

## Pre- and Post-Tensioning

- Cable Trusses
- Concrete Beams
- Stressed Membranes

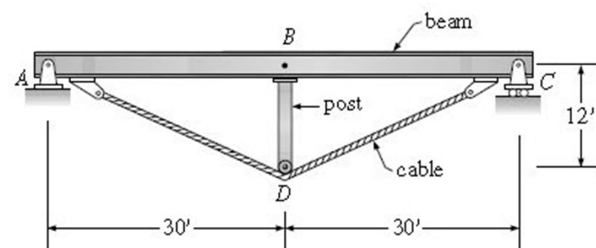


## Cable Trusses

- Reduce flexure stress
- Reduce deflection
- Produces stiffer section with less material
- Lighter weight
- Longer spans possible
- Analysis by combined stress



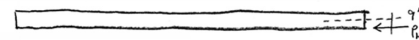
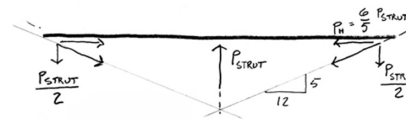
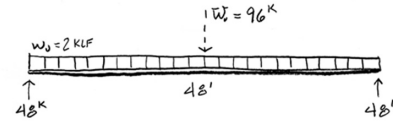
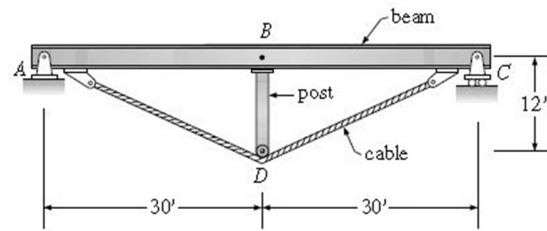
$$f = -\frac{P}{A} \pm \frac{M}{S} \pm \left[ \frac{Pe}{S} \right]$$



# Cable Truss – stress analysis

determine cable prestress

1. Break beam load into 3 FBDs.
  1. applied load
  2. cable + strut
  3. eccentric load (if any)
2. Solve moment for beam at C.L. for applied load
3. Solve C.L. moments for other 2 FBDs in terms of strut force,  $P_s$
4. Equate the moments from the three moment equations to cancel at the CL
5. Solve for the strut and cable forces.
6. Construct moment diagram for the beam with all loadings combined: applied load + cable at ends + struts.
7. Solve combined stress in beam using interaction equation.



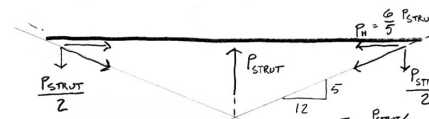
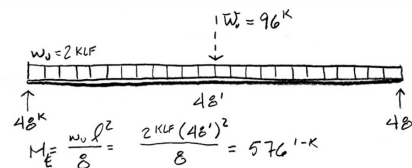
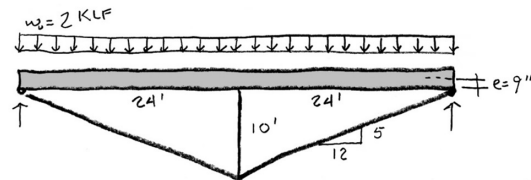
## Cable Truss Analysis

### Example

Given: truss configuration with applied load

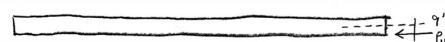
Required: force in the cable which will result in zero moment at the center line, C.L.

1. Divide the truss into 3 Free Body Diagrams:
  1. applied load
  2. cable + strut
  3. eccentric load (if any)



$$M_E = \frac{P_s}{4} = \frac{P_s \cdot 24}{4}$$

$$M_E = P_s (12)$$

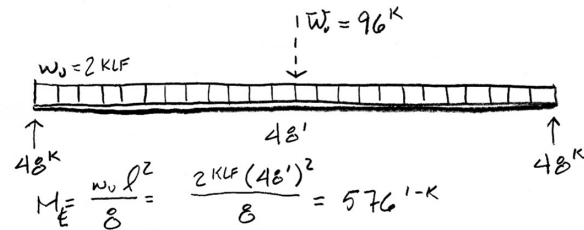


$$M_E = P_e = \frac{6}{5} P_s \times 9$$

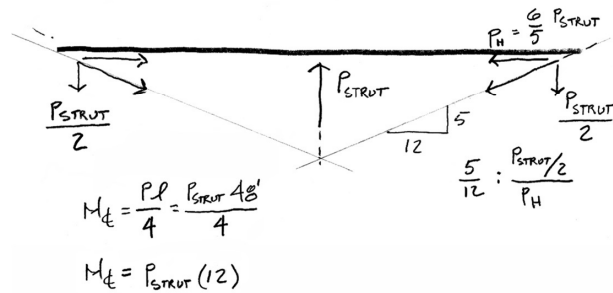
$$= 10.8 P_s$$

# Cable Truss Analysis

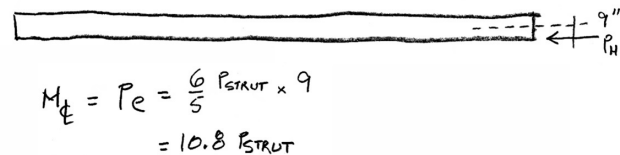
- Find the C.L. moment based on applied load alone.



- Find the C.L. moment for the cable and strut in terms of the strut force,  $P_{\text{strut}}$ . Write the components of the cable force in terms of  $P_{\text{strut}}$

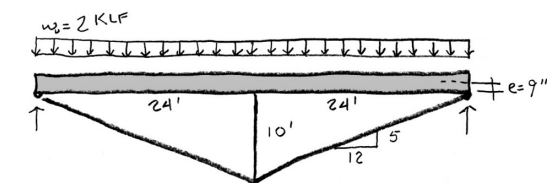


- Find the C.L. moment for the eccentric cable load in terms of  $P_{\text{strut}}$



# Cable Truss Analysis

- Set the sum of the C.L. moments equal to zero and solve for the strut force,  $P_{\text{strut}}$



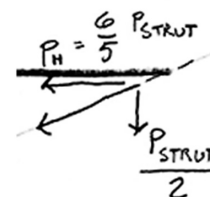
$$+ M_U - M_{\text{strut}} - M_e = 0$$

$$576 - 12 P_{\text{strut}} - 10.8 P_{\text{strut}} = 0$$

$$22.8 P_{\text{strut}} = -576 \text{ K-FT}$$

$$P_{\text{strut}} = 25.26 \text{ K}$$

- Sum the cable components to find the total cable force.

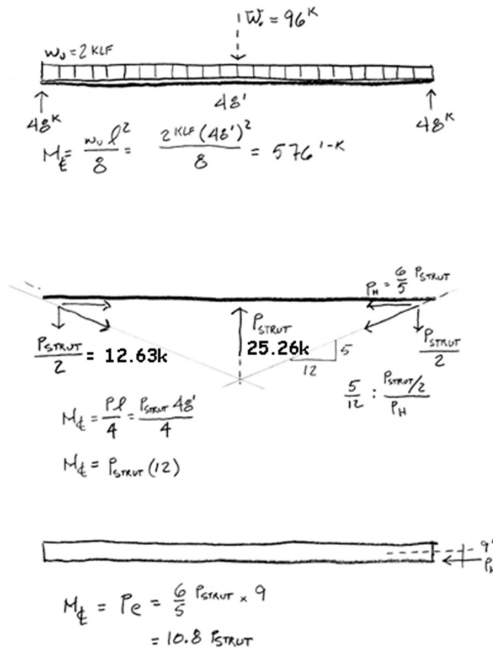


CABLE FORCE

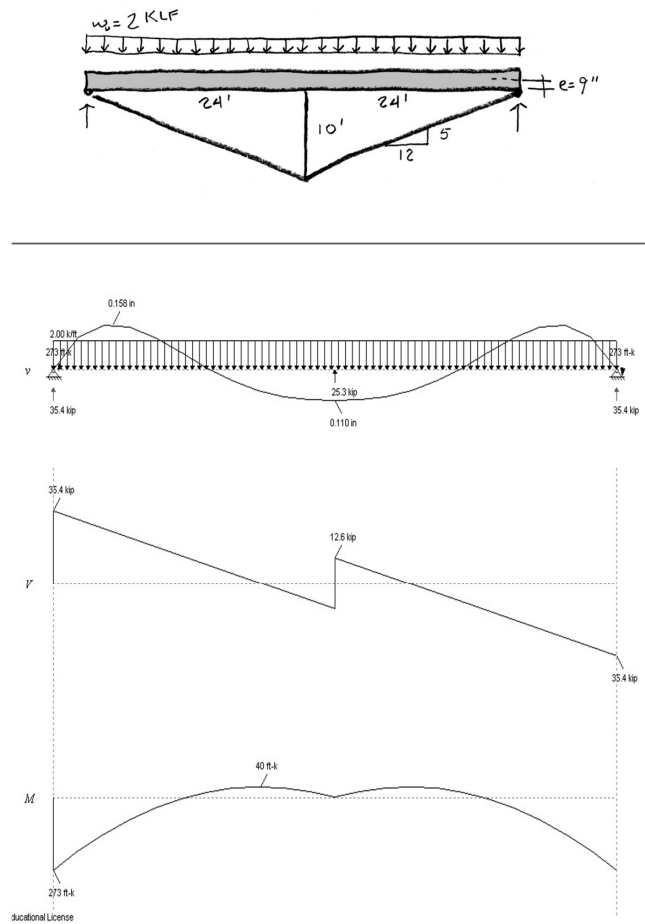
$$\sqrt{\left(\frac{6}{5} P_{\text{strut}}\right)^2 + \left(\frac{P_{\text{strut}}}{2}\right)^2} = 32.84 \text{ K}$$

# Cable Truss Analysis

7. find end reactions and calculate shear & moment



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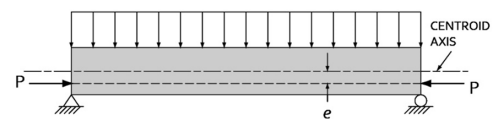
Structures II

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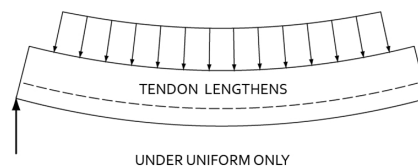
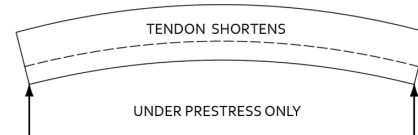
## Pre-stressed Concrete

- More concrete active in resisting moment
- Produces stiffer section with less material
- Lighter weight
- Longer spans possible
- Analysis by combined stress

$$f = -\frac{P}{A} \pm \frac{Pec}{I} \pm \frac{Mc}{I}$$



$$\frac{P}{A} + \frac{Pec}{I} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} + \frac{Mc}{I}$$



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# Pre-stressed Concrete

## Steel:

high strength wires 250 or 270 ksi  
wire diameter 0.105 – 0.276  
used in strands of bundled wire  
most common is 7 wire strand



## Concrete:

higher strength 5 – 10 ksi  
to reduce creep and strain  
reduced cracking  
stiffer sections

Photo by Angelo Marasco

# Pre-stressed Concrete

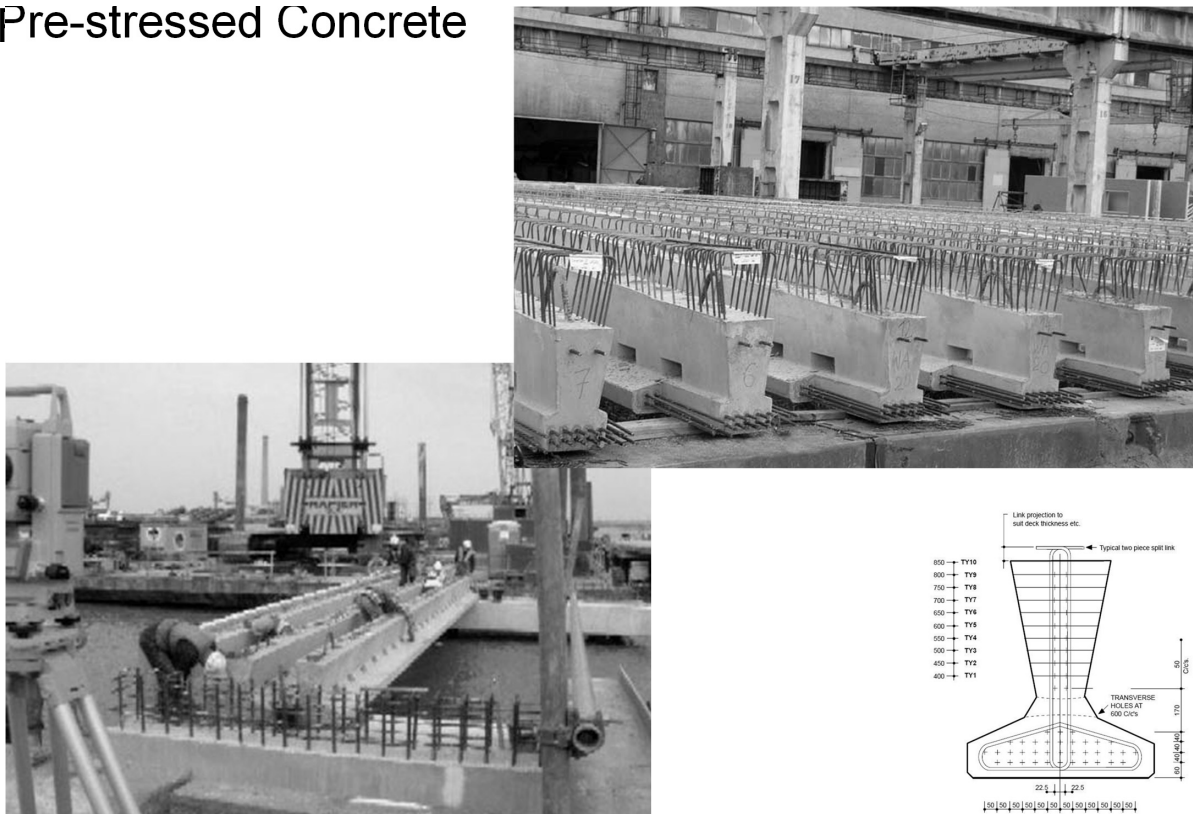
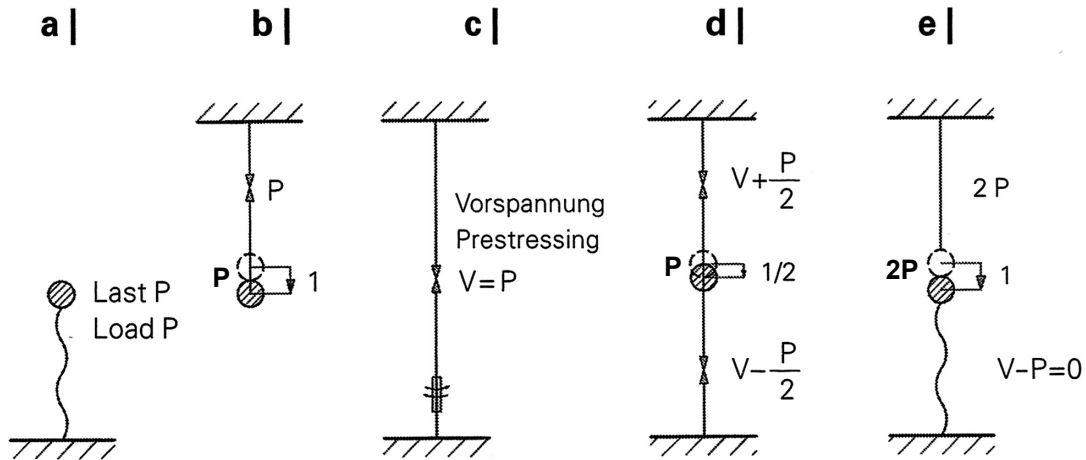


Photo by MACRETE

# Pre-stressing

Reducing deformation

(b) carries  $P$  and deflects 1  
(e) carries  $2P$  and deflects 1  
what makes the difference?



Jörg Schlaich, *Light Structures*

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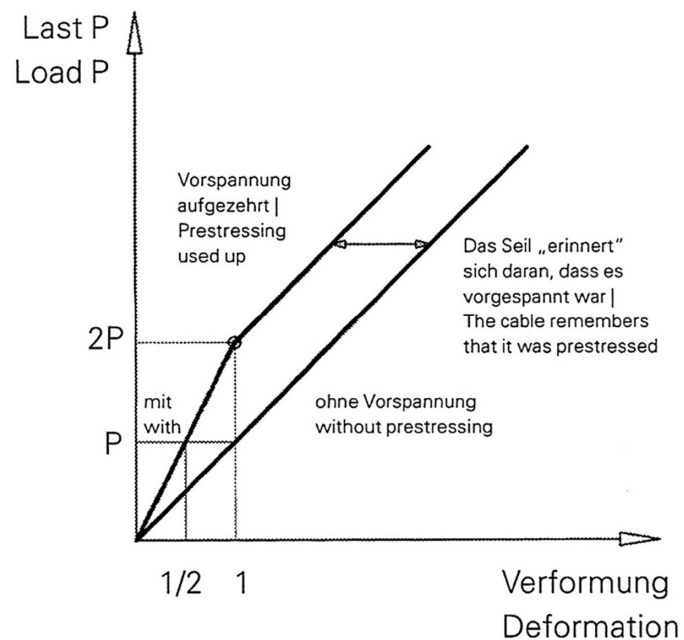
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# Pre-stressing

increasing stiffness

and

reducing deformation



Jörg Schlaich, *Light Structures*

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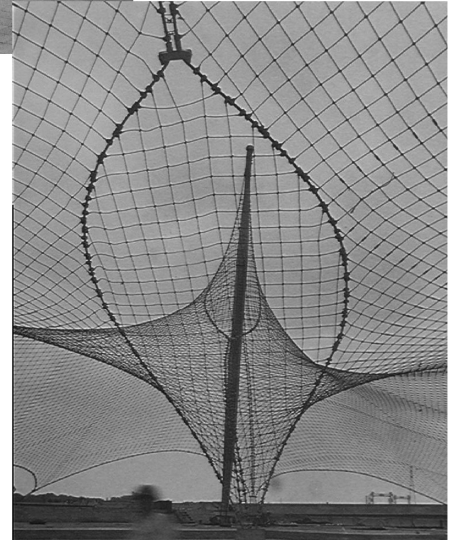
# Expo '67, Montreal

Frei Otto  
German Pavilion



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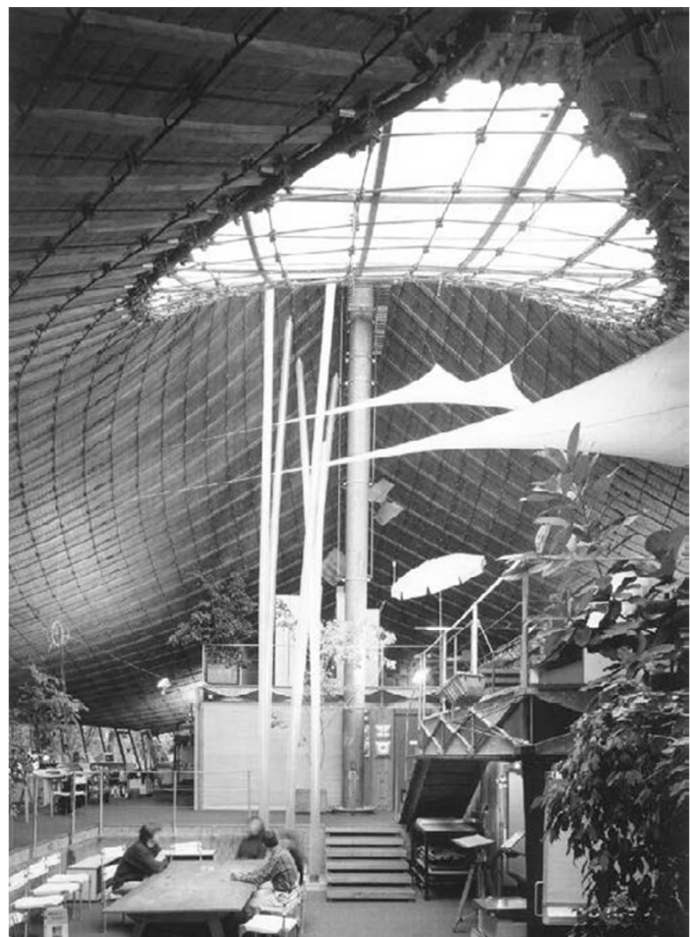
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Institute for Lightweight  
Structures – IL (now ILEK)

University of Stuttgart



Frei Otto, IL building, University of Stuttgart



Structures II

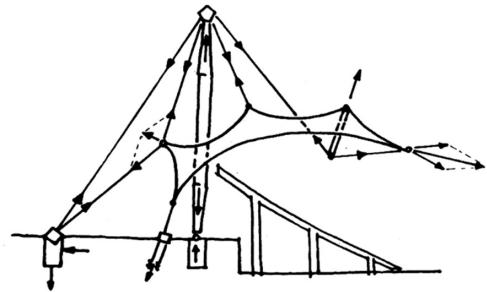
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# Stressed Membrane

Olympic Buildings, Munich 1972  
Eng. Otto, Leonhardt, Schlaich  
Arch: Behnisch

- Opposing curvature
- Stressed by anchors and masts



Frei Otto, Munich Soccer Stadium (from back)

## Stressed Membrane Olympic Stadium, Munich 1972



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## Bundesgartenschau Köln Frei Otto



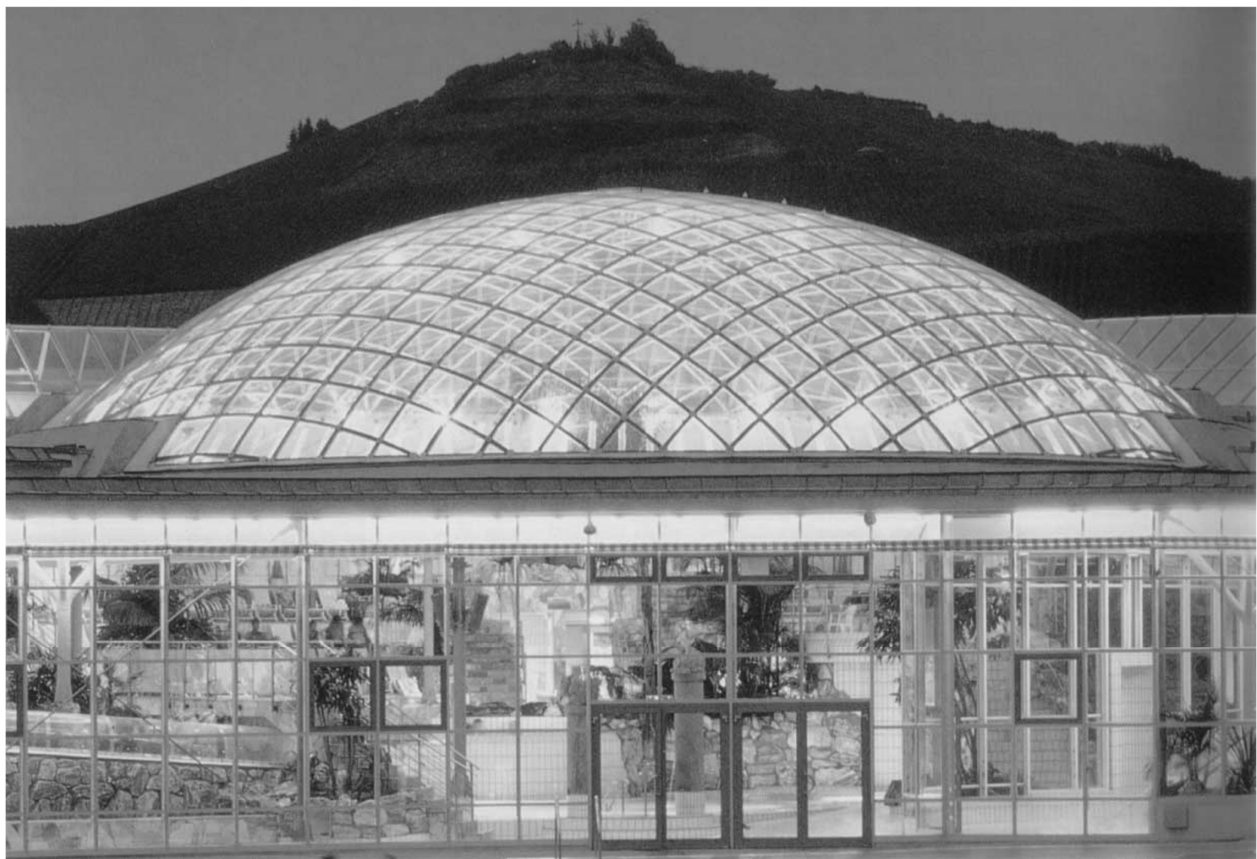
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## Schlaich Bergermann & Partners – Neckarsulm Swimming Pool



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Schlaich Bergermann & Partners

Neckarsulm, 1989



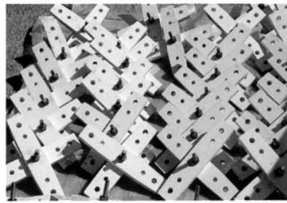
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6.14  
The slats



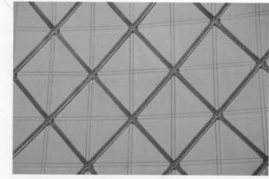
6.15  
The rotatable joints



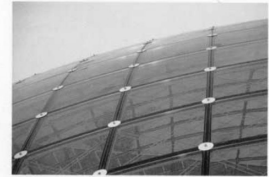
6.16  
Assembly of the grid elements



6.17  
Close-up of the joint assembly  
with diagonal cables installed



6.18  
A segment of the grid showing the double  
pattern formed by the slats and cables



6.19  
A segment of the completed roof  
with the spherically-curved glass panes



6.20  
Water barrels representing  
partial snow load

Schlaich Bergermann & Partners

Neckarsulm Pool

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Schlaich Bergermann & Partners

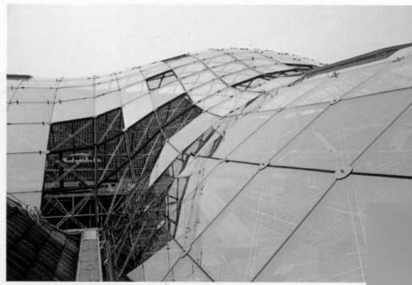
History of Hamburg Museum



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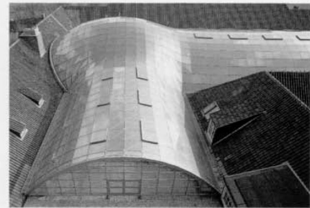
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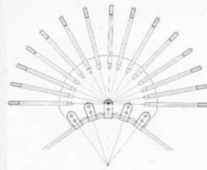


Schlaich Bergemann & Partners

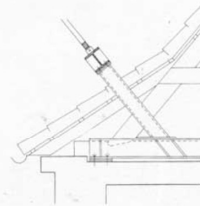
History of Hamburg Museum



6.27-6.31  
The roof under construction



6.32  
Detail of a "spoked wheel"



6.33  
A support point on the old building



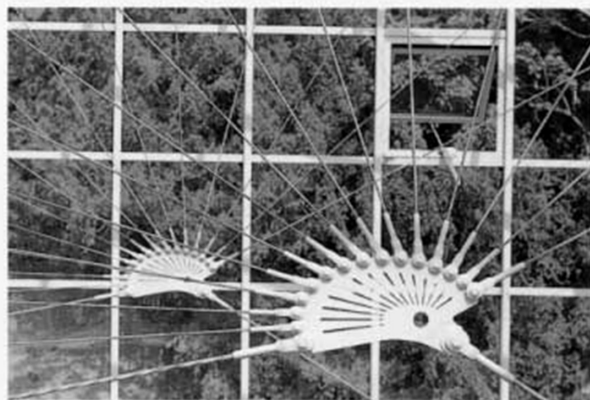
## Schlaich Bergemann & Partners – Pool cover for mineral spa, Bad Cannstatt







**6.41**  
Connections of the pretensioned  
cable "spokes" to the "rim"



**6.40**  
The hub connections (see the dra-  
wing on the cover of this book)

## Stressed Membrane

Bosch-Areals, Stuttgart 2001  
Eng. Schlaich Bergermann + Parteners

- Opposing curvature
- Stressed by cable spokes



# Stressed Membrane

Renaissance Center  
Entrance Pavilion  
Detroit 2004  
SOM

- Point supported glass
- “fish belly” cable truss bacing



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Structures II

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# Stressed Membrane

Renaissance Center  
Entrance Pavilion  
Detroit 2004  
SOM



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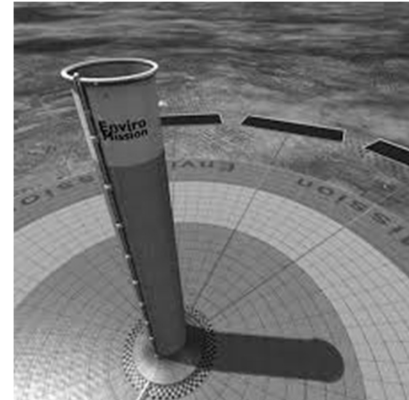
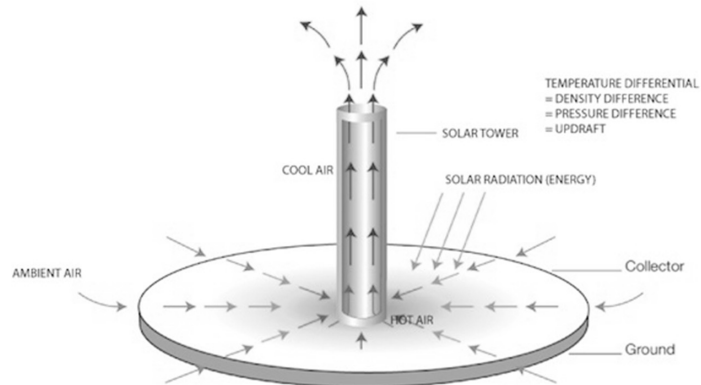
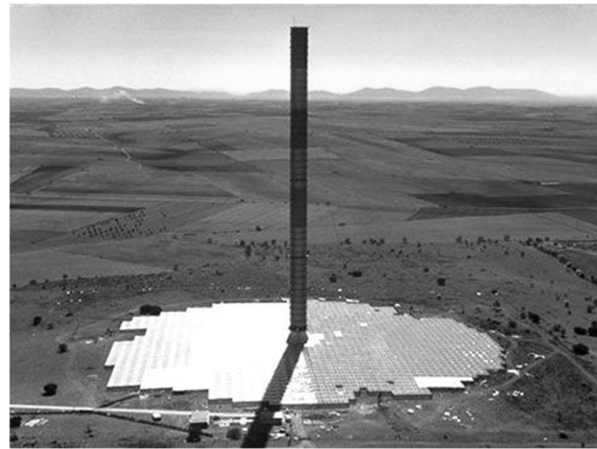
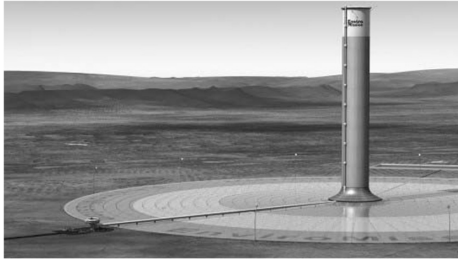


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# Solar Towers

Enviro Mission



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# Solar Towers



Jörg Schlaich, Updraft Solar Chimneys

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