9/16	9/4	2 1/2	14	8.82	54.3	88.6	14.9	4.62	17.4	2.36	1.19	0.753	1.90	0.961	11.7	0.0704	80.4

# Example Problem cont.

For  $Z_{x \text{ req.}} = 23.13 \text{ in}^3$

LOOK UP SECTION IN  $Z_x$  TABLE  
 CHOOSE W12x19 CALC.  
 $Z_x = 24.7 > 23.13$  ✓  
 $\phi M_n = 92.6 > 86.7A$  ✓

$F_y = 50 \text{ ksi}$

**Table 3-2 (continued)**  
**W-Shapes**  
**Selection by  $Z_x$**

$Z_x$

Shape	$Z_x$ in. <sup>3</sup>	$M_{px}/\Omega_b$		$\phi_b M_{px}$		$M_{rx}/\Omega_b$		$\phi_b M_{rx}$		$BF/\Omega_b$		$\phi_b BF$		$L_p$ ft	$L_r$ ft	$I_x$ in. <sup>4</sup>	$V_{nx}/\Omega_v$		$\phi_v V_{nx}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	kips	kips	ASD	LRFD				kips	kips		
W14x26	40.2	100	151	61.7	92.7	5.33	8.11	3.81	11.0	245	70.9	106								
W8x40	39.8	99.3	149	62.0	93.2	1.64	2.46	7.21	29.9	146	59.4	89.1								
W10x33	38.8	96.8	146	61.1	91.9	2.39	3.62	6.85	21.8	171	56.4	84.7								
W12x26	37.2	92.8	140	58.3	87.7	3.61	5.46	5.33	14.9	204	56.1	84.2								
W10x30	36.6	91.3	137	56.6	85.1	3.08	4.61	4.84	16.1	170	63.0	94.5								
W8x35	34.7	86.6	130	54.5	81.9	1.62	2.43	7.17	27.0	127	50.3	75.5								
W14x22	33.2	82.8	125	50.6	76.1	4.78	7.27	3.67	10.4	199	63.0	94.5								
W10x26	31.3	78.1	117	48.7	73.2	2.91	4.34	4.80	14.9	144	53.6	80.3								
W8x31	30.4	75.8	114	48.0	72.2	1.58	2.37	7.18	24.8	110	45.6	68.4								
W12x22	29.3	73.1	110	44.4	66.7	4.68	7.06	3.00	9.13	156	64.0	95.9								
W8x28	27.2	67.9	102	42.4	63.8	1.67	2.50	5.72	21.0	98.0	45.9	68.9								
W10x22	26.0	64.9	97.5	40.5	60.9	2.68	4.02	4.70	13.8	118	49.0	73.4								
<b>W12x19</b>	<b>24.7</b>	<b>61.6</b>	<b>92.6</b>	<b>37.2</b>	<b>55.9</b>	<b>4.27</b>	<b>6.43</b>	<b>2.90</b>	<b>8.61</b>	<b>130</b>	<b>57.3</b>	<b>86.0</b>								
W8x24	23.7	57.6	86.6	36.5	54.9	1.60	2.40	5.69	18.9	82.7	38.9	58.3								
W10x19	21.6	53.9	81.0	32.8	49.4	3.18	4.76	3.09	9.73	96.3	51.0	76.5								
W8x21	20.4	50.9	76.5	31.8	47.8	1.85	2.77	4.45	14.8	75.3	41.4	62.1								
W12x16	20.4	50.1	75.4	29.9	44.9	3.80	5.73	2.73	8.05	103	52.8	79.2								
W10x17	18.7	46.7	70.1	28.3	42.5	2.98	4.47	2.98	9.16	81.9	48.5	72.7								
W12x14	17.4	43.4	65.3	26.0	39.1	3.43	5.17	2.66	7.73	88.6	42.8	64.3								
W8x18	17.0	42.4	63.8	26.5	39.9	1.74	2.61	4.34	13.5	61.9	37.4	56.2								
W10x15	16.6	39.9	60.0	24.1	36.2	2.75	4.14	2.86	8.61	68.9	46.0	68.9								
W8x15	13.8	33.9	51.0	20.6	31.0	1.90	2.85	3.09	10.1	48.0	39.7	59.6								
W10x12	12.6	31.2	46.9	19.0	28.6	2.36	3.53	2.87	8.05	53.8	37.5	56.3								
W8x13	11.4	28.4	42.8	17.3	26.0	1.76	2.67	2.98	9.27	39.6	36.8	55.1								
W8x10	8.87	21.9	32.9	13.6	20.5	1.54	2.30	3.14	8.52	30.8	26.8	40.2								

**BOLD** (pointing to W12x19 row)

ASD LRFD <sup>1</sup>Shape exceeds compact limit for flexure with  $F_y = 50 \text{ ksi}$ ; tabulated values have been adjusted accordingly.  
<sup>2</sup>Shape does not meet the  $H/t_w$  limit for shear in AISC Specification Section G2.1(a) with  $F_y = 50 \text{ ksi}$ ; therefore,  $\phi_v = 0.90$  and  $\Omega_v = 1.67$ .

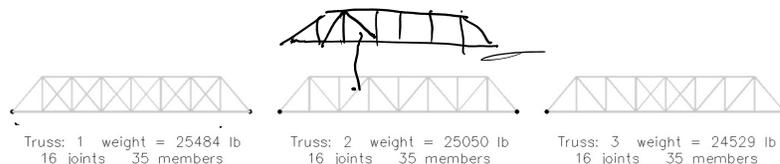
# Structural Optimization

Optimization procedure: Find the "best" solution for a given problem.

- Describe the goal – objectives (single vs. multiple)
- Determine limitations – constraints
- Describe the parameters – variables

Optimization type: What to optimize (objectives)

- Material ✓
- Member (section) ✓
- Geometry ✓
- Topology ✓



TOPOLOGY

## Optimization

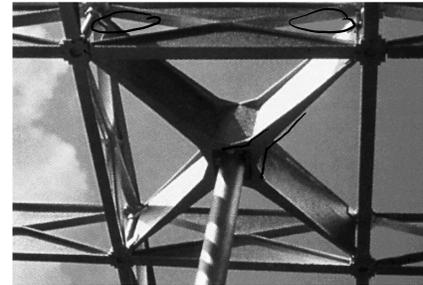
- Material
  - Composites
  - Steel vs. Aluminum
- Member and Geometry
  - Variable Depth or Width
  - Holes and Cut-outs



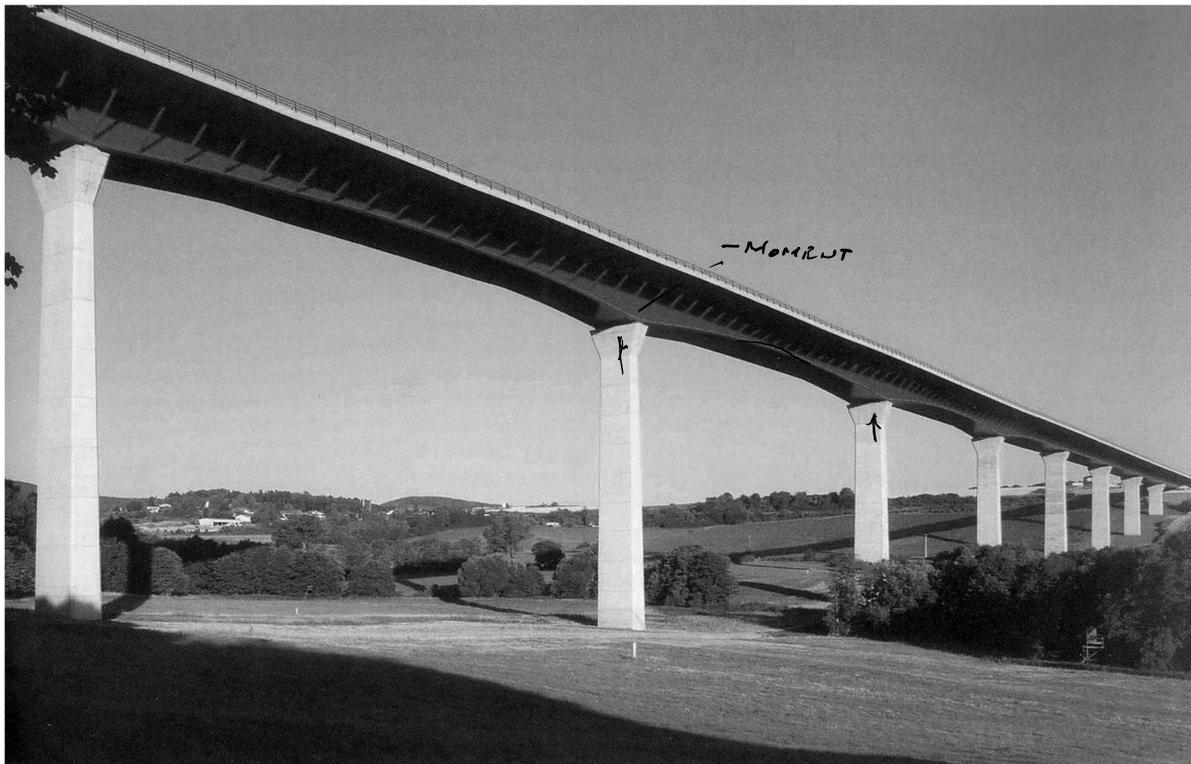
Biesenbach Viaduct, Blumberg Wutachtal Railroad, 1890  
Eng. von Würthenau, Kräuter, Gebhard & Gernet



German Pavilion at Expo 1967, Montreal  
Eng. Frei Otto Arch. Rolf Gutbrot



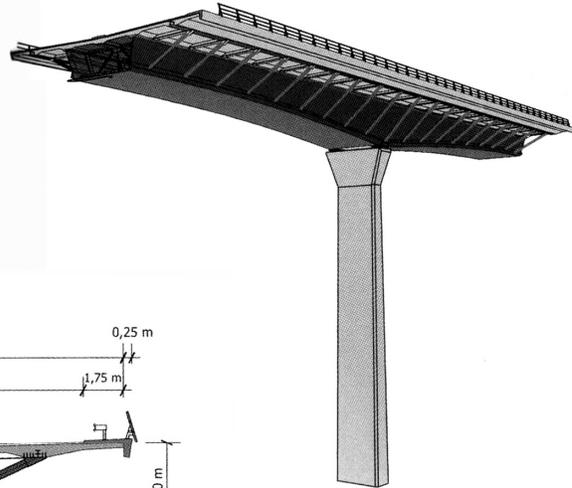
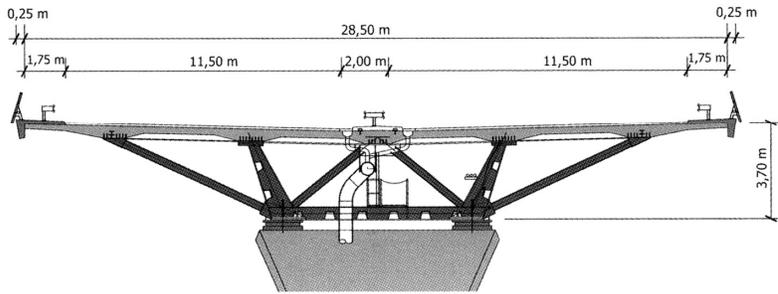
## Section Optimization



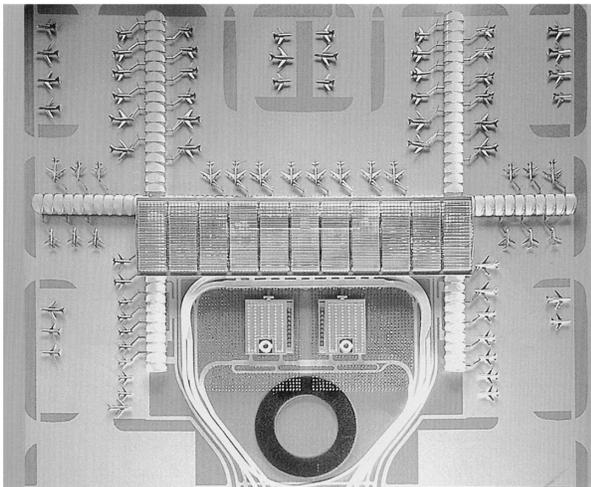
Reichenbach Valley Bridge, 2003  
Eng. Büro Peter + Lochner

# Section Optimization

Reichenbach Valley Bridge, 2003  
Eng. Büro Peter + Lochner



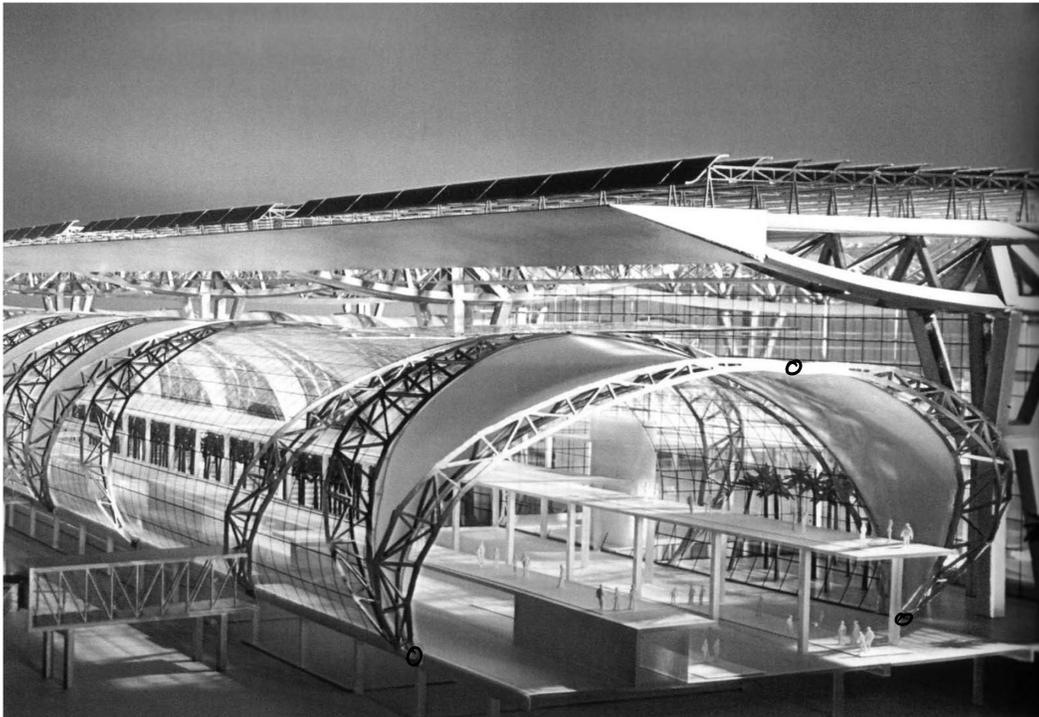
# Geometry Optimization



New Bangkok International Airport, 2003  
Eng. Werner Sobek Arch. Murphy Jahn

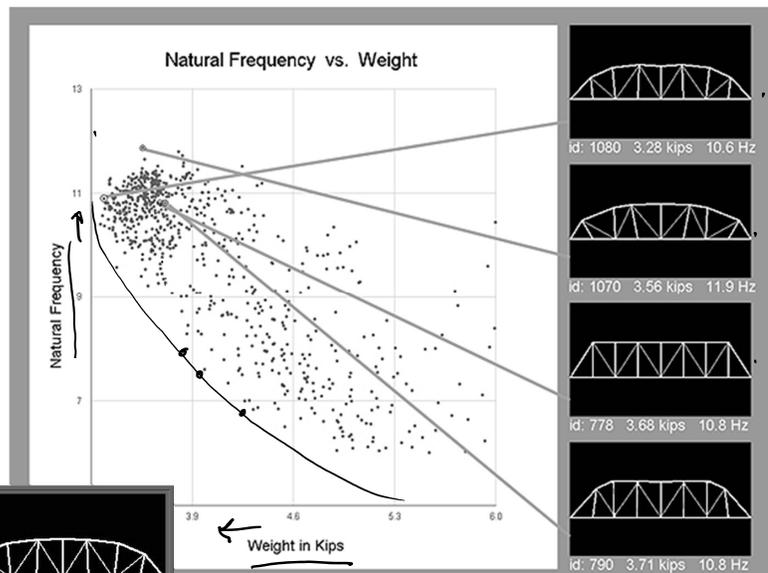
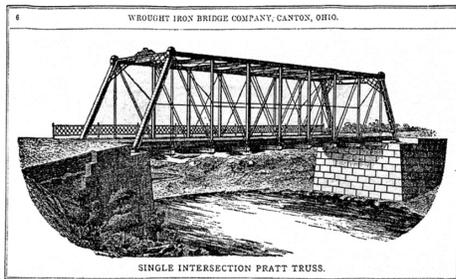


# Geometry Optimization

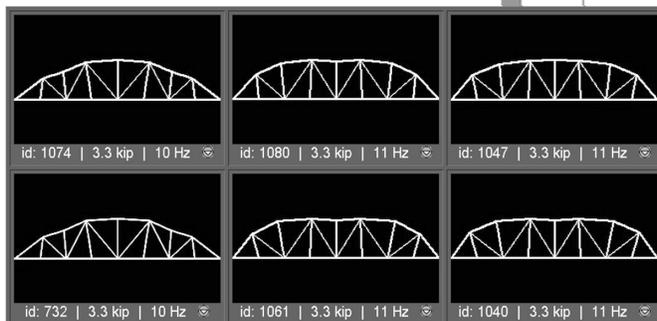


New Bangkok International Airport, 2003  
Eng. Werner Sobek Arch. Murphy Jahn

## Geometry Optimization - Bridges



plot of weight vs natural frequency



lightest solutions

# Topology Optimization - Shukhov towers



Nizhny Novgorod, 1896

