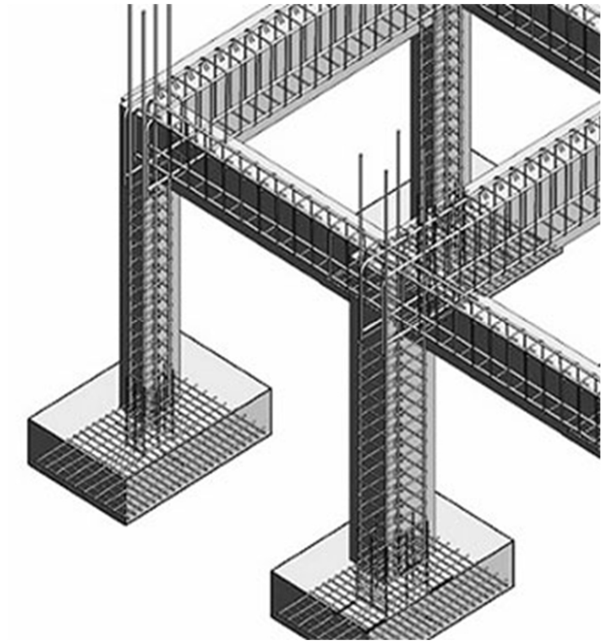
Reinforced Concrete Beams Ultimate Strength Design
(ACI 318-14) – PART III



- Rectangular Beam Design – Method 2
- Non-Rectangular Beam Analysis
- Reinforced Concrete Examples
- 3D-Print Evolution (Video)

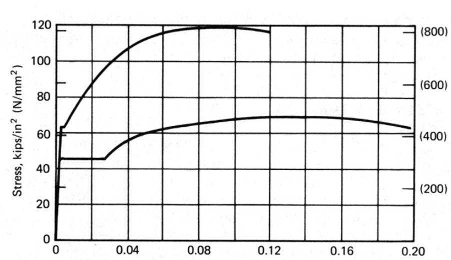
Details of Reinforcement

Size
- Nominal 1/8” increments

Bar size designation	Nominal cross section area, sq. in.	Weight, lb per ft	Nominal diameter, in.
#3	0.11	0.376	0.375 3/8
#4	0.20	0.668	0.500
#5	0.31	1.043	0.625
#6	0.44	1.502	0.750
#7	0.60	2.044	0.875
#8	0.79	2.670	1.000
#9	1.00	3.400	1.128
#10	1.27	4.303	1.270
#11	1.56	5.313	1.410
#14	2.25	7.650	1.693
#18	4.00	13.600	2.257

STIRRUPS

Grade
- 40 (40 ksi) OLD
- 60 (60 ksi) STD
- 75 (75 ksi) HIGH



Details of Reinforcement

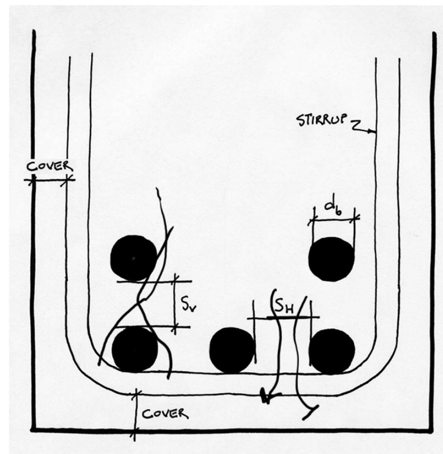
ACI 318 Chapter 25.2
Placement of Reinforcement

- Cover (ACI 20.6.1)
- Horizontal spacing in beams, s_h (ACI 25.2.1)
  - 1 inch
  - d_b
  - 4/3 $d_{agg,max}$
- Vertical spacing in beams (ACI 25.2.2)
  - Min 1 inch

Table 20.6.1.3.1—Specified concrete cover for cast-in-place nonprestressed concrete members

Concrete exposure	Member	Reinforcement	Specified cover, in.
Cast against and permanently in contact with ground	All	All	3
Exposed to weather or in contact with ground	All	No. 6 through No. 18 bars	2
		No. 5 bar, W31 or D31 wire, and smaller	1-1/2
Not exposed to weather or in contact with ground	Slabs, joists, and walls	No. 14 and No. 18 bars	1-1/2
		No. 11 bar and smaller	3/4
	Beams, columns, pedestals, and tension ties	Primary reinforcement, stirrups, ties, spirals, and hoops	1-1/2

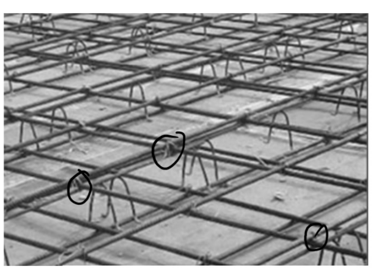
https://www.constructioncost.co/honeycombing-in-concrete.html



Details of Reinforcement

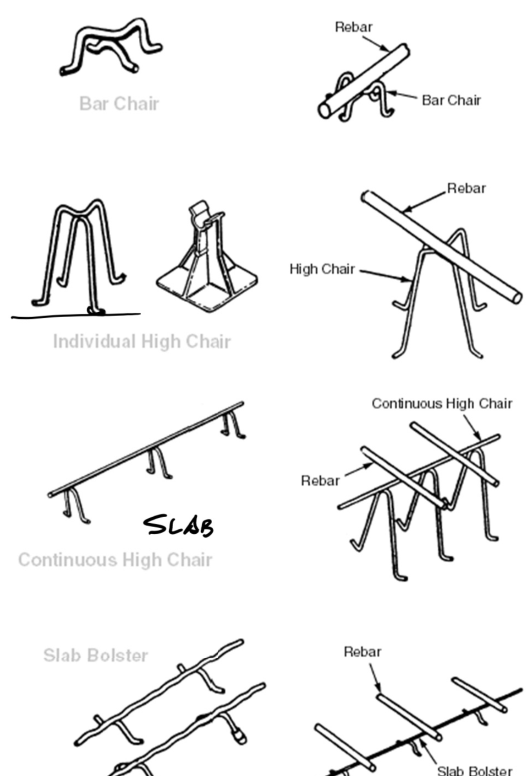
ACI 318 Chapter 25
Placement of Reinforcement

- Chairs
- Bolsters



https://catalog.formtechinc.com

http://contractorsupplymagazine.com



Details of Reinforcement

ACI 318 Chapter 25

Minimum bend diameter
- factor x d_b

Hooks for bars in tension
- ACI Table 25.3.1
- Inside diameter

Bends for stirrups
- ACI Table 25.3.2

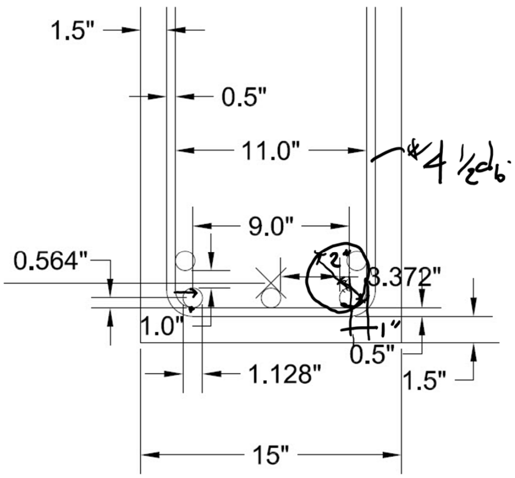
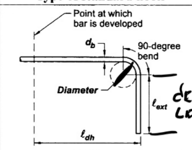
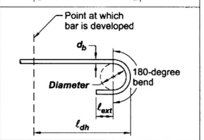
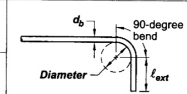
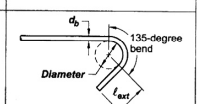
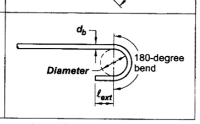


Table 25.3.1—Standard hook geometry for development of deformed bars in tension

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 8	$6d_b$	$12d_b$	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		
180-degree hook	No. 3 through No. 8	$6d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		

[1]A standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Table 25.3.2—Minimum inside bend diameters and standard hook geometry for stirrups, ties, and hoops

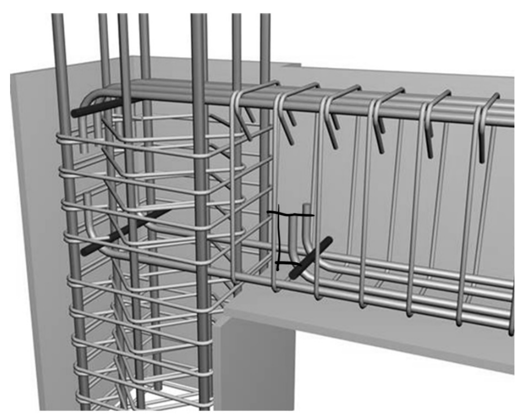
Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] ℓ_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$	$12d_b$	
135-degree hook	No. 3 through No. 5	$4d_b$	Greater of $6d_b$ and 3 in.	
	No. 6 through No. 8	$6d_b$		
180-degree hook	No. 3 through No. 5	$4d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 6 through No. 8	$6d_b$		

[1]A standard hook for stirrups, ties, and hoops includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

Details of Reinforcement

ACI 318 Chapter 25

Development length of bars
- 12" min
- Based on table 25.4.2.2



https://www.buildinghow.com

Table 25.4.2.4—Modification factors for development of deformed bars and deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight λ	Lightweight concrete	0.75
	Lightweight concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] ψ_e	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size ψ_s	No. 7 and larger bars	1.0
	No. 6 and smaller bars and deformed wires	0.8
Casting position[1] ψ_t	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1]The product $\psi_t\psi_e$ need not exceed 1.7.

Table 25.4.2.2—Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less than d_b, clear cover at least d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\left(\frac{f_y\psi_t\psi_e}{25\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{f_y\psi_t\psi_e}{20\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\frac{3f_y\psi_t\psi_e}{50\lambda\sqrt{f'_c}}\right)d_b$	$\left(\frac{3f_y\psi_t\psi_e}{40\lambda\sqrt{f'_c}}\right)d_b$

Other Useful Tables:

Table A.1 Values of Modulus of Elasticity for Normal-Weight Concrete

Customary Units		SI Units	
f'_c (psi)	E_c (psi)	f'_c (MPa)	E_c (MPa)
3,000	3,140,000	20.7	21 650
3,500	3,390,000	24.1	23 373
4,000	3,620,000	27.6	24 959
4,500	3,850,000	31.0	26 545
5,000	4,050,000	34.5	27 924

Jack C McCormac, 1978, *Design of Reinforced Concrete*,

Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

	Customary Units			SI Units		
Bar No.	Diameter (in.)	Cross-sectional Area (in.^{2})	Unit Weight (lb/ft)	Diameter (mm)	Cross-sectional Area (mm^2)	Unit Weight (kg/m)
3	0.375	0.11	0.376	9.52	71	0.560
4	0.500	0.20	0.668	12.70	129	0.994
5	0.625	0.31	1.043	15.88	200	1.552
6	0.750	0.44	1.502	19.05	284	2.235
7	0.875	0.60	2.044	22.22	387	3.042
8	1.000	0.79	2.670	25.40	510	3.973
9	1.128	1.00	3.400	28.65	645	5.060
10	1.270	1.27	4.303	32.26	819	6.404
11	1.410	1.56	5.313	35.81	1006	7.907
14	1.693	2.25	7.650	43.00	1452	11.384
18	2.257	4.00	13.600	57.33	2581	20.238

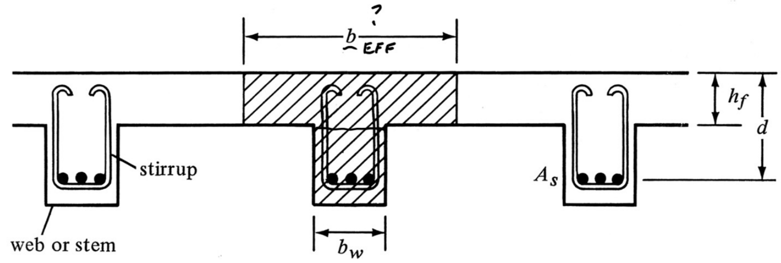
Table A.4 Areas of Groups of StandardBars (in.^{2}) A_s

	Number of Bars												
Bar No.	2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
A 9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

T Beams

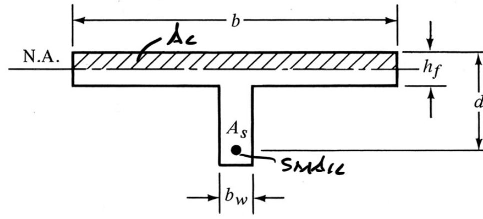
Dimensional limits

Nomenclature



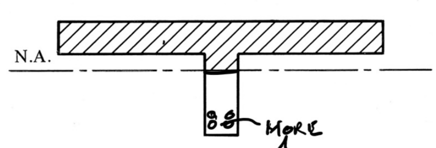
Possible locations of the N.A.:

Within flange – rectangular



(a)

Within stem – non-rectangular



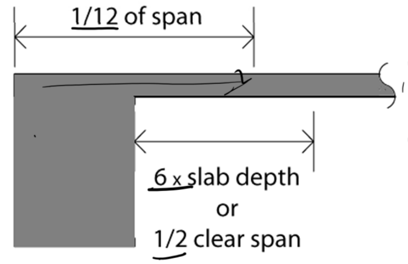
(b)

T Beams - Effective Flange Width, b_e

Slab on one side:

b_e least of either (total width) or (overhang + stem)

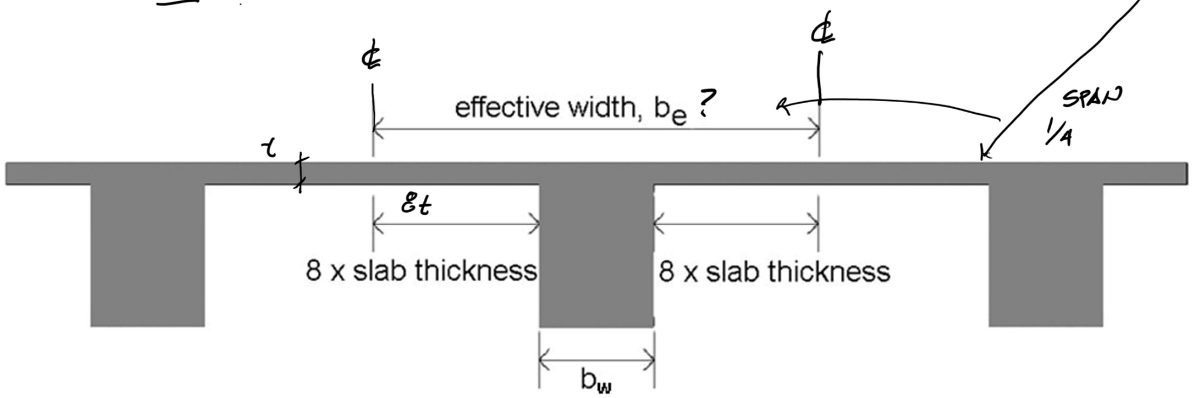
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam



Slab on both sides:

b_e least of either (total width) or (2 x overhang + stem)

- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)



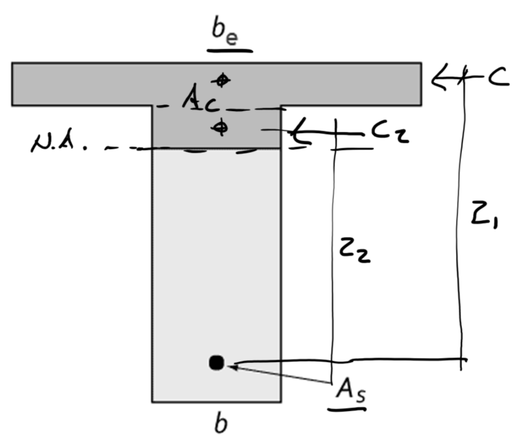
Non-Rectangular Beam Analysis

Data:

- Section dimensions – b, b_e, h, (span)
- Steel area - A_s
- Material properties – f'_c, f_y

Required:

- Required Moment – M_u (or load, or span)

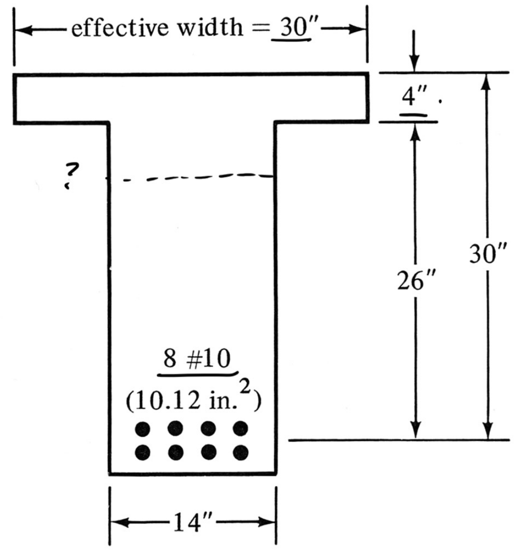


1. Find $T = A_s f_y$ and $C = 0.85f'_c A_c$
2. Set T = C and solve for $A_c = T/(0.85\ f'c)$
3. Draw and label diagrams for section and stress
    1. Determine b effective (for T-beams)
    2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
    Working from the top down,
    add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$
8. Check $A_{s,min} < A_s < A_{s,max}$
9. Check that $\varepsilon_t \geq 0.005$

T Beam Analysis

Given: f'_c = 3000 psi
f_y = 50 ksi
dimensions. Use b_{eff} = 30"

Req'd: Moment capacity, Mu



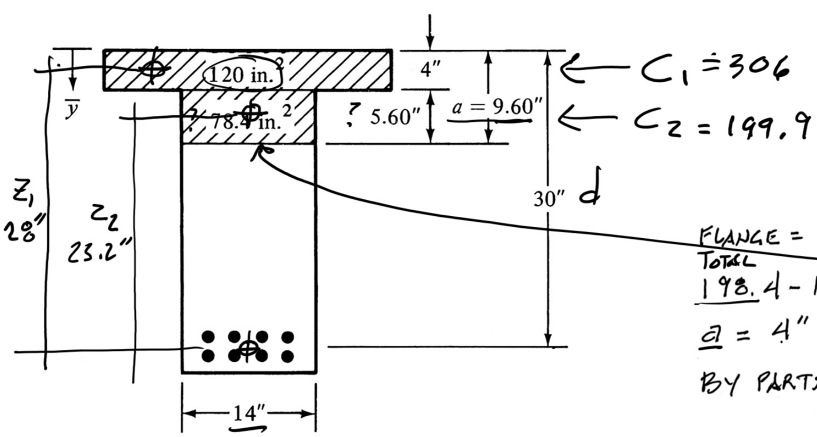
T = C

1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set T = C and solve for $A_c = T/(0.85\ f'c)$

$T = A_s f_y = 10.12\ in^2\ 50^{ksi} = 506^K$

$A_c = \frac{T}{0.85 f'_c} = \frac{506^K}{0.85\ 3^{ksi}} = 198.4\ in^2$

T Beam Analysis (cont.)



FLANGE = 30" × 4" = 120 < 198.4 ∴ NA IN WEB

TOTAL
198.4 − 120 = 78.4 in² = 14" × 5.60"

a = 4" + 5.60" = 9.60"

BY PARTS (FOR EACH AREA)

$Z_1 = 30" - 4"/2 = 28"$ MOMENT ARMS

$Z_2 = 30" - 4" - 5.6/2" = 23.2"$

$C_1 = A_{c1}\ 0.85 f'_c = 120(0.85)(3) = 306^K$

$C_2 = A_{c2}\ 0.85 f'_c = 78.4(0.85)(3) = 199.9^K$

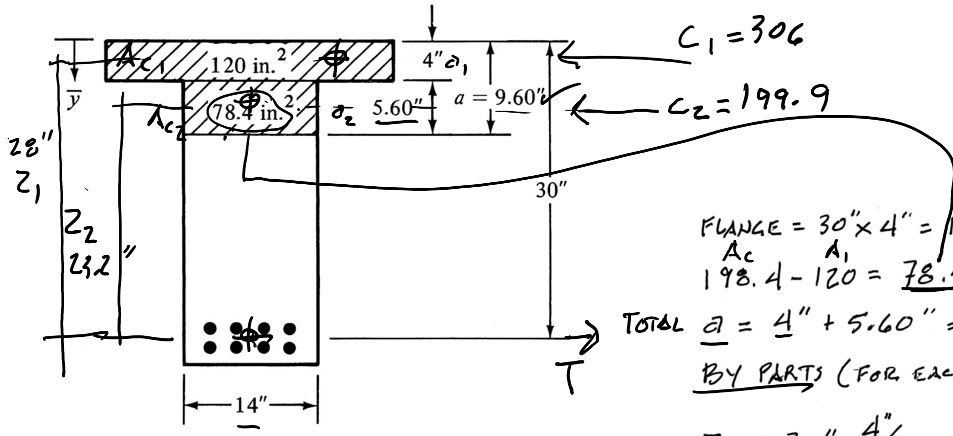
$M_n = \sum C_i z_i = 306^K(28") + 199.9^K(23.2") =$
$8568 + 4638 = 13206^{K-IN}$
$= 1101^{K-FT}$

LOAD
$M_U = \phi M_n = 0.9(1101) = 991^{K-FT}$

3. Draw and label diagrams for section & stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
   Working from the top down,
   add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$

T Beam Analysis (cont.)

$T = C_1 + C_2$



FLANGE = 30″ × 4″ = 120 < 198.4 ∴ NA IN WEB

$A_c - A_1$: 198.4 − 120 = 78.4 in² = 14″ (b_w) × 5.60″

TOTAL a = 4″ + 5.60″ = 9.60″

BY PARTS (FOR EACH AREA)

$Z_1 = 30'' - 4''/2 = 28''$

$Z_2 = 30'' - 4'' - 5.6/2'' = 23.2''$

$C_1 = A_{c1}\, 0.85 f'_c = 120(0.85)(3) = 306^K$

$C_2 = A_{c2}\, 0.85 f'_c = 78.4(0.85)(3) = 199.9^K$

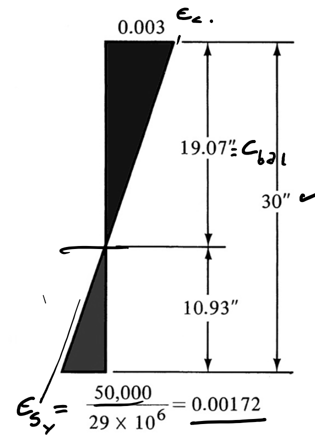
$M_n = \sum C_i z_i = 306(28) + 199.9(23.2) = 8568 + 4638 = 13\,206^{K-IN} = 1101^{K-FT}$

$M_u = \phi M_n = 0.9(1101) = 991^{K-FT}$

3. Draw and label diagrams for section & stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
   Working from the top down,
   add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$

T Beam Analysis (cont.)

ρ_{bal}



$\rho_{max} = 0.75\, \rho_{bal}$

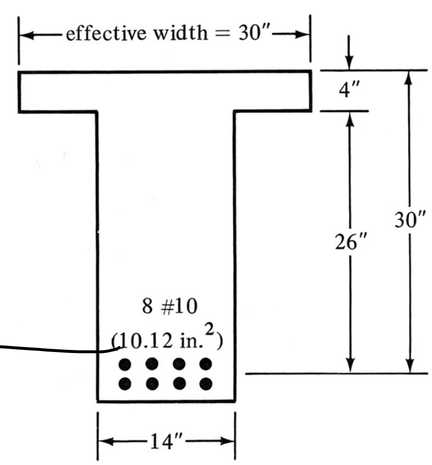
a_{bal} = beta c_{bal} = 0.85 (19.07”) = 16.21” (4 + 12.21 =)

Ac_{bal} = (4”) (30”) + (12.21”) (14”) = 291 in²

C_{bal} = (0.85) (3) (291) = 742^k ($T_{max} = C_{max} = .75 C_{bal}$)

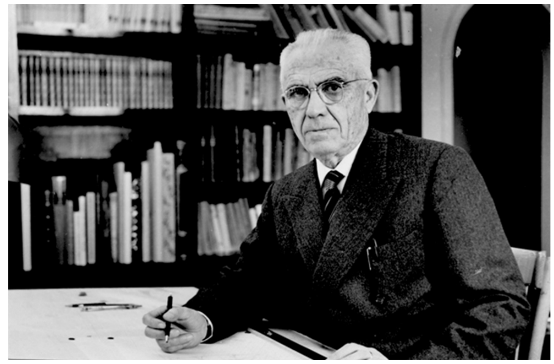
T_{max} = 0.75 C_{bal} = (0.75) (742) = 556^k

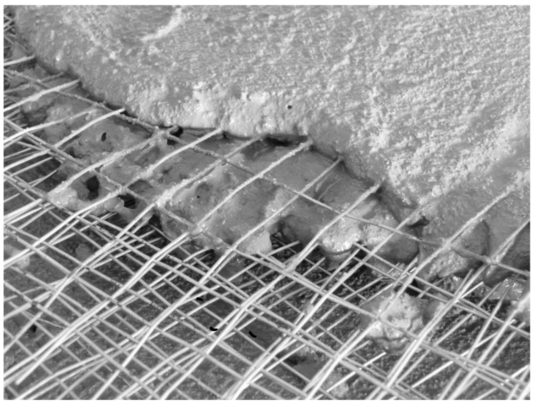
T_{used} = $A_s f_y$ = (10.12) (50) = 506^k < 556^k ok (T_{ACTUAL})

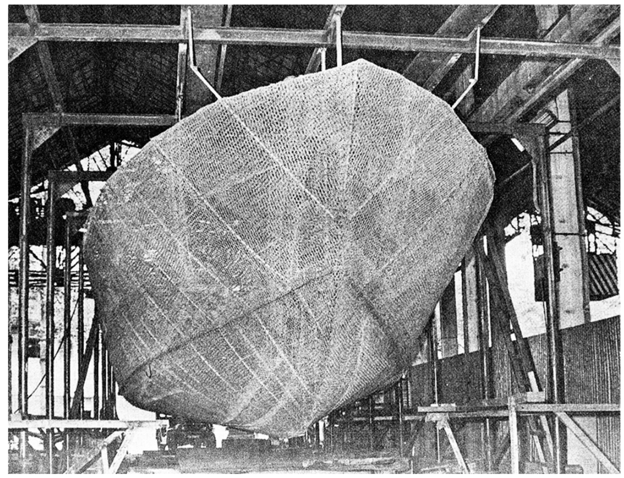


Ferrocement

- Pioneered by Pier Luigi Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking







Ferrocement

- Pioneered by Nervi
- Dense, small gage reinforcement
- More flexible shapes – no formwork
- Well suited for thin shells
- Less cracking
- Low-tech applications

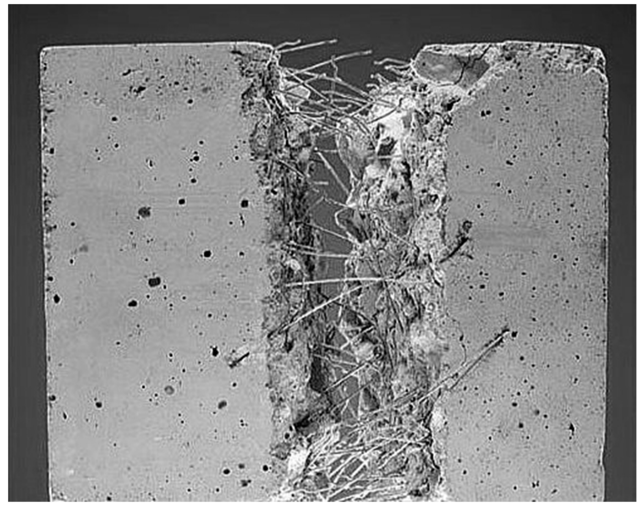
Priory Benedictine Church, Missouri, 1956. Architect Gyo Obata

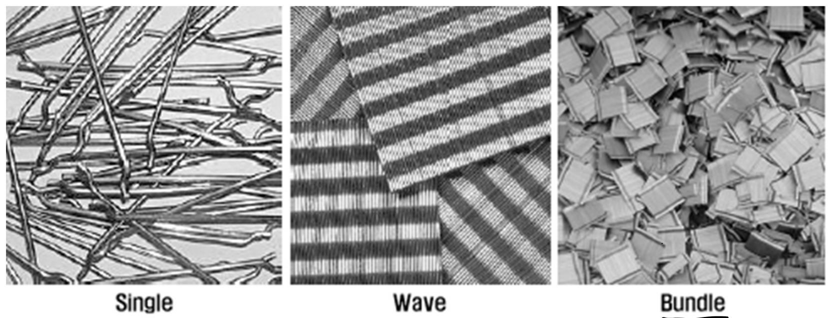
Palazetto dello Sport, Rome, 1957. P.L. Nervi

Fiber Reinforced Concrete

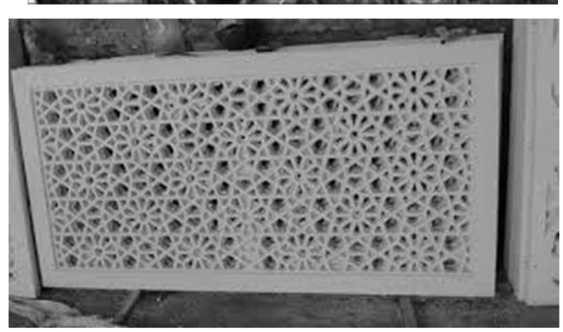
Several different fiber types:
- Steel (SFRC)
- Glass (GFRC)
- Plastic e.g. polypropylene
- Carbon
- Organic e.g. bamboo

Better crack control
Secondary reinforcement





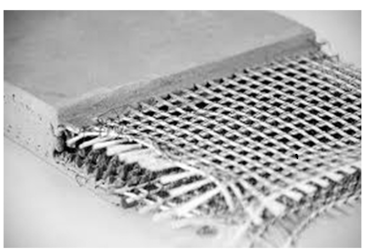
Glass Fiber Reinforced Concrete - GFRC



Carbon Fiber

Bamboo

Textile Reinforced Concrete (TRC)



SPACER

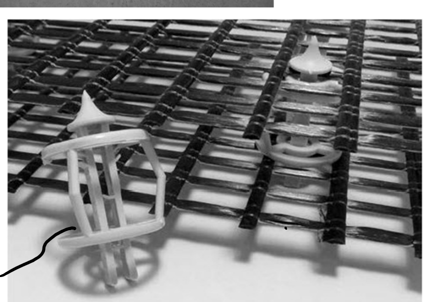


Figure 12: distTEX: special spacers for textile grids [photo: Frank Schladitz, TU Dresden]

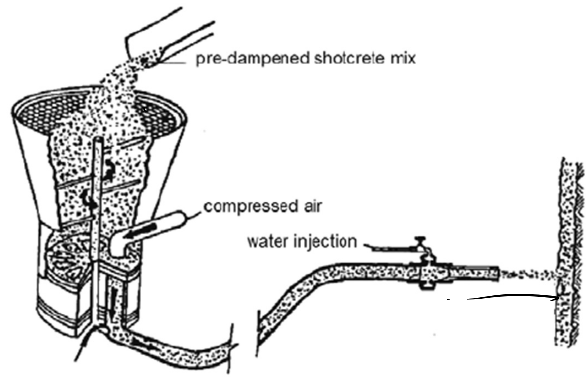
Figure 13: Manufacturing of the TRC hypar-shell layer by layer by shotcrete [photo: © RWTH Aachen], [38]

Figure 10: Demolding of a hardened shell element in the concrete yard in Kahla/Saxony [photo: Daniel Ehlig, TU Dresden]

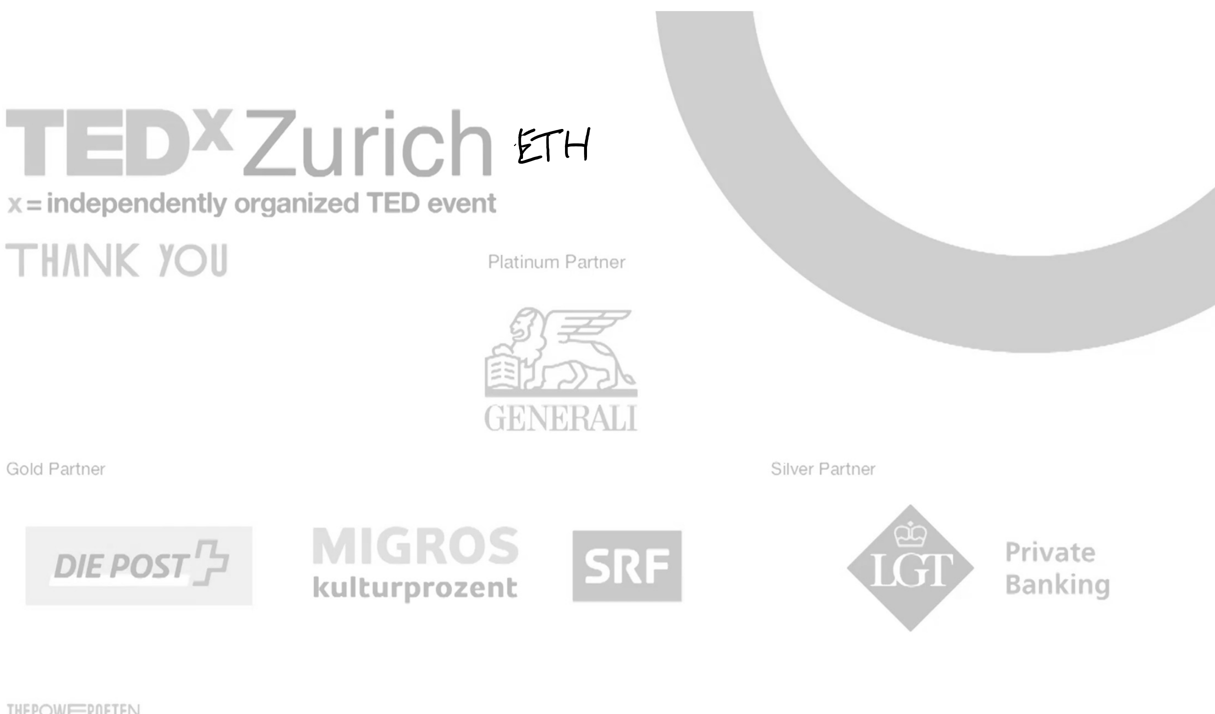
Shotcrete

- Pneumatically applied
- High velocity
- Can include fiber
- Applied to backing
- Reinforced with bars
- Soil stabilization, tunnels

NO VOIDS



3D-Print Evolution



ETH

https://www.youtube.com/watch?v=awpmJriWcEw