Rectangular Beam Design

Two approaches:

**Method 1:**
Data:
- Load and Span
- Material properties – $f'_c$, $f_y$
- All section dimensions: $h$ and $b$

Required:
- Steel area – $A_s$

**Method 2:**
Data:
- Load and Span
- Some section dimensions – $h$ or $b$
- Material properties – $f'_c$, $f_y$
- Choose $\rho$

Required:
- Steel area – $A_s$
- Beam dimensions – $b$ and $h$
Rectangular Beam Design – Method 2

Data:
- Load and Span
- Some section dimensions – b or h
- Material properties – $f'_c$, $f_y$

Required:
- Steel area - $A_s$
- Beam dimensions – b and h

1. Estimate the dead load (estimate h and b)
   ($L/21 \leq h \leq L/8$, $h \approx L/12$ and b:h ≈ 1:2 to 2:3), find $M_u$
2. Choose $\rho$ (equation assumes $\varepsilon_t = 0.0075$)
3. Calculate $bd^2$
4. Choose b and solve for d (or d and solve b)
5. Revise h, weight, $M_u$, and $bd^2$
6. Find $A_g = \rho bd$
7. Choose bars for $A_g$, determine spacing and cover, and revise d
8. Check that $\varepsilon_t \geq 0.005$ (if not, increase h and reduce $A_u$)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

$$M_u = \frac{(\gamma_{DL}w_{DL} + \gamma_{LL}w_{LL})l^2}{8}$$

$$\rho = \frac{\beta f'_c}{4f_y}$$

$$bd^2 = \frac{M_u}{\varphi f_y (1 - 0.59\rho(f_y/f'_c))}$$

$$A_s = \rho bd$$

$$a = \frac{\rho f_y d}{0.85f'_c}$$

Rectangular Beam Design

Data:
- Load and Span
- Material properties – $f'_c$, $f_y$

Required:
- Steel area - $A_g$
- Beam dimensions – b and d

1. Estimate the dead load (self-weight), and find $M_u$ ($h \approx L/12$ and b:h ≈ 1:2 to 2:3)

2. Choose $\rho$ (equation assumes $\varepsilon_t = 0.0075$)

Table 9.3.1.1—Minimum depth of nonprestressed beams

<table>
<thead>
<tr>
<th>Support condition</th>
<th>Minimum $h^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply supported</td>
<td>$6/16$</td>
</tr>
<tr>
<td>One end continuous</td>
<td>$6/18.5$</td>
</tr>
<tr>
<td>Both ends continuous</td>
<td>$6/21$</td>
</tr>
<tr>
<td>Cantilever</td>
<td>$6/8$</td>
</tr>
</tbody>
</table>

$^{(1)}$Expressions applicable for normalweight concrete and $f_y = 60,000$ psi. For other cases, minimum $h$ shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

Assume $h \approx 12 = 300'' / 12 = 30''$
Assume $b/h \approx 12:1 = 12x15''$
Beam PL = 150 15x30

Estimate $M_u$

$$M_u = \frac{Pd^2}{8}$$

$$= 1.2(20)(10') + 1.2(2.469 ksi)(30')^2$$

$$= 320 + 333.3 = 653.3 \text{ k'-f}$$

Choose $\rho$

$$\rho = \frac{\beta f'_c}{4f_y} = \frac{0.85(3)}{0.60} = 0.010$$
Rectangular Beam Design cont.

3. Calculate $bd^2$

\[ bd^2 = \frac{M_u}{\phi \gamma_f \left( 1 - 0.59 \rho \left( \frac{f_{ce}}{f_{c}} \right) \right) \left( \frac{653.3}{12} \right) \left( \frac{0.01(0.9)60}{\left[ 1 - 0.59 (0.01) \left( \frac{60}{3} \right) \right]} \right) } \]

\[ bd^2 = \frac{7840}{0.573 (0.882)} = 15492 \text{ in}^3 \]

Try

<table>
<thead>
<tr>
<th>b</th>
<th>d</th>
<th>h $\approx$ L/12 d</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>14&quot;</td>
<td>33.27&quot;</td>
<td>38&quot; 532</td>
<td></td>
</tr>
<tr>
<td>15&quot;</td>
<td>32.14&quot;</td>
<td>36&quot; 540</td>
<td></td>
</tr>
<tr>
<td>16&quot;</td>
<td>31.11&quot;</td>
<td>35&quot; 560</td>
<td></td>
</tr>
</tbody>
</table>

Choose 15" x 34";

4. Choose $b$ and solve for $d$
(or $d$ and solve for $b$)

$b$ is based on form size – matches column size

$h \approx L/12$, \( b:h \approx 1:2 \text{ to } 2:3 \)

5. Revise $h$, weight, $M_u$, and $bd^2$

6. Find $A_s = \rho bd$

USE 15" x 34"

REVIEW $W = \frac{150 \times 340}{144} = 563 \text{ lb}$

CHECK $M_u$

$M_u = 320 + \frac{1.2 (2.563) 30^2}{8} = 666 \text{ k-in}$

REVIEW $bd$

$bd^2 = \frac{666 (12)}{0.505} = 15814 \text{ in}^3$

For $b = 15"$, $d = 32.5"$

$A_s = \rho bd = (0.01)(15\")(32.5\")$

$A_s = 4.87 \text{ in}^2$
7. Choose bars for $A_s$, determine spacing and cover, and revise $d$.

If bars do not fit in one layer, $d$ is measured to the centroid of the pattern.

$$x = \frac{\sum A \times d_c}{\sum A}$$

### Table A.4 Areas of Groups of Standard Bars (in.²)

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
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<tbody>
<tr>
<td>4</td>
<td>0.39</td>
<td>0.58</td>
<td>0.78</td>
<td>0.98</td>
<td>1.18</td>
<td>1.37</td>
<td>1.57</td>
<td>1.77</td>
<td>1.96</td>
<td>2.16</td>
<td>2.36</td>
<td>2.55</td>
<td>2.75</td>
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<tr>
<td>5</td>
<td>0.61</td>
<td>0.93</td>
<td>1.23</td>
<td>1.53</td>
<td>1.84</td>
<td>2.15</td>
<td>2.45</td>
<td>2.76</td>
<td>3.07</td>
<td>3.37</td>
<td>3.68</td>
<td>3.99</td>
<td>4.30</td>
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<td>6</td>
<td>0.88</td>
<td>1.32</td>
<td>1.77</td>
<td>2.21</td>
<td>2.65</td>
<td>3.09</td>
<td>3.53</td>
<td>3.98</td>
<td>4.42</td>
<td>4.86</td>
<td>5.30</td>
<td>5.74</td>
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<td>1.80</td>
<td>2.41</td>
<td>3.01</td>
<td>3.61</td>
<td>4.21</td>
<td>4.81</td>
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<td>6.61</td>
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<td>3.14</td>
<td>3.93</td>
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<td>5.50</td>
<td>6.30</td>
<td>7.09</td>
<td>7.85</td>
<td>8.64</td>
<td>9.43</td>
<td>10.21</td>
<td>11.00</td>
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<tr>
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<td>5.00</td>
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<td>8.86</td>
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<td>11.39</td>
<td>12.66</td>
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<td>16.45</td>
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<tr>
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<td>9.00</td>
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<td>15.75</td>
<td>18.00</td>
<td>20.25</td>
<td>22.50</td>
<td>24.75</td>
<td>27.00</td>
<td>29.25</td>
<td>31.50</td>
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<tr>
<td>13</td>
<td>6.00</td>
<td>12.00</td>
<td>16.00</td>
<td>20.00</td>
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<td>40.00</td>
<td>44.00</td>
<td>48.00</td>
<td>52.00</td>
<td>56.00</td>
</tr>
</tbody>
</table>

Jack C McCormac, 1978
Design of Reinforced Concrete,

Rectangular Beam Design

7. Choose bars for $A_s$ and determine spacing and cover, recheck $h$ and weight.

Make final check of $M_n$ using final $d$, and check that $M_u \leq \phi M_n$.

8. Check that $\varepsilon_t \geq 0.005$ (if not, increase $h$ and reduce $A_s$).

9. Design shear reinforcement (stirrups)

10. Check deflection, crack control, steel development length.
T Beams

Dimensional limits

Nomenclature

Possible N.A. locations:

Within flange – rectangular

Within stem – non-rectangular

T Beams - Effective Flange Width, $b_e$

Slab on one side:

- $b_e$ least of either (total width) or (overhang + stem)
  - Total width: $1/12$ of the beam span
  - Overhang: $6 \times$ slab thickness
  - Overhang: $1/2$ the clear distance to next beam

Slab on both sides:

- $b_e$ least of either (total width) or ($2 \times$ overhang + stem)
  - Total width: $1/4$ of the beam span
  - Overhang: $8 \times$ slab thickness
  - Overhang: $1/2$ the clear distance to next beam (i.e. the web on center spacing)
Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, b_e, h, (span)
- Steel area - A_s
- Material properties – f'_c, f_y

Required:
- Required Moment – M_u (or load, or span)

1. Find T = A_s f_y and C = 0.85f'_c A_c
2. Set T = C and solve for A_c = T/(0.85 f'_c)
3. Draw and label diagrams for section and stress
   - Determine b effective (for T-beams)
   - Locate T and C (or C_1 and C_2)
4. Determine the location of a
   - Working from the top down, add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find M_n = \sum C_i z_i
7. Find M_u = \phi M_n
8. Check A_{s,min} < A_s < A_{s,max}
9. Check that \varepsilon_t \leq 0.005

T Beam Analysis

Given: 
- f'_c = 3000 psi
- f_y = 50 ksi
- dimensions. Use b_{eff} = 30"

Req'd: Moment capacity, M_u

1. Find T = A_s f_y and C = 0.85f'_c A_c
2. Set T = C and solve for A_c = T/(0.85 f'_c)
3. Draw and label diagrams for section & stress
   1. Determine \( b_{\text{effective}} \) (for T-beams)
   2. Locate \( T \) and \( C \) (or \( C_1 \) and \( C_2 \))
4. Determine the location of \( a \)
   Working from the top down,
   add up area to make \( A_c \)
5. Find the moment arms (\( z \)) for each block of area
6. Find \( M_n = \sum C_i z_i \)
7. Find \( M_u = \phi M_n \)

\[ \rho_{\text{max}} = 0.75 \rho_{\text{bal}} \]
\[ a_{\text{bal}} = \beta \cdot c_{\text{bal}} = 0.85 \cdot (19.07\)"\) = 16.21" \]
\[ A_c = (4\)"\) (30\)"") + (12.21") (14") = 291 \text{ in}^2 \]
\[ C_{\text{bal}} = (0.85) \cdot (3) \cdot (291) = 742\text{k} \]
\[ T_{\text{max}} = 0.75 \cdot C_{\text{bal}} = (0.75) \cdot (742) = 556\text{k} \]
\[ T_{\text{used}} = A_s \cdot f_y = (10.12) \cdot (50) = 506\text{k} < 556\text{k} \text{ ok} \]
Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking

Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata
Fiber Reinforced Concrete

Several different fiber types:
• Steel (SFRC)
• Glass (GFRC)
• Plastic e.g. polypropylene
• Carbon
• Organic e.g. bamboo

Better crack control
Secondary reinforcement
Textile Reinforced Concrete (TRC)

**Figure 12:** distTEX®: special spacer for textile grids
(photo: Frank Schinditz, TU Dresden)

**Figure 13:** Manufacturing of the TRC hyper-shell layer by layer by shotcrete (photo: © RWTH Aachen), [38]

**Figure 10:** Demolding of a hardened shell element in the concrete yard in Kahl/Saxony (photo: Daniel Ehlig, TU Dresden)
Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels

3D-Print Evolution

https://www.youtube.com/watch?v=awpmJriWcEw