Rectangular Beam Design

Two approaches:

**Method 1:**
Data:
- Load and Span
- Material properties – $f_c$, $f_y$
- All section dimensions: $h$ and $b$

Required:
- Steel area – $A_s$

**Method 2:**
Data:
- Load and Span
- Some section dimensions – $h$ or $b$
- Material properties – $f_c$, $f_y$
- Choose $\rho$

Required:
- Steel area – $A_s$
- Beam dimensions – $b$ and $h$
Rectangular Beam Design – Method 2

Data:
- Load and Span
- Some section dimensions – b or h
- Material properties – f'c, fy

Required:
- Steel area - As
- Beam dimensions – b and d

1. Estimate the dead load (estimate h and b)
   \( \frac{L}{21} \leq h \leq \frac{L}{8} \), find Mu
2. Choose \( \rho \) (equation assumes \( \varepsilon_t = 0.0075 \))
3. Calculate bd^2
4. Choose b and solve for d (or d and solve b)
5. Revise h, weight, Mu, and bd^2
6. Find As = \( \rho bd \)
7. Choose bars for As, determine spacing and cover, and revise d
8. Check that \( \varepsilon_t \geq 0.005 \) (if not, increase h and reduce As)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

\[ M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) L^2}{8} \]
\[ \rho = \frac{\beta f_c'}{4 f_y} \]
\[ b d^2 = \frac{M_u}{\varphi p f_y (1 - 0.59 p (f_y / f_c'))} \]
\[ As = \rho bd \]
\[ a = \frac{\rho f_y d}{0.85 f_c'} \]
Rectangular Beam Design cont.

3. Calculate \( bd^2 \)

4. Choose \( b \) and solve for \( d \) (or \( d \) and solve for \( b \))
   \( b \) is based on form size – matches column size
   \( h \approx \frac{L}{12} \), \( b:h \approx 1:2 \) to 2:3

5. Revise \( h \), weight, \( M_u \), and \( bd^2 \)

\[
bd^2 = \frac{M_u}{\phi F_g \left( 1 - 0.59 \rho \left( \frac{3h}{2b} \right) \right)},
\]
\[
bd^2 = \frac{693.3 \text{ (12)}}{0.01(0.9)60 \left[ 1 - 0.59(0.01) \left( \frac{60}{38} \right) \right]},
\]
\[
bd^2 = \frac{7840}{0.573(0.882)} = 15.492 \text{ in}^3
\]

Try

\[
\begin{array}{cccc}
b & d & h \approx \frac{L}{12} & d_g \\
0.14" & 33.27" & 38" & 532 \\
0.15" & 32.14" & 36" & 540 \\
0.16" & 31.11" & 35" & 560 \\
\end{array}
\]

Choose 15 x 36

Rectangular Beam Design cont.

5. Revise \( h \), weight, \( M_u \), and \( bd^2 \)

6. Find \( A_s = \rho bd \)
7. Choose bars for $A_s$, determine spacing and cover, and revise $d$

If bars do not fit in one layer, $d$ is measured to the centroid of the pattern.

$$\bar{d} = \frac{\sum A \times d_i}{\sum A}$$

<table>
<thead>
<tr>
<th>Table A.4 Areas of Groups of Standard Bars (in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Bars</strong></td>
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<tr>
<td>---------------------</td>
</tr>
<tr>
<td>Bar No.</td>
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<td>4</td>
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<td>12</td>
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<td>13</td>
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</tbody>
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8. Check that $\epsilon_t \geq 0.005$ (if not, increase $h$ and reduce $A_s$)

9. Design shear reinforcement (stirrups)

10. Check deflection, crack control, steel development length
Non-Rectangular Beam Analysis

Data:
• Section dimensions – b, b_e, h, (span)
• Steel area - A_s
• Material properties – f'_c, f_y

Required:
• Required Moment – M_u (or load, or span)

1. Find T = A_s f_y and C = 0.85 f'_c A_c
2. Set T = C and solve for A_c
3. Draw and label diagrams for section and stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
   Working from the top down,
   add up area to make A_c
5. Find moment arms (z) for each block of area
6. Find M_n = Σ C_i z_i
7. Find M_u = φ M_n
8. Check A_s,min < A_s < A_s,max
9. Check that ε_t ≥ 0.005

Effective Flange Width, b_e

Slab on one side:
b_e least of either (total width) or (overhang + stem)
• Total width: 1/12 of the beam span
• Overhang: 6 x slab thickness
• Overhang: ½ the clear distance to next beam

Slab on both sides:
b_e least of either (total width) or (2 x overhang + stem)
• Total width: ¼ of the beam span
• Overhang: 8 x slab thickness
• Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)
T-beam Design – Method 1

Given:  
\( f'_c = 3000 \text{ psi} \)  
\( f_y = 60 \text{ ksi} \)  
Applied load, \( M_u \)

Req’d:  
Reinforcement, \( A_s \)

1. Find \( b_{\text{eff}} \) for flange
2. Find \( d \) (estimate bar size)
3. Estimate moment arm, \( z \)  
\( 0.9 \, d \)
4. Estimate \( A_s \)  
Check \( d \) (iterate)
4. Calculate \( A_c \) (total)

Non-Rectangular Section Analysis  
(cont.)

6. Check if \( A_c \) is within flange  
If yes:  
same as rectangular beam  
If no:  
find area below flange
7. Find \( a \)
8. Find centroid of compression  
Area, \( y-bar \)
9. Calculate \( z \)
10. Calculate \( A_s \)  
Check within 2%  
If not – find new \( a \)
11. Choose bars  
Check spacing
Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking

Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

Fiber Reinforced Concrete

Several different fiber types:
- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

Better crack control
Secondary reinforcement

Glass Fiber Reinforced Concrete - GFRC
Textile Reinforced Concrete (TRC)

Figure 12: distTEX: special spacers for textile grids [photo: Frank Schinditz; TU Dresden]

Figure 13: Manufacturing of the TRC hyper-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]

Figure 10: Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]
Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels

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3D-Print Evolution

https://www.youtube.com/watch?v=awpmJriWcEw