Reinforced Concrete Beams
Ultimate Strength Design
(ACI 318-14) – PART II

- Rectangular Slab Analysis
- Reinforcement Detailing
- Rectangular Beam Design – Method I

One-way Slab Analysis

Data:
- Section dimensions – b, h, (span)
- Steel area – As, bar diam. bd, o.c. spacing
- Material properties – f’c, fy

Required:
- Nominal Strength (of beam) Moment - Mn
- Required (by load) Design Moment – Mu
- Load capacity

1. Calculate \( d = h - \text{cover} - \frac{\text{bar}}{2} \)
2. Find \( \frac{\text{As}}{\text{ft}} \). Check \( \text{As} \) min
3. Calculate \( a \)
4. Determine \( c \)
5. Check that \( \varepsilon_t \geq 0.005 \) (tension controlled)
6. Find nominal moment, \( Mn \)
7. Calculate required moment, \( \phi Mn \geq Mu \) (if \( \varepsilon_t \geq 0.005 \) then \( \phi = 0.9 \))
8. Determine max. loading (or span)
Slab Analysis

Data:
- Span = 18 ft
- \( h = 11" \)
- take \( b = 12" \)
- Steel #8 @ 18" o.c.
- \( f_c = 3000 \text{ psi} \)
- \( f_y = 60 \text{ ksi} \)

Required:
- Design moment capacity – \( M_u \)
- Maximum LL in PSF

1. Find \( d \)
2. Find \( A_s \)
   Check \( A_s, \min \)
   \[ A_g = 12" \times 11" = 132 \text{ in}^2 \]
   \[ [0.0018(60)/60] 132 = 0.237 \text{ in}^2 \]
   \[ 0.0014 (132) = 0.1848 \text{ in}^2 \]
   \[ 0.527 > 0.237 \text{ ok} \]

3. Find \( a \)
4. Find \( c = \beta_1 \cdot a \)
5. Check failure mode
   \( \varepsilon_t \geq 0.005 \) for tension controlled
6. Find force \( T \)
7. Find moment arm \( z \)
8. Find nominal strength moment, \( M_n \)
9. Find required moment, $M_u$

10. Find slab DL

11. Determine max. loading

Details of Reinforcement

Size
- Nominal 1/8" increments

Grade
- 40 (40 ksi) 
- 60 (60 ksi) 
- 75 (75 ksi)
Details of Reinforcement

ACI 318 Chapter 25.2
Placement of Reinforcement

• **Cover** (ACI 20.6.1)
• **Horizontal spacing in beams,** $s_h$ (ACI 25.2.1)
  \[
  \frac{1 \text{ inch}}{\frac{4}{3} d_{agg,\text{max}}} 
  \]
• **Vertical spacing in beams** (ACI 25.2.2)
  \[
  \text{Min 1 inch} 
  \]

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**Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members**

<table>
<thead>
<tr>
<th>Concrete exposure</th>
<th>Member</th>
<th>Reinforcement</th>
<th>Specified cover, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cast against and permanently in contact with ground</td>
<td>All</td>
<td>All</td>
<td>3</td>
</tr>
<tr>
<td>Exposed to weather or in contact with ground</td>
<td>All</td>
<td>No. 6 through No. 18 bars</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. 5 bar, W31 or D31 wire, and</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>smaller</td>
<td></td>
</tr>
<tr>
<td>Not exposed to weather or in contact with ground</td>
<td>Slabs, joints, and walls</td>
<td>No. 14 and No. 18 bars</td>
<td>1-1/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>No. 11 bar and smaller</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Beams, columns, pedestals, and tension ties</td>
<td>Primary reinforcement, stirrups, ties, spirals, and hoops</td>
<td>1-1/2</td>
</tr>
</tbody>
</table>

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Details of Reinforcement

ACI 318 Chapter 25
Placement of Reinforcement

• **Chairs**
• **Bolsters**
Details of Reinforcement

ACI 318  Chapter 25

Minimum bend diameter
- factor \( x d_b \)

Hooks for bars in tension
- ACI Table 25.3.1
- Inside diameter

Bends for stirrups
- ACI Table 25.3.2

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Table 25.3.1—Standard hook geometry for development of deformed bars in tension

<table>
<thead>
<tr>
<th>Type of standard hook</th>
<th>Bar size</th>
<th>Minimum inside bend diameter, in.</th>
<th>Straight extension(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-degree hook</td>
<td>No. 3 through No. 9</td>
<td>( 6d_b )</td>
<td>( 12d_b )</td>
</tr>
<tr>
<td></td>
<td>No. 9 through No. 11</td>
<td>( 8d_b )</td>
<td>( 16d_b )</td>
</tr>
<tr>
<td></td>
<td>No. 14 and No. 18</td>
<td>( 10d_b )</td>
<td>( 20d_b )</td>
</tr>
<tr>
<td>180-degree hook</td>
<td>No. 3 through No. 8</td>
<td>( 6d_b )</td>
<td>Greater of ( 4d_b ) and 2.5 ft.</td>
</tr>
<tr>
<td></td>
<td>No. 9 through No. 11</td>
<td>( 8d_b )</td>
<td>Greater of ( 6d_b ) and 3 ft.</td>
</tr>
<tr>
<td></td>
<td>No. 14 and No. 18</td>
<td>( 10d_b )</td>
<td>Greater of ( 8d_b ) and 2.5 ft.</td>
</tr>
</tbody>
</table>

(1) Standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

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Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

<table>
<thead>
<tr>
<th>Type of standard hook</th>
<th>Bar size</th>
<th>Minimum inside bend diameter, in.</th>
<th>Straight extension(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-degree hook</td>
<td>No. 3 through No. 5</td>
<td>( 4d_b )</td>
<td>Greater of ( 6d_b ) and 3 ft.</td>
</tr>
<tr>
<td></td>
<td>No. 6 through No. 8</td>
<td>( 6d_b )</td>
<td>Greater of ( 8d_b ) and 2.5 ft.</td>
</tr>
<tr>
<td>135-degree hook</td>
<td>No. 3 through No. 5</td>
<td>( 4d_b )</td>
<td>Greater of ( 6d_b ) and 3 ft.</td>
</tr>
<tr>
<td></td>
<td>No. 6 through No. 8</td>
<td>( 6d_b )</td>
<td>Greater of ( 8d_b ) and 2.5 ft.</td>
</tr>
<tr>
<td>180-degree hook</td>
<td>No. 3 through No. 5</td>
<td>( 4d_b )</td>
<td>Greater of ( 6d_b ) and 3 ft.</td>
</tr>
<tr>
<td></td>
<td>No. 6 through No. 8</td>
<td>( 6d_b )</td>
<td>Greater of ( 8d_b ) and 2.5 ft.</td>
</tr>
</tbody>
</table>

(1) Standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

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Table 25.4.2.4—Modification factors for development of deformed bars and deformed wires in tension

<table>
<thead>
<tr>
<th>Modification factor</th>
<th>Condition</th>
<th>Value of factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light weight factor</td>
<td>Lightweight concrete</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Lightweight concrete, where ( f_y ) is specified</td>
<td>( \text{In accordance with 19.2.4.3} )</td>
</tr>
<tr>
<td></td>
<td>Normal weight concrete</td>
<td>1.0</td>
</tr>
</tbody>
</table>

- Epoxy(1)
  - Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than \( 6d_b \), or clear spacing less than \( 6d_b \) | 1.5 |
  - Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions | 1.2 |
  - Uncored or zinc-coated (galvanized) reinforcement | 1.0 |

- Size(1)
  - No. 6 and larger bars | 1.0 |
  - No. 6 and smaller bars and deformed wires | 0.8 |

- Casting position(1)
  - More than 12 in. of fresh concrete placed below horizontal reinforcement | 1.3 |
  - Other | 1.0 |

(1) The product \( w_f \), need not exceed 1.7.

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Table 25.4.2.2—Development length for deformed bars and deformed wires in tension

<table>
<thead>
<tr>
<th>Spacing and cover</th>
<th>No. 6 and smaller bars and deformed wires</th>
<th>No. 7 and larger bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear spacing of bars or wires being developed or lap spliced not less than ( d_b ), clear cover at least ( d_b ), and slippage on ties throughout ( d_b ) not less than the Code minimum</td>
<td>( f_{y, c} = \frac{250}{200} f_{y, c} ) ( d_b )</td>
<td>( f_{y, c} = \frac{3 f_{y, c}}{400} f_{y, c} ) ( d_b )</td>
</tr>
<tr>
<td>Clear spacing of bars or wires being developed or lap spliced at least ( 2d_b ) and clear cover at least ( d_b )</td>
<td>( f_{y, c} = \frac{250}{200} f_{y, c} ) ( d_b )</td>
<td>( f_{y, c} = \frac{3 f_{y, c}}{400} f_{y, c} ) ( d_b )</td>
</tr>
<tr>
<td>Other cases</td>
<td>( f_{y, c} = \frac{250}{200} f_{y, c} ) ( d_b )</td>
<td>( f_{y, c} = \frac{3 f_{y, c}}{400} f_{y, c} ) ( d_b )</td>
</tr>
</tbody>
</table>
Rectangular Beam Design

Two approaches:

**Method 1:**

**Data:**
- Load and Span
- Material properties – $f'_{c}$, $f_y$
- All section dimensions: $h$ and $b$

**Required:**
- Steel area – $A_s$

**Method 2:**

**Data:**
- Load and Span
- Some section dimensions – $h$ or $b$
- Material properties – $f'_{c}$, $f_y$
- Choose $\rho$

**Required:**
- Steel area – $A_s$
- Beam dimensions – $b$ or $h$
Rectangular Beam Design – **Method 1**

**Data:**
- Load and Span
- Material properties – $f'_c$, $f_y$
- All section dimensions – $b$ and $h$

**Required:**
- Steel area - $A_s$

1. Calculate the factored load and find factored required moment, $M_{u, \text{req}}$
2. Find $d = h - \text{cover} - \text{stirrup} - \frac{d_b}{2}$
3. Estimate moment arm $z = jd$, for beams $j \approx 0.9$
4. Estimate $A_s$ based on estimate of $jd$.
5. Use $A_s$ to find $a$
6. Use a to find $A_s$ (repeat…until $2\%$ accuracy)
7. Choose bars for $A_s$ and check $A_s$ max & min
8. Check that $\varepsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)
10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

One-way Slab Design

**Method 1**

**Data:**
- Load and Span
- Material properties – $f'_c$, $f_y$
- All section dimensions:
  - $h$ (based on deflection limit)
  - $b = \text{typical 12" width}$

**Required:**
- Steel area – $A_s$

First estimate the slab thickness, $h$.

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.
One-way Slab Slab Design

1. Calculate the dead load and find required $M_u$

$$DL = \frac{11''}{12} (150) = 137.5 \text{ psf}$$

$$LL (4\text{" x}\text{4"}) = 230 \text{ psf}$$

$$w_o = 1.2(137.5) + 1.6(230) = 533 \text{ psf} = \rho CF$$

$$M_u = \frac{w_o \cdot h^2}{8} = \frac{533 \text{ psf} (18')^2}{8} = 21587.12$$

$$\phi M_u = 259'' \cdot k$$

2. Find $d$ based on the estimated $h$ and rebar size (guessing #4)

3. Estimate moment arm

$z \approx 0.95 \cdot d$
One-way Slab Slab Design

4. Estimate $A_s$ based on estimate of $z$

5. Use $A_s$ to find $a$

6. Use $a$ to find $A_s$ (repeat…)

7. Choose bars for $A_s$ required:
   - either choose bars and calculate spacing
   - choose spacing and find bar size
   If the bar size changes, re-calculate to find new $d$. Then, re-calculate $A_s$...

Check $A_{s,min}$
(for slabs $A_{s,min}$ from ACI Table 7.6.1.1)

<table>
<thead>
<tr>
<th>Reinforcement type</th>
<th>$f_y$, psi</th>
<th>$A_{s,min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deformed bars</td>
<td>&lt; 60,000</td>
<td>0.002034</td>
</tr>
<tr>
<td>Deformed bars or welded wire reinforcement</td>
<td>≥ 60,000</td>
<td>Greater of: $0.0018 \times 0.6$ or $0.0014 \frac{s}{t}$</td>
</tr>
</tbody>
</table>
8. Check that $\varepsilon_t \geq 0.005$

$$\varepsilon_t = \frac{E}{E_{ct}}$$

$$\varepsilon_t = \frac{1176}{1050} = 1.1176$$

$$\varepsilon_t = \frac{1176}{1050} = 1.1384$$

$$\varepsilon_t = \frac{9.5\text{in} - 1.1384\text{in}}{1.1384\text{in}} = 0.003$$

$$\varepsilon_t = 0.01759 > 0.005$$

Tension controlled

9. Check $M_u \leq \phi M_n$
   (final condition)

$$A_s = A_{s,\text{used}}$$

$$M_n = Tz$$

10. Add stirrups (no stirrups in slab)

11. Check deflection, crack control, and rebar development length

$$M_u = 0.16(60)(9.5\text{in} - \frac{1176}{2})$$

$$M_u = 36(8.911\text{in}) = 320.8\text{kip}$$

$$M_u = 0.9(320.8) = 288.7\text{kip}$$

$$M_u = 259\text{kip} < 288.7\text{kip}$$

$M_u < 4M_u$