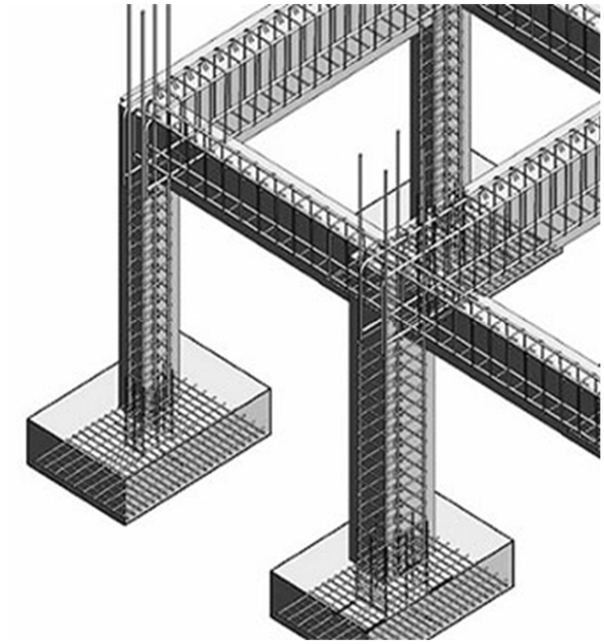
Architecture 324
Structures II

Reinforced Concrete Beams Ultimate Strength Design
(ACI 318-14) – PART III



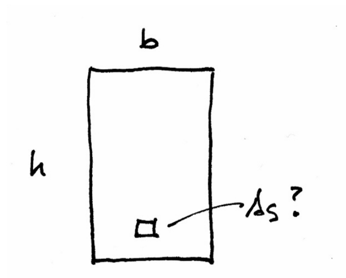
- Rectangular Beam Design – Method 1
- Rectangular Beam Design – Method 2
- Non-Rectangular Beam Analysis
- T-Beams

Rectangular Beam Design
Two approaches:

Method 1:

Data:
- Load and Span
- Material properties – f'_c, f_y
- All section dimensions: h and b

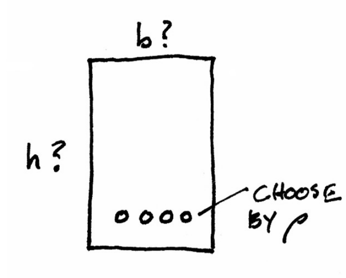
Required:
- Steel area – A_s



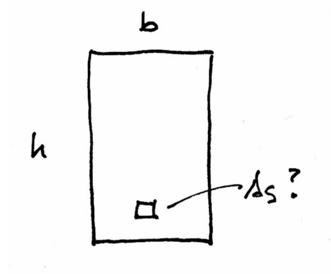
Method 2:

Data:
- Load and Span
- Some section dimensions – h or b
- Material properties – f'_c, f_y
- Choose ρ

Required:
- Steel area – A_s
- Beam dimensions – b or h



Rectangular Beam Design – **Method 1**



Data:
- Load and Span
- Material properties – f'_c, f_y
- All section dimensions – b and h

Required:
- Steel area - A_s

1. Calculate the factored load and find factored required moment, M_u
2. Find d = h – cover – stirrup – $d_b/2$
3. Estimate moment arm z = jd, for beams j ≈ 0.9 for slabs j ≈ 0.95
4. Estimate A_s based on estimate of jd.
5. Use As to find a
6. Use a to find A_s (repeat…until **2%** accuracy)
7. Choose bars for A_s and check A_s max & min
8. Check that $\varepsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)
10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

$$A_s = \frac{M_u}{\varphi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

One-way Slab Design
Method 1

Data:
- Load and Span
- Material properties – f'_c, f_y
- All section dimensions:
- h (based on deflection limit)
- b = typical 12" width

Required:
- Steel area – A_s

First estimate the slab thickness, h.

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

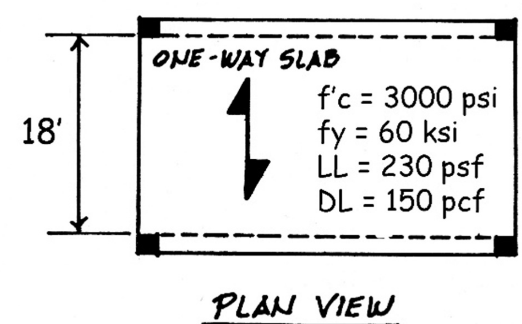


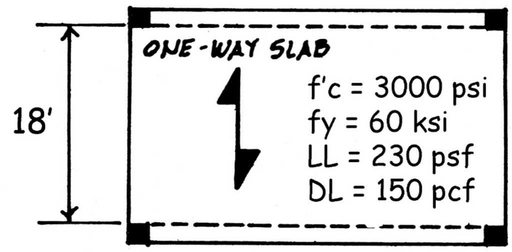
Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum $h^{[1]}$
Simply supported	ℓ/20
One end continuous	ℓ/24
Both ends continuous	ℓ/28
Cantilever	ℓ/10

THICKNESS, h, BASED ON DEFLECTION

$h = \ell/20 = \frac{18 \times 12}{20} = 10.8"$ USE 11"

One-way Slab Slab Design



PLAN VIEW

1. Calculate the dead load and find required M_u

FACTOR LOADS

$DL = \frac{11''}{12}(150) = 137.5 \text{ PSF}$

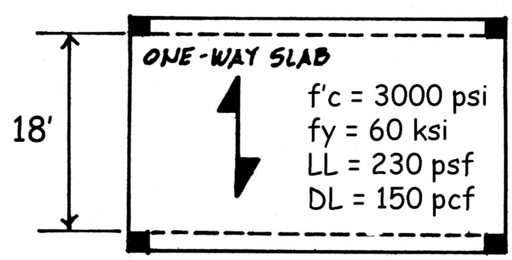
$LL \text{ (GIVEN)} = 230 \text{ PSF}$

$w_u = 1.2(137.5) + 1.6(230) = 533$

$M_u = \frac{w_u \ell^2}{8} = \frac{533 \text{ PLF} (18')^2}{8} = 21\,587 \text{ '-\#}$

$= 259 \text{ ''-K}$

One-way Slab Slab Design



PLAN VIEW

2. Find d based on the estimated h and rebar size (guessing #4)

FOR $j \approx 0.95$, $d = h - \text{COVER} - \frac{1}{2} \text{ BAR}$

$d = 11'' - \frac{3}{4}'' - \frac{1}{2}(\frac{1}{2}'')$

$d = 11'' - 1'' = 10''$

3. Estimate moment arm $z \approx 0.95\ d$

$z \approx jd \approx 0.95(10'') = 9.5''$

One-way Slab Slab Design

4. Estimate A_s based on estimate of z

5. Use A_s to find a

6. Use a to find A_s (repeat…)

TRIAL 1

$$A_s = \frac{M_U}{\phi f_y (z)} = \frac{259''\text{-k}}{0.9(60\text{ KSI})(9.5)''} = 0.505\text{ in}^2$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{0.505(60)}{.85(3)(12)} = 0.99''$$

TRIAL 2

$$A_s = \frac{M_U}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9(60)(10 - \frac{.99}{2})}$$

$A_s = 0.5046\text{ in}^2$ WITHIN 2%

One-way Slab Slab Design

7. Choose bars for A_s required:

either
choose bars and calculate spacing
or
choose spacing and find bar size
If the bar size changes, re-calculate to find new d. Then, re-calculate A_s…

Check $A_{s,min}$

(for slabs $A_{s,min}$ from ACI Table 7.6.1.1)

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y, psi	$A_{s,min}$	
Deformed bars	< 60,000	$0.0020A_g$	
Deformed bars or welded wire reinforcement	≥ 60,000	Greater of:	$\frac{0.0018 \times 60{,}000}{f_y} A_g$
			$0.0014A_g$

CHOOSE BARS

USING #4

$\frac{0.505}{12''} : \frac{0.2}{s}$ $s = 4.75''$

∴ USE 4" o.c. (always round down)

$A_s = 0.60\text{ in}^2/\text{FT} > 0.505$ ✓

ALTERNATE FOR MAX. S = 18"

$\frac{0.505}{12''} : \frac{A_b}{18''}$ $A_b = 0.75\text{ in}^2$

#8 = 0.79

∴ USE #8 @ 18" o.c.

$A_s = 0.526\text{ in}^2/\text{FT} > 0.505$ ✓

Check As,min

$A_{s\,min} = 0.0018\,bh = 0.0018(12)(11'')$

$= 0.24\text{ in}^2 < 0.526\text{ in}^2$ ✓ OK

One-way Slab Slab Design

8. Check that $\varepsilon_t \geq 0.005$

RE-CALC a FOR $A_s = 0.6\ in^2/FT$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d-c}{c} 0.003 =$$

$$= \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$$0.01759 > 0.005$$

∴ TENSION CONTROLLED ✓

One-way Slab Slab Design

9. Check $M_u \leq \phi M_n$ (final condition)

   $A_s = A_{s,used}$
   $M_n = Tz$

10. Add stirrups (no stirrups in slab)

11. Check deflection, crack control, and rebar development length

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 0.6(60)\left(9.5'' - \frac{1.176}{2}\right)$$

$$M_n = 36(8.911'') = 320.8^{k\text{-}''}$$

$$\phi M_n = 0.9(320.8) = 288.7^{k\text{-}''}$$

$$M_u = 259^{k\text{-}''} < 288.7^{k''}$$

$M_u < \phi M_n$ ✓ OK

Rectangular Beam Design – **Method 2**

Data:
- Load and Span
- Some section dimensions – b or h
- Material properties – f'_c, f_y

Required:
- Steel area - A_s
- Beam dimensions – b and h

1. Estimate the dead load (estimate h and b) ($L/8 \le h \le L/21$, $h \approx L/12$ and b:h ≈ 1:2 to 2:3), find M_u
2. Choose ρ (equation assumes $\varepsilon_t = 0.0075$)
3. Calculate bd^2
4. Choose b and solve for d (or d and solve b)
5. Revise h, weight, M_u, and bd^2
6. Find $A_s = \rho bd$
7. Choose bars for A_s, determine spacing and cover, and revise d
8. Check that $\varepsilon_t \ge 0.005$ (if not, increase h and reduce A_s)
9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

$$M_u = \frac{(\gamma_{DL} w_{DL} + \gamma_{LL} w_{LL}) l^2}{8}$$

$$\rho = \frac{\beta_1 f'_c}{4 f_y}$$

$$bd^2 = \frac{M_u}{\varphi \rho f_y \left(1 - 0.59 \rho (fy / f'_c)\right)}$$

$$A_s = \rho bd$$

$$a = \frac{\rho f_y \, d}{0.85 f'_c}$$

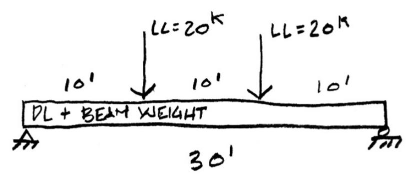
Rectangular Beam Design

Data:
- Load and Span
- Material properties – f'_c, f_y

Required:
- Steel area - A_s
- Beam dimensions – b and d

f'c = 3000 psi
fy = 60 ksi

DL = 2 klf + beam
LL = 2 x 20 k



1. Estimate the dead load (self-weight), and find M_u ($h \approx L/12$ and b:h ≈ 1:2 to 2:3)

Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h[1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

[1]Expressions applicable for normalweight concrete and f_y = 60,000 psi. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

ASSUME $h \approx \frac{L}{12} = \frac{360''}{12} = 30''$

ASSUME b:h ≈ 1:2 ∴ $b \approx 15''$

BEAM DL = $150 \, \frac{15 \times 30}{144} = 469$ PLF

ESTIMATE M_U

$$M_U = Pa + \frac{w\ell^2}{8}$$

$$= 1.6(20^k)(10') + \frac{1.2(2.469 \text{ KLF})(30')^2}{8}$$

$$= 320 + 333.3 = 653.3 \text{ K-'}$$

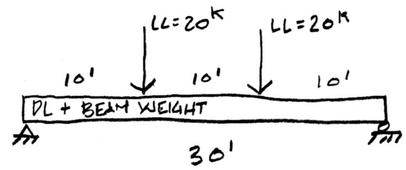
Rectangular Beam Design

Data:
- Load and Span
- Material properties – f'_c, f_y

Required:
- Steel area - A_s
- Beam dimensions – b and d

f'c = 3000 psi
fy = 60 ksi

DL = 2 klf + beam
LL = 2 x 20 k



2. Choose ρ (equation assumes $\varepsilon_t = 0.0075$)

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

CHOOSE ρ

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

Rectangular Beam Design cont.

3. Calculate bd^2

$$bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - 0.59 \rho \left(\frac{f_y}{f'_c}\right)\right)}$$

$$bd^2 = \frac{653.3(12)}{0.9(0.01)60\left[1 - 0.59(0.01)\left(\frac{60}{3}\right)\right]}$$

$$bd^2 = \frac{7840}{0.573(0.882)} = 15492 \text{ in}^3$$

4. Choose b and solve for d
   (or d and solve for b)
   b is based on form size – matches column size
   h ≈ L/12, b:h ≈ 1:2 to 2:3

TRY

b	d	h≈1.12d	A
14"	33.27"	38"	532
15"	32.14"	36"	540
16"	31.11"	35"	560

5. Revise h, weight, M_u, and bd^2

CHOOSE 15 x 36

Rectangular Beam Design cont.

5. Revise h, weight, M_u, and bd^2

USE 15 x 36

REVISE DL = $150 \frac{540}{144} = 563$ PLF

CHECK M_u

$M_u = 320 + \frac{1.2(2{,}563)\,30^2}{8} = 666^{K-1}$

REVISE bd

$bd^2 = \frac{666\,(12)}{0.505} = 15814 \text{ in}^3$

FOR b = 15" d = 32.5"

6. Find $A_s = \rho bd$

$A_s = \rho bd = (0.01)(15")(32.5")$

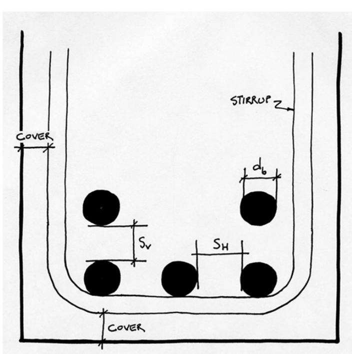
$A_s = 4.87 \text{ in}^2$

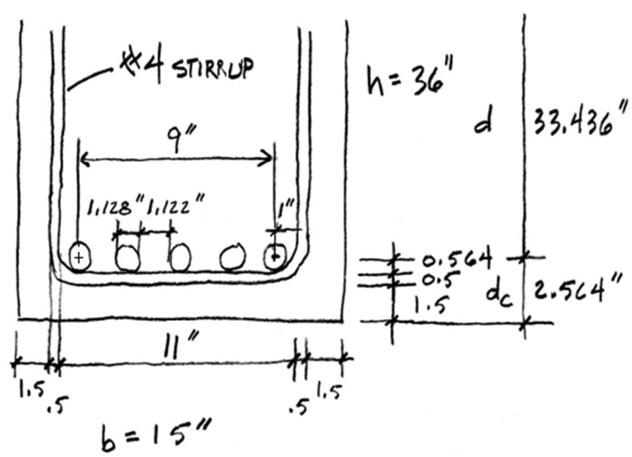
Rectangular Beam Design

7. Choose bars for A_s, determine spacing and cover, and revise d

CHOOSE BARS (SEE TABLE A.4)

TRY 5 x #9 BARS $A_s = 5.0 \text{ in}^2$ > **4.87 in²**





If bars do not fit in one layer, d is measured to the centroid of the pattern.

$$\bar{x} = \frac{\sum \mathbf{A} \times d_x}{\sum \mathbf{A}}$$

Table A.4 Areas of Groups of Standard Bars (in.²)

Bar No.	Number of Bars 2	3	4	5	6	7	8	9	10	11	12	13	14
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00

Jack C McCormac, 1978
Design of Reinforced Concrete,

Rectangular Beam Design

7. Choose bars for A_s and determine spacing and cover, recheck h and weight

$d = h - dc = 36'' - 2.564'' = 33.435''$

$d = 33.436''$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{5(60)}{.85(3)15} = 7.843''$$

Make final check of M_n using final d, and check that $M_u \leq \phi M_n$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 5(60)\left(33.436 - \frac{7.843}{2}\right)$$

$M_n = 8854$ K-'' $= 737.8$ K-'

$\phi M_n = 0.9(737.8) = 664$ K-'

$M_u = 653.3 < 664$ ✓ ok

8. Check that $\varepsilon_t \geq 0.005$ (if not, increase h and reduce A_s)

$$c = \frac{a}{\beta_1} = \frac{7.843''}{0.85} = 9.227''$$

$$\varepsilon_t = \frac{d - c}{c}(0.003)$$

$$\varepsilon_t = \frac{33.436 - 9.227}{9.227}(0.003)$$

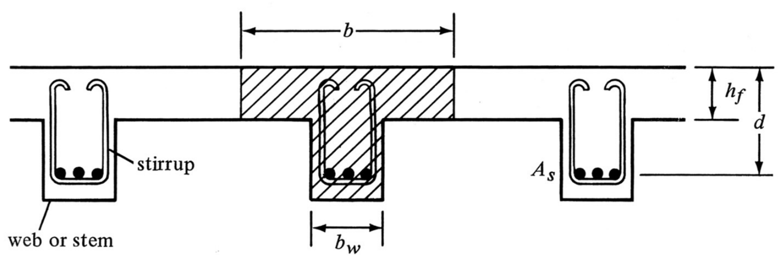
$\varepsilon_t = 0.00787 > 0.005$ ✓ ok

9. Design shear reinforcement (stirrups)
10. Check deflection, crack control, steel development length

T Beams

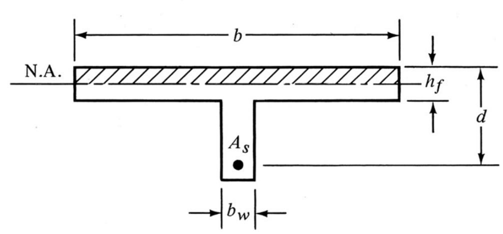
Dimensional limits

Nomenclature



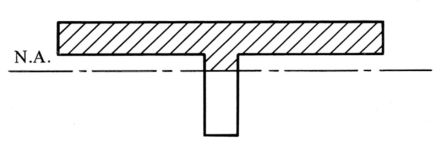
Possible N.A. locations:

Within flange – rectangular



(a)

Within stem – non-rectangular



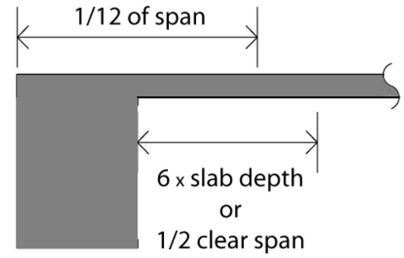
(b)

T Beams - Effective Flange Width, b_e

Slab on one side:

b_e least of either (total width) or (overhang + stem)

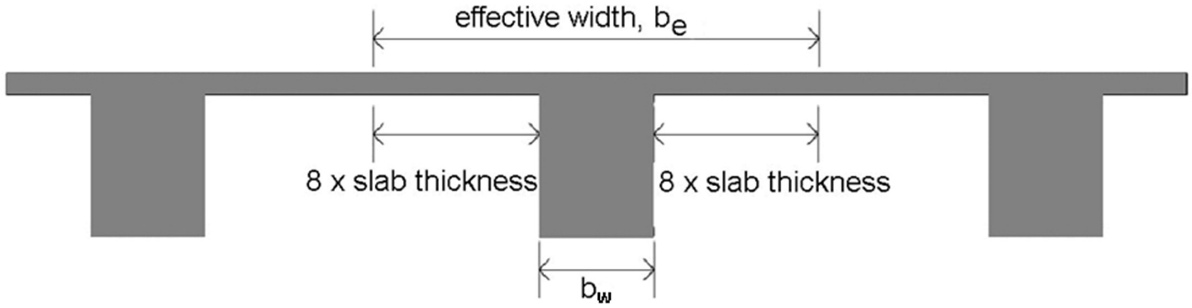
- Total width: 1/12 of the beam span
- Overhang: 6 x slab thickness
- Overhang: ½ the clear distance to next beam



Slab on both sides:

b_e least of either (total width) or (2 x overhang + stem)

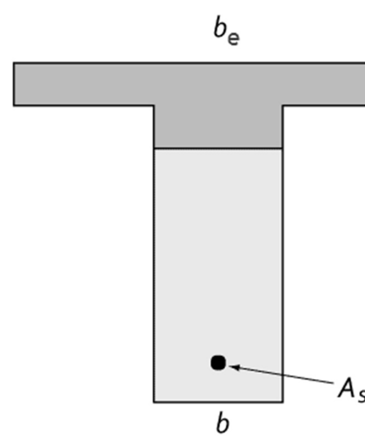
- Total width: ¼ of the beam span
- Overhang: 8 x slab thickness
- Overhang: ½ the clear distance to next beam (i.e. the web on center spacing)



Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, b_e, h, (span)
- Steel area - A_s
- Material properties – f'_c, f_y

Required:
- Required Moment – M_u (or load, or span)

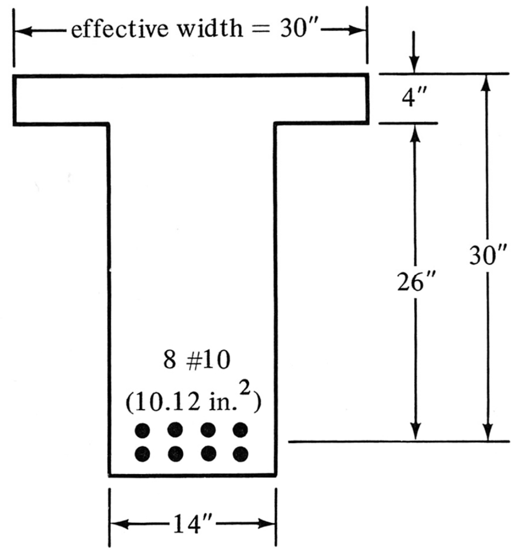


1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set T = C and solve for $A_c = T/(0.85\, f'c)$
3. Draw and label diagrams for section and stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
   Working from the top down,
   add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$
8. Check $A_{s,min} < A_s < A_{s,max}$
9. Check that $\varepsilon_t \geq 0.005$

T Beam Analysis

Given: f'_c = 3000 psi
f_y = 50 ksi
dimensions. Use b_{eff} = 30"

Req'd: Moment capacity, Mu

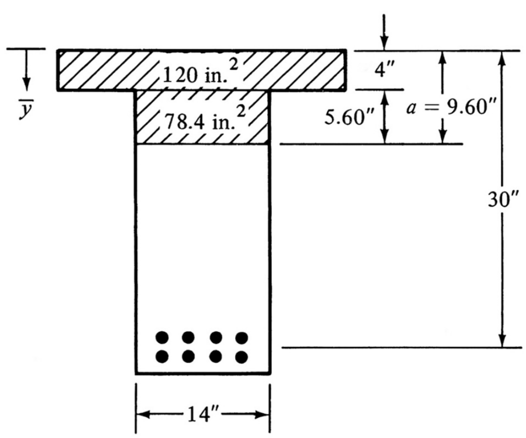


1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set T = C and solve for $A_c = T/(0.85 f'c)$

$T = A_s f_y = 10.12\text{ in}^2\ 50^{ksi} = 506^K$

$A_c = \frac{T}{0.85 f'_c} = \frac{506^K}{0.85\ 3^{ksi}} = 198.4\text{ in}^2$

T Beam Analysis (cont.)



FLANGE = $30" \times 4" = 120 < 198.4$ ∴ NA IN WEB

$198.4 - 120 = 78.4\text{ in}^2 = 14" \times 5.60"$

$a = 4" + 5.60" = 9.60"$

BY PARTS (FOR EACH AREA)

$Z_1 = 30" - 4"/2 = 28"$

$Z_2 = 30" - 4" - 5.6/2" = 23.2"$

$C_1 = A_{c1}\, 0.85 f'_c = 120(0.85)(3) = 306^K$

$C_2 = A_{c2}\, 0.85 f'_c = 78.4(0.85)(3) = 199.9^K$

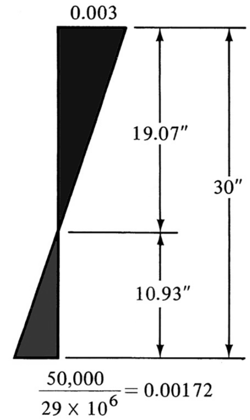
$M_n = \sum C_i z_i = 306(28) + 199.9(23.2) = 8568 + 4638 = 13\,206^{K\text{-}IN} = 1101^{K\text{-}FT}$

$M_u = \phi M_n = 0.9(1101) = 991^{K\text{-}FT}$

3. Draw and label diagrams for section & stress
   1. Determine b effective (for T-beams)
   2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
   Working from the top down,
   add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$

T Beam Analysis (cont.)

ρ_{bal}



$\rho_{max} = 0.75\ \rho_{bal}$

a_{bal} = beta c_{bal} = 0.85 (19.07”) = 16.21”

Ac_{bal} = (4”) (30”) + (12.21”) (14”) = 291 in^2

C_{bal} = (0.85) (3) (291) = 742^k

T_{max} = 0.75 C_{bal} = (0.75) (742) = 556^k

T_{used} = $A_s\ f_y$ = (10.12) (50) = 506^k < 556^k ok

As_{min} = 200 (b_w d) / f_y = 200 (14) (30) / 50 000

As_{min} = 1.68 in^2 < 10.12 in^2 ok