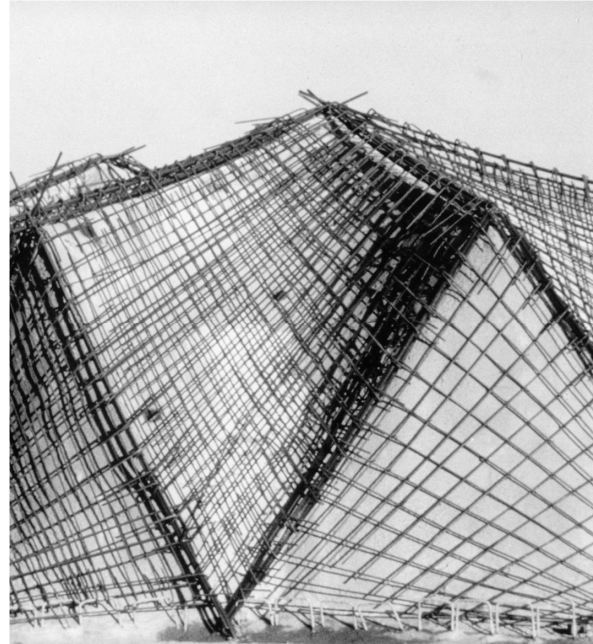


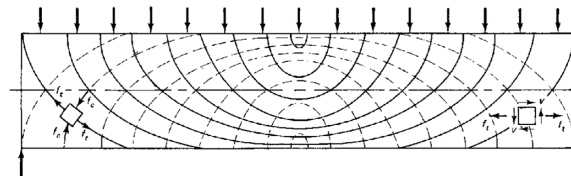
Reinforced Concrete Beams Ultimate Strength Design (ACI 318-19) – PART I

- Flexure in Concrete
- Ultimate Strength Design (LRFD)
- Failure Modes
- Flexure Equations
- Rectangular Beam Analysis



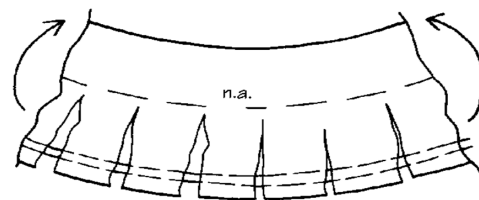
Flexure

The stress trajectories in this simple beam, show principal tension as solid lines.

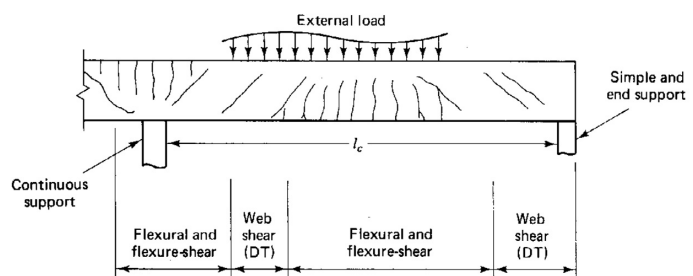
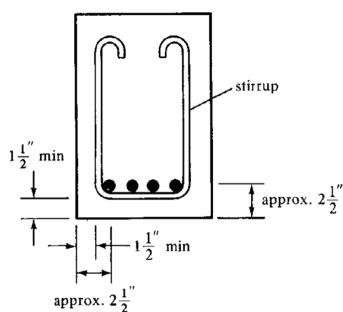
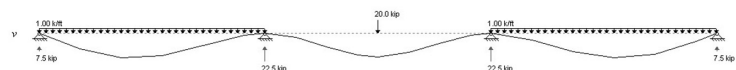


Reinforcement must be placed to resist these tensile forces

In beams continuous over supports, the stress reverses (negative moment).
In such areas, tensile steel is on top.



Shear reinforcement is provided by vertical or sloping stirrups.



Ultimate Strength – (LRFD)

Nominal Strength \geq Design Strength
(strength of member \geq required by loads)

LRFD uses 2 safety factors: γ and ϕ
 ϕ nominal strength $\geq \gamma$ design strength

γ increases the required strength of the member and is placed on the loads

ϕ reduces the member strength capacity and is placed on the calculated force

Loads increased:

γ Factors: DL=1.2 LL=1.6
U is the required strength
U=1.2DL+1.6LL
(factors from ASCE 7)

Strength reduced:

ϕ Factors: e.g. flexure = 0.9
in tension-controlled beams

Table 21.2.1—Strength reduction factors ϕ

Action or structural element	ϕ	Exceptions
(a) Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pre-tensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b) Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquake effects.
(c) Torsion	0.75	—
(d) Bearing	0.65	—
(e) Post-tensioned anchorage zones	0.85	—
(f) Brackets and corbels	0.75	—
(g) Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h) Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i) Plain concrete elements	0.60	—
(j) Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

Ultimate Strength – (ACI 318-14)

Reduced Nominal Strength \geq Factored Load Effects

$$\Phi S_n \geq U$$

γ Factored Loads (see ACSE 7)

- 1) 1.4D
- 2) 1.2D + 1.6L + 0.5(Lr or S or R)
- 3) 1.2D + 1.6(Lr or S or R) + (1.0L or 0.5W)
- 4) 1.2D + 1.0W + 1.0L + 0.5(Lr or S or R)
- 5) 1.2D + 1.0E + 1.0L + 0.2S
- 6) 0.9D + 1.0W
- 7) 0.9D + 1.0E

- D = service dead loads
- L = service live load
- Lr = service roof live load
- S = snow loads
- W = wind loads
- R = rainwater loads
- E = earthquake loads

Strength Reduction Factors, Φ

Mn	Flexural ($\epsilon > 0.005$)	0.90
Vn	Shear	0.75
Pn	Compression (spiral)	0.75
Pn	Compression (other)	0.65
Bn	Bearing	0.65
Tn	Torsion	0.75
Nn	Tension	0.90
Combined stress		0.65 to 0.90

ACI 318 21.2.2



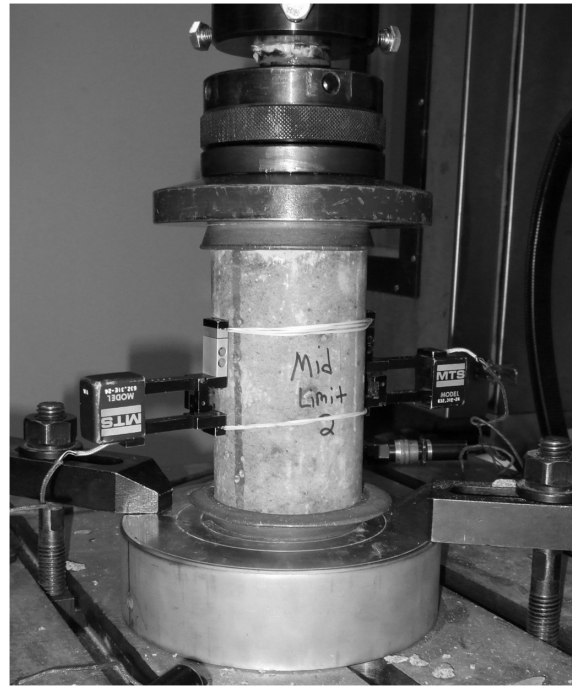
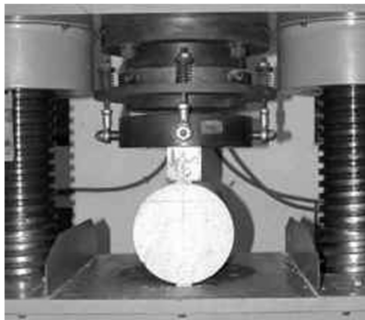
Strength Measurement

- Compressive strength
 - 12" x 6" cylinder
 - 28 day moist cure
 - Ultimate (failure) strength
 - Usable strain $\epsilon_{cu} = 0.003$ (ACI 318)

$$f'_c$$

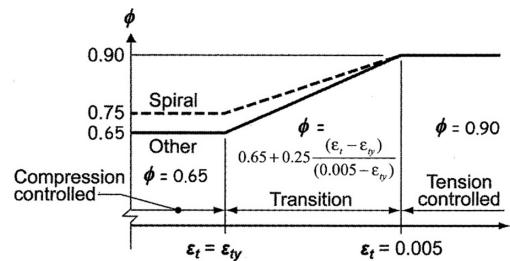
- Tensile strength ASTM C496
 - 12" x 6" cylinder
 - 28 day moist cure
 - Ultimate (failure) strength
 - Split cylinder test
 - ca. 10% of f'_c
 - Neglected in flexure analysis

$$f'_t$$



Failure Modes Based on A_s

- **No Reinforcing**
 - Less than $A_{s,min}$
 - Brittle failure
- **Reinforcing < balance (use this)**
 - Steel yields before concrete fails
 - Ductile failure
 - $\epsilon_t \geq 0.005$ for tension controlled
- **Reinforcing = balance**
 - Concrete fails just as steel yields
 - r_{bal}
- **Reinforcing > balance**
 - Concrete fails before steel yields
 - Low ductility
 - Sudden failure



$A_{s,min}$:
greater of a and b

$$\rho = \frac{A_s}{bd}$$

$$(a) \frac{3\sqrt{f'_c}}{f_y} b_w d$$

$$(b) \frac{200}{f_y} b_w d$$

$$\rho_{max} = 0.75 \rho_b$$

$$\rho_{bal} = \left(\frac{0.85\beta_1 f'_c}{f_y} \right) \left(\frac{87000}{87000 + f_y} \right)$$

$A_s > A_{s,max}$ SuddenDeath!!

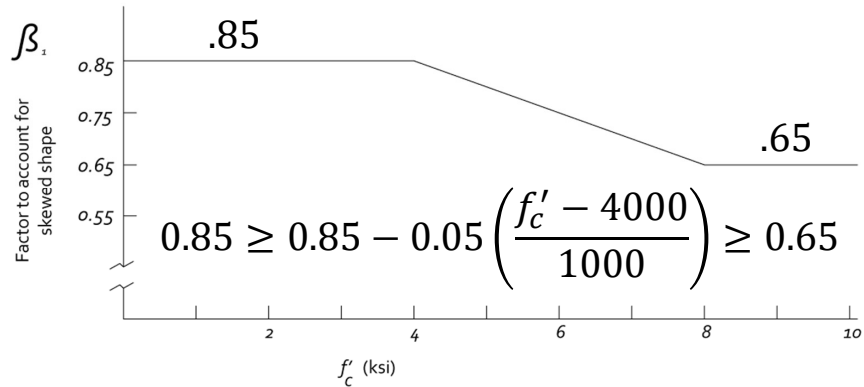
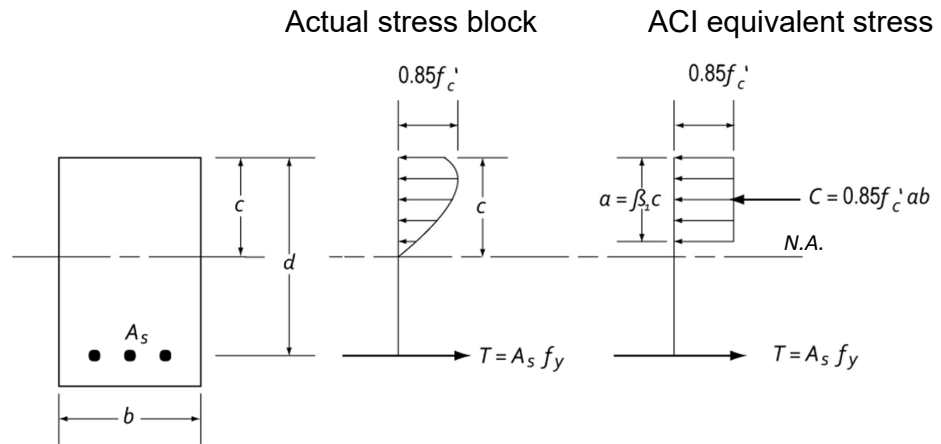
ACI Stress Block

β_1 is a factor to account for the non-linear shape of the compression stress block.

$$a = \beta_1 c$$

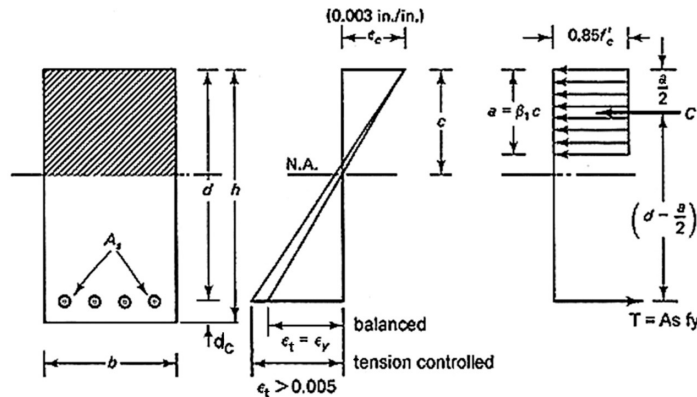
psi

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



Flexure Equations

strain ACI equivalent stress block



$$C = T$$

$$0.85f'_c ab = A_s f_y$$

solving for a ,

$$a = \frac{A_s f_y}{0.85f'_c b} = \frac{\rho f_y d}{0.85f'_c}$$

$$\epsilon_t = \frac{d - c}{c} (0.003)$$

$$\rho = \frac{A_s}{bd}$$

$$M_n = T \left(d - \frac{a}{2} \right) = A_s f_y \left(d - \frac{a}{2} \right)$$

$$M_u = \phi M_n$$

$$M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2} \right)$$

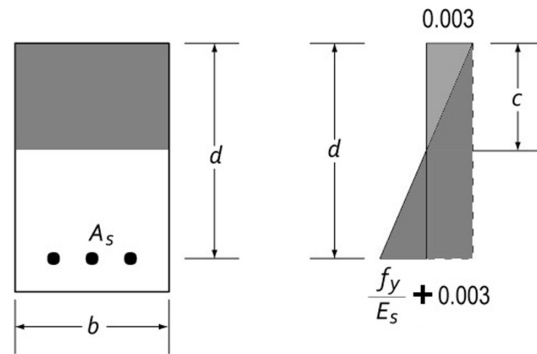
$$M_u = \phi A_s f_y d \left(1 - 0.59 \frac{\rho f_y}{f'_c} \right)$$

Balance Condition

From similar triangles at balance condition:

$$\frac{c}{d} = \frac{0.003}{0.003 + (f_y/E_s)} = \frac{0.003}{0.003 + (f_y/29 \times 10^6)}$$

$$c = \frac{87,000}{87,000 + f_y} d$$



Use equation for a. Substitute into $c = a / \beta_1$

$$a = \frac{\rho f_y d}{0.85 f'_c}$$

$$\rho = \frac{A_s}{bd}$$

Strain diagram for balanced condition.

$$c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f'_c}$$

Table A.8 Balanced Ratio of Reinforcement ρ_b for Rectangular Sections with Tension Reinforcement Only

f_y	f'_c	ρ_b	2,500 psi	3,000 psi	4,000 psi	5,000 psi	6,000 psi
			(17.2 MPa)	(20.7 MPa)	(27.6 MPa)	(34.5 MPa)	(41.4 MPa)
			$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.80$	$\beta_1 = 0.75$
Grade 40 40,000 psi (275.8 MPa)	ρ_b		0.0309	0.0371	0.0495	0.0582	0.0655
	$0.75 \rho_b$		0.0232	0.0278	0.0371	0.0437	0.0492
	$0.50 \rho_b$		0.0155	0.0186	0.0247	0.0291	0.0328
Grade 50 50,000 psi (344.8 MPa)	ρ_b		0.0229	0.0275	0.0367	0.0432	0.0486
	$0.75 \rho_b$		0.0172	0.0206	0.0275	0.0324	0.0365
	$0.50 \rho_b$		0.0115	0.0138	0.0184	0.0216	0.0243
Grade 60 60,000 psi (413.7 MPa)	ρ_b		0.0178	0.0214	0.0285	0.0335	0.0377
	$0.75 \rho_b$		0.0134	0.0161	0.0214	0.0252	0.0283
	$0.50 \rho_b$		0.0089	0.0107	0.0143	0.0168	0.0189
Grade 75 75,000 psi (517.1 MPa)	ρ_b		0.0129	0.0155	0.0207	0.0243	0.0274
	$0.75 \rho_b$		0.0097	0.0116	0.0155	0.0182	0.0205
	$0.50 \rho_b$		0.0065	0.0078	0.0104	0.0122	0.0137

Equate expressions for c:

$$\frac{\rho f_y d}{0.85 \beta_1 f'_c} = \frac{87,000}{87,000 + f_y} d$$

$$\rho_b = \left(\frac{0.85 \beta_1 f'_c}{f_y} \right) \left(\frac{87,000}{87,000 + f_y} \right)$$

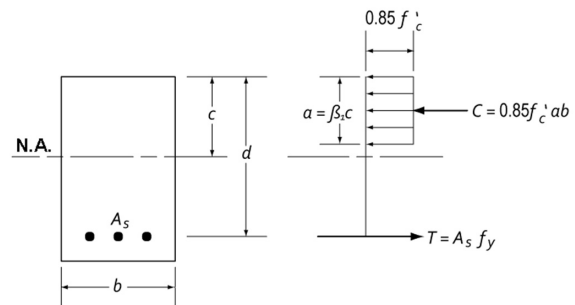
Rectangular Beam Analysis

Data:

- Section dimensions – b, h, (span)
- Steel area - A_s
- Material properties – f'_c , f_y

Required:

- Nominal Strength (of beam) Moment - M_n
- Required (by load) Design Moment – M_u
- Load capacity



$A_{s_{min}}$:
greater of (a) and (b)

$$c = \frac{a}{\beta_1}$$

$$\epsilon_t = \frac{d - c}{c} 0.003 \geq 0.005$$

- Calculate d
- Check A_s min
- Calculate a
- Determine c

$$(a) \frac{3\sqrt{f'_c}}{f_y} b_w d$$

$$(b) \frac{200}{f_y} b_w d$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$\phi M_n \geq M_u$$

- Check that $\epsilon_t \geq 0.005$ (tension controlled)
- Find nominal moment, M_n
- Calculate required moment, $\phi M_n \geq M_u$
(if $\epsilon_t \geq 0.005$ then $\phi = 0.9$)
- Determine max. loading (or span)

$$M_u = \frac{(1.2w_{DL} + 1.6w_{LL})l^2}{8}$$

$$1.6w_{LL} = \frac{M_u 8}{l^2} - 1.2w_{DL}$$

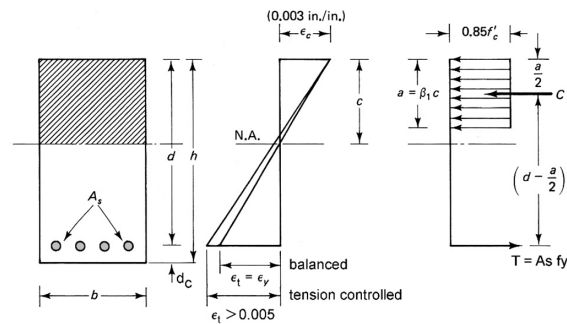
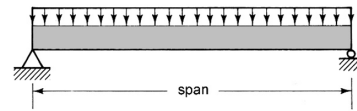
Rectangular Beam Analysis

Data:

- Dimensions – 12"x23"
- Steel $A_s = 4 \times \#6$ $f_y = 60\text{ksi}$
- Concrete $f'_c = 6000\text{psi}$
- Stirrup # 3, Cover 1.5"

Required:

- Required Moment – $\phi M_n = M_u$ (capacity)

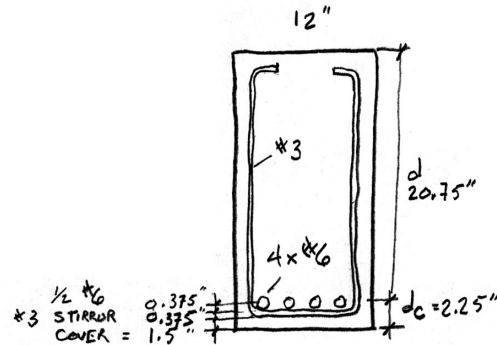


1. Calculate d

$$d_c = \text{COVER} + \#3 + \frac{1}{2}(\#6)$$

$$= 1.5 + 0.375 + \frac{0.75}{2} = 2.25"$$

$$d = h - d_c = 23" - 2.25 = 20.75"$$



Rectangular Beam Analysis cont.

Data:

- Dimensions – 12"x23"
- Steel $A_s = 4 \times \#6 = 1.76\text{in}^2$
- $f_y = 60\text{ksi}$
- Concrete $f'_c = 6000\text{psi}$
- Stirrup # 3, Cover 1.5"

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

Bar No.	Customary Units			SI Units		
	Diameter (in.)	Cross-sectional Area (in. ²)	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm ²)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

2. Check $A_{s\text{min}}$

$A_s \text{ min}$

$$\textcircled{1} \frac{3\sqrt{f'_c}}{f_y} b d = \frac{3\sqrt{6000}}{60000} (12 \times 20.75) = 0.964 \text{ in}^2 \leftarrow \text{controls}$$

$$\textcircled{2} \frac{200 b d}{f_y} = \frac{200 (12) (20.75)}{60000} = 0.83 \text{ in}^2$$

$$\therefore A_{s\text{min}} = 0.964 \text{ in}^2$$

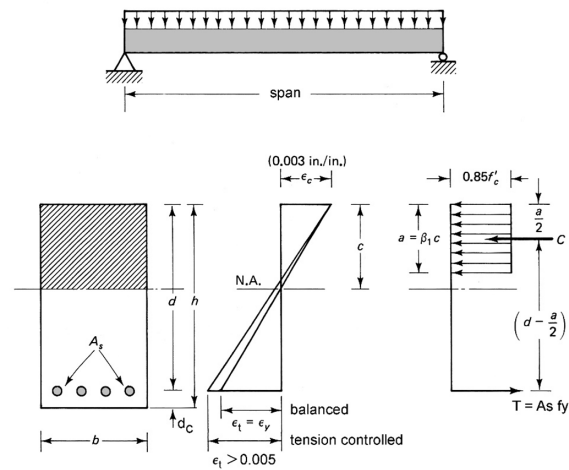
$$A_s = A_b \text{ (N.B.)} = 0.44 (4) = 1.76 \text{ in}^2 \checkmark > 0.964 \text{ in}^2$$

Rectangular Beam Analysis cont.

Data:

- Dimensions – 12"x23"
- Steel $A_s = 4 \times \# 6 = 1.76 \text{ in}^2$
- $f_y = 60 \text{ ksi}$
- Concrete $f'_c = 6000 \text{ psi}$
- Stirrup # 3, Cover 1.5"

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65



Beta1:

$$0.85 \geq$$

$$0.85 - 0.05 \left(\frac{f'_c - 4000}{1000} \right)$$

$$\geq 0.65$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1.76)(60)}{0.85(6)(12)} = 1.725''$$

$$\beta_1 = 0.85 - 0.05 \frac{f'_c - 4000}{1000} = 0.85 - 0.1 = 0.75$$

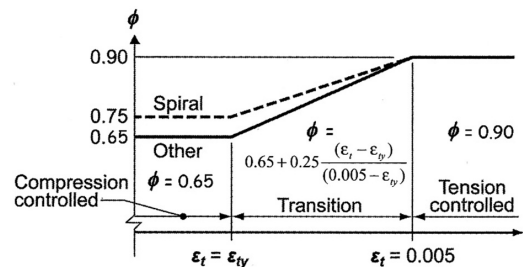
$$c = \frac{a}{\beta_1} = \frac{1.725}{0.75} = 2.300''$$

3. Find a

4. Find c

Rectangular Beam Analysis cont.

$$\varepsilon_t = \frac{d - c}{c} 0.003 \geq 0.005$$



5. Check that $\varepsilon_t \geq 0.005$
(for tension controlled section)
 $\phi = 0.9$

$$\begin{aligned} \varepsilon_t &= \frac{d - c}{c} 0.003 = \frac{20.75 - 2.3}{2.3} 0.003 \\ &= 0.02406 > 0.004 \therefore \text{OK} \\ &= 0.02406 > 0.005 \therefore \text{tension controlled} \end{aligned}$$

6. Find nominal moment, M_n

$$T = A_s f_y = 1.76 (60 \text{ ksi}) = 105.6 \text{ K}$$

$$M_n = T \left(d - \frac{a}{2} \right) = 105.6 \left(20.75 - \frac{1.725}{2} \right)$$

$$M_n = 2100 \text{ K}\cdot\text{in}$$

7. Calculate required moment
 $\phi M_n \geq M_u$

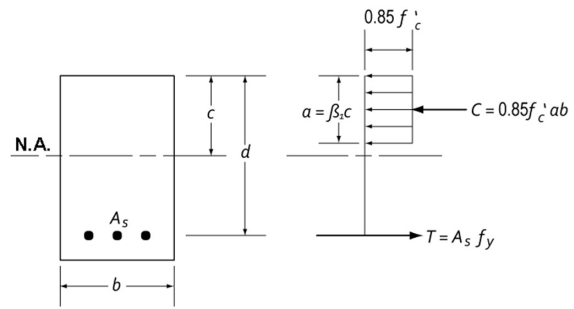
$$\phi M_n = 0.9 (2100) = 1890 \text{ K}\cdot\text{in}$$

$$M_u = \phi M_n = 1890 / 12 = \underline{157.5 \text{ K}\cdot\text{ft}}$$

One-way Slab Analysis

Data:

- Section dimensions – b, h, (span)
- Steel area – A_s , bar diam. b_d , o.c. spacing
- Material properties – f'_c , f_y



Required:

- Nominal Strength (of beam) Moment - M_n
- Required (by load) Design Moment – M_u
- Load capacity

1. Calculate $d = h - \text{cover} - \text{bar}_d/2$

2. Find A_s/ft . Check $A_s \text{ min}$

3. Calculate a

4. Determine c

5. Check that $\epsilon_t \geq 0.005$
(tension controlled)

6. Find nominal moment, M_n

$$A_s/\text{ft} = A_s \times 12/\text{o.c.}$$

7. Calculate required moment,

$$A_g = b \times h$$

$$\phi M_n \geq M_u \text{ (if } \epsilon_t \geq 0.005 \text{ then } \phi = 0.9)$$

8. Determine max. loading (or span)

Table 7.6.1.1— $A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y , psi	$A_{s,min}$	
Deformed bars	< 60,000	0.0020 A_g	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			0.0014 A_g

$$c = \frac{a}{\beta_1}$$

$$\epsilon_t = \frac{d - c}{c} 0.003 \geq 0.005$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$\phi M_n \geq M_u$$

$$M_u = \frac{(1.2W_{DL} + 1.6W_{LL})l^2}{8}$$

$$1.6W_{LL} = \frac{M_u 8}{l^2} - 1.2W_{DL}$$

Slab Analysis

Data:

- Span = 18 ft
- h = 11" take b = 12"
- Steel #8 @ 18" o.c.
- $f'_c = 3000$ psi
- $f_y = 60$ ksi

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257

Required:

- Design moment capacity – M_u
- Maximum LL in PSF

$$d = 11 - \frac{1}{2} - \frac{3}{4} = 9.75''$$

$$A_s = \frac{12''}{18''} (0.79 \text{ in}^2) = 0.5267 \text{ in}^2/\text{FT}$$

1. Find d

2. Find A_s

Check $A_{s,min}$

$$A_g = 12'' \times 11'' = 132 \text{ in}^2$$

$$[0.0018(60)/60] 132 = 0.237 \text{ in}^2$$

$$0.0014 (132) = 0.1848 \text{ in}^2$$

$$0.527 > 0.237 \text{ ok}$$

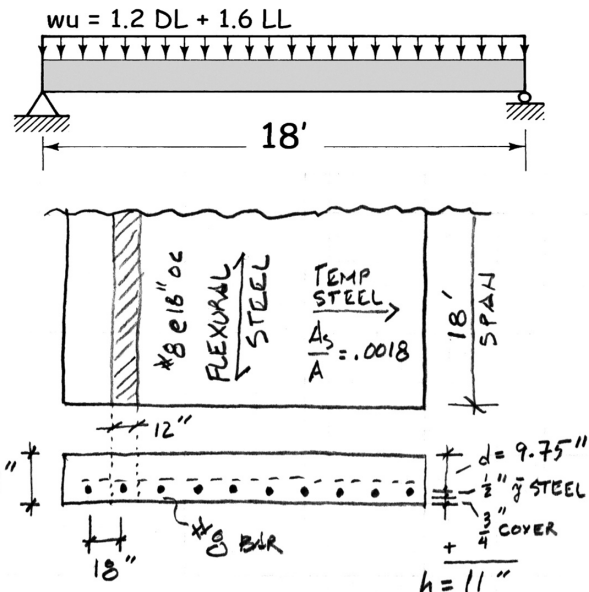


Table 7.6.1.1— $A_{s,min}$ for nonprestressed one-way slabs

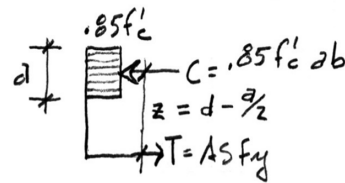
Reinforcement type	f_y , psi	$A_{s,min}$	
Deformed bars	< 60,000	0.0020 A_g	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			0.0014 A_g

Slab Analysis

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

3. Find a

$$d = \frac{A_s f_y}{.85 f'_c b} = \frac{0.5267(60)}{.85(3)(12)} = 1.033''$$



4. Find $c = \beta_1 a$

$$c = \frac{a}{\beta_1} = \frac{1.033''}{0.85} = 1.215''$$

5. Check failure mode

$$\epsilon_t \geq 0.005 \text{ for tension controlled}$$

$$\epsilon_t = \frac{0.003 d}{c} - 0.003$$

$$\epsilon_t = \frac{0.003(9.75'')}{1.215''} - 0.003 = 0.021 \text{ "/>$$

6. Find force T

$$\epsilon_t = 0.021 > 0.005 \therefore \text{TENSION CONTROLLED}$$

7. Find moment arm z

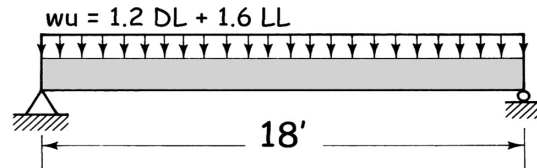
$$T = A_s f_y = 0.5267(60) = 31.6 \text{ K}$$

$$z = d - \frac{a}{2} = 9.75 - \frac{1.033}{2} = 9.23''$$

8. Find nominal strength moment, M_n

$$M_n = T z = 31.6(9.23) = 291.8 \text{ K-ft}$$

Slab Analysis



9. Find required moment, M_u

$$M_u = \phi M_n = 0.9(291.8) \frac{1000}{12} = 21885 \text{ ft-k}$$

10. Find slab DL

$$w_{DL} = \frac{2}{6} \frac{h}{12} = 150 \frac{11}{12} = 137.5 \text{ PSF}$$

11. Determine max. loading

$$M_u = 21885 \text{ ft-k} = \frac{(1.2 w_{DL} + 1.6 w_{LL}) l^2}{8}$$

$$\frac{21885(8)}{(18')^2} = 1.2(137.5) + 1.6(w_{LL})$$

$$540.37 = 165 + 1.6(w_{LL})$$

$$w_{LL} = 234.6 \text{ PSF}$$