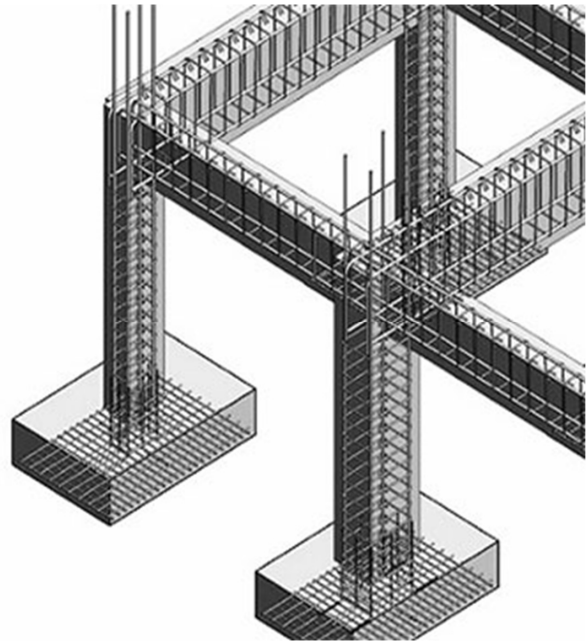


# Reinforced Concrete Beams Ultimate Strength Design (ACI 318-14) – PART III

- Rectangular Beam Design – Method 1
- Rectangular Beam Design – Method 2



## Rectangular Beam Design

Two approaches:

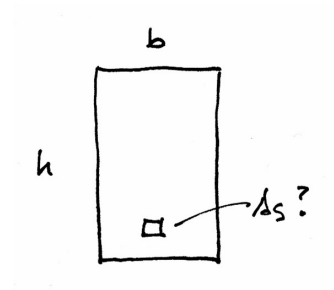
### Method 1:

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions:  $h$  and  $b$

Required:

- Steel area –  $A_s$



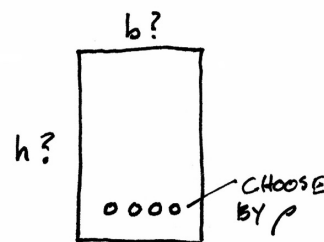
### Method 2:

Data:

- Load and Span
- Some section dimensions –  $h$  or  $b$
- Material properties –  $f'_c$ ,  $f_y$
- Choose  $\rho$

Required:

- Steel area –  $A_s$
- Beam dimensions –  $b$  or  $h$



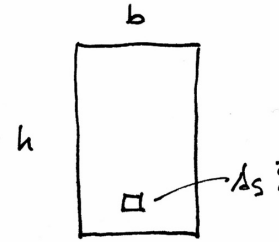
# Rectangular Beam Design – Method 1

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions –  $b$  and  $h$

Required:

- Steel area -  $A_s$



1. Calculate the factored load and find factored required moment,  $M_u$
2. Find  $d = h - \text{cover} - \text{stirrup} - d_b/2$
3. Estimate moment arm  $z = jd$ , for beams  $j \approx 0.9$  for slabs  $j \approx 0.95$
4. Estimate  $A_s$  based on estimate of  $jd$ .
5. Use  $A_s$  to find  $a$
6. Use  $a$  to find  $A_s$  (repeat...until **2%** accuracy)
7. Choose bars for  $A_s$  and check  $A_s$  max & min
8. Check that  $\epsilon_t \geq 0.005$
9. Check  $M_u \leq \phi M_n$  (final condition)

$$M_u = \frac{(\gamma_{DL}W_{DL} + \gamma_{LL}W_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

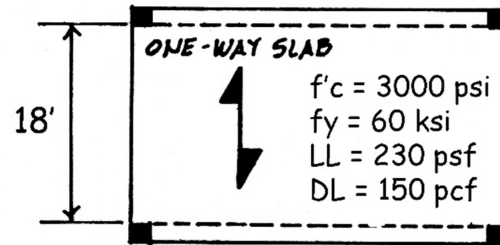
$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

## One-way Slab Design

### Method 1

Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$
- All section dimensions:
- $h$  (based on deflection limit)
- $b = \text{typical } 12'' \text{ width}$



Required:

- Steel area –  $A_s$

First estimate the slab thickness,  $h$ .

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

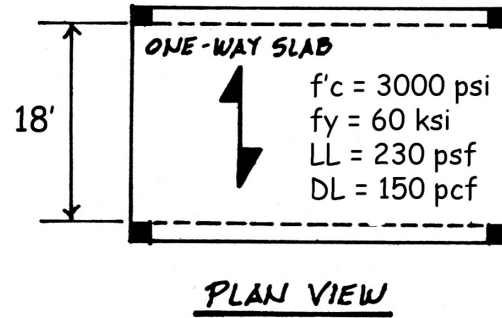
**Table 7.3.1—Minimum thickness of solid nonprestressed one-way slabs**

Support condition	Minimum $h^{(1)}$
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

THICKNESS,  $h$ , BASED ON DEFLECTION

$$h = \frac{\ell}{20} = \frac{18 \times 12}{20} = 10.8'' \text{ USE } 11''$$

## One-way Slab Slab Design



1. Calculate the dead load and find required  $M_u$

Factor Loads

$$DL = \frac{11''}{12} (150) = 137.5 \text{ PSF}$$

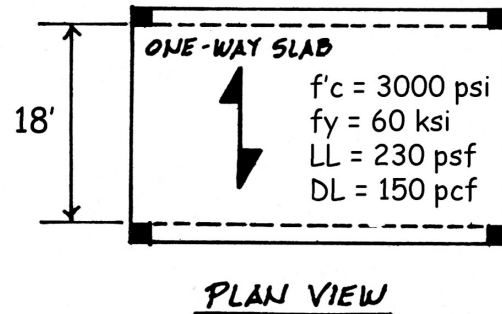
$$LL \text{ (GIVEN)} = 230 \text{ PSF}$$

$$w_u = 1.2(137.5) + 1.6(230) = 533$$

$$M_u = \frac{w_u l^2}{8} = \frac{533 \text{ PLF} (18')^2}{8} = 21587 \text{ '}\cdot\text{K}$$

$$= 259 \text{ '}\cdot\text{K}$$

## One-way Slab Slab Design



2. Find  $d$  based on the estimated  $h$  and rebar size (guessing #4)
3. Estimate moment arm  
 $z \approx 0.95 d$

$$\text{For } j \approx 0.95, \quad d = h - \text{COVER} - \frac{1}{2} \text{ BAR}$$

$$d = 11'' - \frac{3}{4}'' - \frac{1}{2} (\frac{1}{2}'')$$

$$d = 11'' - 1'' = 10''$$

$$z \approx j d \approx 0.95 (10'') = 9.5''$$

# One-way Slab Slab Design

4. Estimate  $A_s$  based on estimate of  $z$

TRIAL 1

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{259 \text{ in}^{\text{-k}}}{0.9(60 \text{ ksi})(9.5 \text{ in})} = 0.505 \text{ in}^2$$

5. Use  $A_s$  to find  $a$

$$a = \frac{A_s f_y}{.85 f_c' b} = \frac{0.505(60)}{.85(3)(12)} = 0.99 \text{ in}$$

6. Use  $a$  to find  $A_s$  (repeat...)

TRIAL 2

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9(60)(10 - \frac{.99}{2})}$$

$$A_s = 0.5046 \text{ in}^2 \quad \text{WITHIN 2\%}$$

# One-way Slab Slab Design

7. Choose bars for  $A_s$  required:

either

**choose bars** and calculate spacing

or

**choose spacing** and find bar size

If the bar size changes, re-calculate to find new  $d$ . Then, re-calculate  $A_s$ ...

CHOOSE BARS

USING #4

$$\frac{0.505}{12 \text{ in}} ; \frac{0.2}{s} \quad s = 4.75 \text{ in}$$

∴ USE 4" o.c. (always round down)

$$A_s = 0.60 \text{ in}^2/\text{FT} > 0.505 \checkmark$$

Check  $A_{s,min}$

(for slabs  $A_{s,min}$  from ACI Table 7.6.1.1)

ALTERNATE FOR MAX. S = 18"

$$\frac{0.505}{12 \text{ in}} ; \frac{A_b}{18 \text{ in}} \quad A_b = 0.75 \text{ in}^2$$

$$\#8 = 0.79$$

∴ USE #8 @ 18" o.c.

$$A_s = 0.526 \text{ in}^2/\text{FT} > 0.505 \checkmark$$

**Table 7.6.1.1— $A_{s,min}$  for nonprestressed one-way slabs**

Reinforcement type	$f_y$ , psi	$A_{s,min}$	
Deformed bars	< 60,000	0.0020 $A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$
			0.0014 $A_g$

Check  $A_{s,min}$

$$A_{s,min} = 0.0018 bh = 0.0018(12)(11 \text{ in})$$

$$= 0.24 \text{ in}^2 < 0.526 \text{ in}^2 \checkmark \text{ OK}$$

## One-way Slab Slab Design

8. Check that  $\epsilon_t \geq 0.005$

$$\text{RE-CALC 2 FOR } A_s = 0.6 \text{ m}^2/\text{ft}$$
$$a = \frac{A_s f_y}{0.85 F'_c b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d-c}{c} 0.003 =$$
$$= \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$$0.01759 > 0.005$$

$\therefore$  TENSION CONTROLLED  $\checkmark$

## One-way Slab Slab Design

9. Check  $M_u \leq \phi M_n$   
(final condition)

$$A_s = A_{s, \text{used}}$$

$$M_n = Tz$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right)$$

$$M_n = 0.6(60) \left( 9.5'' - \frac{1.176}{2} \right)$$

$$M_n = 36(8.911'') = 320.8 \text{ k-in}$$

$$\phi M_n = 0.9(320.8) = 288.7 \text{ k-in}$$

10. Add stirrups (no stirrups in slab)

$$M_u = 259 \text{ k-in} < 288.7 \text{ k-in}$$

11. Check deflection, crack control,  
and rebar development length

$$M_u < \phi M_n \checkmark \text{ OK}$$

## Rectangular Beam Design – Method 2

### Data:

- Load and Span
- Some section dimensions – b or h
- Material properties –  $f'_c$ ,  $f_y$

### Required:

- Steel area -  $A_s$
  - Beam dimensions – b and h
1. Estimate the dead load (estimate h and b) ( $L/8 \leq h \leq L/21$ ,  $h \approx L/12$  and  $b:h \approx 1:2$  to  $2:3$ ), find  $M_u$
  2. Choose  $\rho$  (equation assumes  $\epsilon_t = 0.0075$ )
  3. Calculate  $bd^2$
  4. Choose b and solve for d (or d and solve b)
  5. Revise h, weight,  $M_u$ , and  $bd^2$
  6. Find  $A_s = \rho bd$
  7. Choose bars for  $A_s$ , determine spacing and cover, and revise d
  8. Check that  $\epsilon_t \geq 0.005$  (if not, increase h and reduce  $A_s$ )
  9. Design shear reinforcement (stirrups)
  10. Check deflection, crack control, steel development length

$$M_u = \frac{(\gamma_{DL}W_{DL} + \gamma_{LL}W_{LL})l^2}{8}$$

$$\rho = \frac{\beta_1 f'_c}{4f_y}$$

$$bd^2 = \frac{M_u}{\phi \rho f_y (1 - 0.59\rho(f_y/f'_c))}$$

$$A_s = \rho bd$$

$$a = \frac{\rho f_y d}{0.85f'_c}$$

## Rectangular Beam Design

### Data:

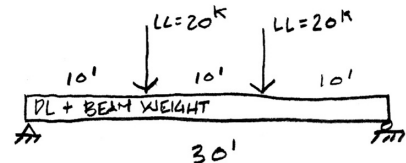
- Load and Span
- Material properties –  $f'_c$ ,  $f_y$

### Required:

- Steel area -  $A_s$
- Beam dimensions – b and d

$f'_c = 3000$  psi  
 $f_y = 60$  ksi

DL = 2 klf + beam  
LL = 2 x 20 k



1. Estimate the dead load (self-weight), and find  $M_u$  ( $h \approx L/12$  and  $b:h \approx 1:2$  to  $2:3$ )

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum $h^{[1]}$
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

<sup>[1]</sup>Expressions applicable for normalweight concrete and  $f'_c = 60,000$  psi. For other cases, minimum  $h$  shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

$$\text{ASSUME } h \approx \frac{L}{12} = \frac{360''}{12} = 30''$$

$$\text{ASSUME } b:h \approx 1:2 \therefore b \approx 15''$$

$$\text{BEAM DL} = 150 \frac{15 \times 30}{144} = 469 \text{ PLF}$$

ESTIMATE  $M_u$

$$M_u = P \omega + \frac{w l^2}{8}$$

$$= 1.6(20^k)(10') + \frac{1.2(2.469 \text{ KLF})(30')^2}{8}$$

$$= 320 + 333.3 = 653.3 \text{ K-1}$$

# Rectangular Beam Design

## Data:

- Load and Span
- Material properties –  $f'_c$ ,  $f_y$

## Required:

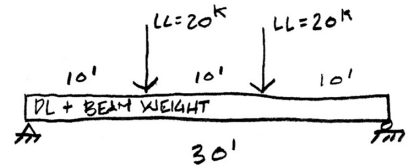
- Steel area -  $A_s$
- Beam dimensions –  $b$  and  $d$

$$f'_c = 3000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

$$DL = 2 \text{ klf} + \text{beam}$$

$$LL = 2 \times 20 \text{ k}$$



2. Choose  $\rho$  (equation assumes  $\epsilon_t = 0.0075$ )

$f'_c$	$\beta_1$
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

Choose  $\rho$

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

# Rectangular Beam Design cont.

3. Calculate  $bd^2$

$$bd^2 = \frac{M_u}{\phi \rho f_y (1 - 0.59 \rho (f_y / f'_c))}$$

$$bd^2 = \frac{653.3 (12)}{0.9 (0.01) 60 [1 - 0.59 (0.01) (\frac{60}{3})]}$$

$$bd^2 = \frac{7840}{0.573 (0.882)} = 15492 \text{ in}^3$$

4. Choose  $b$  and solve for  $d$   
(or  $d$  and solve for  $b$ )

$b$  is based on form size – matches column size  
 $h \approx L/12$ ,  $b:h \approx 1:2$  to  $2:3$

TRY

$b$	$d$	$h \approx 1.12 d$	$A$
14"	33.27"	38"	532
15"	32.14"	36"	540
16"	31.11"	35"	560

5. Revise  $h$ , weight,  $M_u$ , and  $bd^2$

CHOOSE 15 x 36

# Rectangular Beam Design cont.

5. Revise  $h$ , weight,  $M_u$ , and  $bd^2$

USE 15 x 36

$$\text{REVISE } PL = 150 \frac{540}{144} = 563 \text{ PLF}$$

CHECK  $M_u$

$$M_u = 320 + \frac{1.2(2.563)30^2}{8} = 666 \text{ K-1}$$

REVISE  $bd$

$$bd^2 = \frac{666(12)}{0.505} = 15814 \text{ in}^3$$

$$\text{FOR } b = 15'' \quad d = 32.5''$$

6. Find  $A_s = \rho bd$

$$A_s = \rho bd = (0.01)(15'')(32.5'')$$

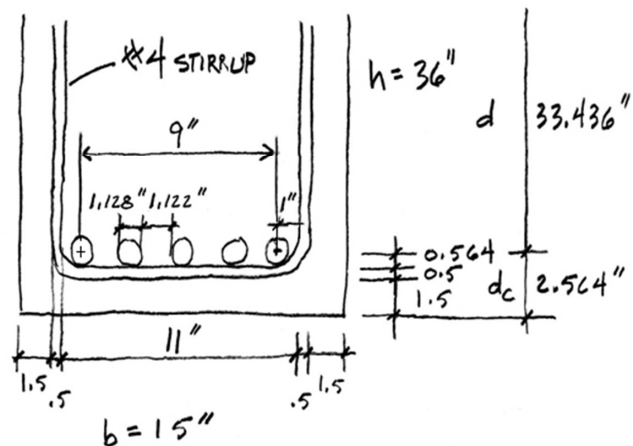
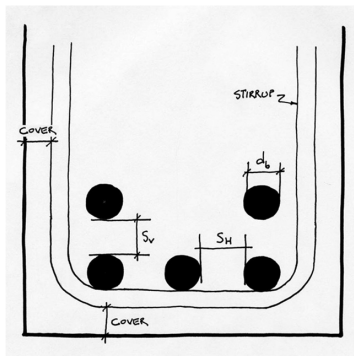
$$A_s = 4.87 \text{ in}^2$$

# Rectangular Beam Design

7. Choose bars for  $A_s$ , determine spacing and cover, and revise  $d$

CHOOSE BARS (SEE TABLE A.4)

$$\text{TRY } 5 \times \#9 \text{ BARS } A_s = 5.0 \text{ in}^2 > 4.87 \text{ in}^2$$



If bars do not fit in one layer,  $d$  is measured to the centroid of the pattern.

$$\bar{x} = \frac{\sum A \times d_x}{\sum A}$$

Table A.4 Areas of Groups of Standard Bars (in.<sup>2</sup>)

Bar No.	Number of Bars													
	2	3	4	5	6	7	8	9	10	11	12	13	14	
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75	
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30	
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19	
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42	
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00	
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72	
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87	
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50	
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	

## Rectangular Beam Design

7. Choose bars for  $A_s$  and determine spacing and cover, recheck  $h$  and weight

Make final check of  $M_n$  using final  $d$ , and check that  $M_u \leq \phi M_n$

$$d = h - dc = 36'' - 2.564'' = 33.435''$$

$$d = 33.436''$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{5(60)}{.85(3)15} = 7.843''$$

$$M_n = A_s f_y \left( d - \frac{a}{2} \right) = 5(60) \left( 33.436 - \frac{7.843}{2} \right)$$

$$M_n = 8854 \text{ K}\cdot\text{ft} = 737.8 \text{ K}\cdot\text{ft}$$

$$\phi M_n = 0.9(737.8) = 664 \text{ K}\cdot\text{ft}$$

$$M_u = 653.3 < 664 \quad \checkmark \text{ OK}$$

8. Check that  $\epsilon_t \geq 0.005$  (if not, increase  $h$  and reduce  $A_s$ )

$$c = \frac{a}{\beta_1} = \frac{7.843''}{0.85} = 9.227''$$

$$\epsilon_t = \frac{d - c}{c} (0.003)$$

$$\epsilon_t = \frac{33.436 - 9.227}{9.227} (0.003)$$

$$\epsilon_t = 0.00787 > 0.005 \quad \checkmark \text{ OK}$$

9. Design shear reinforcement (stirrups)  
10. Check deflection, crack control, steel development length