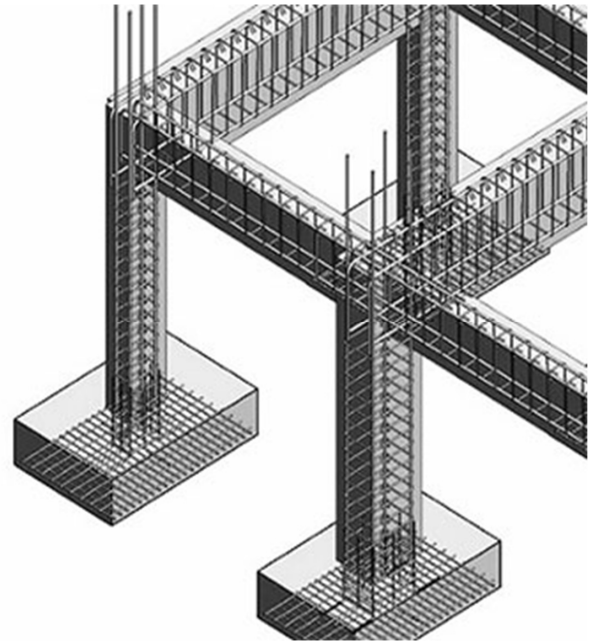


Reinforced Concrete Beams Ultimate Strength Design (ACI 318-14) – PART III

- Rectangular Beam Design – Method 1
- Rectangular Beam Design – Method 2
- Non-Rectangular Beam Analysis
- T-Beams



Rectangular Beam Design

Two approaches:

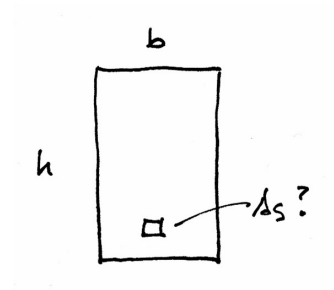
Method 1:

Data:

- Load and Span
- Material properties – f'_c , f_y
- All section dimensions: h and b

Required:

- Steel area – A_s



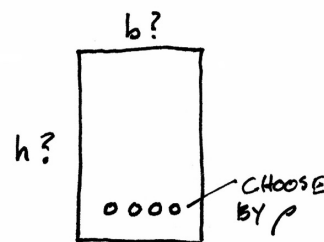
Method 2:

Data:

- Load and Span
- Some section dimensions – h or b
- Material properties – f'_c , f_y
- Choose ρ

Required:

- Steel area – A_s
- Beam dimensions – b or h



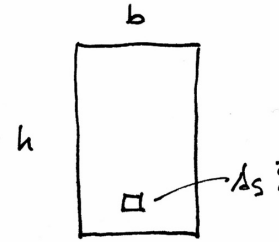
Rectangular Beam Design – Method 1

Data:

- Load and Span
- Material properties – f'_c , f_y
- All section dimensions – b and h

Required:

- Steel area - A_s



1. Calculate the factored load and find factored required moment, M_u
2. Find $d = h - \text{cover} - \text{stirrup} - d_b/2$
3. Estimate moment arm $z = jd$, for beams $j \approx 0.9$ for slabs $j \approx 0.95$
4. Estimate A_s based on estimate of jd .
5. Use A_s to find a
6. Use a to find A_s (repeat...until **2%** accuracy)
7. Choose bars for A_s and check A_s max & min
8. Check that $\epsilon_t \geq 0.005$
9. Check $M_u \leq \phi M_n$ (final condition)

$$M_u = \frac{(\gamma_{DL}W_{DL} + \gamma_{LL}W_{LL})l^2}{8}$$

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

10. Design shear reinforcement (stirrups)
11. Check deflection, crack control, rebar development length

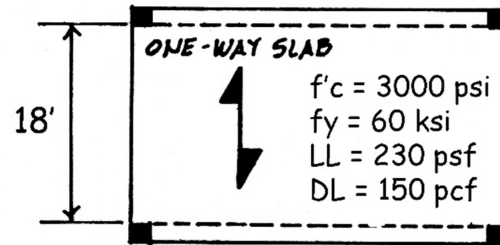
$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

One-way Slab Design

Method 1

Data:

- Load and Span
- Material properties – f'_c , f_y
- All section dimensions:
- h (based on deflection limit)
- $b = \text{typical } 12'' \text{ width}$



Required:

- Steel area – A_s

First estimate the slab thickness, h .

Try first the recommended minimum.

Deeper sections require less steel, but of course more concrete.

ACI

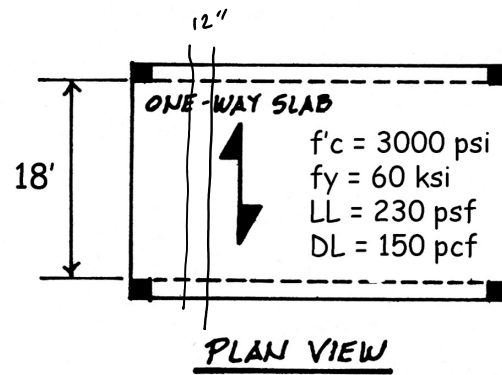
Table 7.3.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum $h^{(1)}$
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

THICKNESS, h , BASED ON DEFLECTION

$$h = \frac{\ell}{20} = \frac{18 \times 12}{20} = 10.8'' \text{ USE } \underline{\underline{11''}}$$

One-way Slab Slab Design



1. Calculate the dead load and find required M_u

FACTOR LOADS PCF

$$DL = \frac{11}{12} (150) = 137.5 \text{ PSF/LF}$$

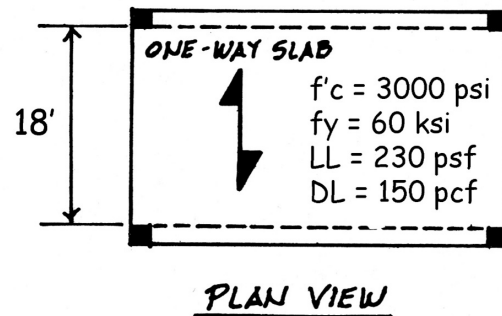
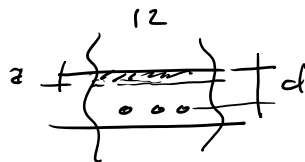
$$LL \text{ (GIVEN)} = 230 \text{ PSF}$$

$$w_u = 1.2 \left(\frac{DL}{\gamma} \right) + 1.6 \left(\frac{LL}{\gamma} \right) = 533$$

$$\underline{M_u} = \frac{w_u l^2}{8} = \frac{533 \text{ PLF} (18')^2}{8} = 21587 \text{ '}-\text{K}$$

$$= 259 \text{ ''}-\text{K} \text{ TARGET}$$

One-way Slab Slab Design



2. Find d based on the estimated h and rebar size (guessing #4)

FOR SLAB $\bar{z} \approx 0.95$, FIND -STIRUP CENTROID

BEAM ≈ 0.9 $d = h - \text{COVER} - \frac{1}{2} \text{ BAR}$

$$d = 11 - \frac{3}{4} - \frac{1}{2} \left(\frac{1}{2} \right)$$

$$\underline{d} = 11 - 1 = 10''$$

3. Estimate moment arm

$$\underline{z} \approx 0.95 d$$

$$\underline{z} \approx \bar{z} d \approx 0.95 (10'') = 9.5''$$

One-way Slab Slab Design

4. Estimate A_s based on estimate of z
5. Use A_s to find a
6. Use a to find A_s (repeat...)

TRIAL 1

$$A_s = \frac{M_u}{\phi f_y (z)} = \frac{259 \text{ in}^{\text{-k}}}{0.9 (60 \text{ ksi}) (9.5 \text{ in})} = 0.505 \text{ in}^2$$

$$a = \frac{A_s f_y}{.85 f_c' b} = \frac{0.505 (60)}{.85 (3) (12 \text{ in})} = 0.99 \text{ in}$$

TRIAL 2

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{259}{0.9 (60) (10 - \frac{.99}{2})}$$

$$A_s = 0.5046 \text{ in}^2 \quad \text{WITHIN 2\%}$$

One-way Slab Slab Design

7. Choose bars for A_s required:
 - either
 - choose bars** and calculate spacing
 - or
 - choose spacing** and find bar size
 If the bar size changes, re-calculate to find new d . Then, re-calculate A_s ...

Check $A_{s,min}$
 (for slabs $A_{s,min}$ from ACI Table 7.6.1.1)

Table 7.6.1.1— $A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y , psi		$A_{s,min}$
Deformed bars	< 60,000		$0.0020A_g$
Deformed bars or welded wire reinforcement	$\geq 60,000$	Greater of:	$\frac{0.0018 \times 60,000}{f_y} A_g$ $\frac{0.0018 \times 60,000}{60,000} A_g$ $0.0014A_g$

CHOOSE BARS

TRY USING #4

$$\frac{0.505}{12 \text{ in}} ; \frac{0.2A_b}{s} \quad s = 4.75 \text{ in}$$

PER 12" → 5?

∴ USE #4 @ 5" o.c. (always round down)

$$A_s = \frac{(0.60 \text{ in})^2}{\text{FT}} > 0.505 \checkmark$$

ALTERNATE FOR MAX. S = 18"

$$\frac{0.505}{12 \text{ in}} ; \frac{A_b}{18 \text{ in}} \quad A_b = 0.75 \text{ in}^2$$

$$\#8 = 0.79 \text{ in}^2$$

∴ USE #8 @ 18" o.c. COULD

$$A_s = 0.526 \text{ in}^2 / \text{FT} > 0.505 \checkmark$$

Check $A_{s,min}$

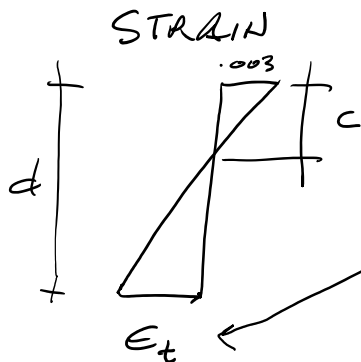
$$A_{s,min} = \frac{0.0018}{f_y} b h = \frac{0.0018 (12) (11 \text{ in})}{60,000}$$

$$= 0.24 \text{ in}^2 < 0.526 \text{ in}^2 \checkmark \text{ OK}$$

OR 0.60 ✓

One-way Slab Slab Design

8. Check that $\epsilon_t \geq 0.005$



* 4 BARS \rightarrow
 RE-CALC 2 FOR $A_s = 0.6 \text{ m}^2/\text{ft}$
 $a = \frac{A_s f_y}{0.85 F'_c b} = \frac{0.6(60)}{0.85(3)(12)} = 1.176''$

$$c = \frac{a}{\beta_1} = \frac{1.176}{0.85} = 1.384''$$

$$\epsilon_t = \frac{d-c}{c} 0.003 = \frac{9.5'' - 1.384''}{1.384''} 0.003 = 0.01759$$

$$0.01759 > 0.005 \quad \checkmark$$

\therefore TENSION CONTROLLED \checkmark

One-way Slab Slab Design

9. Check $M_u \leq \phi M_n$
 (final condition)

$$A_s = A_{s, \text{used}}$$

$$M_n = Tz$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$M_n = 0.6(60) \left(9.5'' - \frac{1.176}{2} \right)$$

$$M_n = 36(8.911'') = 320.8 \text{ K-''}$$

$$\phi M_n = 0.9(320.8) = 288.7 \text{ K-''}$$

10. Add stirrups (no stirrups in slab)

$$M_u = 259 \text{ K-''} < 288.7 \text{ K-''} \quad \checkmark$$

11. Check deflection, crack control,
 and rebar development length

$$M_u < \phi M_n \quad \checkmark \text{ OK}$$

Rectangular Beam Design – Method 2

Data:

- Load and Span
- Some section dimensions – b or h ?
- Material properties – f'_c, f_y ✓

Required:

- Steel area A_s
 - Beam dimensions – b and h
1. Estimate the dead load (estimate h and b)
✓ ($L/8 \leq h \leq L/21$, $h \approx L/12$ and $b:h \approx 1:2$ to $2:3$), find M_u
 2. Choose ρ (equation assumes $\epsilon_t = 0.0075$)
 3. Calculate bd^2
 4. Choose b and solve for d (or d and solve b)
 5. Revise h , weight, M_u , and bd^2
 6. Find $A_s = \rho bd$
 7. Choose bars for A_s , determine spacing and cover, and revise d
 8. Check that $\epsilon_t \geq 0.005$ (if not, increase h and reduce A_s)
 9. Design shear reinforcement (stirrups) ✓
 10. Check deflection, crack control, steel development length ✓

$$M_u = \frac{(\gamma_{DL}W_{DL} + \gamma_{LL}W_{LL})l^2}{8}$$

$$\rho = \frac{A_s}{bd} \quad \rho = \frac{\beta_1 f'_c}{4f_y}$$

$$bd^2 = \frac{M_u \text{ (k-ft)}}{\phi \rho f_y (1 - 0.59 \rho (f_y / f'_c))}$$

$$A_s = \rho bd$$

$$a = \frac{\rho f_y d}{0.85 f'_c}$$

Rectangular Beam Design

Data:

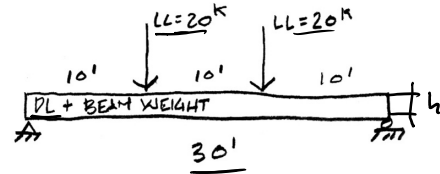
- Load and Span
- Material properties – f'_c, f_y

Required:

- Steel area - A_s
- Beam dimensions – b and d

$f'_c = 3000$ psi
 $f_y = 60$ ksi

DL = 2 klf + beam
LL = 2 x 20 k



1. Estimate the dead load (self-weight), and find M_u ($h \approx L/12$ and $b:h \approx 1:2$ to $2:3$)

ACI
Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h ^[1]
Simply supported	$l/16$
One end continuous	$l/18.5$
Both ends continuous	$l/21$
Cantilever	$l/8$

^[1]Expressions applicable for normalweight concrete and $f'_c = 60,000$ psi. For other cases, minimum h shall be modified in accordance with 9.3.1.1.1 through 9.3.1.1.3, as appropriate.

ASSUME $h \approx \frac{L}{12} = \frac{360}{12} = 30''$ ✓
 ASSUME $b:h \approx 1:2 \therefore b \approx 15''$
 BEAM DL = $\frac{150}{144} \frac{15 \times 30}{144} = 4.69$ PLF

ESTIMATE M_u
 $M_u = \frac{P_{LL} l}{2} + \frac{w_{DL} l^2}{8}$
 $= \frac{1.6(20)(10')}{2} + \frac{1.2(2.469 \text{ KLF})(30')^2}{8}$

$M_u = 320 + 333.3 = 653.3 \text{ K-ft}$

Rectangular Beam Design

Data:

- Load and Span
- Material properties – f'_c , f_y

Required:

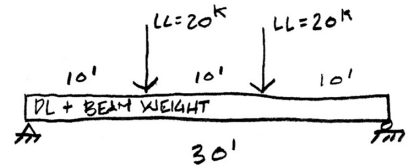
- Steel area - A_s
- Beam dimensions – b and d

$$f'_c = 3000 \text{ psi}$$

$$f_y = 60 \text{ ksi}$$

$$DL = 2 \text{ klf} + \text{beam}$$

$$LL = 2 \times 20 \text{ k}$$



2. Choose ρ (equation assumes $\epsilon_t = 0.0075$)

f'_c	β_1
0	0.85
1000	0.85
2000	0.85
3000	0.85
4000	0.85
5000	0.8
6000	0.75
7000	0.7
8000	0.65
9000	0.65
10000	0.65

Choose ρ

$$\rho = \frac{\beta_1 f'_c}{4 f_y} = \frac{0.85(3)}{4(60)} = 0.010$$

Rectangular Beam Design cont.

3. Calculate bd^2

$$bd^2 = \frac{M_u}{\phi \rho f_y (1 - 0.59 \rho (f_y / f'_c))}$$

$$bd^2 = \frac{653.3(12)}{0.9(0.01)60 [1 - 0.59(0.01)(\frac{60}{3})]}$$

$$bd^2 = \frac{7840}{0.573(0.882)} = 15492 \text{ in}^3$$

4. Choose b and solve for d
(or d and solve for b)

b is based on form size – matches column size
 $h \approx L/12$, $b:h \approx 1:2$ to $2:3$

TRIAL b	d	ROUND $h \approx 1.12d$	A_c
14"	33.27"	38"	532
15"	32.14"	36"	540
16"	31.11"	35"	560

5. Revise h , weight, M_u , and bd^2

CHOOSE 15 x 36

Rectangular Beam Design cont.

5. Revise h, weight, M_u , and bd^2

USE 15 x 36

REVISE $DL = 150 \frac{540}{144} = 563 \text{ PLF}$

CHECK M_u

$$M_u = 320 + \frac{1.2(2 \times 563) 30^2}{8} = 666 \text{ K-ft}$$

REVISE bd

$$bd^2 = \frac{666(12)}{0.505} = 15814 \text{ in}^3$$

FOR $b = 15''$ $d = 32.5''$

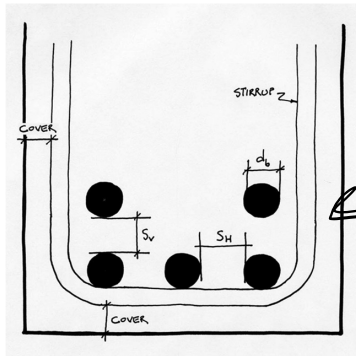
6. Find $A_s = \rho bd$

$$A_s = \rho bd = (0.01)(15'')(32.5'')$$

$$A_s = 4.87 \text{ in}^2$$

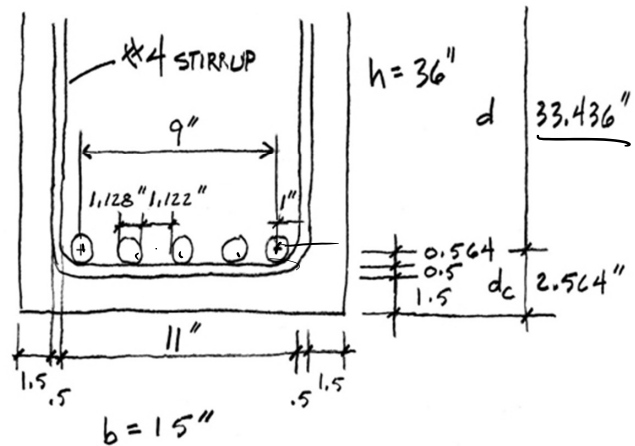
Rectangular Beam Design

7. Choose bars for A_s , determine spacing and cover, and revise d



CHOOSE BARS (SEE TABLE A.4)

TRY 5 x #9 BARS $A_s = 5.0 \text{ in}^2 > 4.87 \text{ in}^2$



If bars do not fit in one layer, d is measured to the centroid of the pattern.

$$\bar{x} = \frac{\sum A \times d_x}{\sum A}$$

Table A.4 Areas of Groups of Standard Bars (in.²)

Bar No.	Number of Bars													
	2	3	4	5	6	7	8	9	10	11	12	13	14	
4	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75	
5	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30	
6	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19	
7	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42	
8	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00	
9	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	
10	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	17.72	
11	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87	
14	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50	24.75	27.00	29.25	31.50	
18	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	

Rectangular Beam Design

- Choose bars for A_s and determine spacing and cover, recheck h and weight

Make final check of M_n using final d , and check that $M_u \leq \phi M_n$

$$d = h - dc = 36" - 2.564" = 33.435"$$

$$d = 33.436"$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5(60)}{0.85(3)15} = 7.843"$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) = 5(60) \left(33.436 - \frac{7.843}{2} \right)$$

$$M_n = 88.54 \text{ K-ft} = 737.8 \text{ K-in}$$

$$\phi M_n = 0.9(737.8) = 664 \text{ K-in}$$

$$M_u = 653.3 < 664 \quad \checkmark \text{ OK} \quad \checkmark$$

- Check that $\epsilon_t \geq 0.005$ (if not, increase h and reduce A_s)

$$c = \frac{a}{\beta_1} = \frac{7.843}{0.85} = 9.227"$$

$$\epsilon_t = \frac{d - c}{c} (0.003)$$

$$\epsilon_t = \frac{33.436 - 9.227}{9.227} (0.003)$$

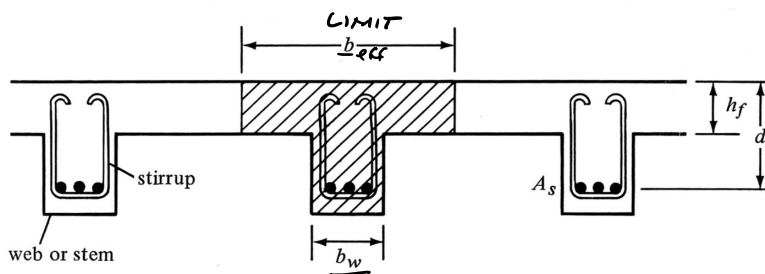
$$\epsilon_t = 0.00787 > 0.005 \quad \checkmark \text{ OK} \quad \checkmark$$

- Design shear reinforcement (stirrups)
- Check deflection, crack control, steel development length

T Beams

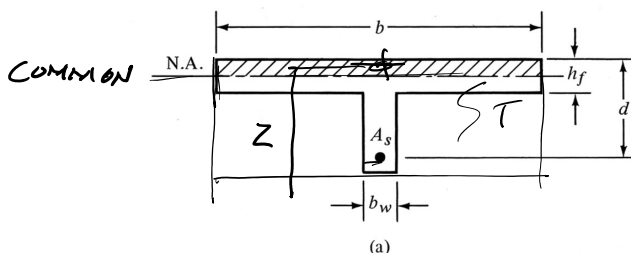
Dimensional limits

Nomenclature

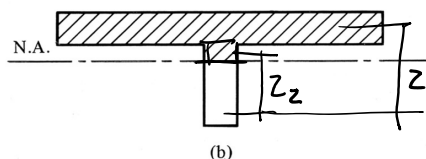


Possible N.A. locations:

Within flange – rectangular



Within stem – non-rectangular

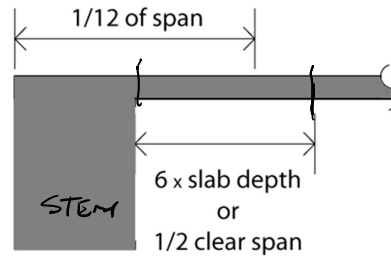


T Beams - Effective Flange Width, b_e

Slab on one side:

b_e least of either (total width) or (overhang + stem)

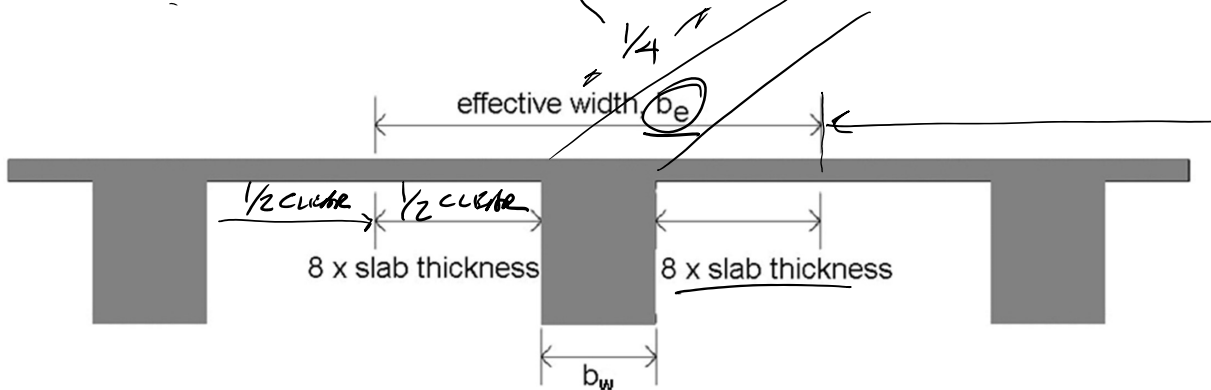
- Total width: $\frac{1}{12}$ of the beam span
- Overhang: $6 \times$ slab thickness
- Overhang: $\frac{1}{2}$ the clear distance to next beam



Slab on both sides:

b_e least of either (total width) or (2 x overhang + stem)

- Total width: $\frac{1}{4}$ of the beam span
- Overhang: $8 \times$ slab thickness
- Overhang: $\frac{1}{2}$ the clear distance to next beam (i.e. the web on center spacing)



Non-Rectangular Beam Analysis

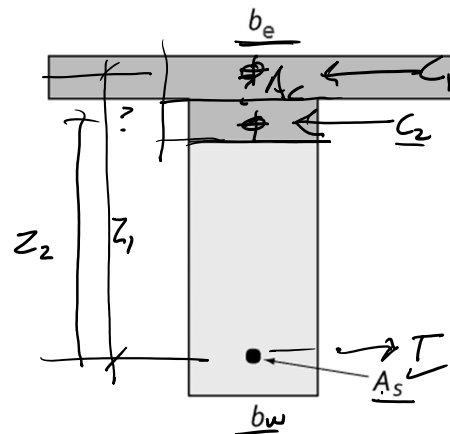
Data:

- Section dimensions – b , b_e , h , (span)
- Steel area - A_s ✓
- Material properties – f'_c , f_y ✓

Required:

- Required Moment – M_u (or load, or span)

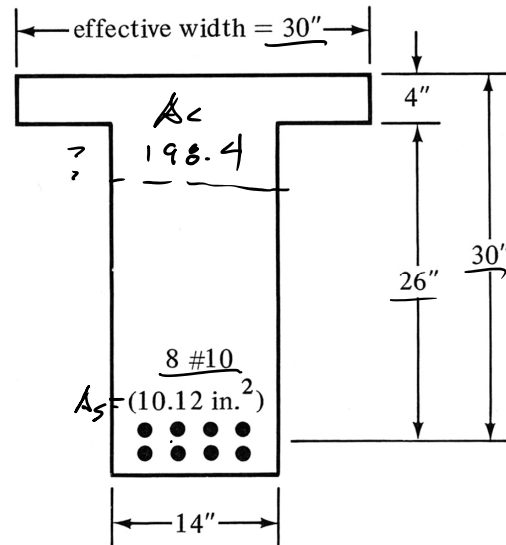
1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set $T = C$ and solve for $A_c = T / (0.85 f'_c)$
3. Draw and label diagrams for section and stress
 1. Determine b effective (for T-beams)
 2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
Working from the top down,
add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$
8. Check $A_{s,min} < A_s < A_{s,max}$
9. Check that $\epsilon_t \geq 0.005$ ✓



T Beam Analysis

Given: $f'_c = 3000$ psi
 $f_y = 50$ ksi
 dimensions. Use $b_{eff} = 30''$

Req'd: Moment capacity, M_u

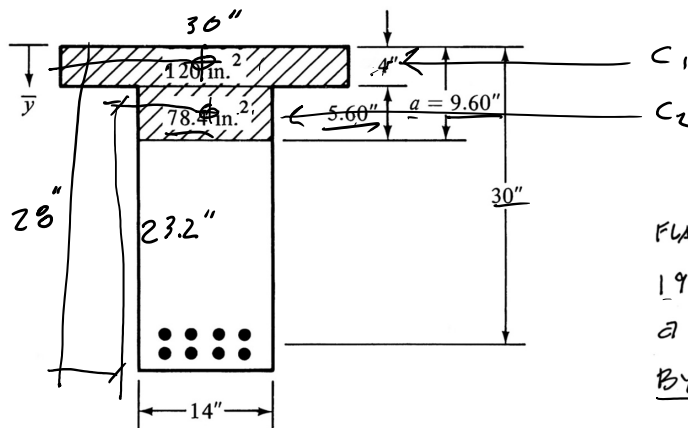


1. Find $T = A_s f_y$ and $C = 0.85 f'_c A_c$
2. Set $T = C$ and solve for $A_c = T / (0.85 f'_c)$

$$T = A_s f_y = 10.12 \text{ in}^2 \cdot 50 \text{ ksi} = 506 \text{ K}$$

$$A_c = \frac{T}{0.85 f'_c} = \frac{506 \text{ K}}{0.85 \cdot 3 \text{ ksi}} = 198.4 \text{ in}^2$$

T Beam Analysis (cont.)



3. Draw and label diagrams for section & stress
 1. Determine b effective (for T-beams)
 2. Locate T and C (or C_1 and C_2)
4. Determine the location of a
Working from the top down,
add up area to make A_c
5. Find the moment arms (z) for each block of area
6. Find $M_n = \sum C_i z_i$
7. Find $M_u = \phi M_n$

TOTAL A_c

FLANGE = $30'' \times 4'' = 120 < 198.4 \therefore$ NA IN WEB
 $198.4 - 120 = 78.4 \text{ in}^2 = 14'' \times 5.60''$
 $a = 4'' + 5.60'' = 9.60''$

BY PARTS (FOR EACH AREA)

$$z_1 = 30'' - 4''/2 = 28''$$

$$z_2 = 30'' - 4'' - 5.60''/2 = 23.2''$$

$$C_1 = A_{c1} \cdot 0.85 f'_c = 120(0.85)(3) = 306 \text{ K}$$

$$C_2 = A_{c2} \cdot 0.85 f'_c = 78.4(0.85)(3) = 199.9 \text{ K}$$

$$M_n = \sum C_i z_i = 306(28) + 199.9(23.2) = 8568 + 4638 = 13206 \text{ K-FT}$$

$$M_u = \phi M_n = 0.9(1101) = 991 \text{ K-FT}$$

T Beam Analysis (cont.)

CHECK ϵ_T

$$\rho_{\max} = 0.75 \rho_{\text{bal}} \quad \beta_1 \quad c$$

$$a_{\text{bal}} = \beta_1 c_{\text{bal}} = 0.85 (19.07'') = 16.21''$$

$$A_{c_{\text{bal}}} = (4'') (30'') + (12.21'') (14'') = 291 \text{ in}^2$$

$$C_{\text{bal}} = (0.85) (3) (291) = 742^k$$

$$T_{\max} = 0.75 C_{\text{bal}} = (0.75) (742) = 556^k$$

$$T_{\text{used}} = A_s f_y = (10.12) (50) = 506^k < 556^k \text{ ok}$$

$$A_{s_{\min}} = 200 (b_w d) / f_y = 200 (14) (30) / 50\,000$$

$$A_{s_{\min}} = 1.68 \text{ in}^2 < 10.12 \text{ in}^2 \text{ ok}$$

